VI. The quark model: hadron quantum numbers, resonances

Characteristics of a hadron:

1) Mass
2) Quantum numbers arising from space symmetries : J, P, C. Common notation:
   - J^P (e.g. for proton: $\frac{1}{2}^+$), or
   - J^{PC} if a particle is also an eigenstate of C-parity (e.g. for π^0 : 0^{-+})
3) Internal quantum numbers: Q and B (always conserved), S, C, $\tilde{B}$, T (conserved in electromagnetic and strong interactions)

- How do we know what are quantum numbers of a newly discovered hadron?

- How do we know that mesons consist of a quark-antiquark pair, and baryons – of three quarks?

Some *a priori* knowledge is needed:

Particle	Mass (Gev/c2)	Quark composition	Q	B	S	C	$\tilde{B}$
p	0.938	uud	1	1	0	0	0
n	0.940	udd	0	1	0	0	0
K^-	0.494	$s\bar{u}$	-1	0	-1	0	0
D^-	1.869	$d\bar{c}$	-1	0	0	-1	0
B^-	5.279	$b\bar{u}$	-1	0	0	0	-1

For the lightest 3 quarks (u, d, s), possible 3-quark and 2-quark states will be ($q_{i,j,k}$ are u- or d- quarks):

	sss	ssq_i	sq_iq_j	$q_iq_jq_k$	$s\bar{s}$	$s\bar{q}_i$	$\bar{s}q_i$	$q_i\bar{q}_i$	$q_i\bar{q}_j$
S	-3	-2	-1	0	0	-1	1	0	0
Q	-1	0; -1	1; 0; -1	2; 1; 0; -1	0	0; -1	1; 0	0	-1; 1

- Hence restrictions arise: for example, mesons with S = -1 and Q = 1 are *forbidden*

- Particles which fall out of above restrictions are called *exotic* particles (like $dd\bar{u}s$, $uuu\bar{d}s$ etc.)

From observations of **strong interaction** processes, quantum numbers of many particles can be deduced:

	$p + p$	$\rightarrow p + n$	$+\ \pi^+$
Q=	2	1	1
S=	0	0	0
B=	2	2	0

	$p + p$	$\rightarrow p + p$	$+\ \pi^0$
Q=	2	2	0
S=	0	0	0
B=	2	2	0

	p	$+\ \pi^-$	$\rightarrow \pi^0 + n$
Q=	1	-1	0
S=	0	0	0
B=	1	0	1

Observations of pions confirm these predictions, ensuring that pions are non-exotic particles.

Assuming that K^- is a strange meson, one can predict quantum numbers of Λ-baryon:

$$K^- + p \rightarrow \pi^0 + \Lambda$$

	$K^- + p$	π^0	Λ
Q=	0	0	0
S=	-1	0	-1
B=	1	0	1

And further, for K^+-meson:

$$\pi^- + p \rightarrow K^+ + \pi^- + \Lambda$$

	$\pi^- + p$	K^+	$\pi^- + \Lambda$
Q=	0	1	-1
S=	0	1	-1
B=	1	0	1

- All of the more than 200 hadrons of certain existence satisfy this kind of predictions
  - It so far confirms validity of the quark model, which suggests that only quark-antiquark and 3-quark (or 3-antiquark) states can exist

Pentaquark observation

- In 1997, a theoretical model predicted *pentaquark* possibility with mass 1.54 GeV
- Since 2003, LEPS/SPring-8 experiment in Japan reports observation of a particle with precisely this mass, and having structure consistent with pentaquark

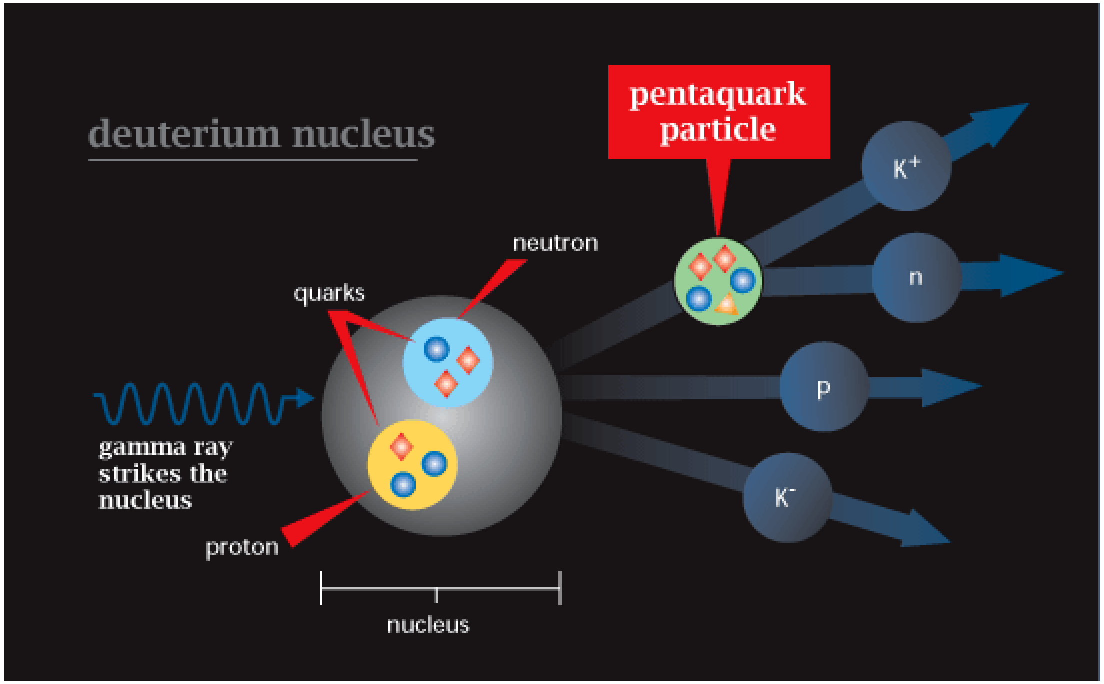

Figure 89: Pentaquark production and observation at JLab

- Reported Θ^+ particle composition: $uudd\bar{s}$, $B = +1$, $S = +1$, spin = 1/2

LEPS/SPring-8 experimental setup:

$$\gamma + n \rightarrow \Theta^{+}(1540) + K^{-} \rightarrow K^{+} + K^{-} + n$$

- Laser beam was shot to a target made of liquid deuterium

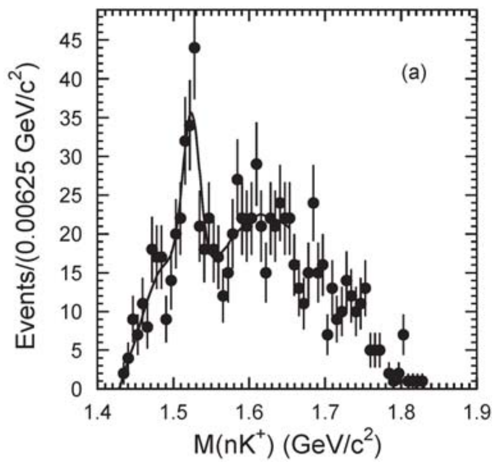

Figure 90: New particle signal (the peak) published by LEPS in 2009

- A reference target of liquid hydrogen (only protons) showed no such signal
- Many experiments reported similar observations

- Still, many dedicated precision experiments show no signal
  - Main problem: how to estimate the background. Search continues...

Even more quantum numbers...

It is convenient to introduce some more quantum numbers, which are conserved in strong and electromagnetic interactions:

- Sum of all internal quantum numbers, except of Q,

$$\textit{hypercharge}\ \mathrm{Y} \equiv \mathrm{B} + S + C + \tilde{B} + T$$

- Instead of Q :

$$\mathrm{I}_3 \equiv \mathrm{Q} - \mathrm{Y}/2$$

...which is to be treated as a projection of a new vector:

- *Isospin*

$$\mathrm{I} \equiv (\mathrm{I}_3)_{max}$$

so that I_3 takes 2I+1 values from -I to I

- I_3 is a good quantum number to denote up- and down- quarks, and it is convenient to use notations for particles as: $\mathrm{I}(J^P)$ or $\mathrm{I}(J^{PC})$

	B	S	C	$\tilde{B}$	T	Y	Q	I_3
u	1/3	0	0	0	0	1/3	2/3	1/2
d	1/3	0	0	0	0	1/3	-1/3	-1/2
s	1/3	-1	0	0	0	-2/3	-1/3	0
c	1/3	0	1	0	0	4/3	2/3	0
b	1/3	0	0	-1	0	-2/3	-1/3	0
t	1/3	0	0	0	1	4/3	2/3	0

- Hypercharge Y, isospin I and its projection I_3 are additive quantum numbers, thus quantum numbers for hadrons can be deduced from those of quarks:

$$Y^{a+b} = Y^a + Y^b \; ; \; I_3^{a+b} = I_3^a + I_3^b$$

$$I^{a+b} = I^a + I^b, I^a + I^b - 1, \ldots, \left|I^a - I^b\right|$$

- Proton and neutron both have isospin of 1/2, and also very close masses:

p(938) = uud ; n(940) = udd : $I(J)^P = \frac{1}{2}\left(\frac{1}{2}\right)^+$

proton and neutron are said to belong to *isospin doublet*

Other examples of *isospin multiplets*:

$$K^+(494) = u\bar{s}\ ;\ K^0(498) = d\bar{s} : I(J)^P = \frac{1}{2}(0)^-$$

$$\pi^+(140) = u\bar{d}\ ;\ \pi^-(140) = d\bar{u} : I(J)^P = 1(0)^- \text{ and}$$

$$\pi^0(135) = (u\bar{u}-d\bar{d})/\sqrt{2} : I(J)^{PC} = 1(0)^{-+}$$

- Principle of *isospin symmetry*: it is a good approximation to treat u- and d-quarks as having same masses

Particles with I=0 are called *isosinglets* :

$$\Lambda(1116) = uds,\ I(J)^P = 0\left(\frac{1}{2}\right)^+$$

- By introducing isospin, we get more criteria for non-exotic particles:

	sss	ssq_i	sq_iq_j	$q_iq_jq_k$	$s\bar{s}$	$s\bar{q}_i$	$\bar{s}q_i$	$q_i\bar{q}_i$	$q_i\bar{q}_j$
S	-3	-2	-1	0	0	-1	1	0	0
Q	-1	0; -1	1; 0; -1	2; 1; 0; -1	0	0; -1	1; 0	0	-1; 1
I	0	1/2	0; 1	3/2; 1/2	0	1/2	1/2	0; 1	0; 1

In all observed interactions (save pentaquarks) isospin-related criteria are satisfied as well, confirming once again the quark model.

- This allows predictions of possible multiplet members: suppose we observe production of the Σ^+ baryon in a strong interaction:

$$K^- + p \rightarrow \pi^- + \Sigma^+$$

which then decays weakly :

$$\Sigma^+ \rightarrow \pi^+ + n$$
$$\Sigma^+ \rightarrow \pi^0 + p$$

It follows that Σ^+ baryon quantum numbers are: B = 1, Q = 1, S = -1 and hence Y = 0 and $I_3 = 1$.

- Since $I_3 > 0 \Rightarrow I \neq 0$ and there are more multiplet members!

- When a baryon has $I_3=1$, the only possibility for isospin is $I=1$, and we have a triplet:

$$S^+, S^0, S^-$$

Indeed, all such particles have been observed:

$$K^- + p \to \pi^0 + \Sigma^0$$
$$\hookrightarrow \Lambda + \gamma$$
$$K^- + p \to \pi^+ + \Sigma^-$$
$$\hookrightarrow \pi^- + n$$

Masses and quark composition of Σ-baryons are:

$$\Sigma^+(1189) = uus\ ;\ \Sigma^0(1193) = uds\ ;\ \Sigma^-(1197) = dds$$

It indicates that d-quark is heavier than u-quark, under following assumptions:

(a) strong interactions between quarks do not depend on their flavour and give contribution of M_o to the baryon mass

(b) electromagnetic interactions contribute as $\delta\sum e_i e_j$, where e_i are quark charges and δ is a constant

The simplest attempt to calculate mass difference of up- and down-quarks:

$$M(\Sigma^-) = M_0 + m_s + 2m_d + \delta/3$$

$$M(\Sigma^0) = M_0 + m_s + m_d + m_u - \delta/3$$

$$M(\Sigma^+) = M_0 + m_s + 2m_u$$

$$\Downarrow$$

$$m_d - m_u = [\, M(\Sigma^-) + M(\Sigma^0) - 2M(\Sigma^+) \,] / 3 = 3.7 \text{ MeV/c}^2$$

❖ NB : this is a very simplified model, as under these assumptions $M(\Sigma^0) = M(\Lambda)$, while their mass difference $M(\Sigma^0) - M(\Lambda) \approx 77$ Mev/c^2 .

Generally, combining other methods:

$$2 \leq m_d - m_u \leq 4 \text{ (MeV/c}^2 \text{)}$$

which is negligible comparing to hadron masses (but not if compared to estimated u and d masses themselves)

Resonances

- *Resonances* are highly unstable particles that decay by strong interaction (lifetimes about 10^{-23} s)

$$L=0 \qquad \begin{array}{cc} ^1S_0 & ^3S_1 \\ \uparrow\ \downarrow & \uparrow\ \uparrow \\ u\ \ \bar{d} & u\ \ \bar{d} \\ I(J^P)=1(0^-) & I(J^P)=1(1^-) \\ \text{ground state} & \text{resonance} \end{array}$$

Figure 91: Example of a $q\bar{q}$ system in ground and first excited states

- If a ground state is a member of an isospin multiplet, then resonant states will form a corresponding multiplet too

Since resonances have very short lifetimes, they can only be detected by registering their *decay products*:

$$\pi^- + p \rightarrow n + X$$
$$X \rightarrow A + B$$

- Invariant mass of a particle is measured via energies and masses of its decay products (see 4-vectors in Chapter I.):

$$W^2 \equiv (E_A + E_B)^2 - (\vec{p}_A + \vec{p}_B)^2 = E^2 - \vec{p}^2 = M^2 \qquad (102)$$

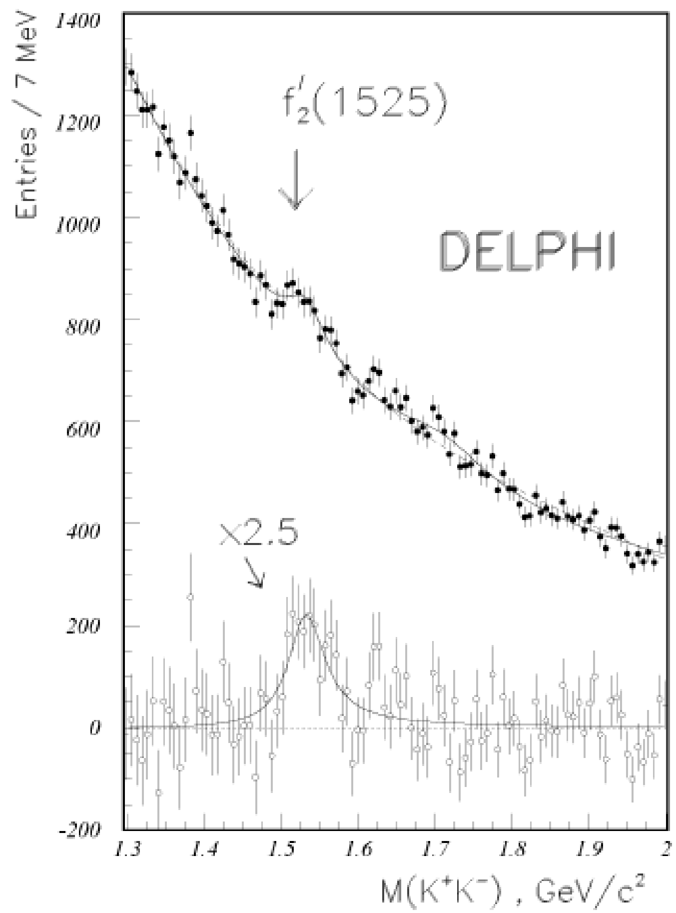

Figure 92: A typical resonance peak in K^+K^- invariant mass distribution

❖ Resonance peak shapes are approximated by the *Breit-Wigner* formula:

$$N(W) = \frac{K}{(W - W_0)^2 + \Gamma^2 / 4} \qquad (103)$$

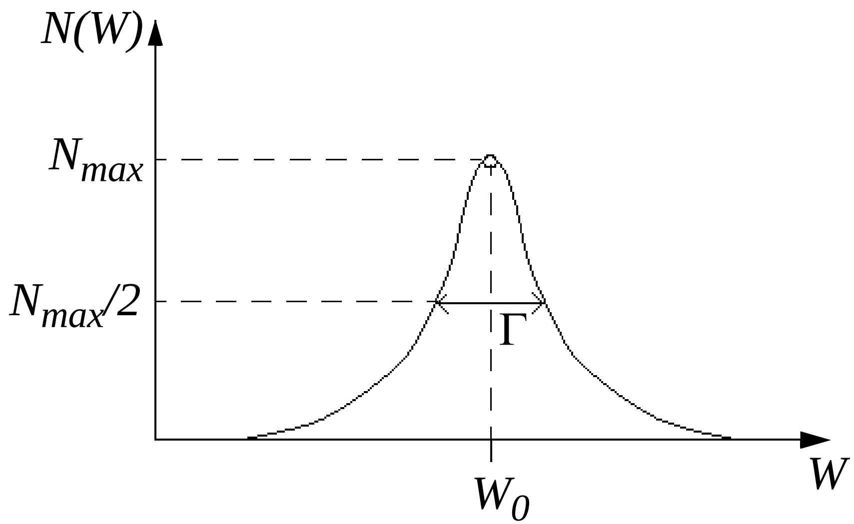

Figure 93: Breit-Wigner shape

- Mean value of the Breit-Wigner shape is the mass of a resonance: $M = W_0$
- Γ is the width of a resonance, and it has the meaning of inverse mean lifetime of particle at rest: $\Gamma \equiv 1/\tau$

Internal quantum numbers of resonances are also derived from their decay products:

$$X^0 \to \pi^+ + \pi^-$$

for such X^0: $B = 0; S = C = \tilde{B} = T = 0; Q = 0 \Rightarrow$ Y=0 and I_3=0.

- When I_3=0, to determine whether I=0 or I=1, searches for isospin multiplet partners have to be done.

Example: $\rho^0(769)$ and $\rho^0(1700)$ both decay to $\pi^+\pi^-$ pair and have isospin partners ρ^+ and ρ^- :

$$\pi^{\pm} + p \to p + \rho^{\pm}$$
$$\hookrightarrow \pi^{\pm} + \pi^0$$

By measuring angular distribution of a $\pi\pi$ pair, the relative orbital angular momentum of the pair L can be determined, and hence spin and parity of the resonance X^0 are (S=0):

$$J = L; P = P_\pi^2(-1)^L = (-1)^L; C = (-1)^L$$

- Some excited states of pion:

resonance	$I(J^{PC})$
$\rho^0(769)$	$1(1^{--})$
$f_2^0(1275)$	$0(2^{++})$
$\rho^0(1700)$	$1(3^{--})$

- B=0 : *meson resonances*, B=1 : *baryon resonances*.

Many baryon resonances can be produced in pion-nucleon scattering:

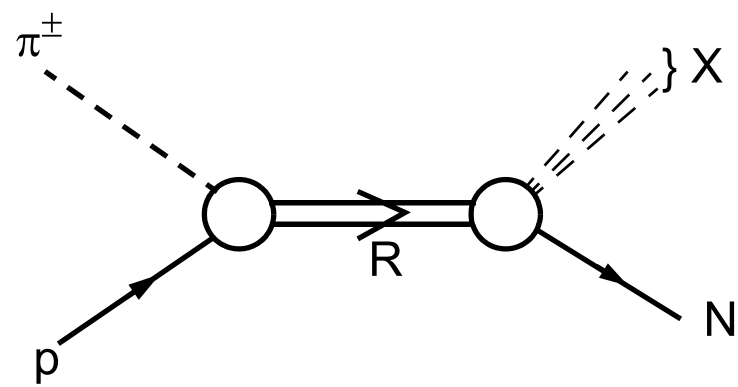

Figure 94: Formation of a resonance R and its decay into a nucleon N

- Peaks in the observed total cross-section of the $\pi^\pm p$-reaction correspond to resonance formation

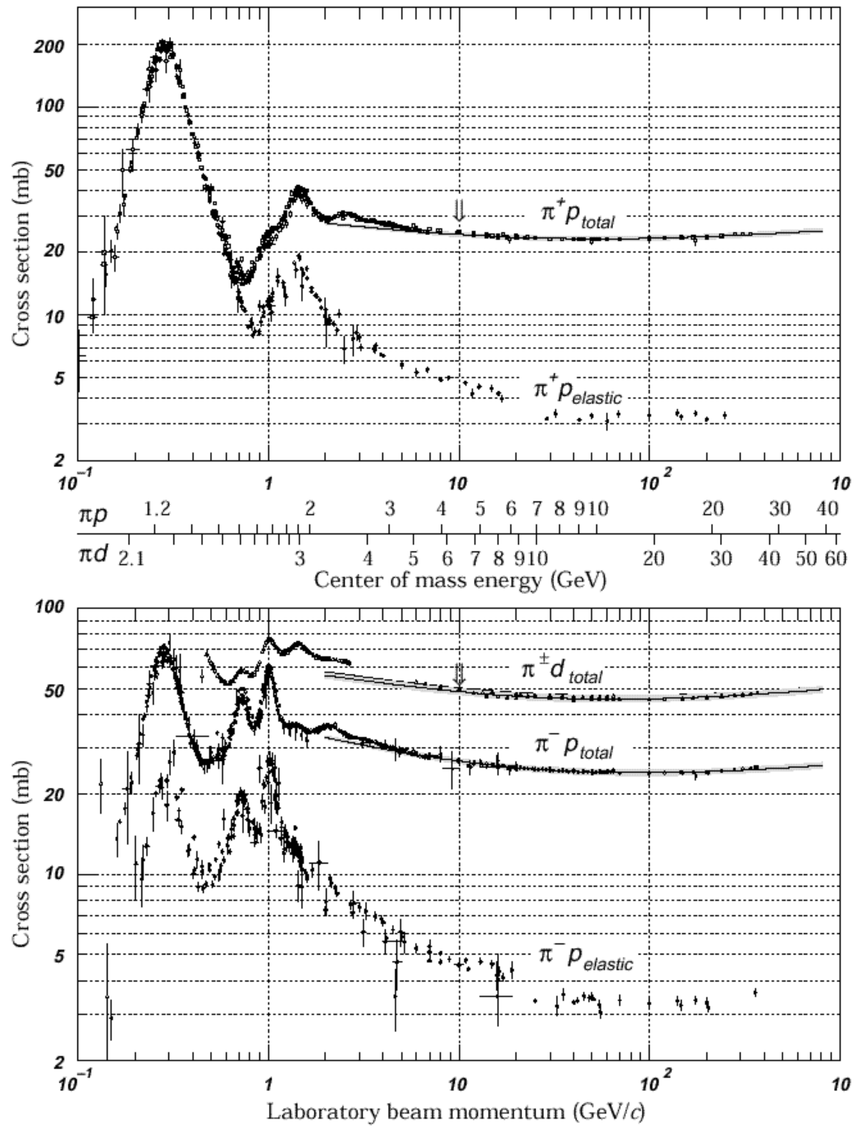

Figure 95: Scattering of π^+ and π^- on proton

All the resonances produced in pion-nucleon scattering have the same internal quantum numbers as the initial state:

$$B = 1; S = C = \tilde{B} = T = 0$$

and thus Y=1 and $Q=I_3+1/2$

Possible isospins are $I=1/2$ or $I=3/2$, since for pion $I=1$ and for nucleon $I=1/2$

- $I=1/2 \Rightarrow$ N-resonances (N^0, N^+)
- $I=3/2 \Rightarrow$ Δ-resonances ($\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$)

Figure 95: peaks at ≈1.2 GeV/c^2 correspond to Δ^{++} and Δ^0 resonances:

$$\pi^+ + p \to \Delta^{++} \to \pi^+ + p$$

$$\pi^- + p \to \Delta^0 \to \pi^- + p$$

$$\hookrightarrow \pi^0 + n$$

- Fits by the Breit-Wigner formula show that both Δ^{++} and Δ^{0} have approximately same mass of ≈1232 MeV/c^2 and width ≈120 MeV/c^2.
  - Studies of angular distributions of decay products show that $I(J^P) = \frac{3}{2}\left(\frac{3}{2}\right)^+$
  - Remaining members of the multiplet are also observed: Δ^+ and Δ^-
- There is no lighter state with these quantum numbers $\Rightarrow$ Δ is a *ground state*, although observed as a resonance.

Quark diagrams

- Quark diagrams are convenient way of illustrating strong interaction processes

Consider an example:

$$\Delta^{++} \to p + \pi^+$$

- The only 3-quark state consistent with Δ^{++} quantum numbers (Q=2) is (uuu), while p=(uud) and π^+=($u\bar{d}$)

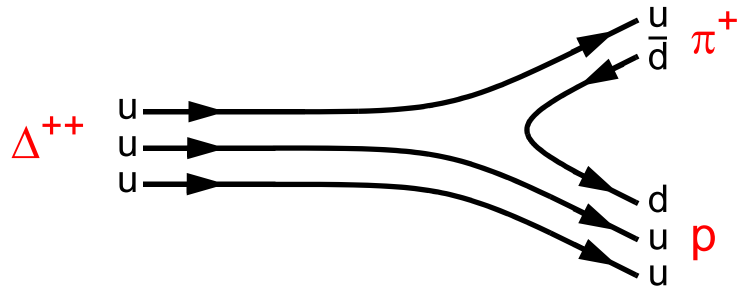

Figure 96: Quark diagram of the reaction $\Delta^{++} \rightarrow p + \pi^+$

Analogously to Feinman diagrams:

- arrow pointing rightwards denotes a particle, and leftwards – antiparticle
- time flows from left to right

Allowed resonance formation process:

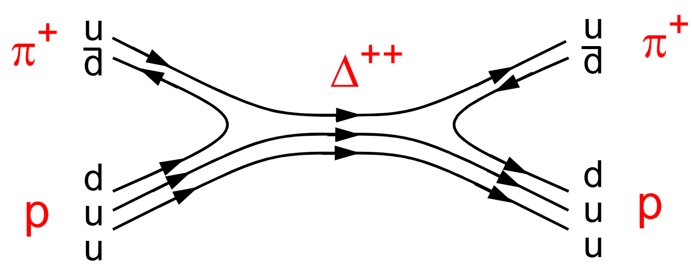

Figure 97: Formation and decay of Δ^{++} resonance in π^+p elastic scattering

Hypothetical exotic resonance:

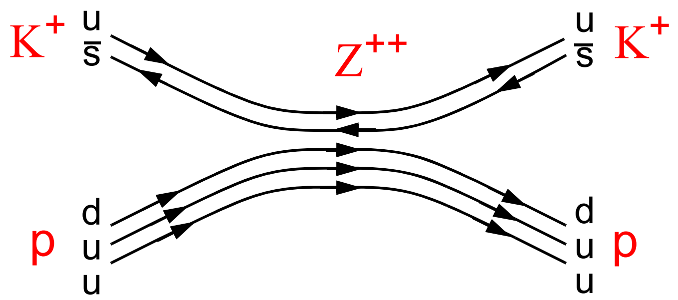

Figure 98: Exotic resonance Z^{++} in K^+p elastic scattering

- Quantum numbers of such a particle Z^{++} are exotic. There are no resonance peaks in the corresponding cross-section, but data are scarce:

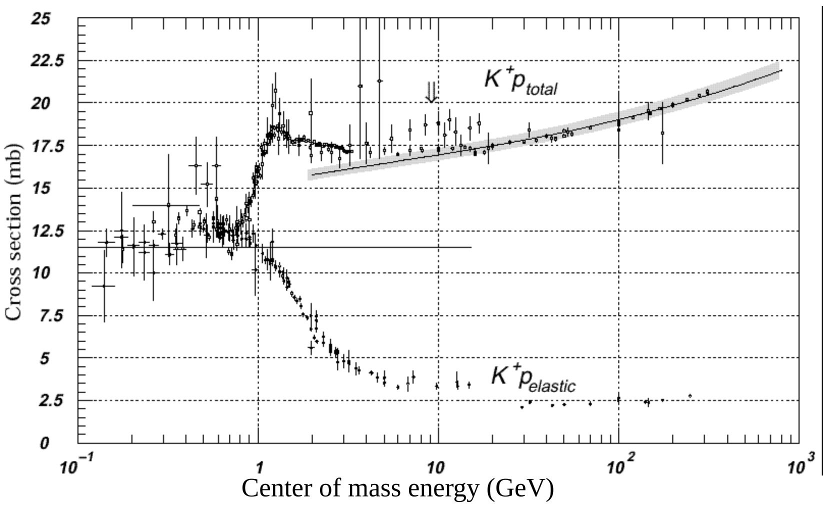

Figure 99: Cross-section for K^+p scattering

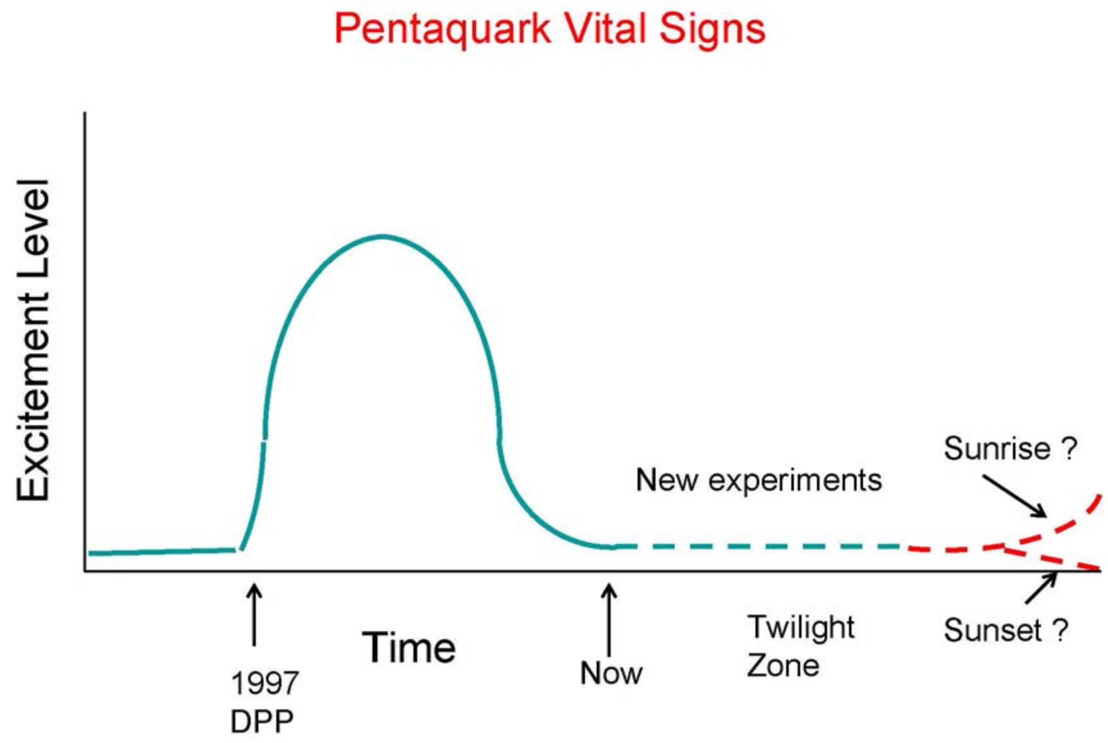

Figure 100: Pentaquark searches status as of October 2005, by Paul Stoler