Chapters 5-7; return by February 15

1) In a fixed target experiment, a π^{-} beam is used on a proton target and the process

 $\pi^- + p \rightarrow \Delta^0 \rightarrow \pi^0 + n$ can occur.

a) Draw a quark diagram for this process and estimate the mean distance travelled by the Δ^0 before it decays, assuming it was produced with $\gamma = E/m \approx 10$.

b) Using four-vectors, compute the π^- beam energy required to produce the above process at the Δ^0 resonance, m(Δ^0)=1230 MeV.

c) Show that, if the π^0 and n are produced with an angle $\theta = \pi/2$ between them, they can only obtain the energies $E(n), E(\pi^0) = E(\pi^-)$ and $E(\pi^0), E(n) = m(p)$, assuming that $m(\pi^-) = m(\pi^0)$ and m(n) = m(p).

2) Resonance Δ^{++} has a barion number B=1, electric charge Q=2, and S = C = \tilde{B} = T = 0. Explain why such particle can not exist unless color charge is introduced. Could a baryon with three down quarks exist?

3) The Coulomb potential represents a point charge. When an electrostatic potential is instead represented by a spherically symmetric charge density $\rho(\mathbf{r})$, the differential scattering cross section differs from the Rutherford cross section by a form factor squared, $G_E^2(q^2)$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{R}} \mathrm{G}_{\mathrm{E}}^{2}(\mathrm{q}^{2})$$

where

$$G_{E}(q^{2}) = \int \rho(r) e^{i\vec{q}\cdot\vec{x}} d^{3}\vec{x}$$

Perform the angular integration of the form factor and show that:

- $G_E^2(q^2)$ is a function of q^2 only
- the mean squared radius of $\rho(r)$ equals

$$\overline{r^2} = \int r^2 \rho(r) d^3 \dot{\vec{x}} = -6 \frac{dG_E(q^2)}{dq^2} \bigg|_{q^2 = 0}$$

Bonus problem (not mandatory, but you can get an extra point):

Explain the effect on the differential cross section (w.r.t. the scattering angle θ) when a point charge (infinitely narrow distribution) is replaced by a charge density $\rho(r)$, represented by a Gaussian (normal) distribution.

Note that the Fourier transform of a "narrow" Gaussian becomes a "wide" Gaussian distribution (and vice versa). Both charge distributions are normalized to 1.