VI. Quark states and colours

→ Forces acting between quarks in hadrons can be investigated by studying a simple quark-antiquark system

→ Systems of heavy quarks, like $c\bar{c}$ (*charmonium*) and $b\bar{b}$ (*bottomonium*), are essentially non-relativistic (masses of quarks are much bigger than their kinetic energies)

- Charmonium and bottomonium are analogous to a hydrogen atom in a sense that they consist of many energy levels
- While the hydrogen atom is governed by the electromagnetic force, the charmonium system is dominated by the strong force

Like in a hydrogen atom, energy states of a quarkonium can be labelled by their principal quantum number n, and J, L, S, where $L \leq n-1$ and S can be either 0 or 1 (a meson).

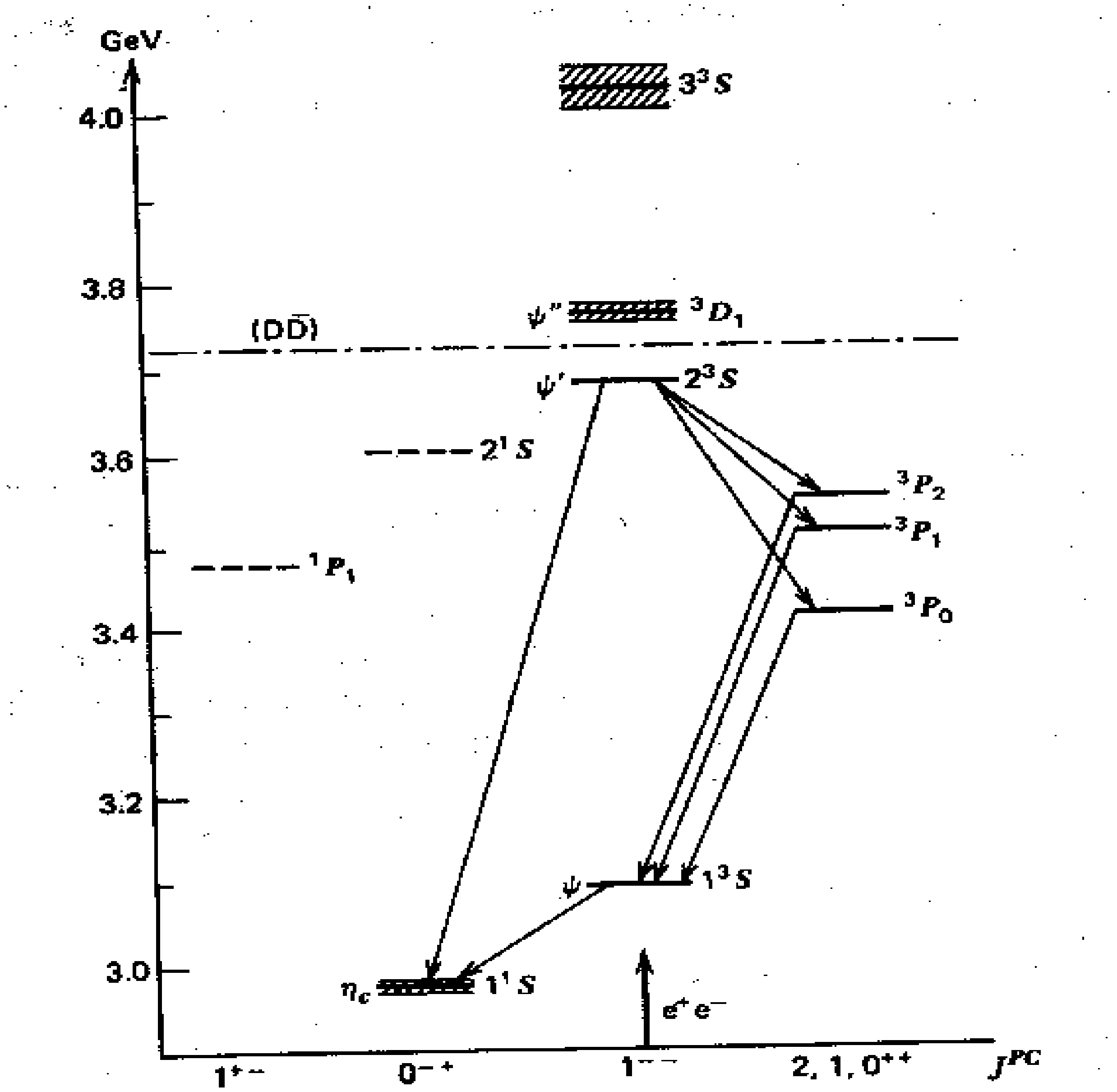

Fig. 2.13 The observed charmonium spectrum. The transitions shown have all been observed. The $^1P_1$ and 2^1S states await discovery. The particle widths are shown by shaded bands. The dot-dash line shows the $D\bar{D}$ threshold; states below this line cannot decay into charmed mesons. The states with $J^{PC} = 1^{--}$ can be directly produced by e^+e^- collisions.

Figure 54: The charmonium spectrum

From Equations (70) and (81), parity and C-parity of a quarkonium are:

$$P=P_q P_{\bar{q}}(-1)^L=(-1)^{L+1} \;\; ; \;\; C=(-1)^{L+S}$$

Remember the spectroscopic notation (Eq. 59): ${}^{2S+1}L_J$. Predicted and observed charmonium and bottomium states for n=1 and n=2:

		J^{PC}	$c\bar{c}$ state	$b\bar{b}$ state
$n=1$	${}^1S_0$	0^{-+}	$\eta_c(2980)$	–
$n=1$	${}^3S_1$	1^{--}	$J/\psi(3097)$	$\Upsilon(9460)$
$n=2$	${}^1S_0$	0^{-+}	–	–
$n=2$	${}^3S_1$	1^{--}	$\psi(3686)$	$\Upsilon(10023)$
$n=2$	${}^3P_0$	0^{++}	$\chi_{c0}(3415)$	$\chi_{b0}(9860)$
$n=2$	${}^3P_1$	1^{++}	$\chi_{c1}(3511)$	$\chi_{b1}(9892)$
$n=2$	${}^3P_2$	2^{++}	$\chi_{c2}(3556)$	$\chi_{b2}(9913)$
$n=2$	${}^1P_1$	1^{+-}	–	

- States J/ψ and ψ have the same J^{PC} quantum numbers as a photon: 1^{--}, and the most common way to form them is through e^+e^--annihilation, where virtual photon converts to a charmonium state

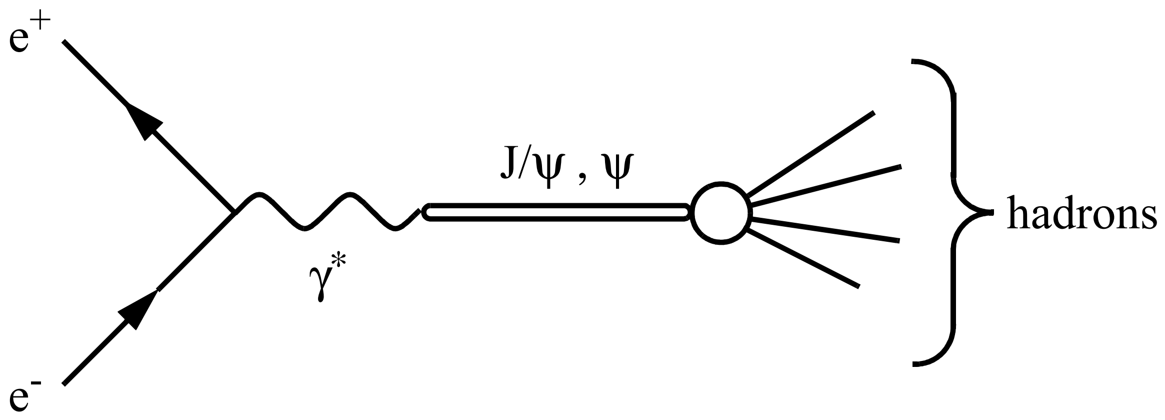

Figure 55: Formation and decay of J/ψ (ψ) mesons in e^+e^- annihilation

- If centre-of-mass energy of incident e^+ and e^- is equal to the quarkonium mass, formation of the latter is highly probable, which leads to the peak in the cross-section $\sigma(e+e- \to \text{hadrons})$.

Cross-section is defined through

$$N = \sigma \times L, \tag{84}$$

where N is the number of reactions (events), and L is the integrated luminosity (describing the density of colliding particles integrated over a time).

$[\sigma]$ = 1 barn (1 b) $\equiv 10^{-24}$ cm^2, $[L]$ = cm^{-2} or 1 barn^{-1} (1 b^{-1}).

For example, at LHC (pp-collider), the instantaneous luminosity is $L=10^{34}$ cm^{-2}s^{-1}, integrated luminosity is $L=10^{34}$ cm^{-2}s^{-1} x 10^7 s = 10^{41} cm^{-2} = 100 fb^{-1} = 100/ (10^{-15} x 10^{-24} cm^2) (assuming a collider running time of 10^7 s).

The total production cross-section for $b\bar{b}$-pairs is about 500 μb → in 10^7 s (about 1/3 of a year), the number of produced events is $N=\sigma \times L$ = 500 μb x100 fb^{-1} = 5 x 10^{13}

Convenient way to represent cross-sections in e^+e^- annihilation: normalize the hadron cross-section to the muon cross-section.

$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{85}$$

→ sharp peaks in R at E_{cm}=3.097 GeV (J/Ψ), 3.686 GeV (Ψ)

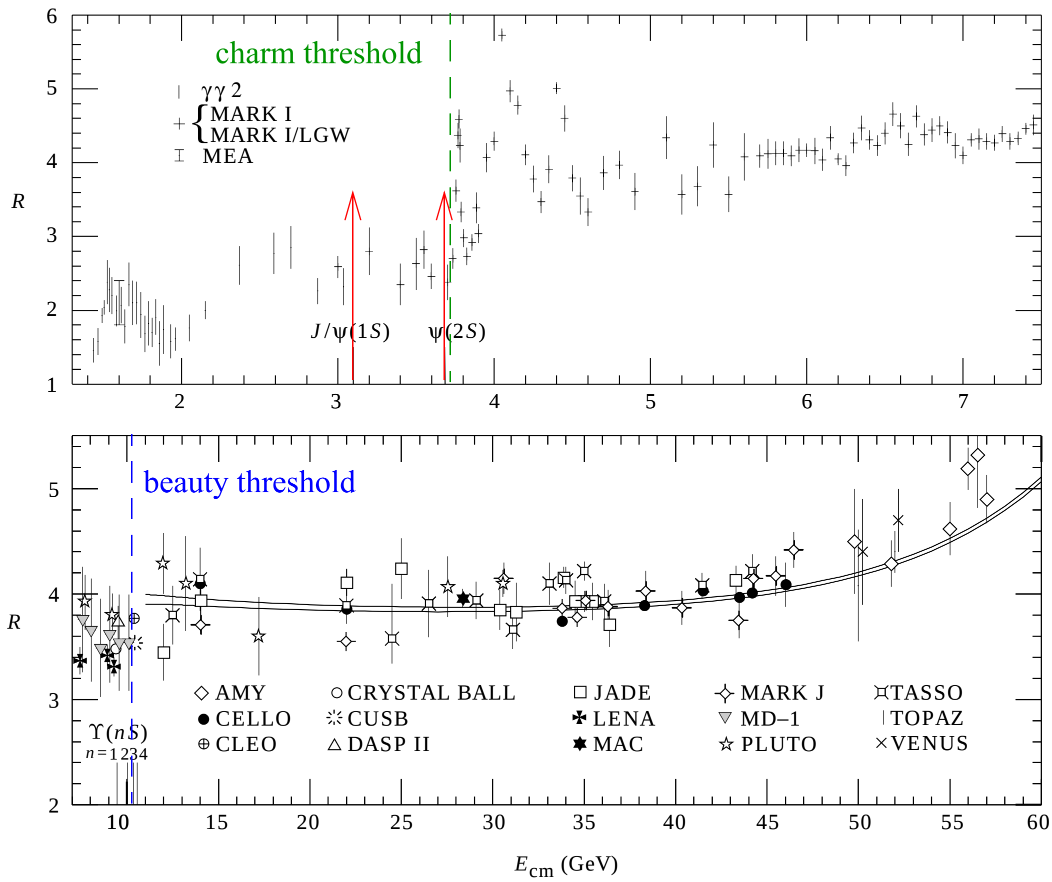

Figure 56: Cross-section ratio R in e^+e^- collision. Peaks at 3.097 and 3.686 in the upper plot are marked with the arrows, since they go out of the y-scale.

- The $\mu^+\mu^-$ pair production cross-section depends only on the E_{cm} (smooth function), and the coupling constant α:

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2} \tag{86}$$

❖ Charm threshold (3730 MeV): $E_{cm} = 2m_D$. Pair production of $e^+e^- \Rightarrow D\bar{D}$ becomes possible.

→ Wide peaks above charm threshold: short-living resonances

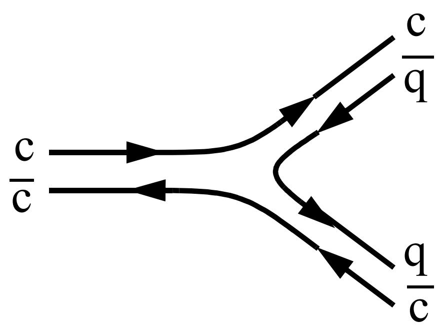

Figure 57: Charmonium resonance decay to charmed mesons

→ Narrow J/ψ and ψ peaks below charm threshold: can not decay by the mechanism on Fig.57 due to the energy conservation ($E_{cm}<2m_D$). J/ψ and ψ can only decay to light hadrons (containing u, d, s), or to e^+e^-, or to $\mu^+\mu^-$. J/ψ and ψ have therefore very long lifetimes ($\tau=1/\Gamma$). Annihilation of a heavy quark-antiquark is thus *suppressed* as opposed to light quark-antiquark pairs.

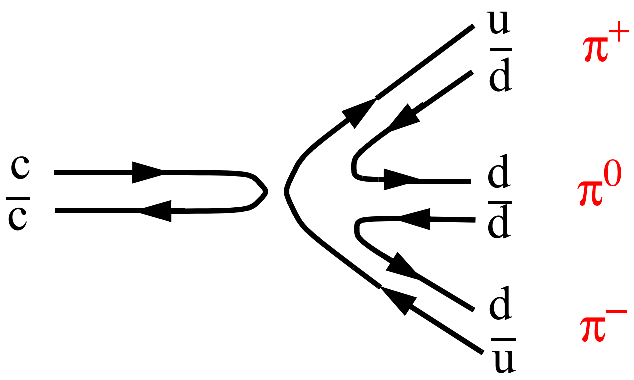

Figure 58: Charmonium decay to light non-charmed mesons

→ Charmonium states with quantum numbers different of those of photon can not be produced as in Fig.55, but can be found in radiative decays of J/ψ or ψ:

$$\psi(3686) \to \chi_{ci} + \gamma \qquad (i{=}0,1,2) \tag{87}$$

$$\psi(3686) \to \eta_c(2980) + \gamma \tag{88}$$

$$J/\psi(3097) \to \eta_c(2980) + \gamma \tag{89}$$

❖ Bottomonium spectrum is observed in much the same way as the charmonium one

❖ Beauty threshold is at 10560 MeV/c^2 (twice mass of the B meson)

→ Similarities between spectra of bottomonium and charmonium suggest similarity of forces acting in the two systems

The quark-antiquark potential

❖ Assume that the $q\bar{q}$ potential is a central one, *V(r)*, and the system is non-relativistic

In the centre-of-mass frame of the $q\bar{q}$ pair, the Schrödinger equation is

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{x}) + V(r)\psi(\vec{x}) = E\psi(\vec{x}) \qquad (90)$$

Here $\mu = m_q/2$ is the *reduced mass* of the quarks, and $r = |\vec{x}|$ is the distance between quarks.

Mass of a quarkonium state in this framework is

$$M(q\bar{q}) = 2m_q + E \qquad (91)$$

In the case of a Coulomb-like potential $V(r) \propto r^{-1}$, energy levels depend only on the principal quantum number n:

$$E_n = -\frac{\mu\alpha^2}{2n^2}$$

For a harmonic oscillator potential $V(r) \propto r^2$, the degeneracy of energy levels is broken - the energy

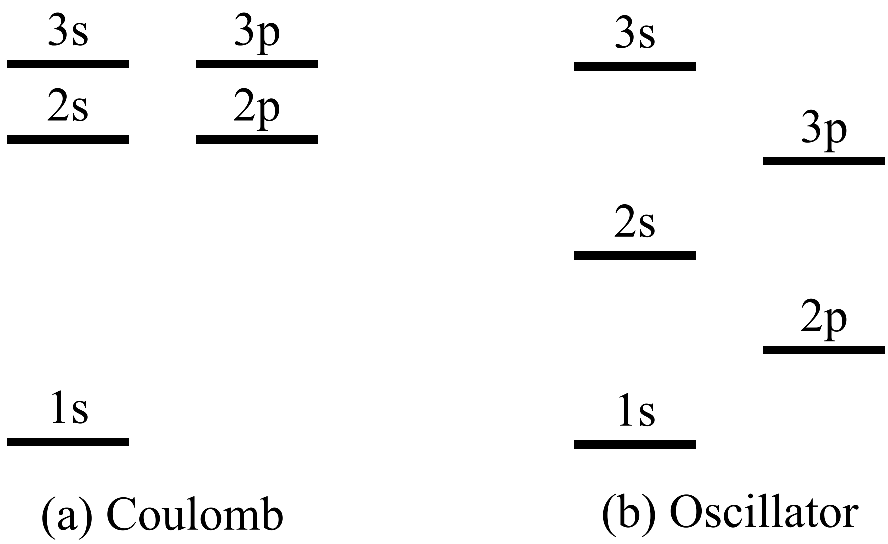

Figure 59: Energy levels $E(nL)$ arising from Coulomb and harmonic oscillator potentials for n=1,2,3. $E(3s)$ states are set the same.

levels depend on both n and L.

Comparing Fig. 59 with Fig.54, one can see that heavy quarkonia spectra are in between the two possibilities. The potential can be fitted by:

$$V(r) = -\frac{a}{r} + br \tag{92}$$

The potential behaves as 1/*r* at small *r*, and like *r* at large r.

Coefficients *a* and *b* are determined by solving Equation (90) and fitting results to data

→ *a*=0.48, *b*=0.18 GeV^2

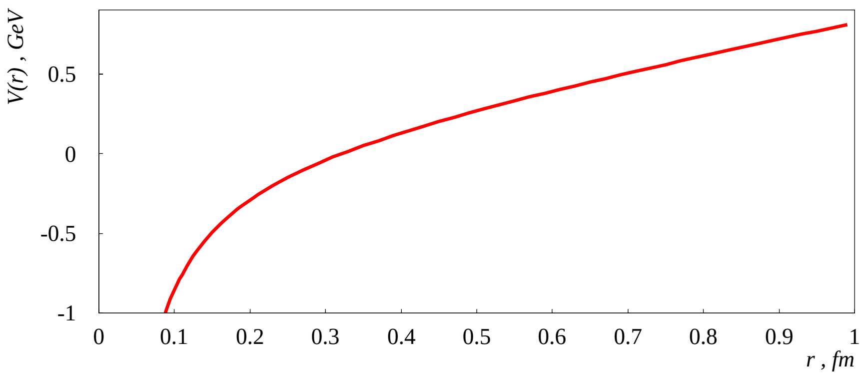

Figure 60: Modified Coulomb potential (92)

Other forms of the potential can give equally good fits, for example

$$V(r) = a\,ln(br) \tag{93}$$

where parameters appear to be

→ a=0.7 GeV b=0.5 GeV

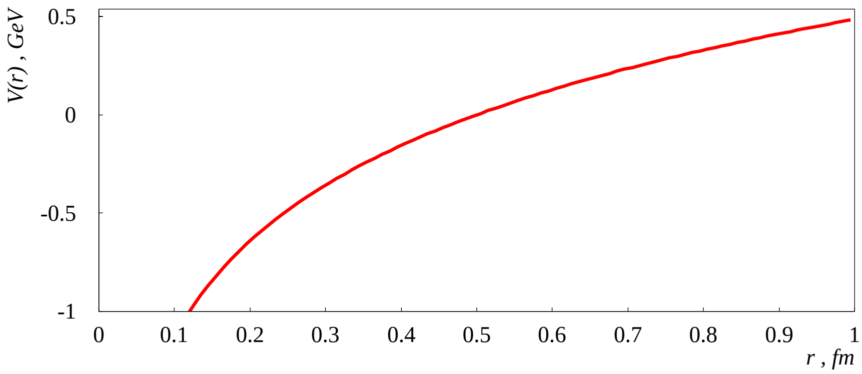

Figure 61: Logarithmic potential (93)

→ In the range $0.2 \le r \le 0.8$ fm potentials like (92) and (93) are in good agreement ⇒ in this region the quark-antiquark potential is well-defined

→ Simple non-relativistic Schrödinger equation explains the existence of several energy states for a given quark-antiquark system

Light mesons; nonets

- Mesons with spin J=0 are called *"pseudoscalar mesons"* (quark spins have opposite directions)
- Mesons with spin J=1 are *"vector mesons"* (quark spins point to the same direction)

There are nine possible $q\bar{q}$ combinations containing the lightest quarks (u,d,s).

– *Pseudoscalar meson nonet*: 9 mesons with J^P=0^-

– *Vector meson nonet*: 9 mesons with J^P=1^-

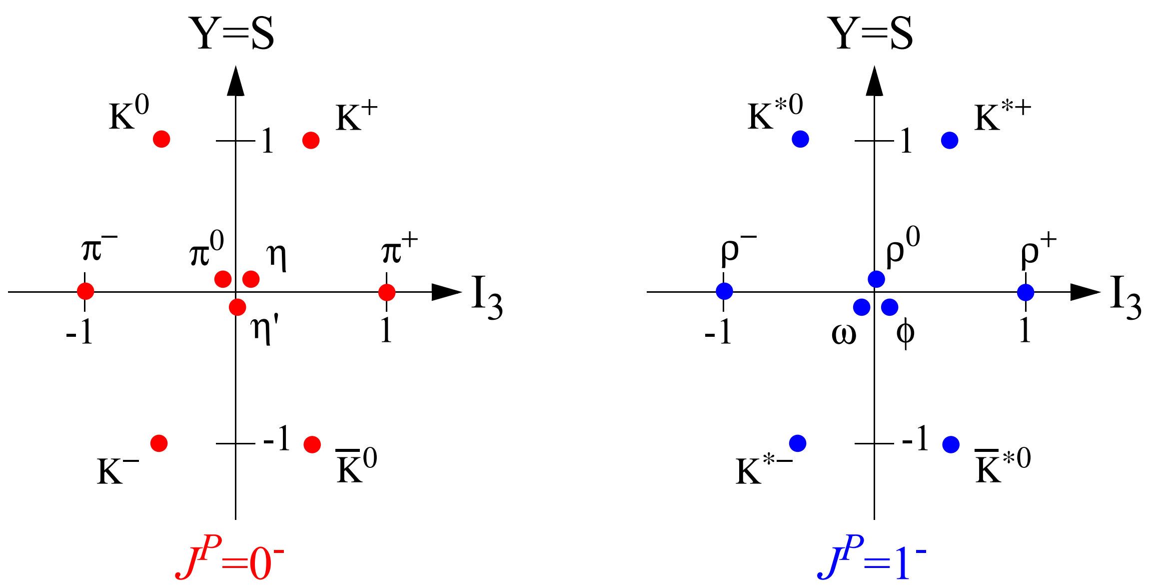

Figure 62: Light meson nonets in (I_3, Y) space ("weight diagrams")

In each nonet, there are 3 particles with equal quantum numbers Y=S=I_3=0. They correspond to a $q\bar{q}$ pair ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$), or a *linear combination* of these states (follows from an isospin operator analysis):

$$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I = 1, I_3 = 0 \tag{94}$$

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad I = 0, I_3 = 0 \tag{95}$$

→ π^0 and ρ^0 mesons are linear combinations of $u\bar{u}$ and $d\bar{d}$ states (94): $(u\bar{u} - d\bar{d})/(\sqrt{2})$

→ ω meson is the linear combination of (95): $(u\bar{u} + d\bar{d})/(\sqrt{2})$

Inclusion of an $s\bar{s}$ pair leads to further combinations:

$$\eta(547) = \frac{(d\bar{d} + u\bar{u} - 2s\bar{s})}{\sqrt{6}} \qquad I = 0, I_3 = 0 \tag{96}$$

$$\eta'(958) = \frac{(d\bar{d} + u\bar{u} + s\bar{s})}{\sqrt{3}} \qquad I = 0, I_3 = 0 \tag{97}$$

→ Meson $\phi(1019)$ is a quarkonium $s\bar{s}$, having I=0 and I_3=0

Light baryons

→ Three-quark states of the lightest quarks (u,d,s) form baryons, which can be arranged in *supermultiplets* (*singlets, octets* and *decuplets*).

→ The lightest baryon supermultiplets are an octet of $J^P = \frac{1}{2}^+$ particles and a decuplet of $J^P = \frac{3}{2}^+$ particles

Weight diagrams of baryons can be deduced from the quark model under the assumption that the **combined** space-spin wavefunctions are *symmetric* under the interchange of **like** quarks. Antisymmetric wavefunction would predict a $1/2^+$ octet but a $3/2^+$ singlet. Observations: $3/2^+$ are a decuplet.

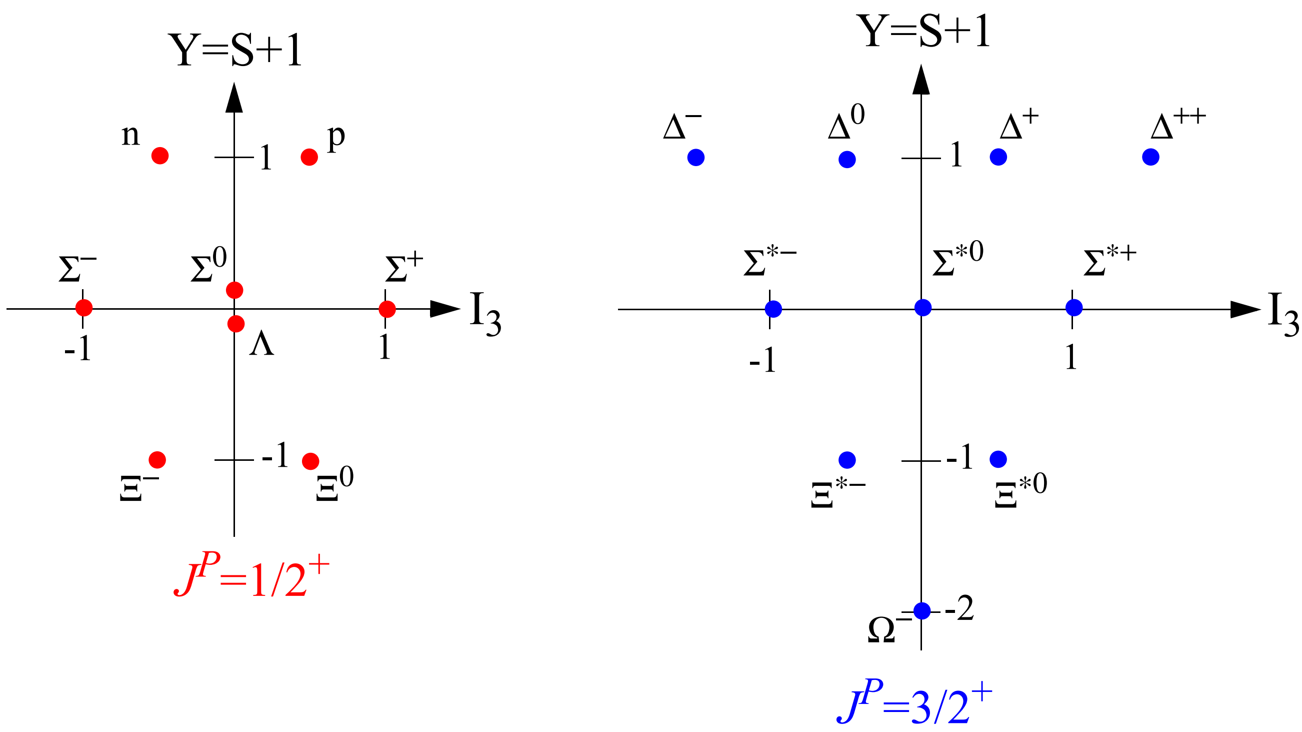

Figure 63: Weight diagrams for light baryons

– Parity of a 3-quark state $q_iq_jq_k$ is $P=P_iP_jP_k=1$

– Spin of such a state is the sum of the quark spins

– Assuming a symmetry under the exchange of like quarks, any pair of like quarks qq must have spin 1.

$$\Downarrow$$

→ there are six distinct combination of the form $q_iq_iq_j$: uud, uus, ddu, dds, ssu, ssd each of them can have spin J=1/2 or J=3/2

→ three combinations of the form $q_iq_iq_i$ are possible: uuu, ddd, sss

spins of all like-quarks have to be parallel (symmetry presumption), hence J=3/2 only

→ the remaining combination is uds, with two states having spin values J=1/2 and one state with J=3/2

→ By adding up the number of states, one gets 8 states with $J^P=1/2^+$ and 10 states with $J^P=3/2^+$, exactly what is shown by the weight diagrams

❖ Measured masses of baryons show that the mass difference is much smaller between members of the same isospin multiplet than between members of different isospin multiplets

In what follows, equal masses of isospin multiplet members are assumed, e.g.,

$$m_p = m_n \equiv m_N$$

Experimentally, particles with more s-quarks are heavier:

Ξ^0(1315)=(uss);Σ^+(1189)=(uus); p(938)=(uud)

Ω^-(1672)=(sss); Ξ^{*0}(1532)=(uss);
Σ^{*+}(1383)=(uus); Δ^{++}(1232)=(uuu)

→ There is evidence that the main contribution to large mass differences comes from the s-quark

Knowing the masses of baryons, one can calculate 6 estimates of the mass difference between the s-quark and the light quarks (u,d):

From the $3/2^+$ decuplet one obtains:

$$M_\Omega - M_\Xi = M_\Xi - M_\Sigma = M_\Sigma - M_\Delta = m_s - m_{u,d}$$

and from the $1/2^+$ octet:

$$M_\Xi - M_\Sigma = M_\Xi - M_\Lambda = M_\Lambda - M_N = m_s - m_{u,d}$$

Average value of those differences is

$$m_s - m_{u,d} \approx 160\ MeV \tag{98}$$

→ BUT baryons are spin-1/2 (3/2) particles ⇒ fermions ⇒ their wavefunctions must be *antisymmetric*, otherwise all the discussion above **contradicts the Pauli principle!**

COLOUR

- Experimental data confirm the predictions based on the assumption of symmetric space-spin wave functions

→ That means that apart from the space and spin degrees of freedom, quarks must have yet another attribute, which makes the total wavefunction antisymmetric

In 1964-1965, Greenberg and Nambu proposed a new property – the *colour* – with THREE possible states. Colour is associated with the corresponding antisymmetric wavefunction χ^C:

$$\Psi = \psi(\vec{x})\chi\chi^C \qquad (99)$$

- Conserved quantum numbers associated with χ^C are *colour charges* – in strong interactions they play an analogous role to the electric charge in e.m. interactions

- Hadrons can exist only in *colour singlet* states, with the total colour charge equal to zero

- Quarks have to be *confined* within the hadrons, since non-zero colour states are forbidden

- The three independent colour wavefunctions are represented by *"colour spinors"* :

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{100}$$

- They are acted on by eight independent *"colour operators"* which are represented by a set of 3x3 matrices (analogues of Pauli matrices)

- Colour charges I_3^C and Y^C are the eigenvalues of the corresponding operators

Values of I_3^C and Y^C for the colour states of quark and antiquarks are:

Quarks			Antiquarks		
	I_3^C	Y^C		I_3^C	Y^C
r ("red")	1/2	1/3	$\bar{\mathbf{r}}$	-1/2	-1/3
g ("green")	-1/2	1/3	$\bar{\mathbf{g}}$	1/2	-1/3
b ("blue")	0	-2/3	$\bar{\mathbf{b}}$	0	2/3

- *Colour hypercharge* Y^C and *colour isospin charge* I_3^C are <u>additive</u> quantum numbers, having an opposite sign for quarks and antiquarks

The confinement condition for the total colour charge of a hadron is:

$$I_3^C = Y^C = 0 \tag{101}$$

The most general colour wavefunction for a baryon is a linear superposition of six possible combinations:

$$\begin{aligned}\chi_B^C = {} & \alpha_1 r_1 g_2 b_3 + \alpha_2 g_1 r_2 b_3 + \alpha_3 b_1 r_2 g_3 \\ & + \alpha_4 b_1 g_2 r_3 + \alpha_5 g_1 b_2 r_3 + \alpha_6 r_1 b_2 g_3\end{aligned} \tag{102}$$

where α_i are constants.

The colour wavefunction of a baryon has to be totally antisymmetric:

$$\chi_B^C = \frac{1}{\sqrt{6}}(r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3) \tag{103}$$

The colour confinement principle (101) implies certain requirements for states containing both quarks and antiquarks:

– consider combination $q^m\bar{q}^n$ of *m* quarks and *n* antiquarks, $m \geq n$

– for a particle with α quarks in the r-state, β quarks in the g-state, γ quarks in the b-state ($\alpha+\beta+\gamma=m$), and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ antiquarks in corresponding antistates ($\bar{\alpha}+\bar{\beta}+\bar{\gamma}=n$), the colour wavefunction is

$$r^{\alpha} g^{\beta} b^{\gamma} \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}} \tag{104}$$

Adding up the colour charges and applying the confinement requirement gives:

$$I_3^C = \alpha \cdot \frac{1}{2} + \bar{\alpha} \cdot \frac{-1}{2} + \beta \cdot \frac{-1}{2} + \bar{\beta} \cdot \frac{1}{2} = 0$$

$$Y^C = \alpha \cdot \frac{1}{3} + \bar{\alpha} \cdot \frac{-1}{3} + \beta \cdot \frac{1}{3} + \bar{\beta} \cdot \frac{-1}{3} + \gamma \cdot \frac{-2}{3} + \bar{\gamma} \cdot \frac{2}{3} = 0$$

$$\Downarrow$$

$$\alpha - \bar{\alpha} = \beta - \bar{\beta} = \gamma - \bar{\gamma} \equiv p$$

Here p is a non-negative integer, and hence m-n=3p

The only combination $q^m\bar{q}^n$ allowed by the colour confinement principle is

$$(3q)^p(q\bar{q})^n\ , \qquad p,n \geq 0 \tag{105}$$

- Equation (105) forbids states with fractional electric charges
- However, it allows exotic combinations like $qq\bar{q}\bar{q}$, $qqqq\bar{q}$. The observed pentaquark is a quark-state $Z^+ = uudd\bar{s}$, which is allowed by Eq.(105).

SUMMARY

- Forces acting between quarks can be investigated through heavy quark systems, like $c\bar{c}$ and $b\bar{b}$.

- Non-relativistic system → the Schrödinger equation. The observed $c\bar{c}$ and $b\bar{b}$ spectra seem to follow the potential $V(r) = -\frac{a}{r} + br$ or $V(r) = a\ln(br)$. The potentials behave as $1/r$ at small r, and like r at large r. In the range $0.2 \le r \le 0.8$ fm both potentials are in good agreement ⇒ in this region the quark-antiquark potential is well-defined. Coefficients *a* and *b* are determined from data. The energy levels depend on both *n* and *L*.

- $c\bar{c}$ and $b\bar{b}$ states: parity and C-parity are:

$$P = P_q P_{\bar{q}} (-1)^L = (-1)^{L+1} \quad ; \quad C = (-1)^{L+S}$$

- States J/ψ and ψ: J^{PC} same as a photon (1^{--}) → formed through e^+e^--annihilation, where virtual photon converts to a charmonium state. J/ψ and ψ can not decay to $D\bar{D}$ due to the energy conservation ($E_{cm}<2m_D$). J/ψ and ψ can only decay to light hadrons (containing *u, d, s*), or to e^+e^-, or to $\mu^+\mu^-$. J/ψ and ψ have therefore very long lifetimes ($\tau=1/\Gamma$). Annihilation of a heavy quark-antiquark is thus *suppressed*.

- Cross-section σ is defined as $N = \sigma \times L$.

- Hadronic cross-section in e^+e^- annihilation is often normalized to the muon cross-section. Sharp peaks at E_{cm}=3.097 GeV (=m(J/Ψ)), 3.686 GeV (=m(Ψ)). The $\mu^+\mu^-$ pair production cross-section depends only on the E_{cm}.

- Charm threshold (3730 MeV): $E_{cm} = 2m_D$. Pair production of $e^+e^- \Rightarrow D\bar{D}$ becomes possible. Wide peaks above charm threshold: short-living resonances.

- Light meson multiplets: Pseudoscalar meson nonet: $J^P=0^-$, vector meson nonet $J^P=1^-$ The lightest baryon supermultiplets: octet $J^P=1/2^+$ and decuplet $J^P=3/2^+$.

- Observations: the **combined**

space-spin wavefunctions of baryons are *symmetric* under the interchange of **like** quarks. Therefore quarks must have yet another property, which makes the total wavefunction antisymmetric ⇒ COLOUR.

- *Colour* has 3 possible states. Colour corresponds to the antisymmetric wavefunction χ^C so that the total wavefunction is antisymmetric: $\Psi = \psi(\vec{x})\chi\chi^C$

- The three colour wavefunctions are represented by *"colour spinors"* $\chi^C = r$, *g* and *b*.

- Conserved quantum numbers associated with χ^C are *the colour isospin charge* I_3^C and *the colour hypercharge* Y^C.

- Hadrons can exist only in *colour singlet* states, with the total colour charge equal to zero. Therefore quarks have to be *confined* within the hadrons, since non-zero colour states are forbidden. The colour confinement principle forbids states with fractional electric charges, but allows exotic combinations like $qq\bar{q}\bar{q}$, $qqqq\bar{q}$.