X. Charge conjugation and parity

Parity and charge conjugation transformations are defined as (see Chapter IV.):

- Parity transformation is the transformation by reflection, reversing the *coordinate* $\vec{r}$ and *momentum* $\vec{p}$:

$$\vec{x}_i \to \vec{x}'_i = -\vec{x}_i$$

  - Parity transformation does not change $\vec{L} = \vec{r} \times \vec{p}$ or spin

- Charge conjugation (C-parity) replaces particles with their antiparticles, reversing *charges* and *magnetic moments*

- While conserved in strong and electromagnetic interactions, parity and C-parity are violated in weak processes

In 1956, T.D.Lee & C.N.Yang suggested non-conservation of parity in weak processes; can be shown by studying e.g. kaon decays

- Some known decays of K^+ are:

$$K^+ \to \pi^0 + \pi^+ \qquad \text{and} \qquad K^+ \to \pi^+ + \pi^+ + \pi^-$$

Intrinsic parity of a pion P_π=-1, and for the $\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ states parities are

$$P_{0+} = P_\pi^2(-1)^L = 1, \qquad P_{++-} = P_\pi^3(-1)^{L_{12}+L_3} = -1$$

where L=0 since kaon has spin-0.

- One of the K^+ decays **violates** parity!

- 1957: Mrs. Wu carried out dedicated studies of parity violation in β-decay

– ^{60}Co β-decay into $^{60}Ni^*$ was studied

– ^{60}Co was cooled to 0.01 K to prevent thermal disorder

– Sample was placed in a magnetic field $\Rightarrow$ nuclear spins were aligned along the field direction

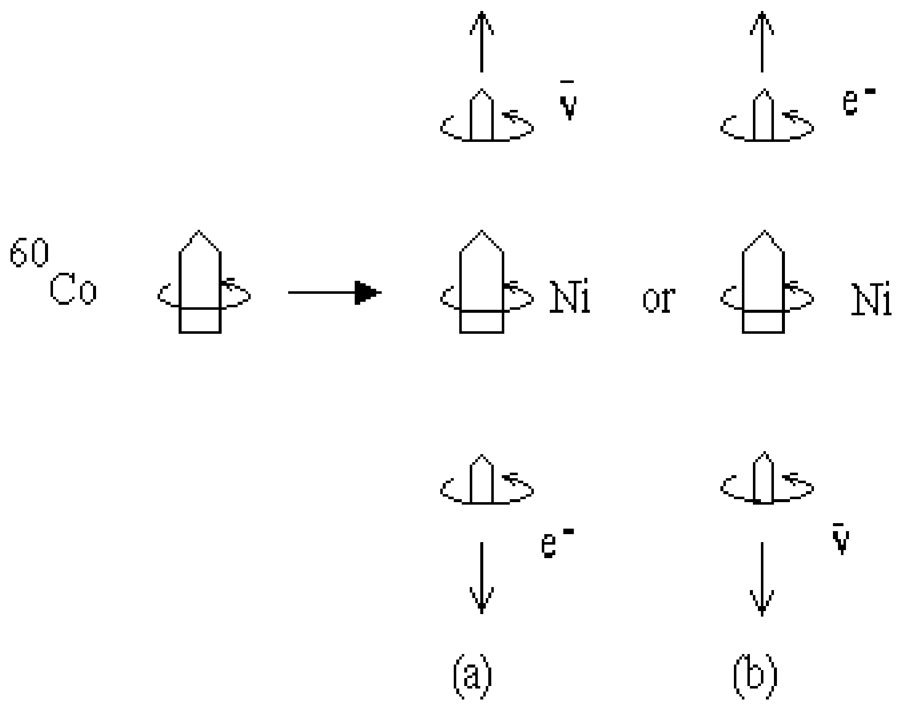

Figure 129: Possible β-decays of ^{60}Co: case (a) is preferred.

– If parity is conserved, processes (a) and (b) must have equal rates

- *Electrons were emitted predominantly in the direction opposite the ^{60}Co spin*

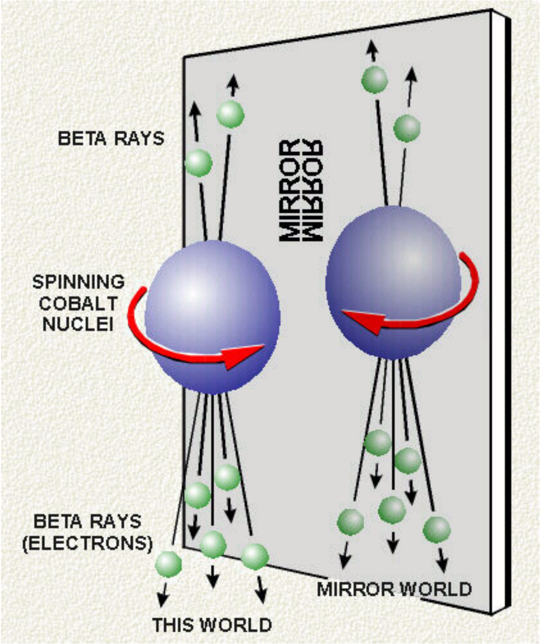

Figure 130: Representation of parity not being conserved. In the mirror, the direction of spin is reversed, while the direction in which most beta rays are emitted remains unchanged. Parity transformation would turn the mirror upside down, to return the spin to the original direction. But then the electrons will be emitted upwards, which is not the case: parity is not conserved.

❖ Another case of parity and C-parity violation was observed in muon decays:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \tag{195}$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \tag{196}$$

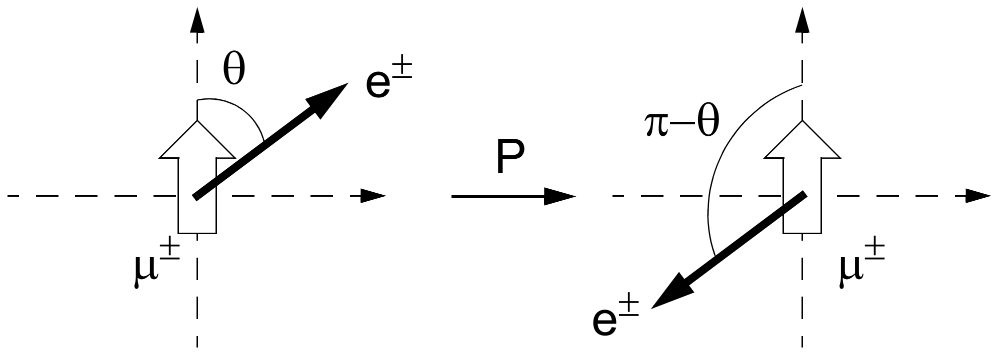

Figure 131: Effect of a parity transformation on the muon decays (195) and (196)

Angular distribution of e^- (e^+) emitted in μ^- (μ^+) decay has a form of:

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1 - \frac{\xi_\pm}{3}\cos\theta\right)$$

– here $\xi_\pm$ are constants – "*asymmetry parameters*", and $\Gamma_\pm$ are total decay rates (thus inverse lifetimes)

$$\Gamma_{\pm} = \int_{-1}^{1} \Gamma_{\pm}(\cos\theta)d\cos\theta \equiv \frac{1}{\tau_{\pm}} \tag{197}$$

- If the process is invariant under charge conjugation (C-invariance) $\Rightarrow$

$$\Gamma_{+} = \Gamma_{-} \qquad \xi_{+} = \xi_{-} \tag{198}$$

(rates and angular distributions are the same for e^- and e^+)

- If the process is P-invariant, then angular distributions in forward and backward directions are the same:

$$\Gamma_{\mu^{\pm}}(\cos\theta) = \Gamma_{\mu^{\pm}}(-\cos\theta) \qquad \xi_{+} = \xi_{-} = 0 \tag{199}$$

- Experimental results:

$$\Gamma_{+} = \Gamma_{-} \qquad \xi_{+} = -\xi_{-} = 1.00 \pm 0.04 \tag{200}$$

Both C- and P-invariance are violated, on daily basis!

Solution: combined operation CP is conserved

$$\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(-\cos\theta) \qquad (201)$$

$$\Downarrow$$

$$\Gamma_+ = \Gamma_- \qquad \xi_+ = -\xi_- \qquad (202)$$

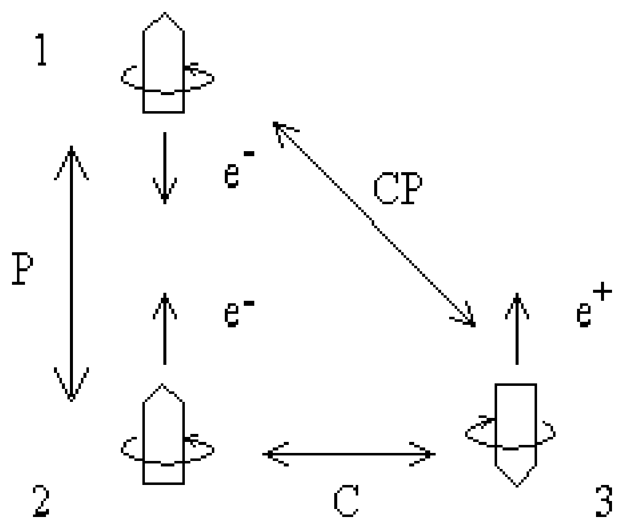

Figure 132: P-, C- and CP-transformation of an electron

- It appears that electrons prefer to be emitted with momentum opposite to their spin

Corresponding observable: *helicity* – projection of particle's spin to its direction of motion

$$\Lambda = \frac{\vec{J}\vec{p}}{|\vec{p}|} = \frac{\vec{s}\vec{p}}{|\vec{p}|} \qquad (203)$$

Eigenvalues of helicity are h=-s,-s+1,...,+s, $\Rightarrow$ for spin-1/2 electron it can be either -1/2 or 1/2

Figure 133: Helicity states of spin-1/2 particle

Helicity of neutrino

- 1958: Goldhaber et al. measured helicity of neutrino using the reaction of electron capture in Eu:

$$e^- + {}^{152}Eu\ (J=0) \rightarrow {}^{152}Sm^*\ (J=1) + \nu_e \qquad (204)$$

$$ {}^{152}Sm^*\ (J=1) \rightarrow {}^{152}Sm\ (J=0) + \gamma \qquad (205)$$

In the reaction (204), ${}^{152}Sm^*$ and ν_e recoil in opposite directions

Figure 134: Spin of ${}^{152}Sm^*$ has to be opposite to the neutrino spin (parallel to the electron spin)

In the initial state, electron has spin-1/2, ${}^{152}Eu$ – spin-0, in final state: ${}^{152}Sm^*$ has spin-1 and ν_e – spin-1/2 $\Rightarrow$ spin of ${}^{152}Sm^*$ is parallel to the electron spin and opposite to the neutrino spin.

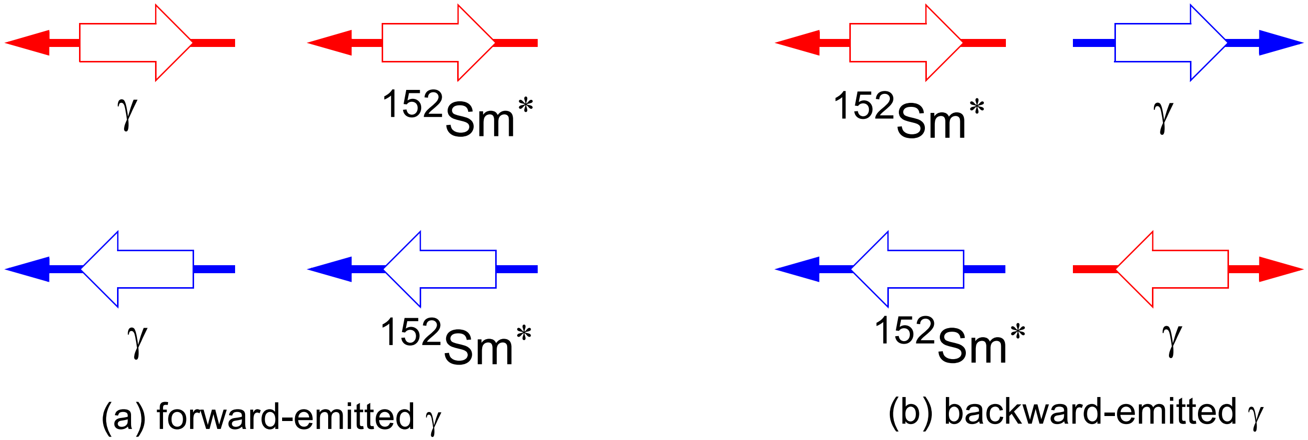

Figure 135: Forward-emitted γ has the same helicity as ν_e

Events with γ emitted in the direction of motion of $^{152}Sm^*$ were selected.

– Polarization of photons was determined by studying their absorption in magnetized iron.

- It turned out that neutrinos can be only **left-handed**!
- Antineutrinos were found to be always right-handed.

V-A interaction

- V-A interaction theory was introduced by Fermi as an analytic description of spin dependence of charged current interactions.
  - It denotes "polar Vector - Axial vector" interaction

– *Polar vector* is any which direction is reversed by parity transformation: momentum $\vec{p}$

– *Axial vector* is that which direction is not changed by parity transformation: spin $\vec{s}$ or orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$

  - Weak current has both vector and axial components, hence parity is not conserved in weak interactions

- Main conclusion: if $v \approx c$, only left-handed fermions ν_L, e^-_L etc. are emitted, and right-handed antifermions.

- *The very existence of preferred states violates both C- and P-invariance*

- Neutrinos (antineutrinos) are always relativistic and hence always are left(right)-handed
- For other (massive) fermions, preferred states are left-handed, and right-handed states are not completely forbidden, but *suppressed* by factors

$$\left(1-\frac{v}{c}\right) \approx \frac{m^2}{2E^2} \qquad (206)$$

Consider pion decay modes (pion at rest):

$$\pi^+ \to l^+ + \nu_l \qquad (l=e,\ \mu) \qquad (207)$$

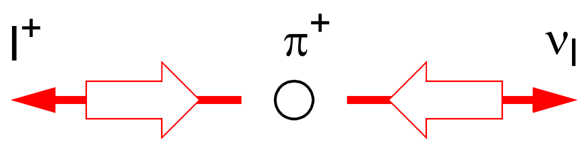

Figure 136: Helicities of leptons emitted in a pion decay

– π^+ has spin-0, $\Rightarrow$ spins of charged lepton and neutrino must be opposite

– Neutrino is always left-handed $\Rightarrow$ charged lepton has to be left-handed as well. BUT: e^+ and μ^+ as antileptons ought to be right-handed!

- It follows that a relativistic charged lepton can not be emitted in a pion decay!
  - Muons are rather heavy $\Rightarrow$ non-relativistic $\Rightarrow$ can be left-handed (see Eq.(206))
  - Decays of pions to positrons ought to be *suppressed* by a factor of 10^{-5}

Measured ratio:

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \times 10^{-4} \qquad (208)$$

- Muons emitted in pion decays are always polarized (μ^+ are left-handed)

This can be used to measure muon decay (195), (196) symmetries by detecting highest-energy (relativistic) electrons with energy

$$E = \frac{m_\mu}{2}\left(1 + \frac{m_e^2}{m_\mu^2}\right) \gg m_e \qquad (209)$$

- Highest-energy electrons are emitted in decays when both ν_μ and $\bar{\nu}_e$ are emitted in the direction opposite to e^-:

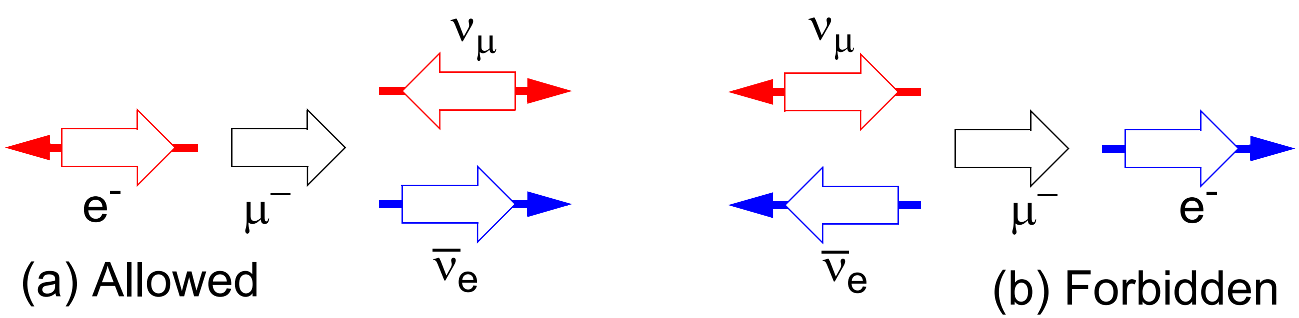

Figure 137: Muon decays with highest-energy electron emission

- Electron must have spin parallel to the muon spin $\Rightarrow$ configuration (a) is strongly preferred $\Rightarrow$ observed experimentally forward-backward asymmetry (200)

Neutral kaons

- CP symmetry apparently can be violated in weak interactions
  - Neutral kaons $K^0(498)=(d\bar{s})$ and $\bar{K}^0(498)=(s\bar{d})$ can be converted into each other because they have same quantum numbers

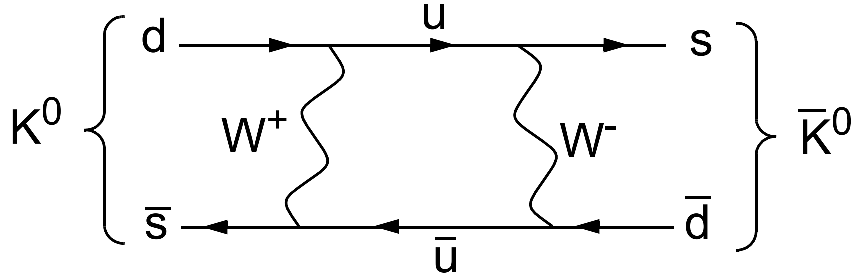

Figure 138: Example of a process converting K^0 to $\bar{K}^0$.

- Phenomenon of *K^0–$\bar{K}^0$ mixing* : observed physical particles are linear combinations of K^0 and $\bar{K}^0$, since there is no conserved quantum number to distinguish them
  - The same is true for neutral B-mesons: $B^0 = d\bar{b}$, $\bar{B}^0= b\bar{d}$, $B_s=s\bar{b}$ and $\bar{B}_s=b\bar{s}$, and for neutral D-mesons $D^0=c\bar{u}$ and $\bar{D}^0=u\bar{c}$.

C-transformation changes a quark into antiquark $\Rightarrow$

$$C|K^0, \vec{p}\rangle = -|\bar{K}^0, \vec{p}\rangle \quad and \quad C|\bar{K}^0, \vec{p}\rangle = -|K^0, \vec{p}\rangle \tag{210}$$

– Here signs are chosen for further convenience and do not affect physical predictions

Intrinsic parity of a kaon is P_K=-1 $\Rightarrow$ for $\vec{p} = (0, 0, 0)$

$$P|K^0, \vec{p}\rangle = -|K^0, \vec{p}\rangle \quad and \quad P|\bar{K}^0, \vec{p}\rangle = -|\bar{K}^0, \vec{p}\rangle \tag{211}$$

and the CP transformation is

$$CP|K^0, \vec{p}\rangle = |\bar{K}^0, \bar{p}\rangle \quad and \quad CP|\bar{K}^0, \vec{p}\rangle = |K^0, \vec{p}\rangle \tag{212}$$

$\Rightarrow$ there are two CP eigenstates :

$$|K_1^0, \vec{p}\rangle = \frac{1}{\sqrt{2}}\{|K^0, \vec{p}\rangle + |\bar{K}^0, \vec{p}\rangle\} \tag{213}$$

$$|K_2^0, \vec{p}\rangle = \frac{1}{\sqrt{2}}\{|K^0, \vec{p}\rangle - |\bar{K}^0, \vec{p}\rangle\} \tag{214}$$

such that

$$CP|K_1^0, \vec{p}\rangle = |K_1^0, \vec{p}\rangle \;\; and \;\; CP|K_2^0, \vec{p}\rangle = -|K_2^0, \vec{p}\rangle$$

- Experimentally observed are two types of neutral kaons: K_S^0 ("S" for "short", lifetime $\tau = 0.9 \times 10^{-10} s$) and K_L^0 ("long", $\tau = 5 \times 10^{-8} s$).
  - K_S^0 is identified with K_1^0 CP-eigenstate, and K_L^0 – with K_2^0

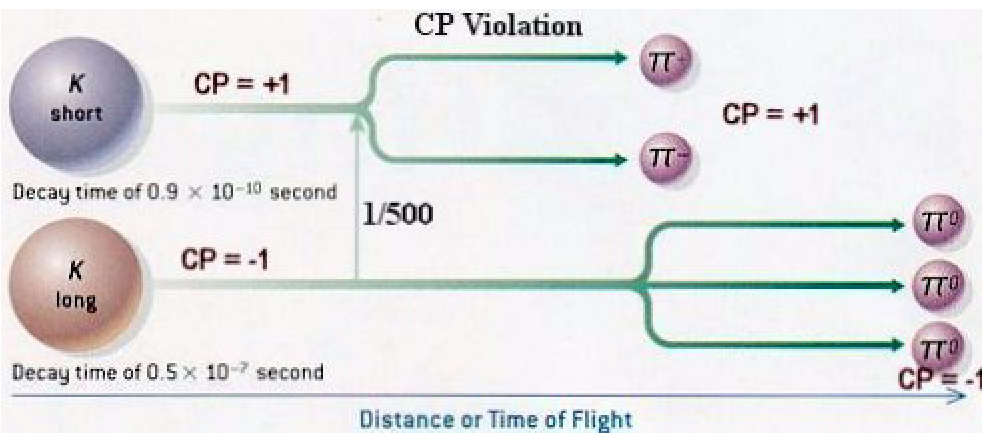

Figure 139: Decays of K-short and K-long

- If CP-invariance holds for neutral kaons, K^0_S should decay only to states with CP=1, and K^0_L – to states with CP=-1:

$$K^0_S \to \pi^+\pi^-, \qquad K^0_S \to \pi^0\pi^0 \tag{215}$$

– Parity of a two-pion state is $P = P^2_\pi(-1)^L = 1$ (kaon has spin-0)

– C-parity of $\pi^0\pi^0$ state is $C = (C_{\pi^0})^2 = 1$, and of a $\pi^+\pi^-$ state: $C = (-1)^L = 1$,

$\Rightarrow$ for final states in (215) CP=1

$$K^0_L \to \pi^+\pi^-\pi^0, K^0_L \to \pi^0\pi^0\pi^0 \tag{216}$$

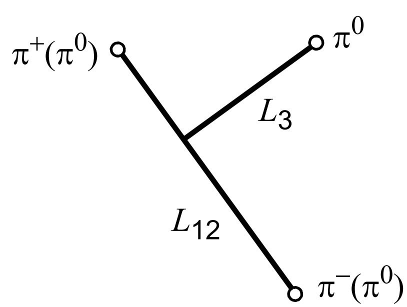

Figure 140: Angular momenta in a 3-pion system

– Parity of a 3-pion state is $P = P_{\pi}^{3}(-1)^{L_{12}+L_{3}} = -1$

– C-parity of $\pi^0\pi^0\pi^0$ is $C = (C_{\pi^0})^3 = 1$, and of the state $\pi^+\pi^-\pi^0$:

$C = C_{\pi^0}(-1)^{L_{12}} = (-1)^{L_{12}}$. L_{12} can be defined experimentally: L_{12}=0 $\Rightarrow$ for final states in (216) CP=-1

- However, the *CP-violating* decay

$$K_L^0 \to \pi^+\pi^- \tag{217}$$

was observed in 1964, with a branching ratio B≈10^{-3}.

  - In general, physical states K_S^0 and K_L^0 don't have to correspond to CP-eigenstates K_1^0 and K_2^0: K_S^0 has admixture of K_2^0 and K_L^0 – of K_1^0.
  - There can be different mechanisms for CP-violation, esp. in B^0-$\overline{B}^0$ systems; dedicated experiments (BaBar, Belle) are observing this

- CP violation in B systems is even larger than for kaons!

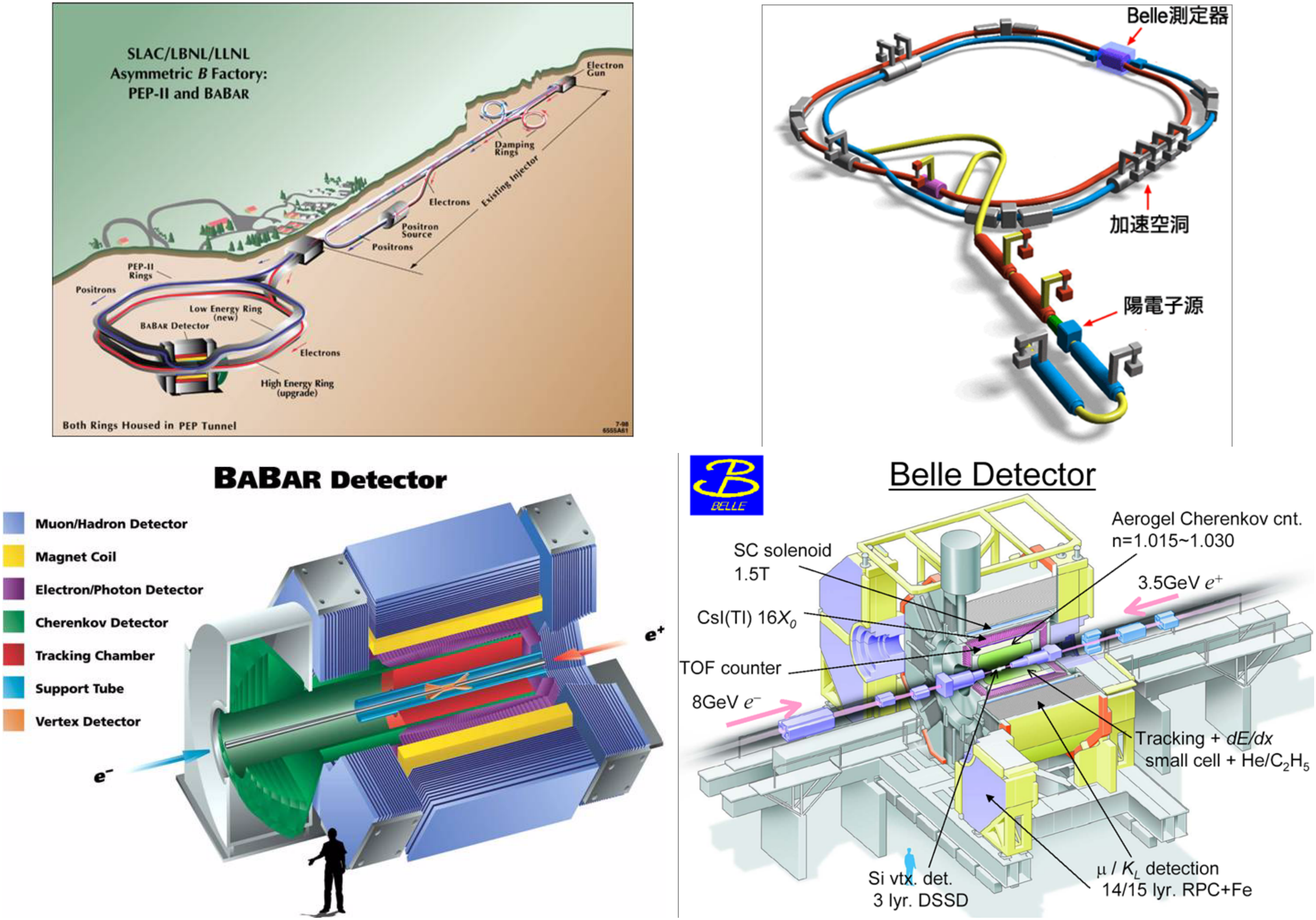

Figure 141: Today's B-physics experiments: BaBar and Belle

Strangeness oscillation

Kaons created at t=0 can be represented as:

$$K^0(0) = \frac{1}{\sqrt{2}}\{K^0_S(0) + K^0_L(0)\} \qquad \bar{K}^0(0) = \frac{1}{\sqrt{2}}\{K^0_S(0) - K^0_L(0)\} \qquad (218)$$

With time, the “mixture” changes as:

$$K^0(t) = \frac{1}{\sqrt{2}}\left(e^{-iE_S t} e^{-\Gamma_S t/2} K^0_S(0) + e^{-iE_L t} e^{-\Gamma_L t/2} K^0_L(0)\right) \qquad (219)$$

$$\bar{K}^0(t) = \frac{1}{\sqrt{2}}\left(e^{-iE_S t} e^{-\Gamma_S t/2} K^0_S(0) - e^{-iE_L t} e^{-\Gamma_L t/2} K^0_L(0)\right) \qquad (220)$$

- If the time is measured in the restframe of the kaons, then $E_S=m_S$ and $E_L=m_L$ are the masses of the K_S and K_L and Γ_S and Γ_L are the decay rates with $\Gamma=1/\tau$ as usual

This gives:

$$K^0(t) = A(t)K^0(0) + B(t)\bar{K}^0(0) \qquad (221)$$

❖ Strangeness oscillation: a K^0 beam partly turns into a $\overline{K}^0$ beam

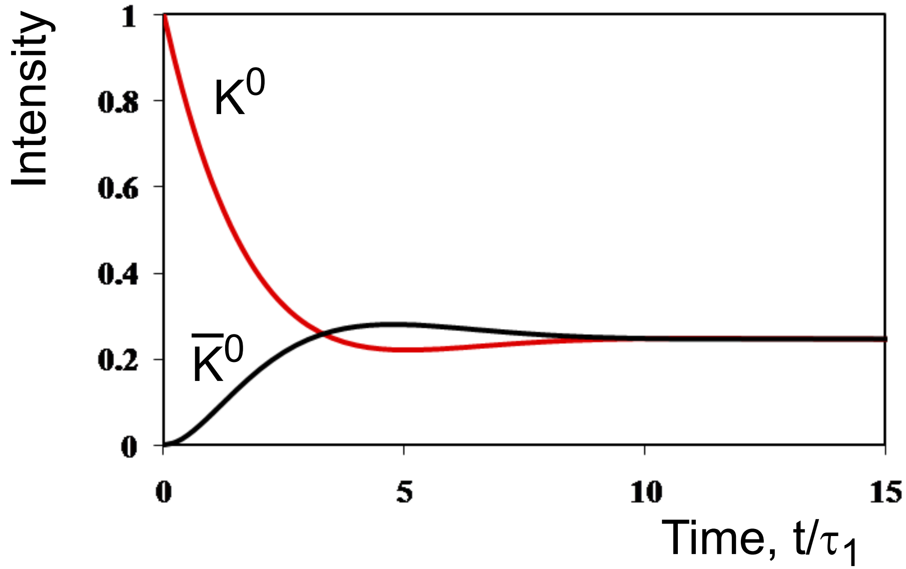

Figure 142: Strangeness oscillation in neutral kaons

The same effect is readily observed in B mesons (even faster oscillations)