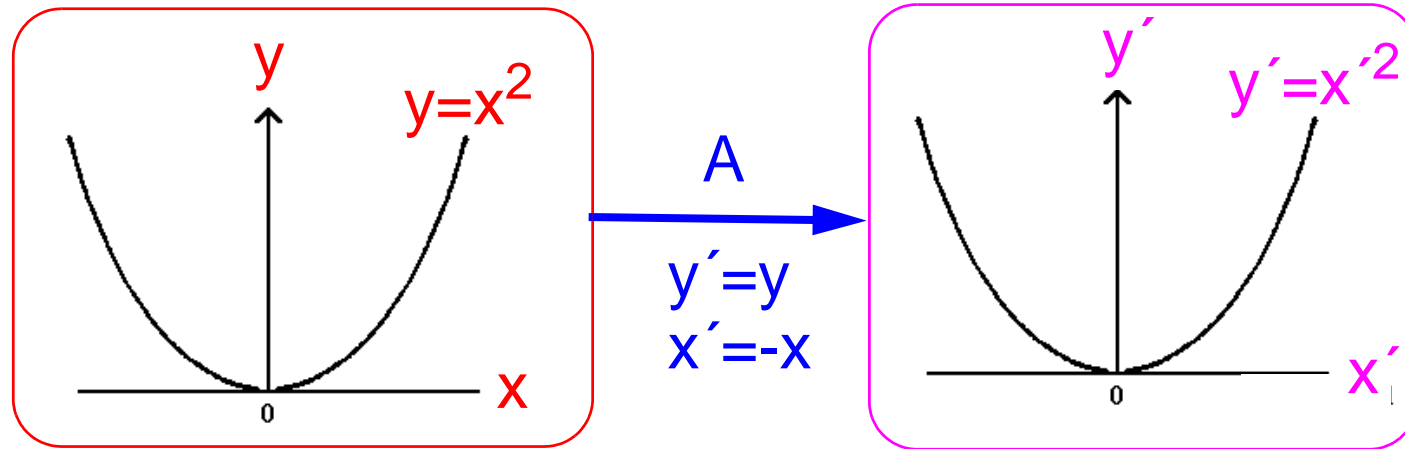


The electroweak theory

➔ Gauge invariant theories.



The equation $y=x^2$ is **symmetric** or **invariant** under the **transformation A**, i.e. it looks the same before and after the transformation.

- **Gauge invariant** theories \Rightarrow the main equations do not change when a gauge transformation is performed.
- Requiring gauge invariance \Rightarrow **deduce** the various **interactions**.

The electroweak theory

➔ What is a gauge transformation ?

- Different **gauge transformations** \Leftrightarrow different interactions.

Example: Non-relativistic electromagnetism

- The equation of motion: for a **free non-relativistic particle**:

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2m}\nabla^2\Psi$$

where $\Psi(\mathbf{r},t)$ is the wavefunction of the particle and m is its mass.

The free particle Schrödinger equation

- **Goal:** modify this equation to describes particles that **interact electromagnetically**.

The electroweak theory

- **Assumption:** the new equation should be invariant under a **U(1) phase transformation:**

$$\psi \rightarrow \psi' = e^{iq\alpha} \psi$$

were $\alpha(r,t)$ is an arbitrary continuous function and q is the charge of the particle.

- **Result:**

$$i\frac{\partial \psi'}{\partial t} = \left[\frac{1}{2m}(\nabla + iq\alpha) - 2mq\frac{d\alpha}{dt} \right] \psi'$$

new terms



- **Conclusion:** ψ' is **not a solution** to the Schrödinger equation.

The electroweak theory

- **Electromagnetism:** A vector potential \vec{A} and a scalar potential V defined by

$$\vec{E} = -\nabla V - \frac{d\vec{A}}{dt}$$

$$\vec{B} = \nabla \times \vec{A}$$

where E is the electrical field and B the magnetic field.

- **Question:** How to add \vec{A} and V to the Schrödinger equation while keeping it invariant under $\psi \rightarrow \psi' = e^{iq\alpha} \psi$?

- **Introduce interaction:** require that the Schrödinger equation is also invariant under a **gauge transformation** of type

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla \alpha$$

$$V \rightarrow V' = V - \frac{\partial \alpha}{\partial t}$$

The electroweak theory

- **Result:** U(1) phase transformation of the wave function + gauge transformations of A and V is fulfilled by

⇒ **new equation**
$$i\frac{\partial\Psi}{\partial t} = \left[\frac{1}{2m}(\nabla - q\vec{A}) + qV \right] \Psi$$

The equation for a non-relativistic particle with charge q moving in an electromagnetic field.

The Gauge principle

- **Gauge principle:** to keep the invariance condition satisfied it is necessary to add a minimal field to the Schroedinger equation
⇒ **an interaction** will have to be **introduced**.

The electroweak theory

➔ The electro weak theory by Glashow, Weinberg and Salam.

● The EW or GWS model is a **quantum field theory** for both weak interactions and electromagnetic interactions.

● **Weak isospin charge** $\Rightarrow I_3^W$
Weak hypercharge $\Rightarrow Y^W$
Electric charge $\Rightarrow Q$ } $Q = I_3^W + Y^W/2$

● **Gauge invariance** \Rightarrow **massless** gauge particles $\Rightarrow W^+, W^-, W^0, B^0$

● The gauge particles interact with **massless fermions**.

The electroweak theory

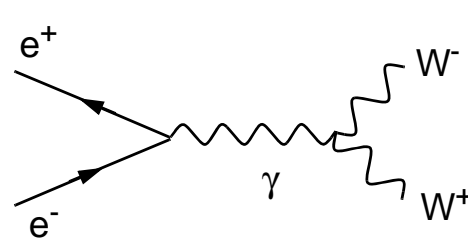
- The Higgs field \Rightarrow generates mass to gauge bosons and fermions.
- W^+ and W^- \Rightarrow weak radioactive decay
- W^0 and B^0 \Rightarrow not observed experimentally
- Photon \Rightarrow gauge boson for the electromagnetic interaction
 Z^0 \Rightarrow gauge boson for the weak neutral current interaction

$$\begin{aligned}\gamma &= B^0 \cos \theta_W + W^0 \sin \theta_W \\ Z^0 &= -B^0 \sin \theta_W + W^0 \cos \theta_W\end{aligned}$$

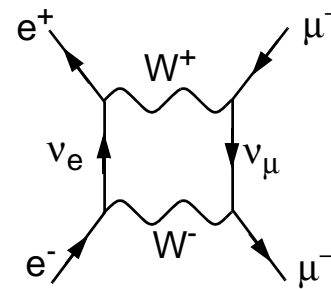
- θ_W \Rightarrow the weak mixing angle \Rightarrow a parameter (not predicted).

The electroweak theory

- Only W -exchange \Rightarrow divergent processes \Rightarrow infinities

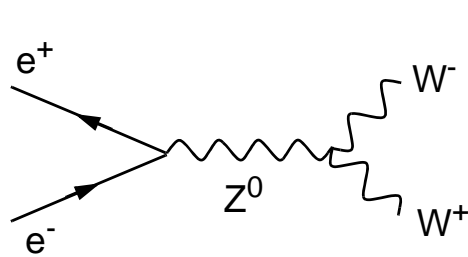


Non-divergent integrals

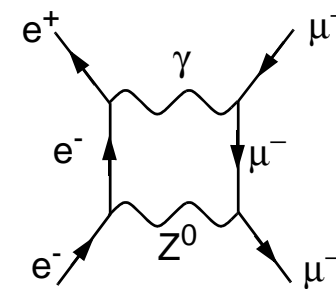
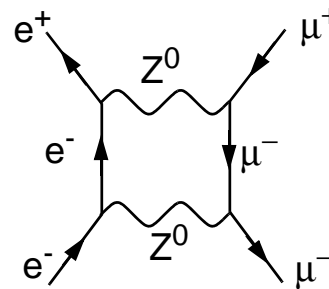


Divergent integrals

- The Z^0 -boson \Rightarrow new diagrams cancel out the divergencies



Non-divergent integrals



Divergent integrals

The electroweak theory

- Coupling constants in EW $\Rightarrow e, g_W$ and g_Z

- "Strength parameters"

$$\text{QED: } \alpha = \frac{e^2}{4\pi} \approx \frac{1}{137} \quad \text{QCD: } \alpha_s = \frac{g_s^2}{4\pi} \approx \frac{1}{9} \quad \text{EW: } \alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{250} \quad \alpha_Z = \frac{g_Z^2}{4\pi} \approx \frac{1}{850}$$

- The coupling constants are not independent (in order for all the infinities to cancel out)

The unification condition.

$$\begin{array}{ccc}
 \boxed{e/\sqrt{8} = g_W \sin \theta_W = g_Z \cos \theta_W} & \xrightarrow{\text{Alternative}} & \boxed{
 \begin{array}{l}
 e = g \sin \theta_W = g' \cos \theta_W \\
 g = \sqrt{8} g_W \quad g' = \sqrt{8} g_Z
 \end{array}
 }
 \end{array}$$

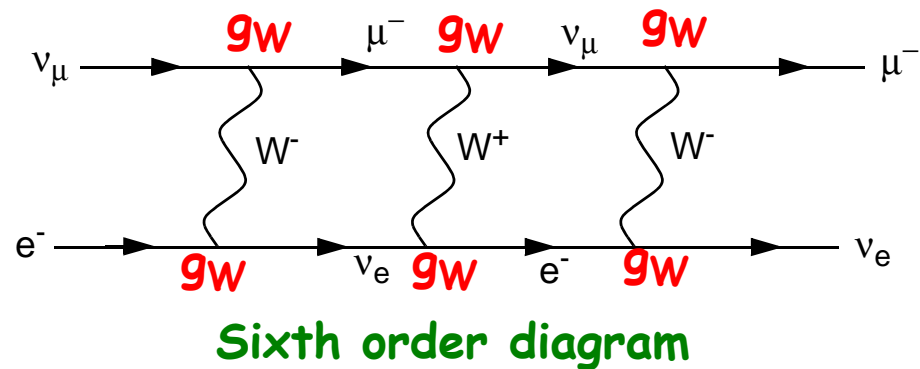
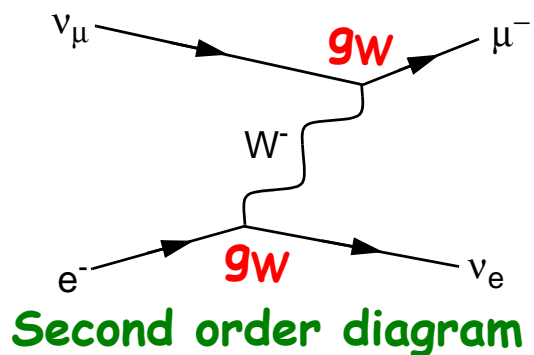
- The weak mixing angle (Weinberg angle) $\Rightarrow \cos \theta_W = \frac{M_W}{M_Z}$

The electroweak theory

- The **strength parameters** \Rightarrow contribution of different processes (diagrams) to the **cross-section**.

- **Example: muon-scattering on electrons** $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$

Diagrams with increasing complexity can contribute:

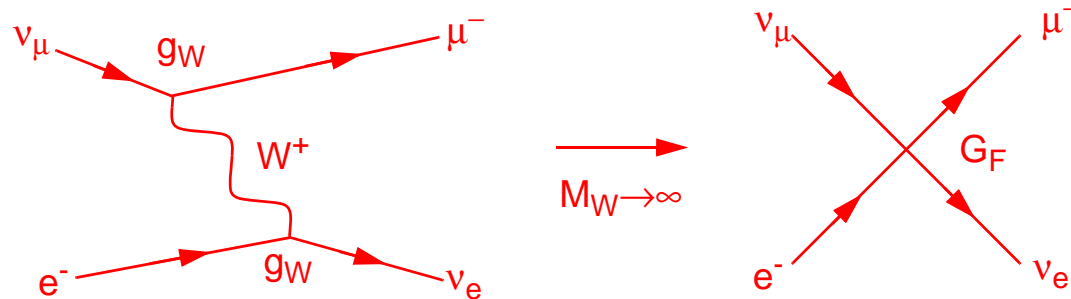


- **Second order diagram** \Rightarrow cross section is proportional to a_W^2
- **Sixth order diagram** \Rightarrow cross section is proportional to a_W^6

The electroweak theory

➔ Point-like interactions and low-energy measurements

- Fermi's theory of weak interactions \Rightarrow four-fermion point-like interactions (without W and Z exchange).
- W bosons are heavy \Rightarrow charged current interactions can be **approximated** by a **zero-range interaction** at low energy



- Fermi coupling constant (G_F) \Rightarrow

The strength of the zero-range interactions

Relationship between g_W and G_F :

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} = \frac{4\pi\alpha_W}{M_W^2}$$

next slide \rightarrow

The electroweak theory

- A bit of algebra:

The unification condition

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} \quad \text{and} \quad \sqrt{\frac{\pi \cdot \alpha_{em}}{2}} = g_W \sin \theta_W \quad \text{gives} \quad M_W^2 = \frac{g_W^2 \sqrt{2}}{G_F} = \frac{\pi \alpha_{em}}{\sqrt{2} G_F \sin^2 \theta_W}$$

$$\cos \theta_W = \frac{M_W}{M_Z} \quad \text{and} \quad M_W^2 = \frac{\pi \alpha_{em}}{\sqrt{2} G_F \sin^2 \theta_W} \quad \text{gives} \quad M_Z^2 = \frac{\pi \alpha_{em}}{\sqrt{2} G_F \cos^2 \theta_W \sin^2 \theta_W}$$

Definition Weinberg angle

- $G_Z \Rightarrow$ neutral current coupling constant in the low energy zero-range approximation

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2} \quad \text{which gives} \quad \frac{G_Z}{G_F} = \frac{\sqrt{2} g_Z^2 M_W^2}{\sqrt{2} g_W^2 M_Z^2} = \frac{\frac{\pi \alpha}{2 \cos^2 \theta_W} \cos^2 \theta_W}{\frac{\pi \alpha}{2 \sin^2 \theta_W}} = \sin^2 \theta_W$$

The electroweak theory

- Measurements of **weak interaction rates** at low energy \Rightarrow
 G_Z and $G_F \Rightarrow G_Z/G_F = \sin^2(\theta_W) \Rightarrow$

$$\sin^2 \theta_W = 0,277 \pm 0.014$$

- Measurement of the weak mixing angle \Rightarrow
predict the masses of the W and Z:

$$M_W = 78.3 \pm 2.4 \text{ GeV}/c^2$$

$$M_Z = 89.0 \pm 2.0 \text{ GeV}/c^2$$

- Discovery at CERN of W and Z at predicted masses \Rightarrow
confirmation that the electroweak theory is correct.

The electroweak theory

- Modern estimation of the Weinberg angle (from many experiments)

$$\sin^2 \theta_W = 0.2255 \pm 0.0021$$

- This value and the previous formulas give:

$$M_W = 78.5 \text{ GeV}/c^2$$

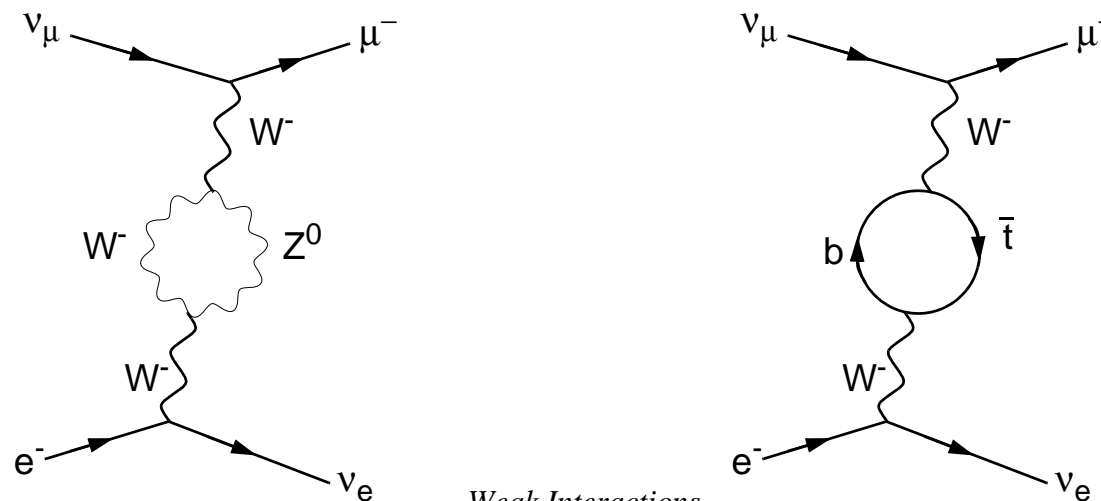
$$M_Z = 89.0 \text{ GeV}/c^2$$

while direct measurements give

$$M_W = 80.4 \text{ GeV}/c^2$$

$$M_Z = 91.2 \text{ GeV}/c^2$$

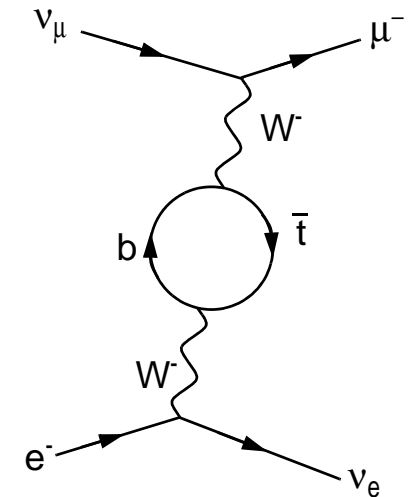
- The **difference** is caused by higher-order diagrams (not taken into account in the low-energy formulas)



The electroweak theory

- Higher-order corrections with top-quarks \Rightarrow
measurement of electroweak processes \Rightarrow
predict the top-quark mass

$$m_t = 170 \pm 30 \text{ GeV}/c^2$$

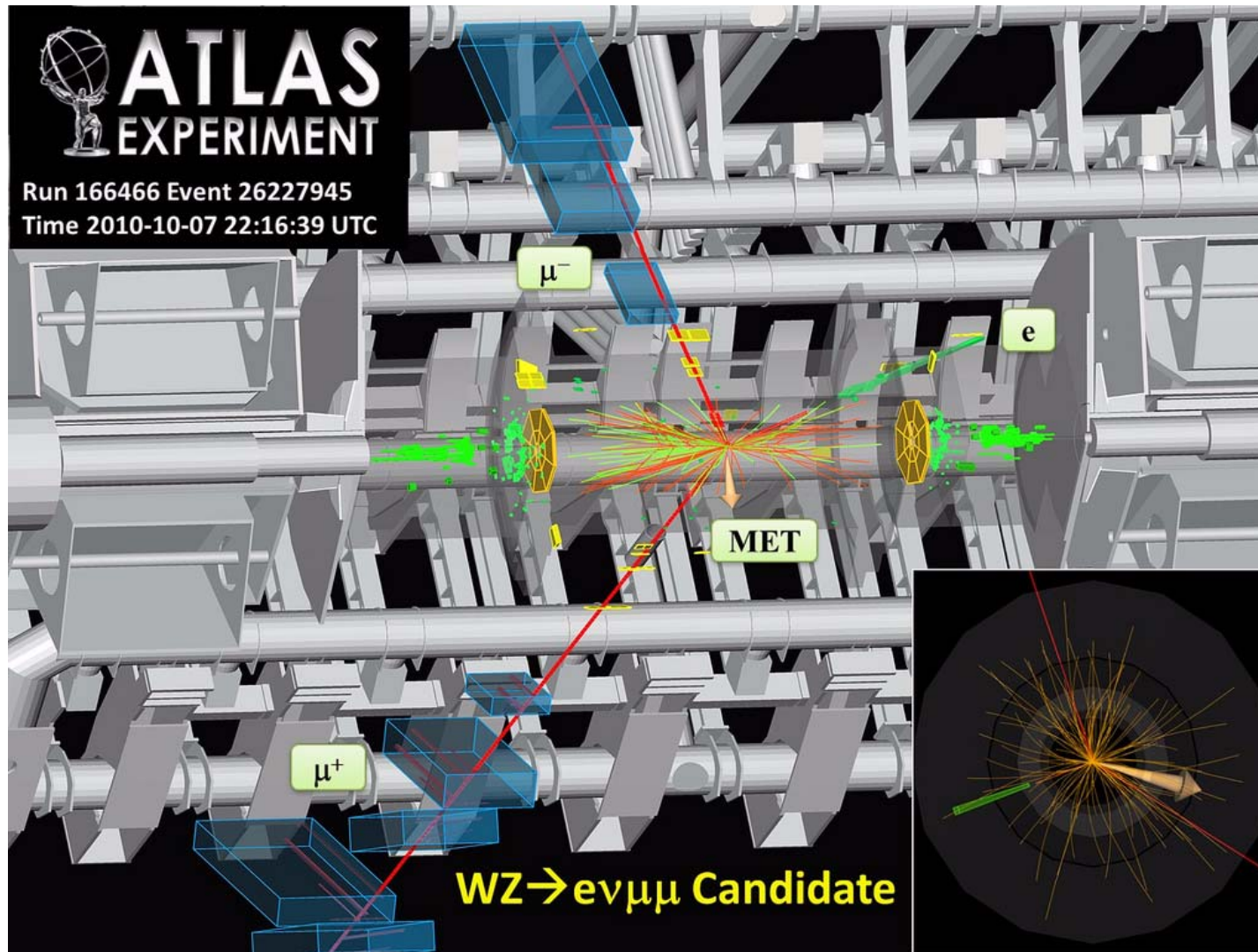


- The **directly measured mass** of the top quark at Fermilab by the CDF experiment gave a value

$$m_t = 176 \pm 5 \text{ GeV}/c^2$$

Perfect agreement with the low-energy prediction !

The W and Z bosons



The W and Z bosons

- ➔ The force carriers in weak interactions are spin-1 bosons (as in QED and QCD) that couple to quarks and leptons.
- W^+ , W^- and Z^0 \Rightarrow Intermediate vector bosons
The force carriers of weak interactions
 - W^+ , W^- and Z^0 bosons are very massive particles
 $m_W = 80.4 \text{ GeV}$ and $m_Z = 91.2 \text{ GeV}$ \Rightarrow
Weak interactions have a very short range ($2 \times 10^{-3} \text{ fm}$).
 - All observed low-energy weak processes (e.g. β -decay) \Rightarrow
charged current reactions \Rightarrow mediated by W^+ or W^- bosons.
 - Electroweak prediction \Rightarrow neutral current reactions
caused by the Z^0 boson should exist.

Discovery of the neutral current

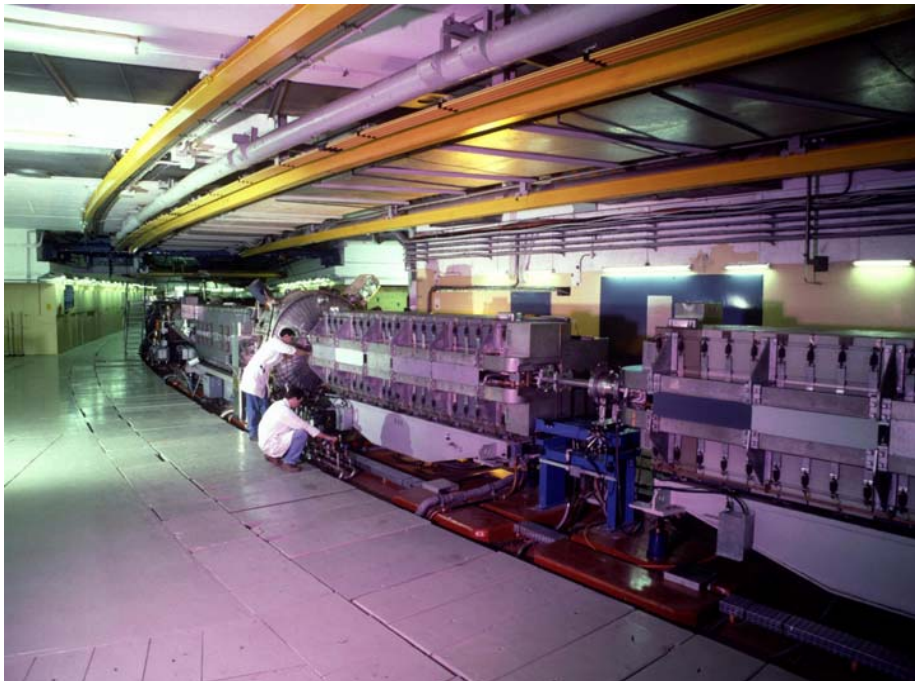
➔ Accelerator: The Proton Synchrotron (PS) at CERN

The PS accelerator: Length = 628 m, Number of magnets = 277,
Proton beam energy = 28 GeV

Neutrino beams:

Step 1. Intense proton beam hits a target

Step 2. Charged pions and kaons decay $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$.



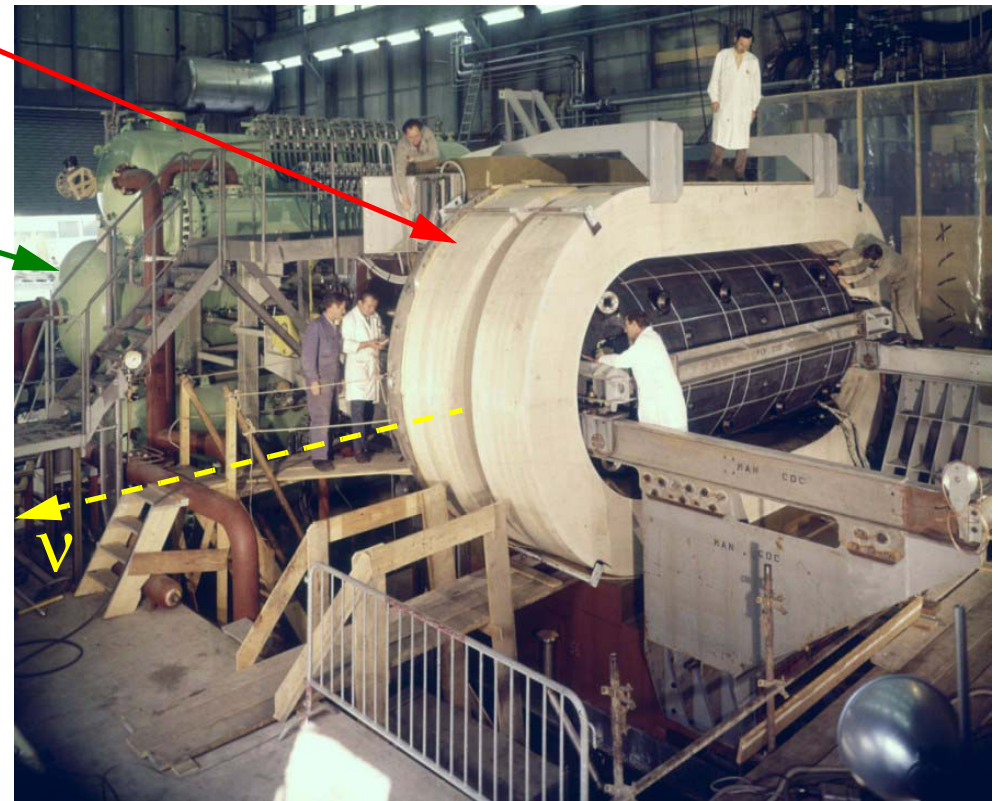
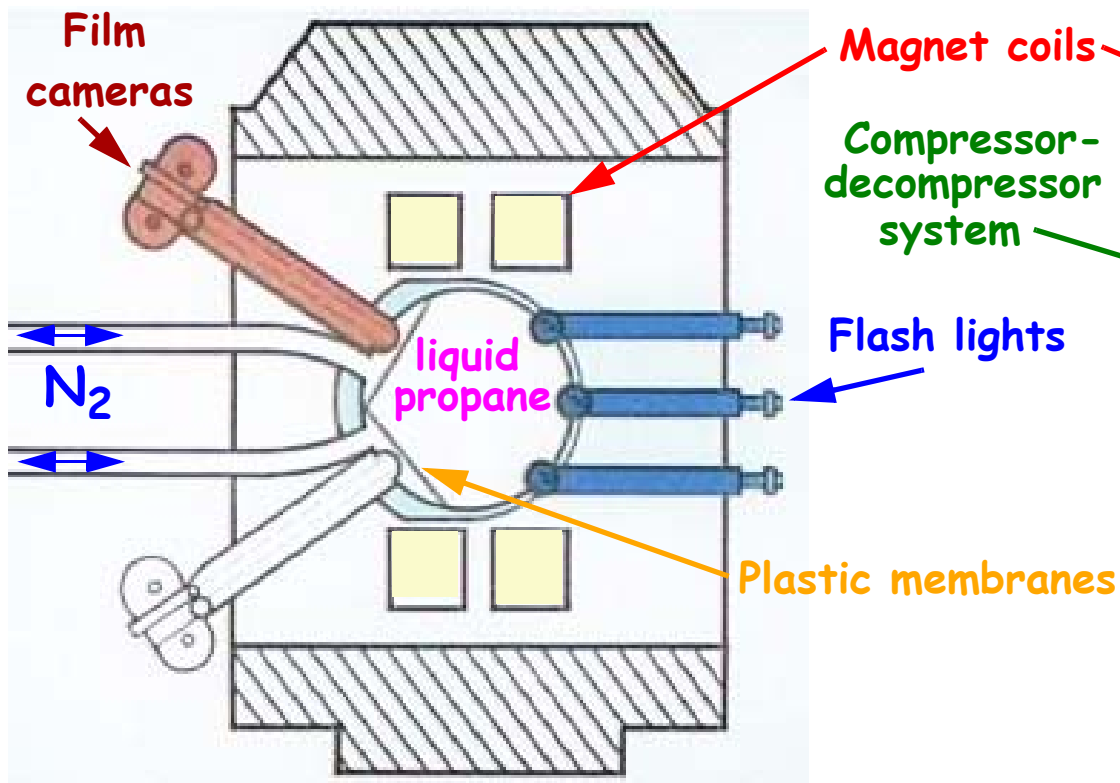
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Weak Interactions

Discovery of the neutral current

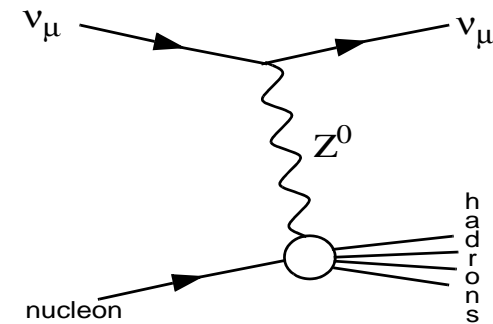
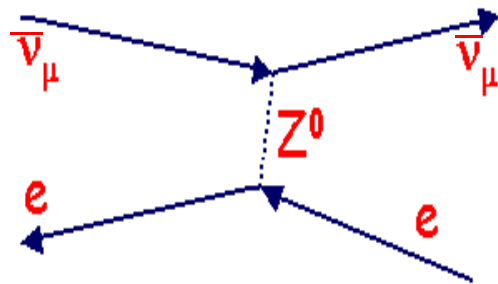
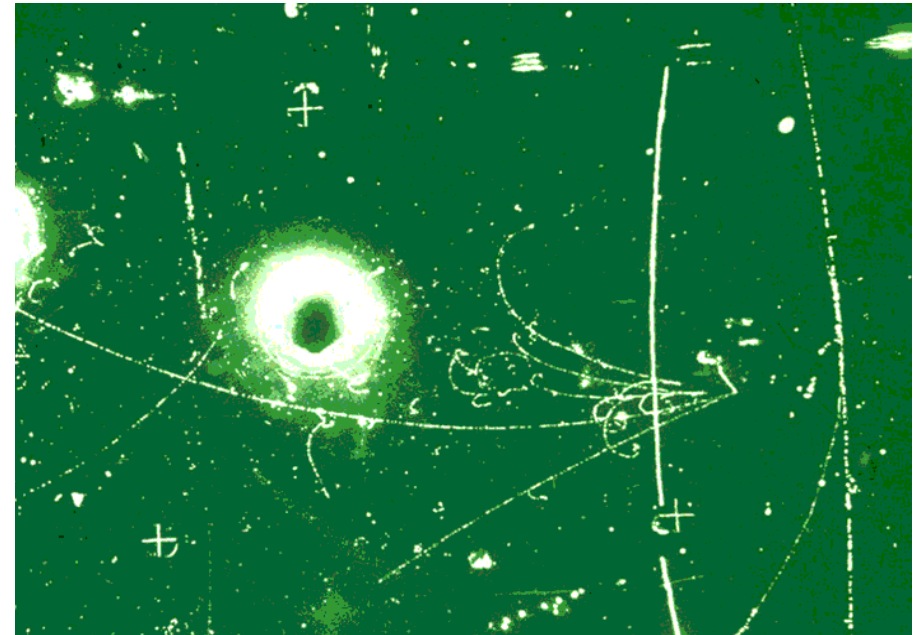
➔ Neutral current events was first observed by the Gargamelle experiment at CERN in 1973.



- Step 1. The liquid propane is at a temperature below its boiling point.
- Step 2. When the V enters the propane, its pressure is lowered to make it superheated.
- Step 3. Charged tracks ionize the propane and these ions create bubbles in the liquid.
- Step 4. The bubble tracks are photographed by film cameras.

Discovery of the neutral current

➔ Elastic and inelastic neutral current reactions possible.



● Main background ➔ neutron - nucleon interactions.

Discovery of the neutral current



One of the first neutral current reactions seen by the Gargamelle experiment.

The discovery of the W and Z bosons

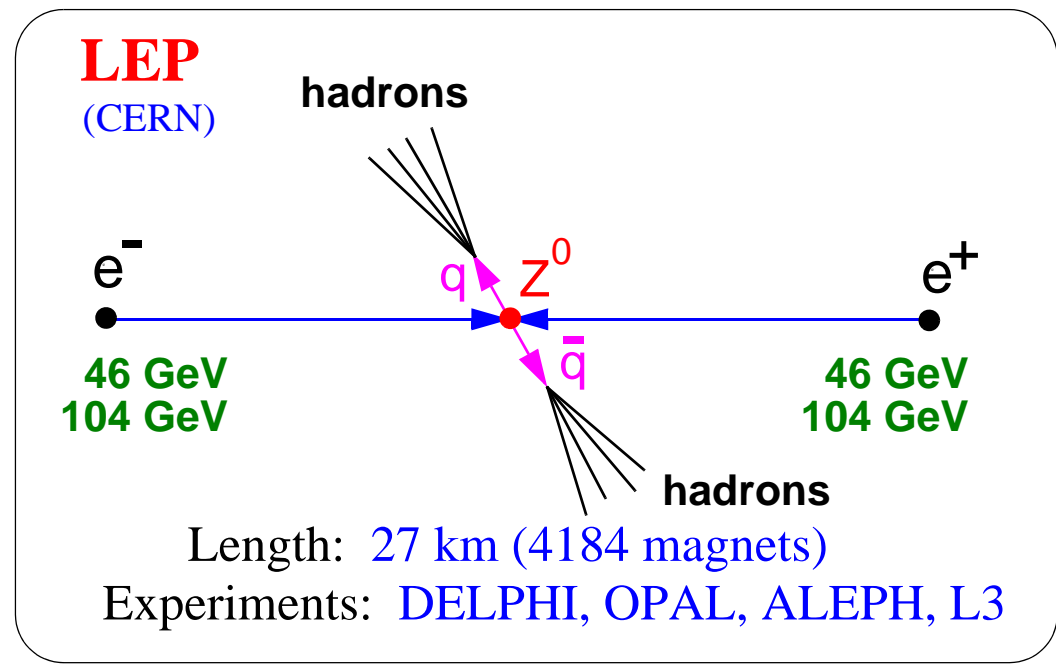
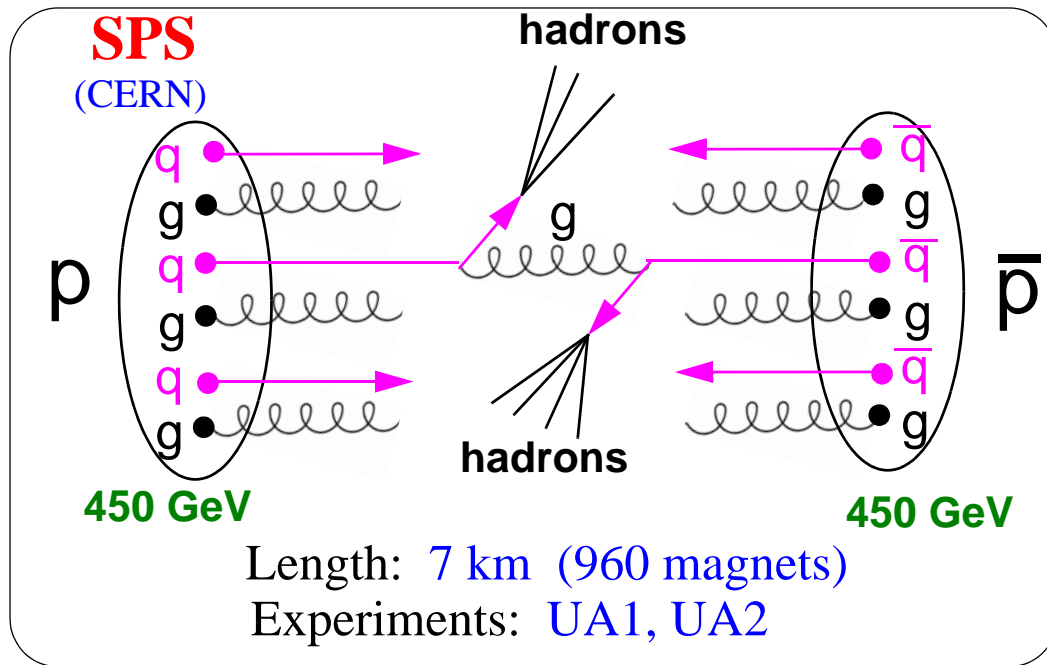
➔ The accelerator: The Super Proton Synchrotron

- The W and Z were discovered by the **UA1 and UA2 experiments**.

The SPS accelerator: Length = 6.9 km, Number of magnets = 960,
Fixed target = 400 → 450 GeV
 $p\bar{p}$ -collider = 540 → 630 → 900 GeV

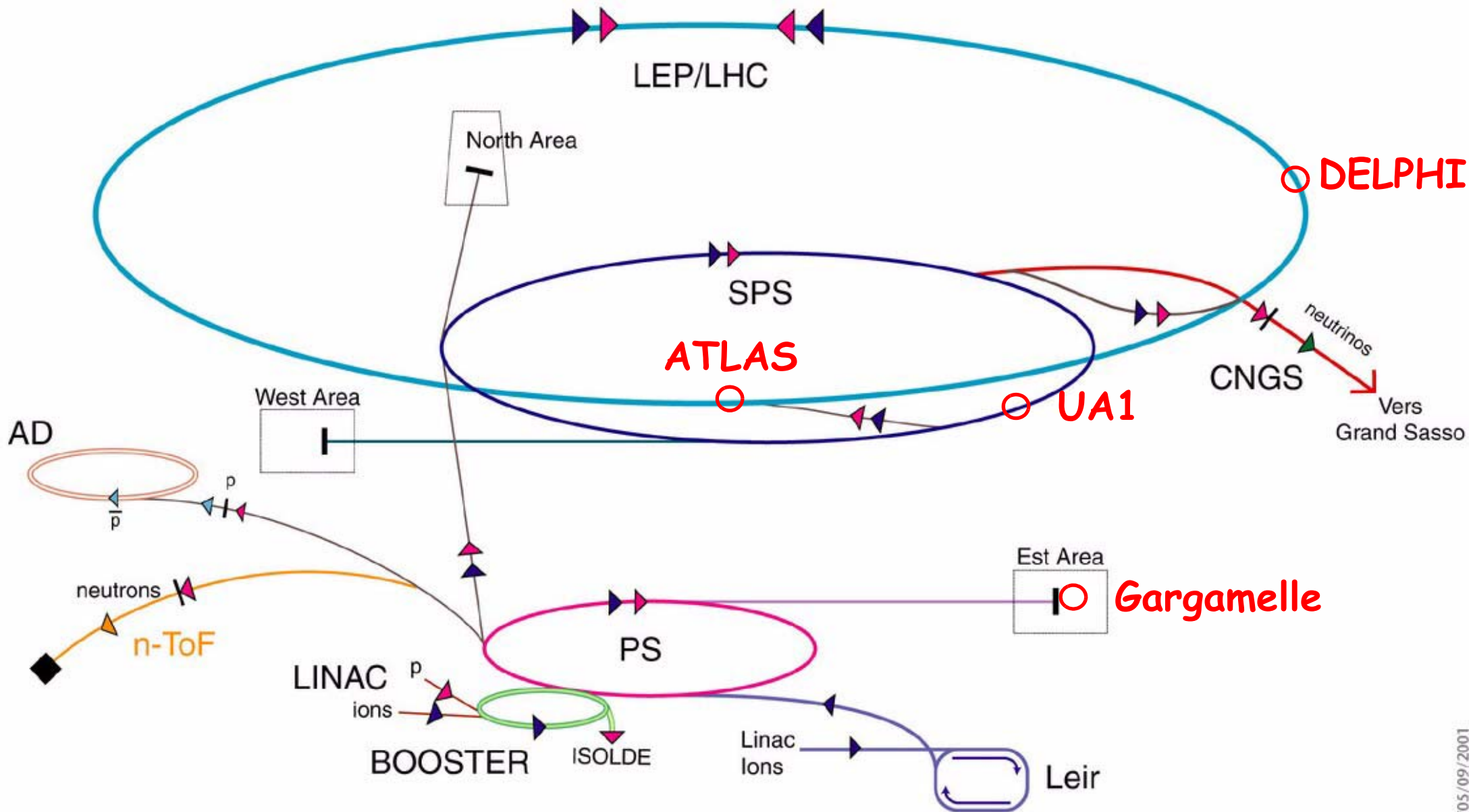


The discovery of the W and Z bosons



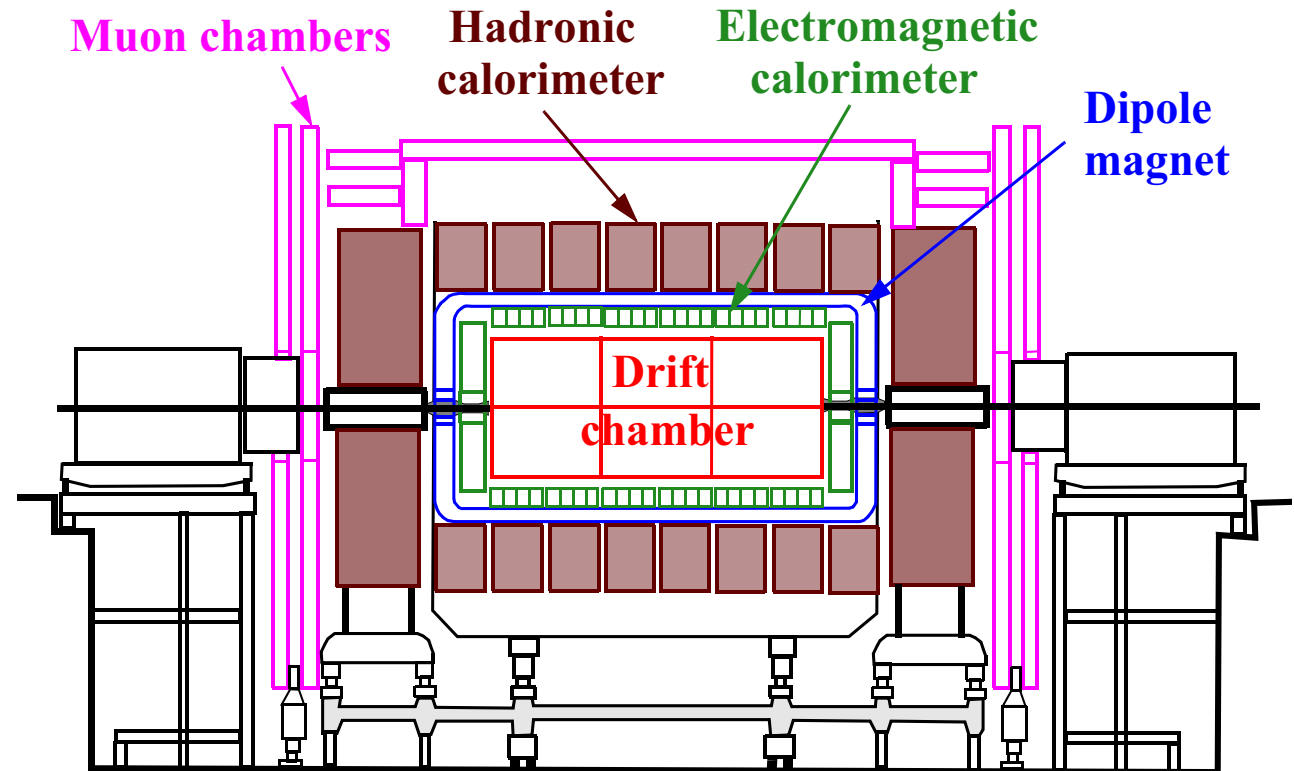
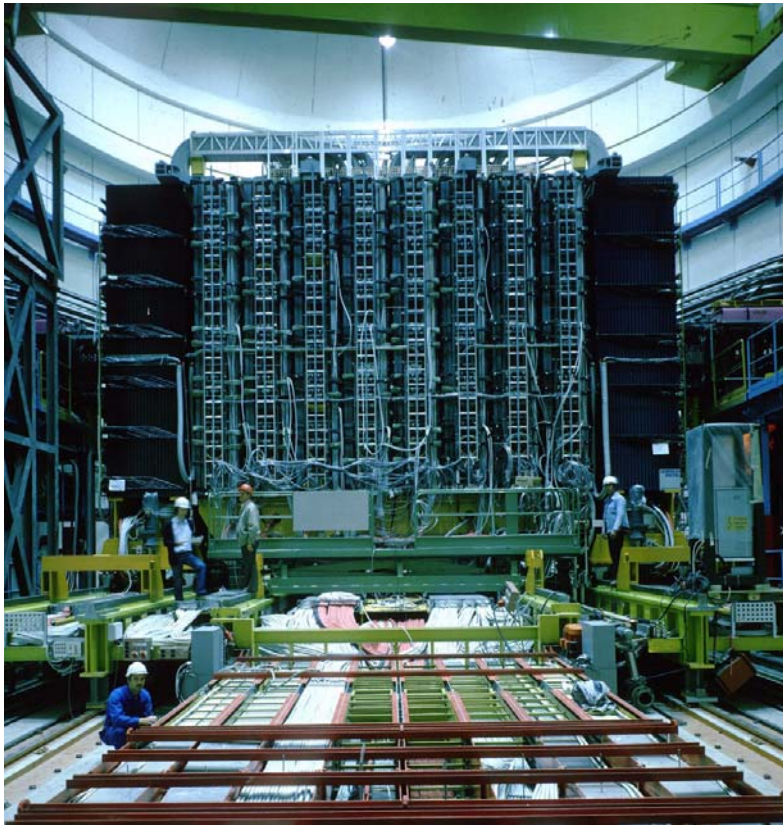
- The energy of the quarks and gluons carry only a **fraction of the proton energy**.
- To produce W- and Z-bosons with a mass of 80-90 GeV \Rightarrow a collider with a beam energy of 270 GeV was needed.
- The beam energy was later increased to 450 GeV.

The discovery of the W and Z bosons



The discovery of the W and Z bosons

→ The experiment: UA1



Tracking detector: Central wire chamber with 0.7 T dipole magnet.

Electromagnetic calorimeter: Lead/scintillator sandwich

Hadronic calorimeter: Iron/Scintillator sandwich

Muon detector: 8 planes of drift chambers.

The discovery of the W and Z bosons

➔ Production of W and Z bosons

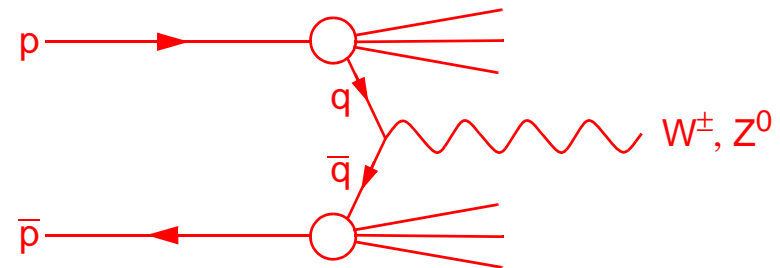
- The W and Z bosons are in proton colliders produced by **quark-antiquark annihilations**:

$$u + \bar{d} \rightarrow W^+$$

$$d + \bar{u} \rightarrow W^-$$

$$u + \bar{u} \rightarrow Z^0$$

$$d + \bar{d} \rightarrow Z^0$$



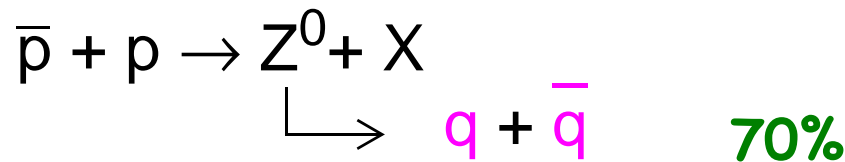
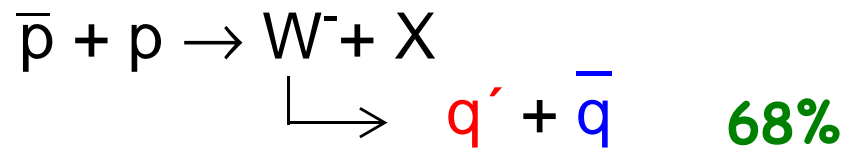
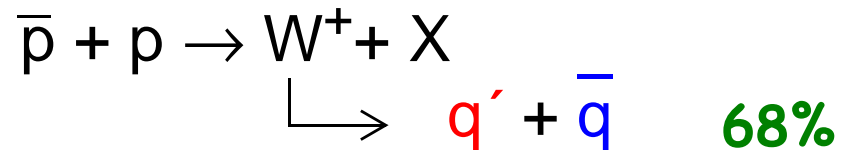
- The **lifetime** of both the W and the Z is about 3×10^{-25} s
➔ they cannot be seen directly in the experiments.

The discovery of the W and Z bosons

→ The decay of W and Z bosons

hadronic decays

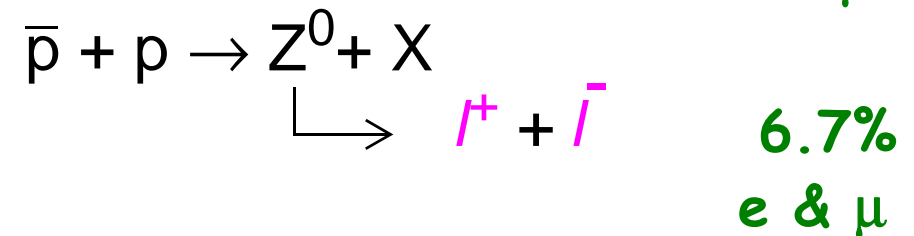
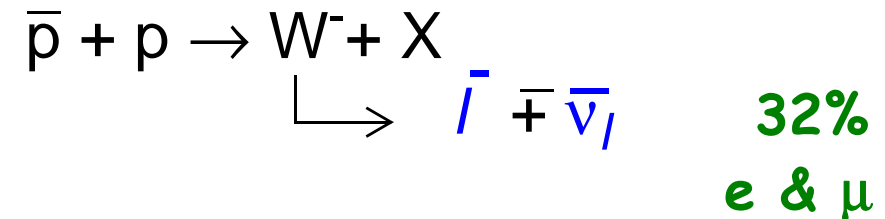
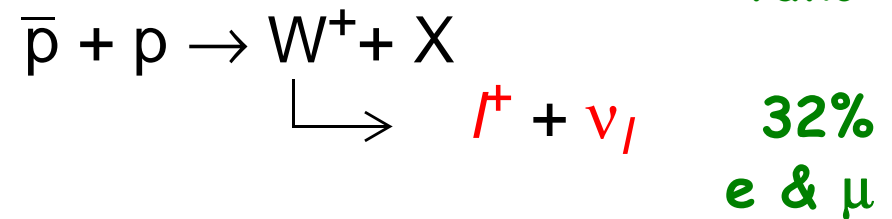
Branching
ratio



Cannot be found among
all other hadrons produced.

leptonic decays

Branching
ratio

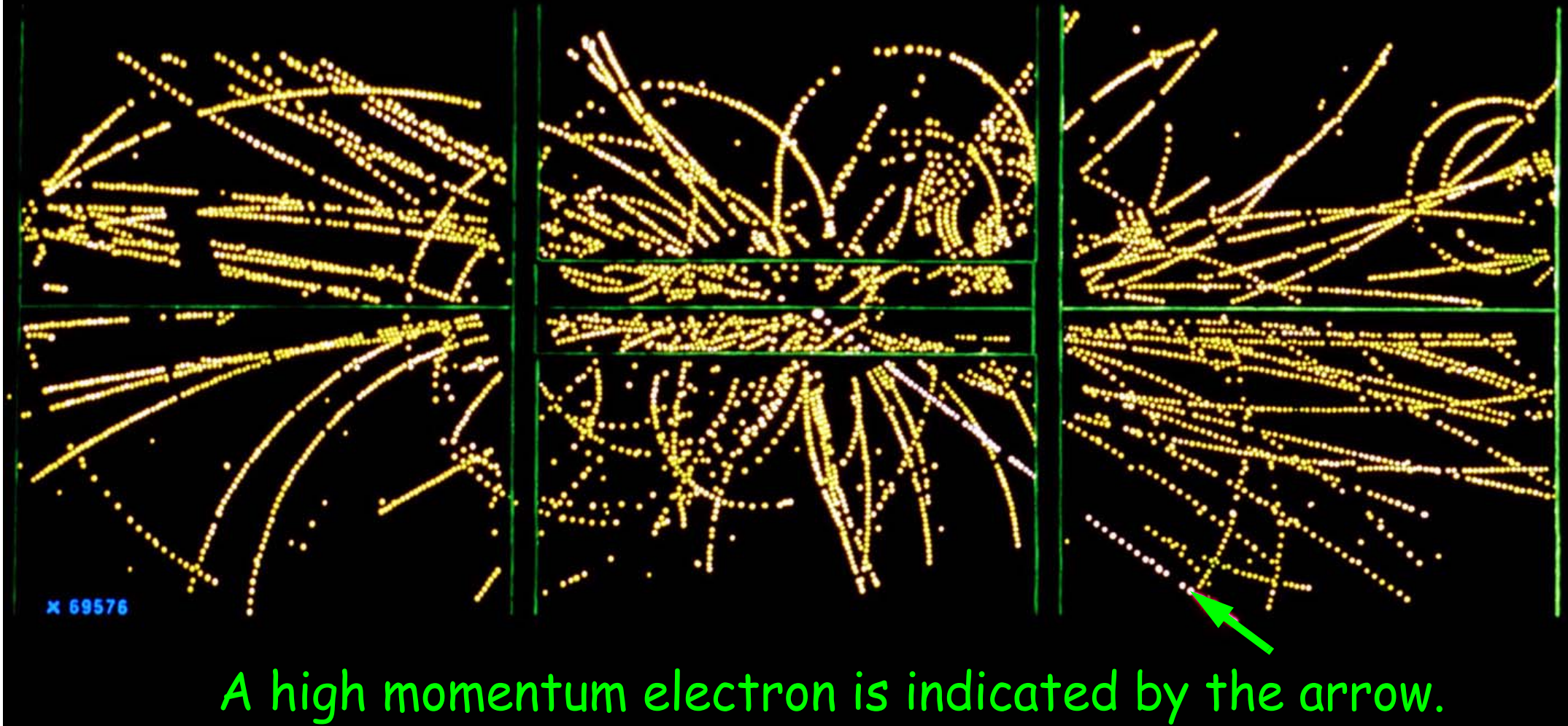


The decays to leptons
are easy to identify

The discovery of the W and Z bosons

EVENT 2958. 1279. → Analysis of W-events in UA1

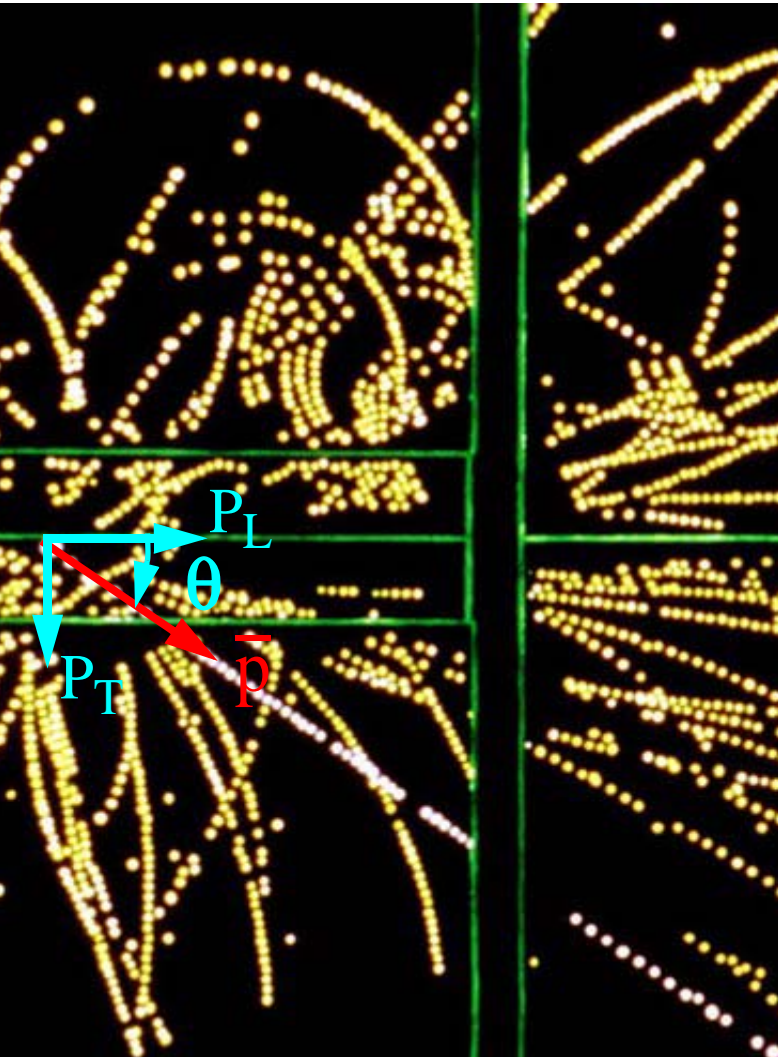
The analysis searched for events with a charged lepton + neutrino



A high momentum electron is indicated by the arrow.

The discovery of the W and Z bosons

→ Transverse momentum



- The **transverse momentum** and energy of a particle:

$$P_T = P \sin(\theta)$$
$$E_T = E \sin(\theta)$$

The angle to the beam

- $E_T = P_T$ if the mass of the particle is small since $E^2 = P^2 + m^2$
- The **total momentum is zero** of all the particles in a collision.
- Neutrinos are not detectable \Rightarrow
if the total momentum $\neq 0$ \Rightarrow
the event has **missing momentum**
(or missing energy).

The discovery of the W and Z bosons

➔ Analysis of W-events in UA1

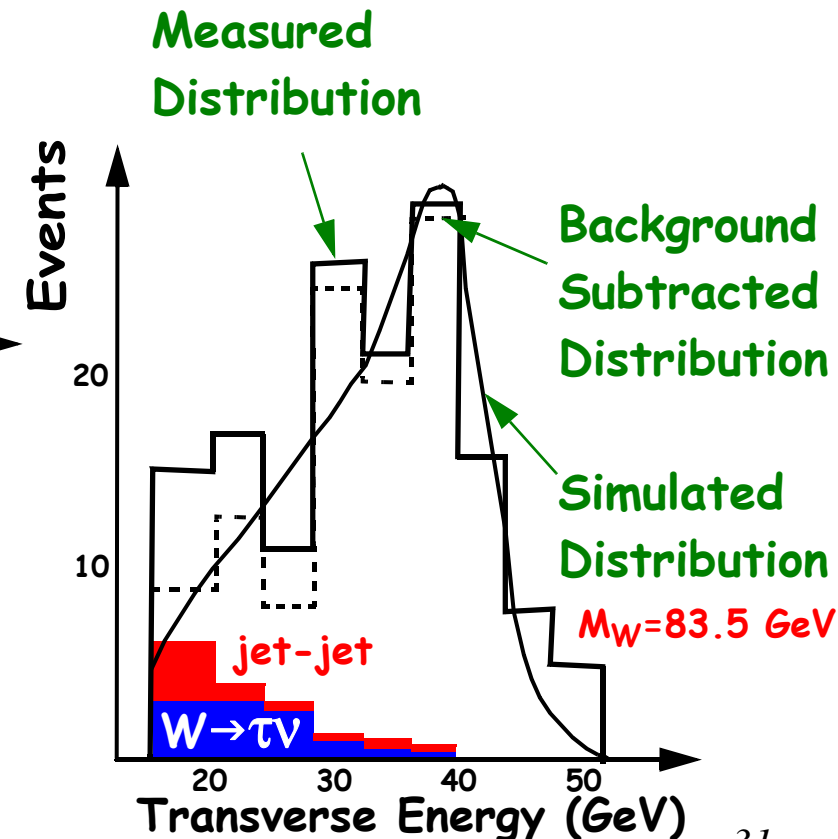
● The main selection criteria in the UA1 W-analysis was:

- A charged **electron or muon** with a large momentum (**>10 GeV/c**)
- This lepton should be emitted at a wide **angle** to the beam (**>5°**)
- There should be **large missing transverse momentum** in the event

The expected distribution of the transverse energy of the selected electrons was compared with the measured distribution.

From the first 148 electron and 47 muon events it was estimated that:

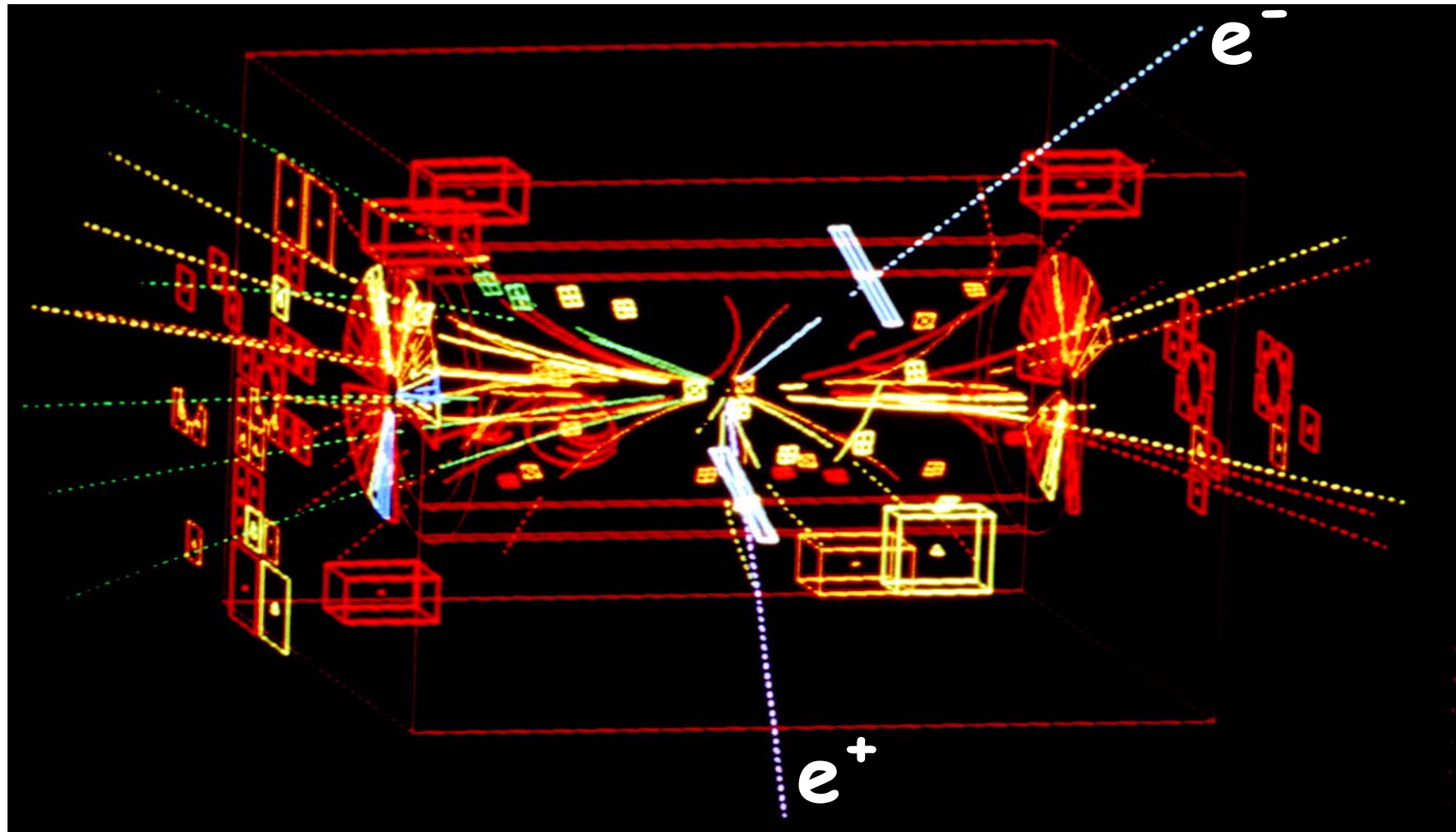
$$M_W = 83,5 \pm 2,8 \text{ GeV} \quad \Gamma_W \leq 6,5 \text{ GeV}$$



The discovery of the W and Z bosons

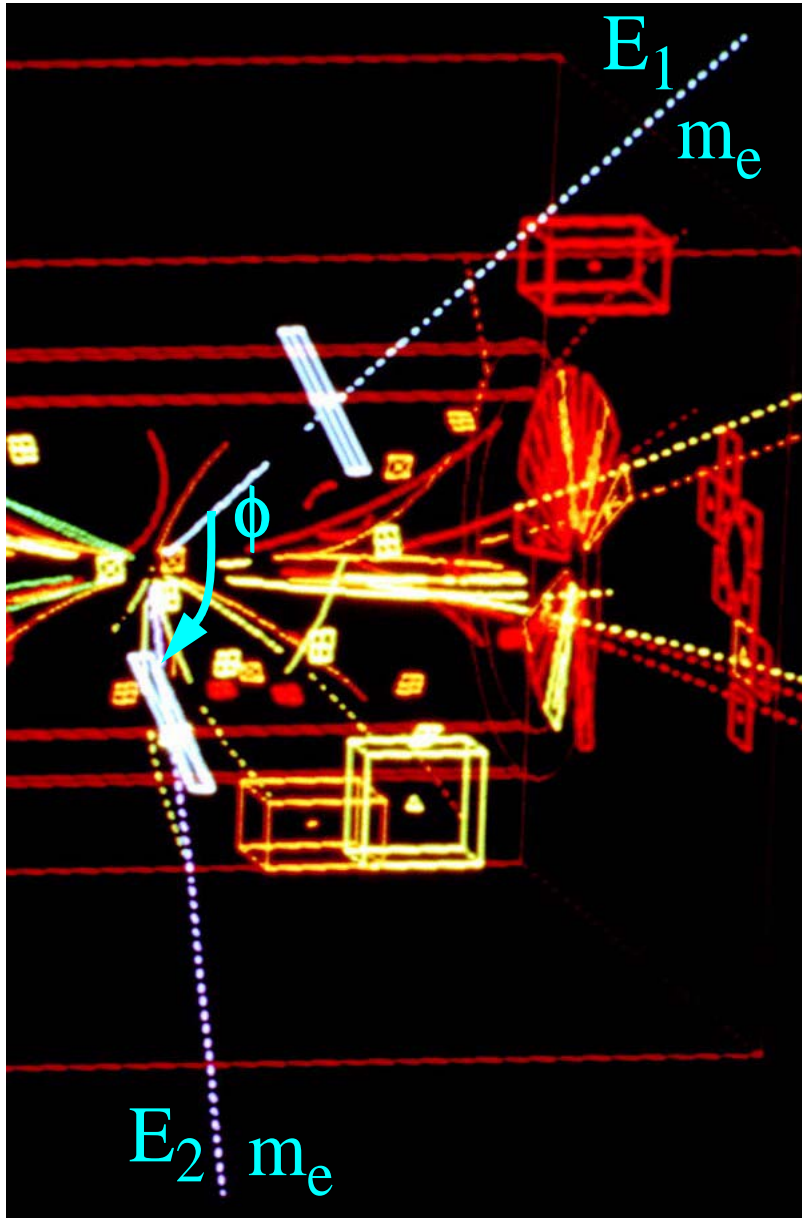
➔ Analysis of Z-events in UA1

- The analysis was looking for a pair of charged leptons.

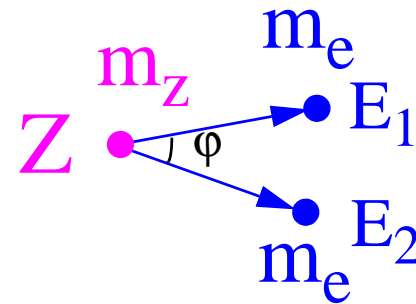


The discovery of the W and Z bosons

→ Invariant mass



- The **invariant mass** of a particle that decays to two other particles:



$$m_Z^2 = (\vec{\mathbf{P}}_1 + \vec{\mathbf{P}}_2)^2 \quad (4\text{-vectors})$$

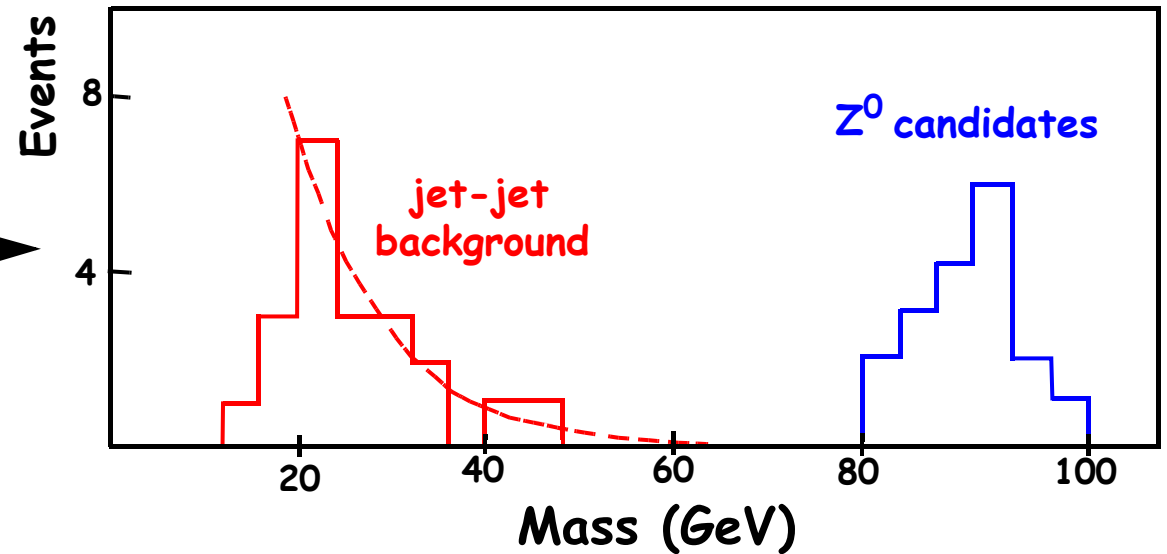
$$m_Z^2 = \overset{\text{if } m=0}{\rightarrow} 2 E_1 E_2 (1 - \cos\phi)$$

The discovery of the W and Z bosons

→ Analysis of Z-events in UA1

- Main search criteria → require **pair of electrons or muons** with a **large transverse energy**.

The mass distribution of pairs of electrons where each electron has $E_T > 8$ GeV.



The first 18 electron pairs and 10 muon pairs in UA1

$$M_Z = 93,0 \pm 1,4 \text{ GeV}$$

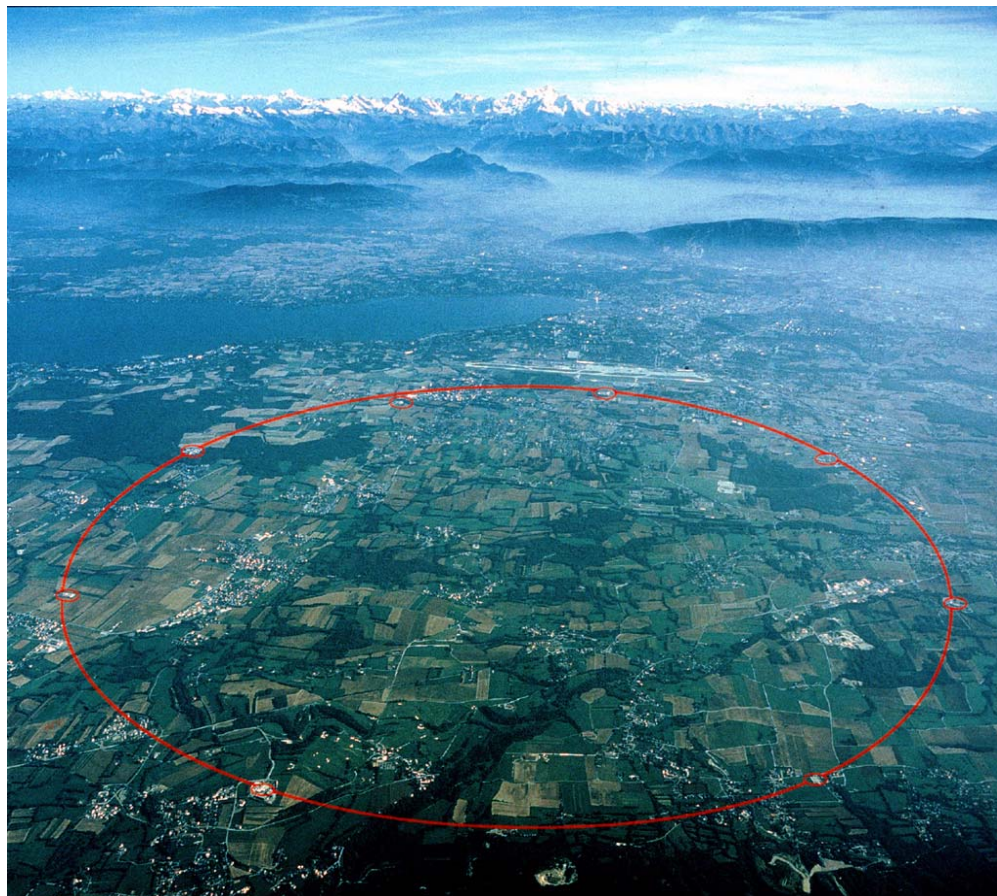
$$\Gamma_Z \leq 8,1 \text{ GeV}$$

Precision studies of the W and Z bosons

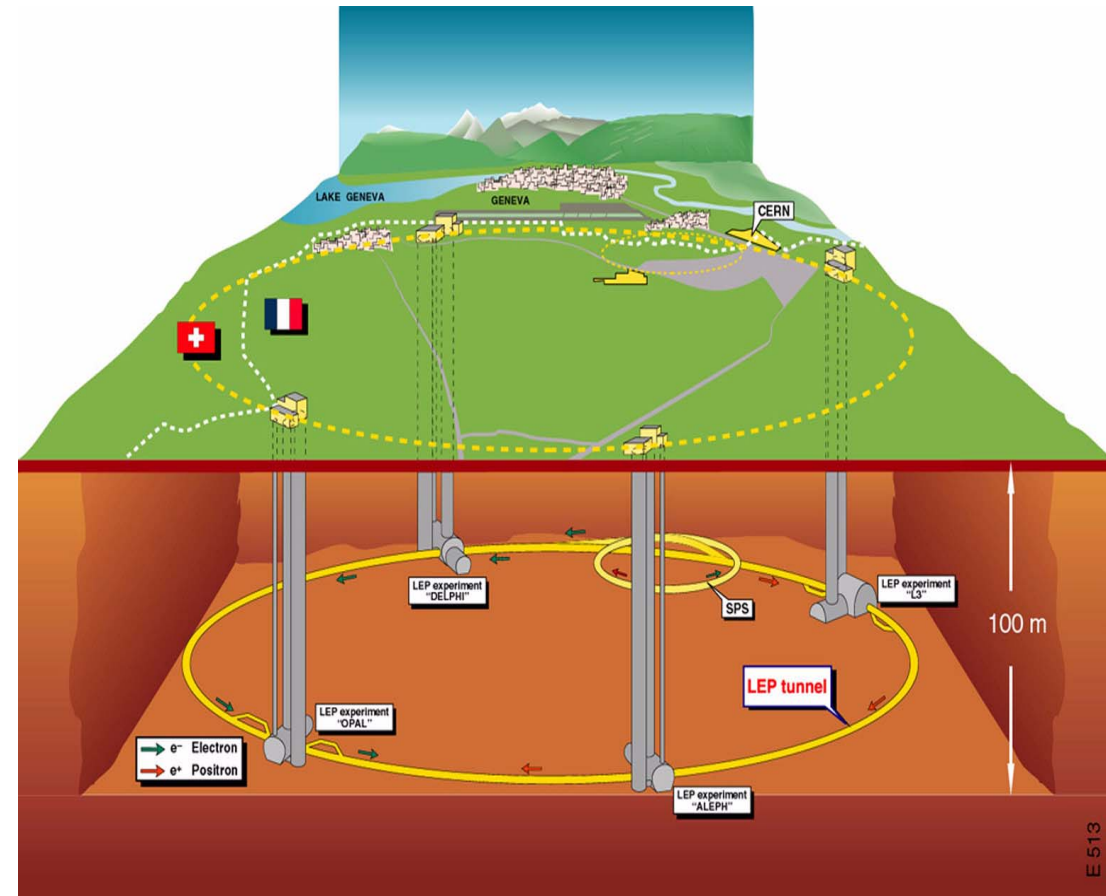
➔ The accelerator: The Large Electron Positron Collider

- **Electrons-positrons** collisions in four experiments.

- Collision energy: 91 GeV $\xrightarrow{\quad}$ 209 GeV
Z mass \uparrow maximum
288 superconducting cavities.

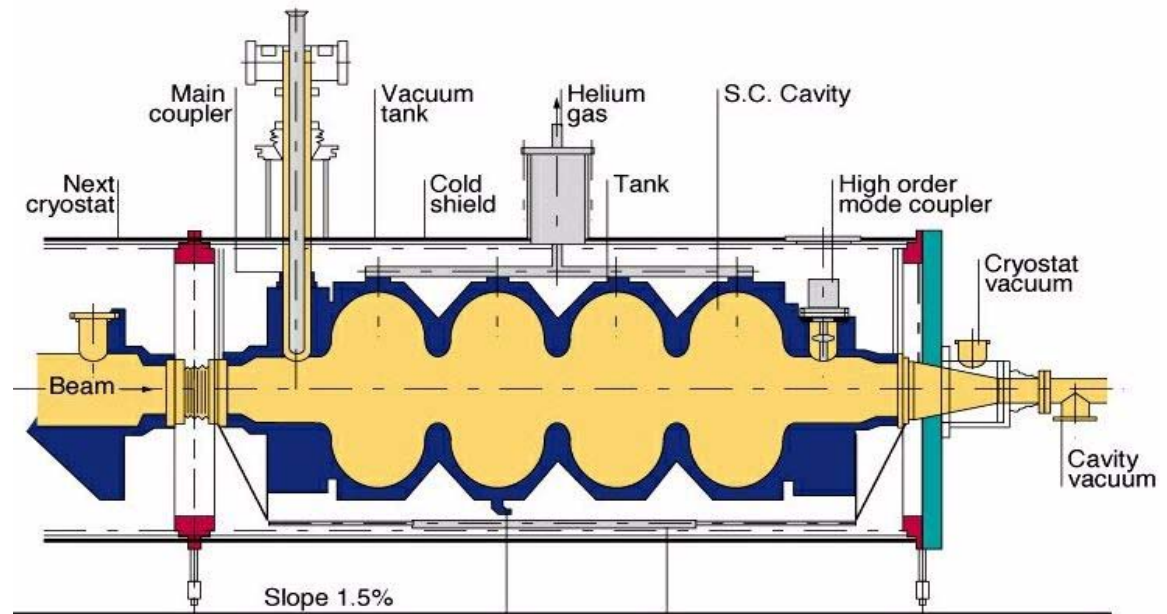
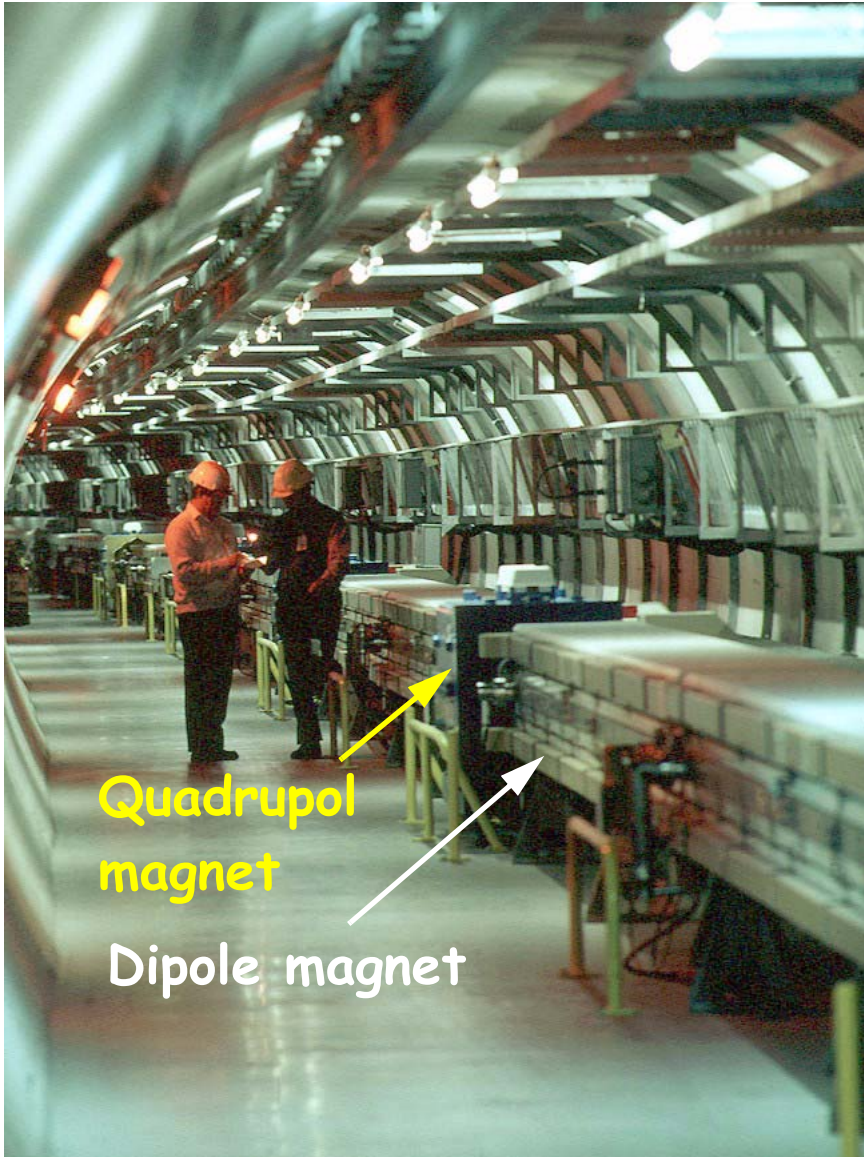


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Weak Interactions

Precision studies of the W and Z bosons



Precision studies of the W and Z bosons

LEP
(CERN)

hadrons

e^- 46 GeV
 104 GeV

q Z^0 \bar{q}

e^+ 46 GeV
 104 GeV

hadrons

Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

HERA
(DESY)

e^+ 30 GeV

920 GeV

e^+ γ

hadrons

q g q g q g

p

Length: 6 km (1650 magnets)
Experiments: H1, ZEUS

TEVATRON
(FERMILAB)

hadrons

q g q g q g

p

g g g g

hadrons

\bar{q} g \bar{q} g \bar{q} g

\bar{p}

980 GeV 980 GeV

hadrons

Length: 6 km (990 magnets)
Experiments: CDF, D0

LHC
(CERN)

hadrons

q g q g q g

p

g g g g

hadrons

q g q g q g

p

7000 GeV 7000 GeV

hadrons

Length: 27 km (1232 + 386 magnets)
Experiments: ATLAS, CMS, LHCb, ALICE

Precision studies of the W and Z bosons

➔ Differences between proton and electron colliders.

- Synchrotron radiation:

Total energy loss is proportional to $1/\text{mass}^4$
Total energy loss is proportional to E_{beam}^4
Total energy loss is proportional to $1/\text{Radius}$

- Energy loss in an electron accelerator is 10^{13} times larger.

- LEP: 344 cavities, accelerating voltage = 3630 MV, energy = 104 GeV

LHC: 16 cavities, accelerating voltage = 16 MV, energy = 7000 GeV

- Magnetic bending field $\sim \frac{\text{Beam momentum}}{\text{Length of bending field}}$

- LEP: 0.12 Tesla bending field
- LHC: 8.38 Tesla bending field

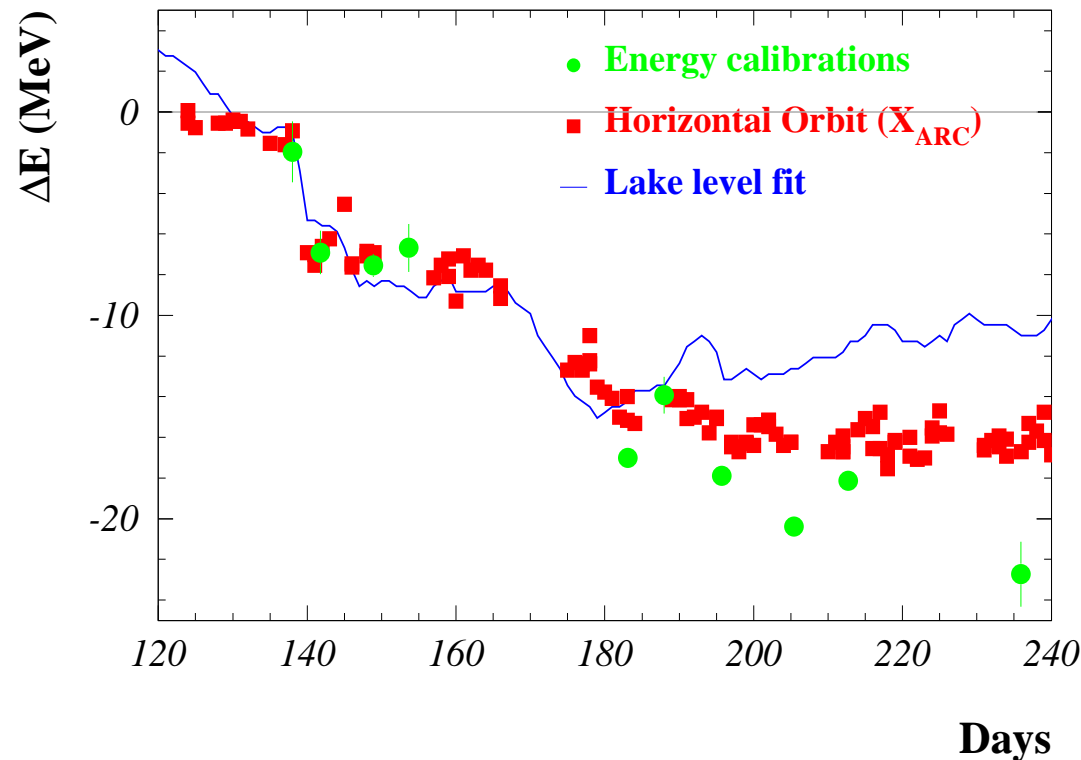
} factor 70

Precision studies of the W and Z bosons

➔ The accelerator: The Large Electron Positron Collider

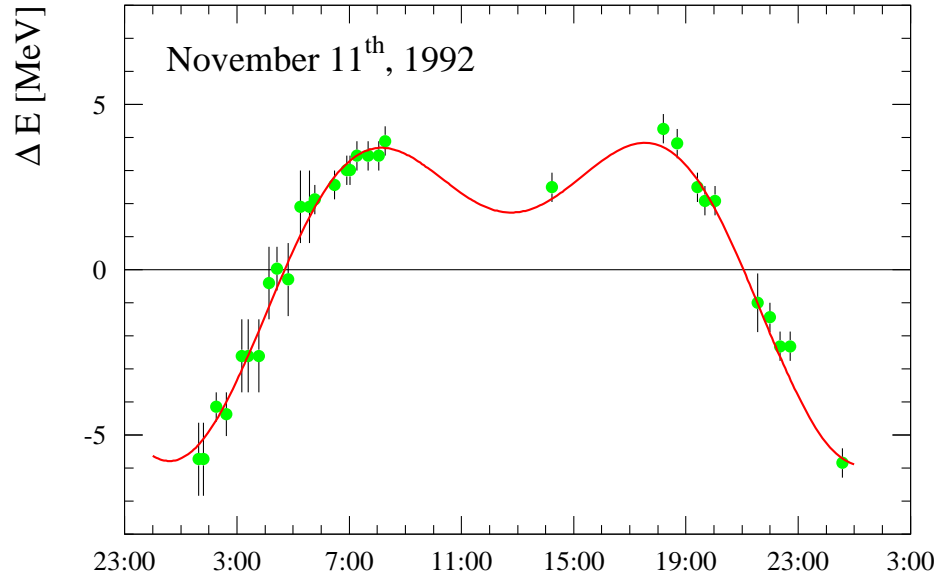
- Electron-positron collider: Precise measurement of \sqrt{s} important !
- Problem: During 1993 the LEP energy was changing with time !

➔ Geological shifts



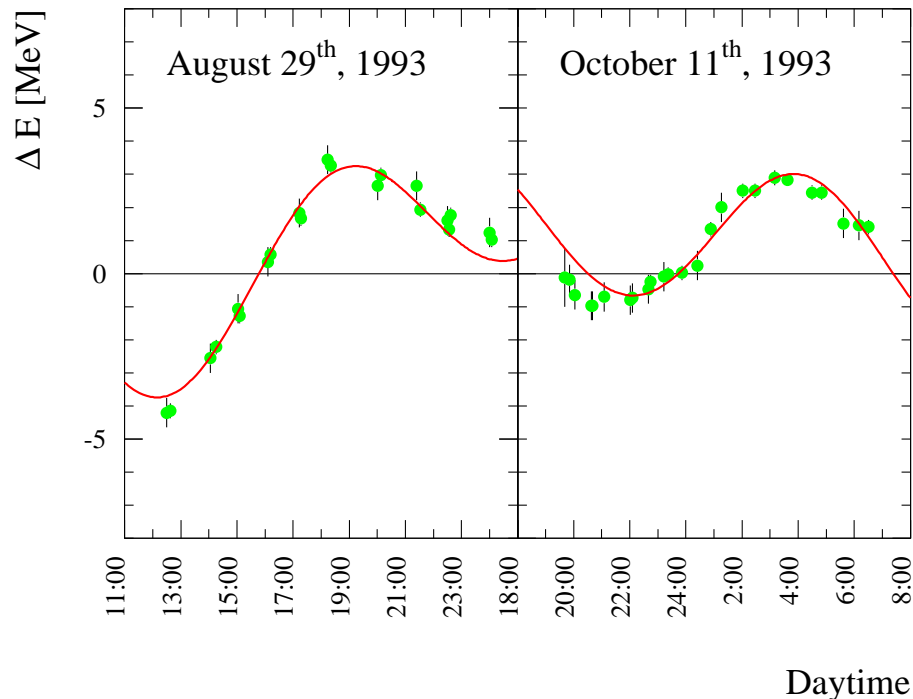
- The water level in lake Geneva
- Rainfalls and the water table.

Precision studies of the W and Z bosons



→ Tides

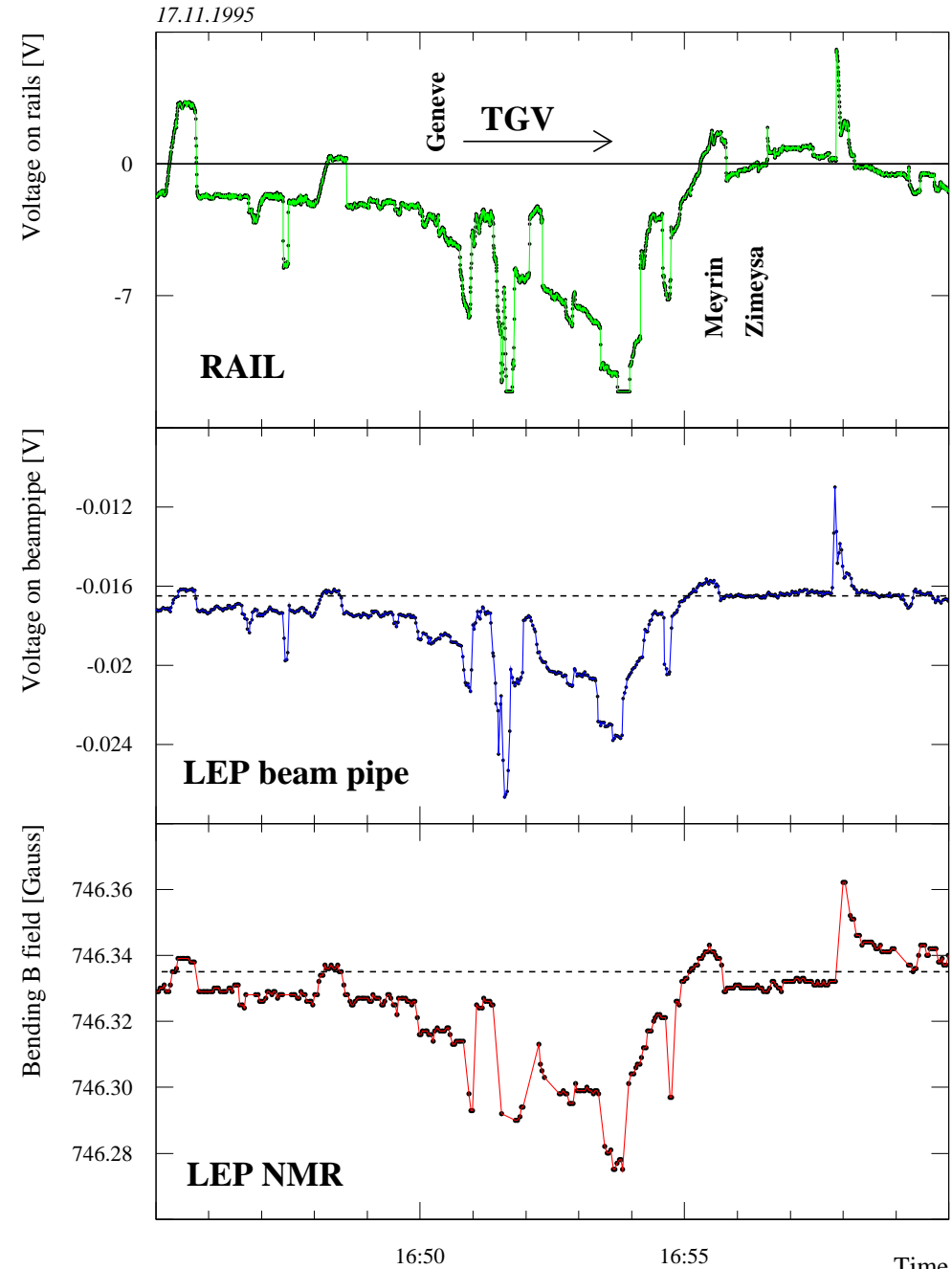
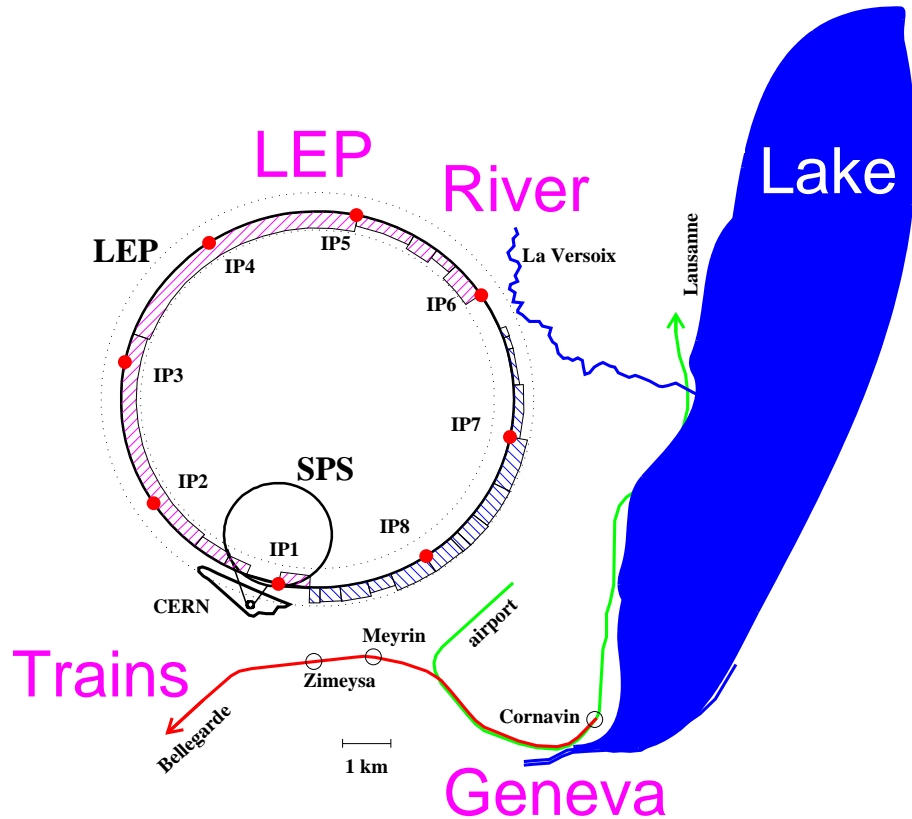
- Earth tides caused by the moon.
- The electron's orbit changes.
- 1 mm ⇒ energy change of 10 MeV



Precision studies of the W and Z bosons

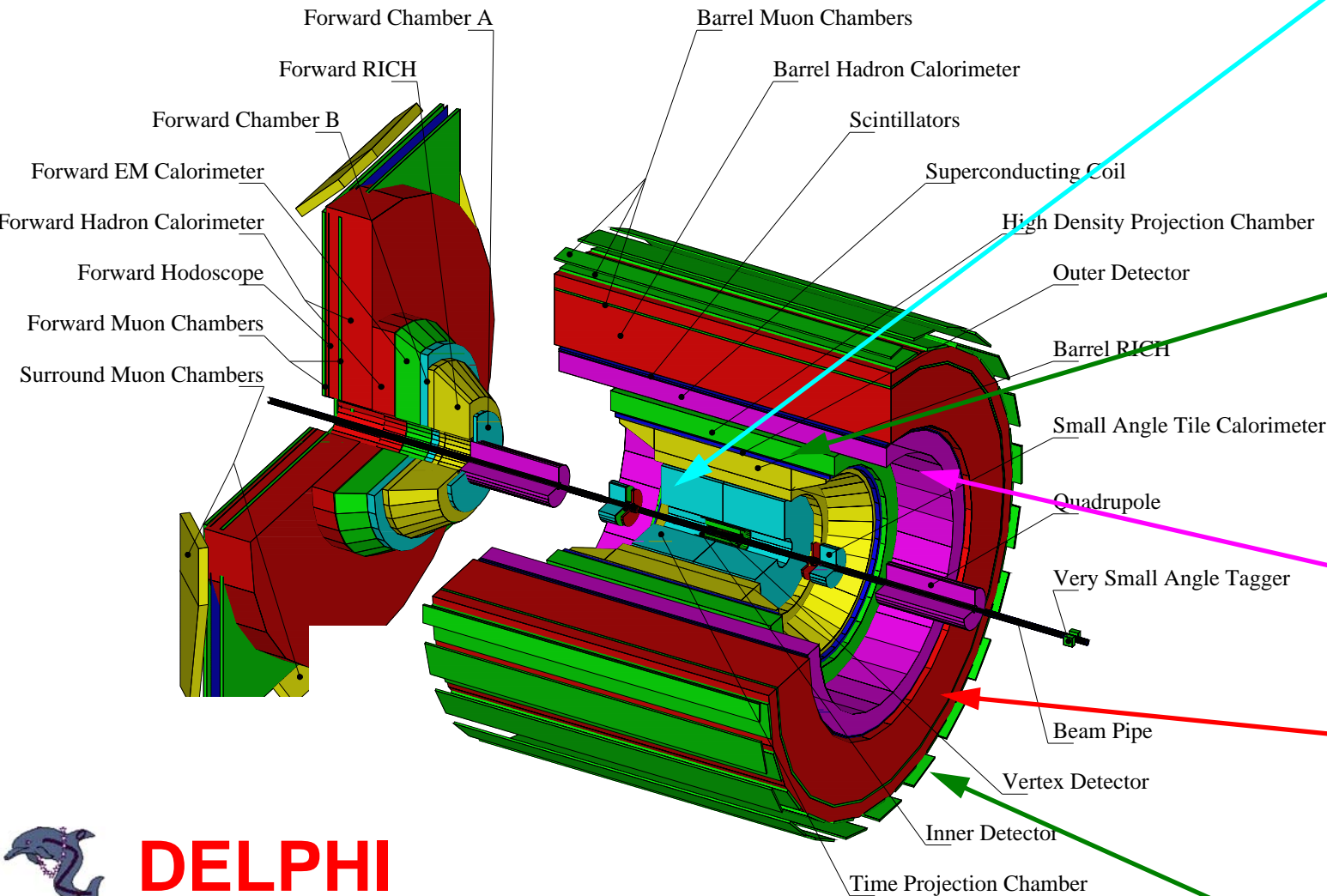
➔ Beampipe current

- Trains ➔ parasitic currents
- Currents ➔ the magnetic field
- Mag. field ➔ electron's orbit
- Orbit ➔ electron's energy



Precision studies of the W and Z bosons

→ The DELPHI Experiment



Tracking detector:
Time projection chamber

Electromagnetic calorimeter:
Lead absorber

Magnet

Hadronic calorimeter
Iron absorber

Muon detectors

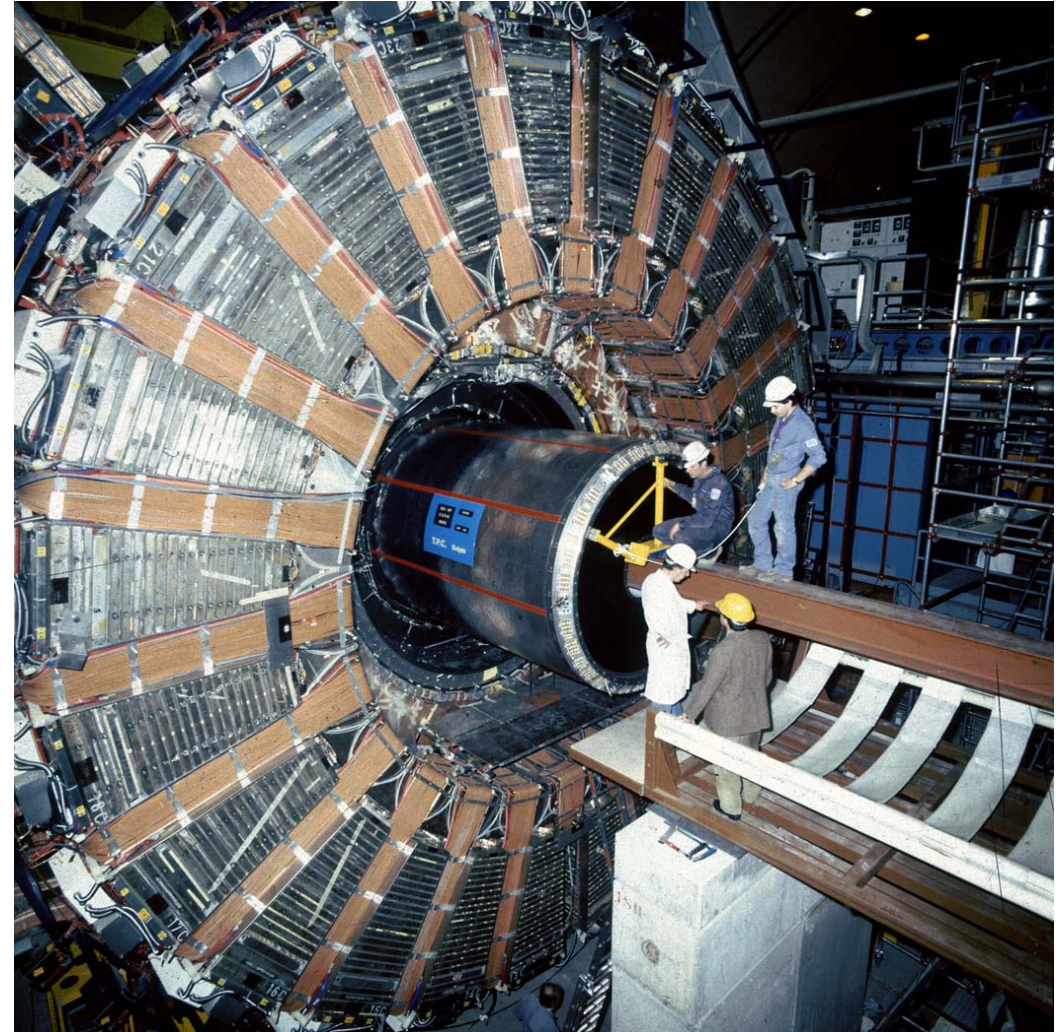


Precision studies of the W and Z bosons

→ The DELPHI Experiment



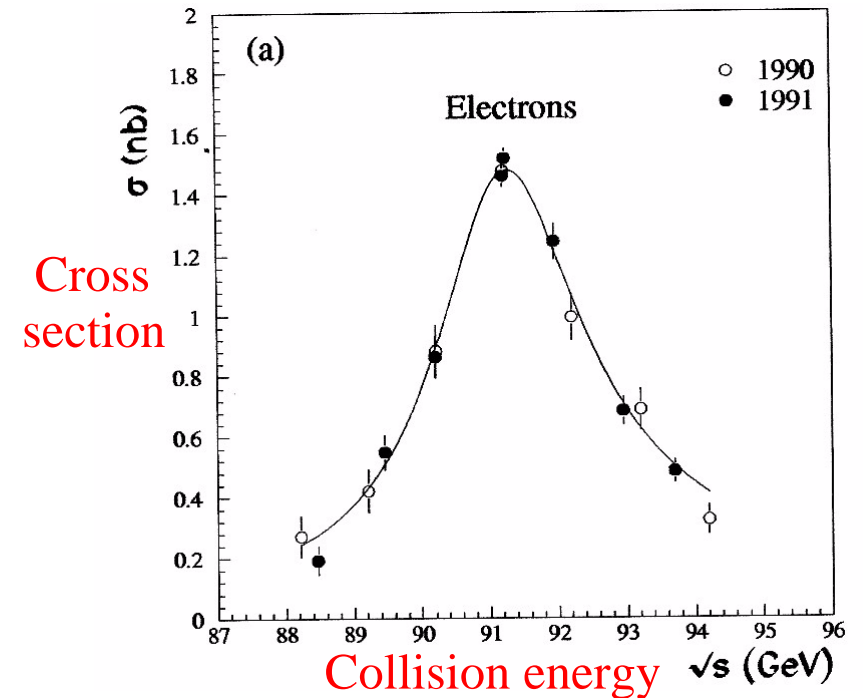
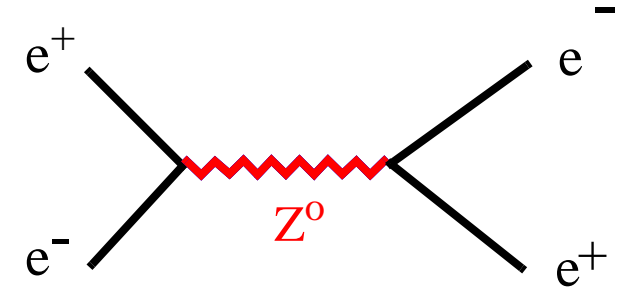
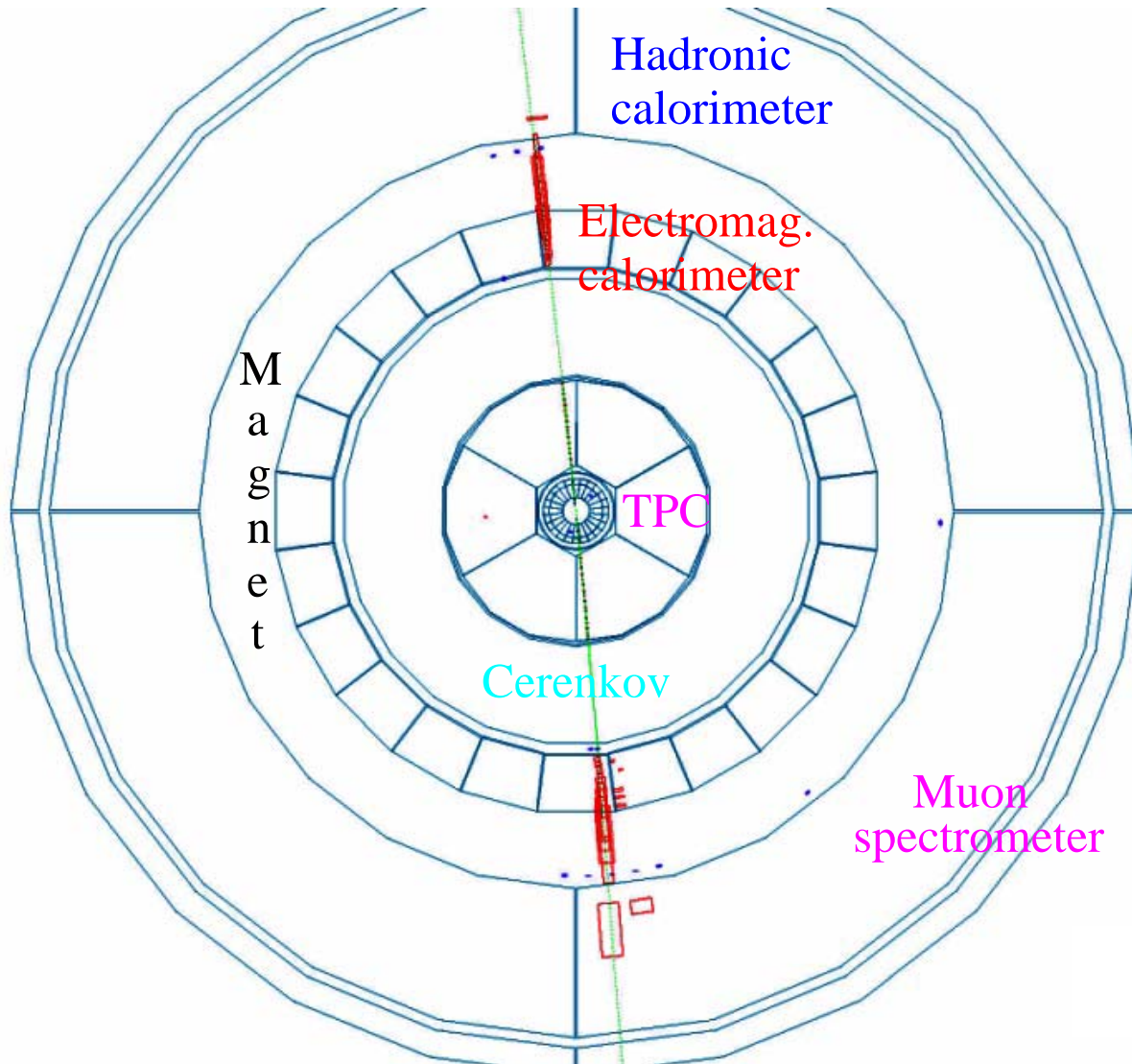
The DELPHI cavern



The Time Projection chamber

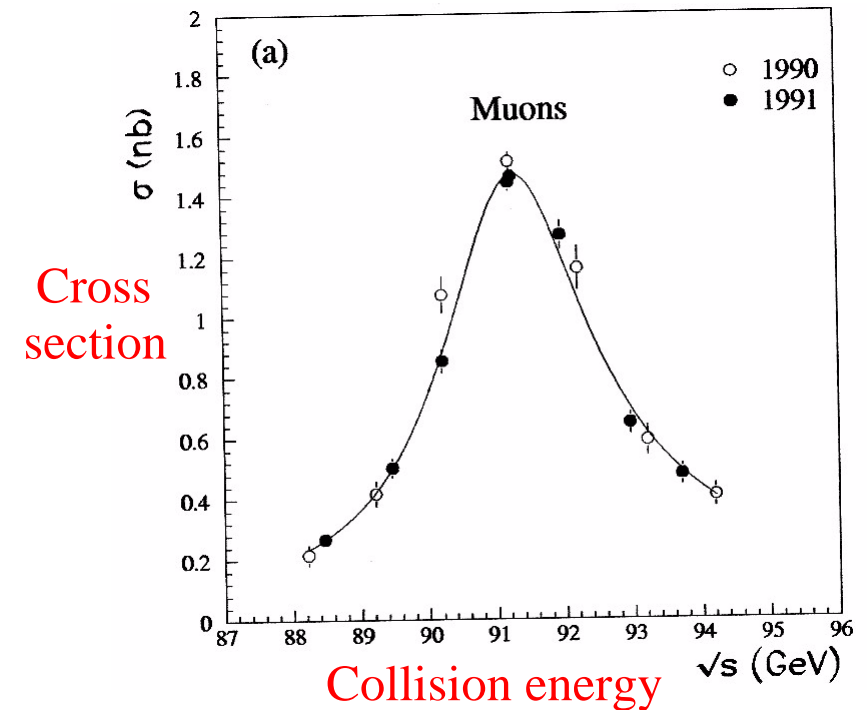
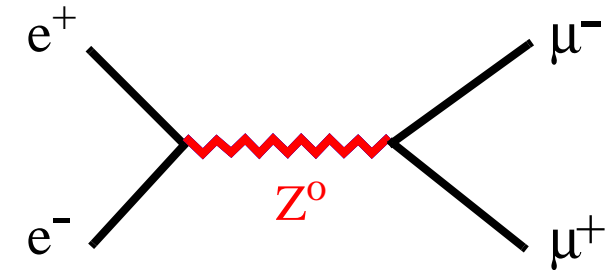
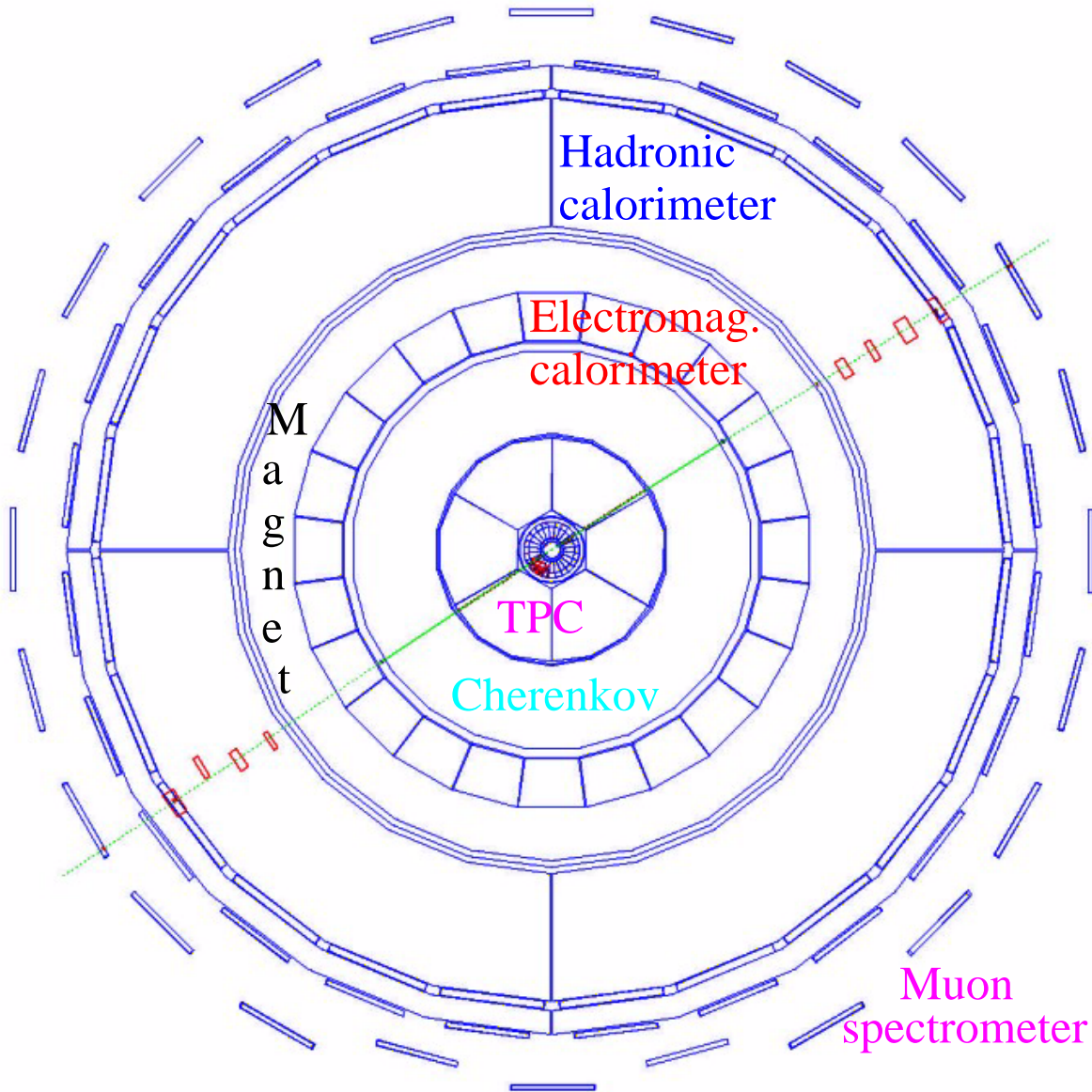
Precision studies of the W and Z bosons

➔ Studies of the Z-boson



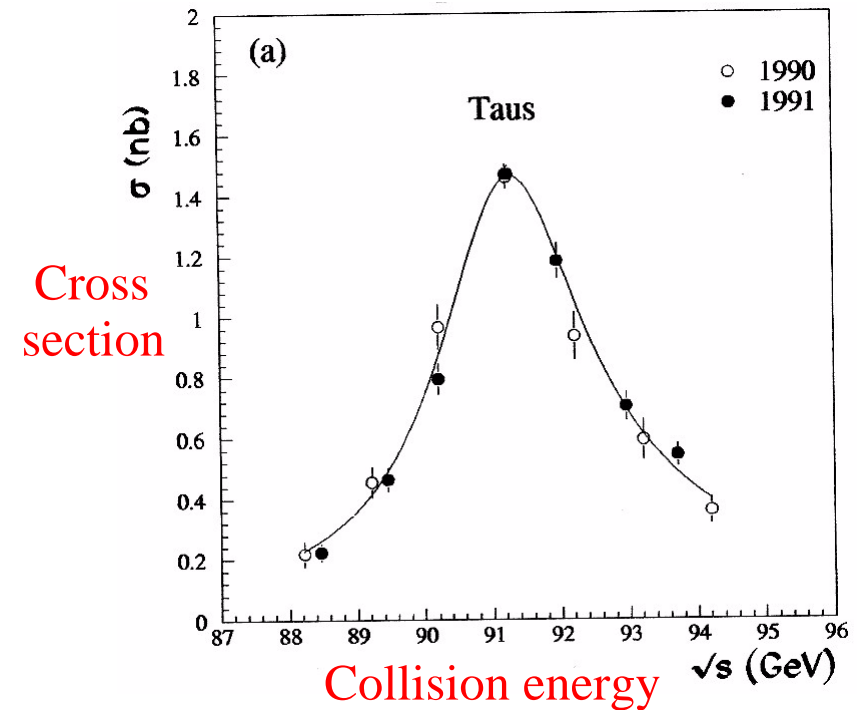
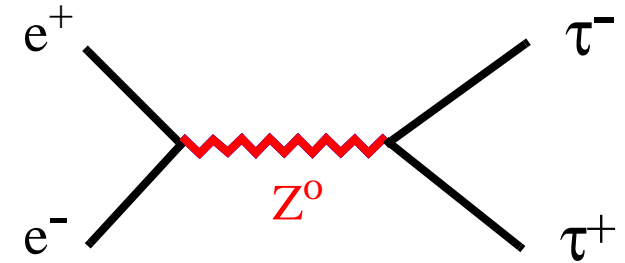
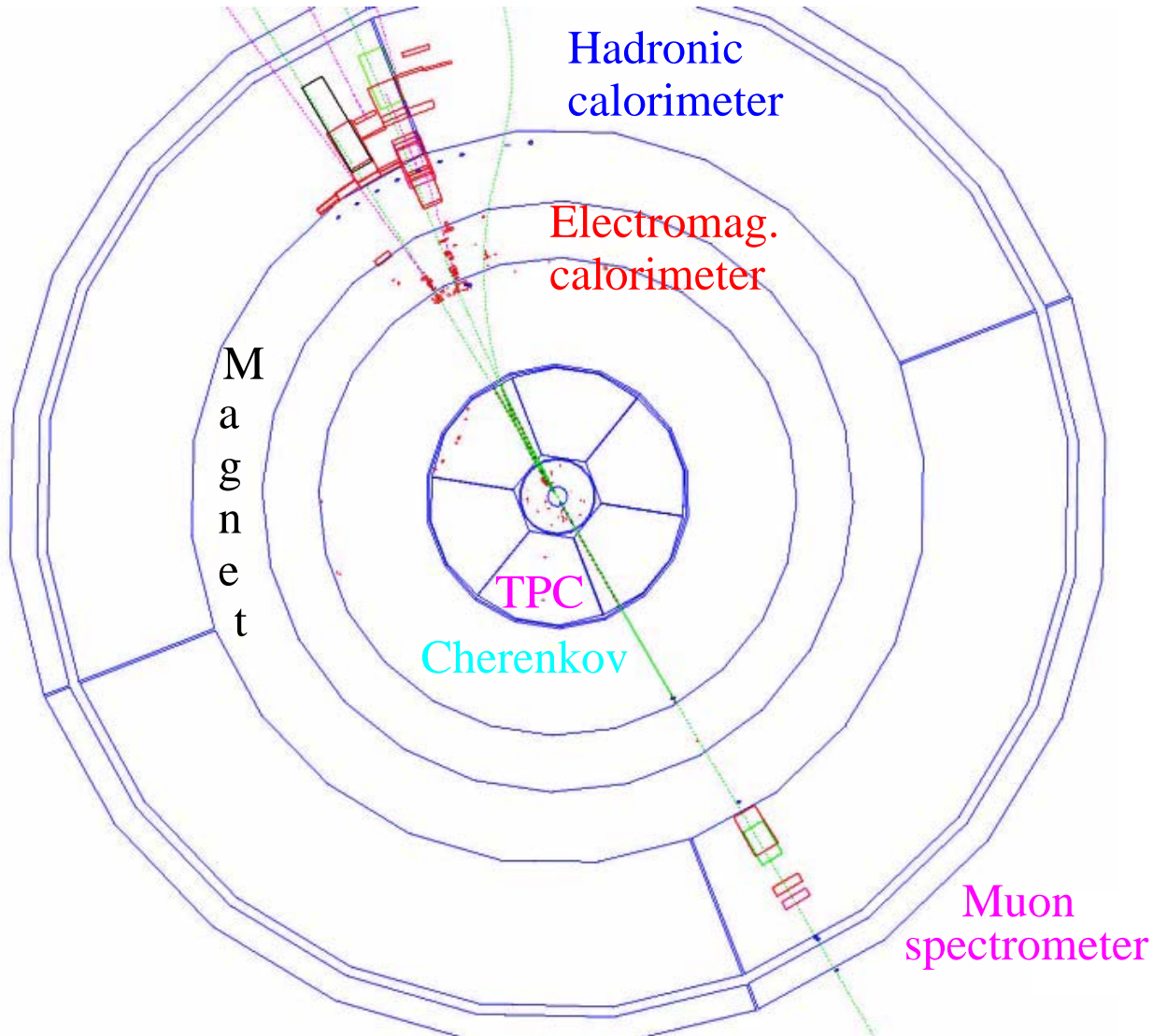
Precision studies of the W and Z bosons

➔ Studies of the Z-boson

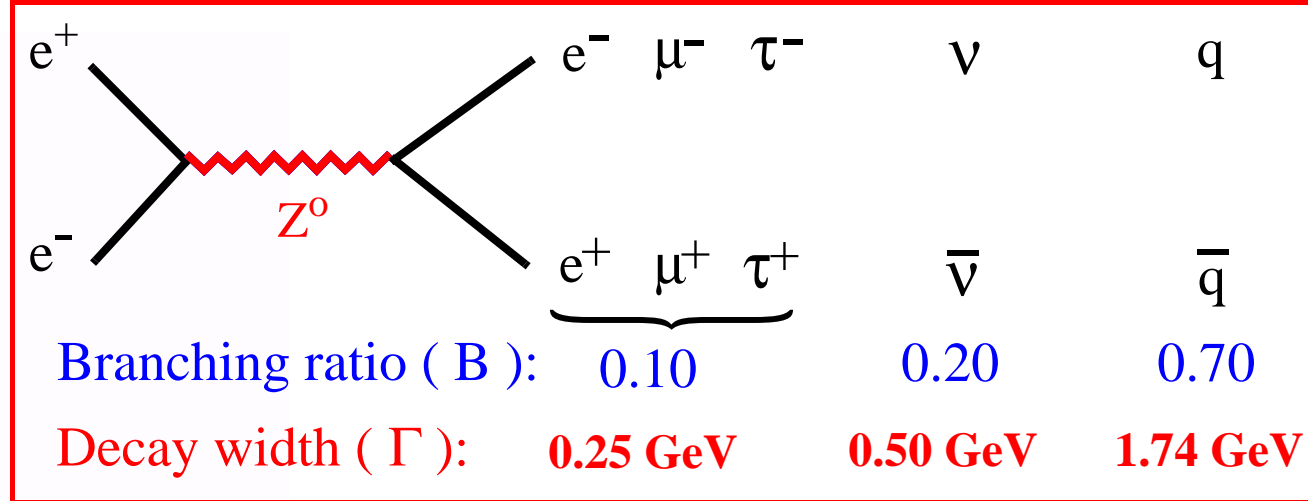
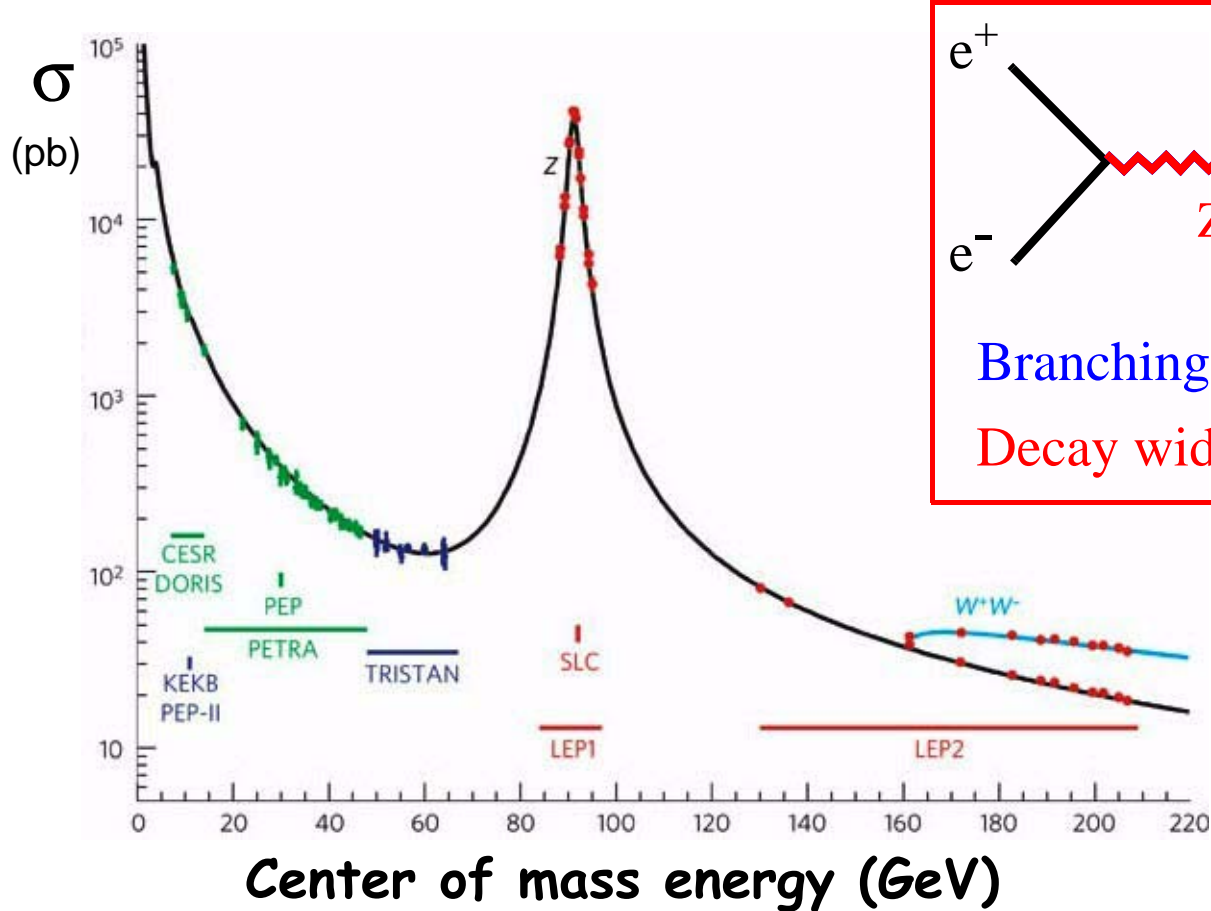


Precision studies of the W and Z bosons

➔ Studies of the Z-boson



The number of neutrino families



Partial decay width:

$$\Gamma = \frac{B}{\tau}$$

← Branching ratio

← Decay time

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3 \times 10^{-25} s$$

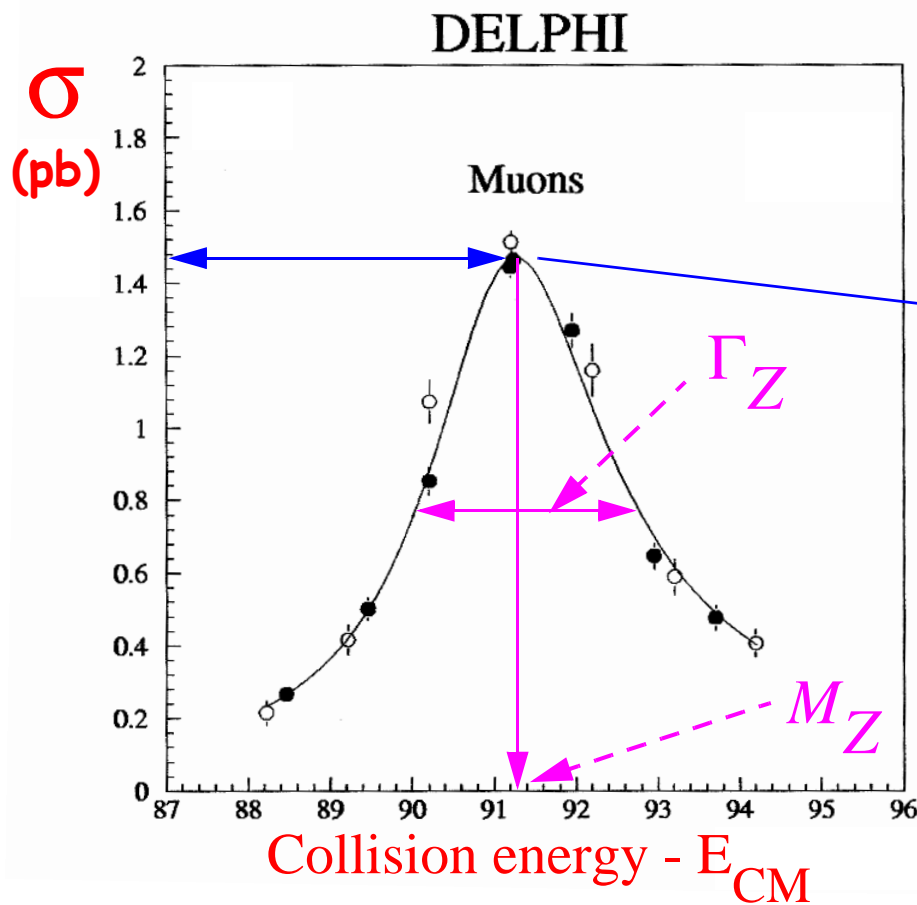
The total decay width →

The partial decay widths →

$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had} + B_{ll} + B_{\nu\nu}}{\Gamma_{had} + \Gamma_{ll} + \Gamma_{\nu\nu}}$$

The number of neutrino families

- Breit-Wigner \Rightarrow the muon **partial decay widths** of the Z^0



$$\sigma(e^+e^- \rightarrow \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2} \left[\frac{\Gamma(Z \rightarrow ee)\Gamma(Z \rightarrow \mu\mu)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]$$

σ at $E_{cm} = M_Z$ give $\Gamma(Z^0 \rightarrow \mu\mu)$ because then

The decay rate to ee The decay rate to $\mu\mu$

$$\sigma(e^+e^- \rightarrow \mu\mu) = \frac{12\pi}{M_Z^2} \left[\frac{\Gamma(Z \rightarrow ee)\Gamma(Z \rightarrow \mu\mu)}{\Gamma_Z^2} \right]$$

The mass of the Z^0 The total Z^0 decay rate

In reality the measurements are a bit more complicated because the measured peaks have to be corrected for initial state radiation (photons radiated off incoming electrons).

The number of neutrino families

- The fitted parameters:

$$M_Z = 91.187 \pm 0.007 \text{ GeV}/c^2$$

$$\Gamma_Z = 2.490 \pm 0.007 \text{ GeV}$$

$$\Gamma(Z^0 \rightarrow \text{hadrons}) = 1.741 \pm 0.006 \text{ GeV}$$

$$\Gamma(Z^0 \rightarrow l^+ l^-) = 0.0838 \pm 0.0003 \text{ GeV}$$

- Neutrinos cannot be measured in the experiments.

$$\Gamma_Z = \Gamma(Z^0 \rightarrow \text{hadrons}) + 3\Gamma(Z^0 \rightarrow l^+ l^-) + N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l)$$

$$N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0.498 \pm 0.009 \text{ GeV}$$

Decay width to neutrinos

Number of neutrinos

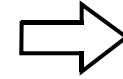
The number of neutrino families

● **Measurement:**

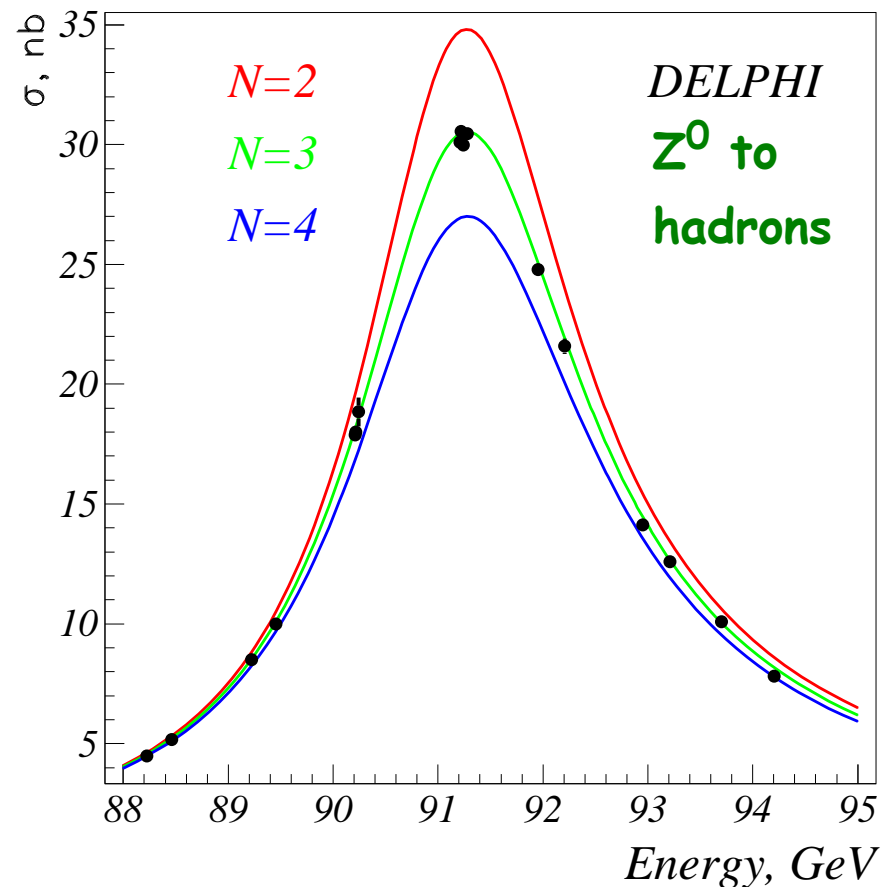
$$N_{\nu} \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0.498 \text{ GeV}$$

Calculation:

$$\Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0.166 \text{ GeV}$$



$$N_{\nu} = 2.994 \pm 0.011$$



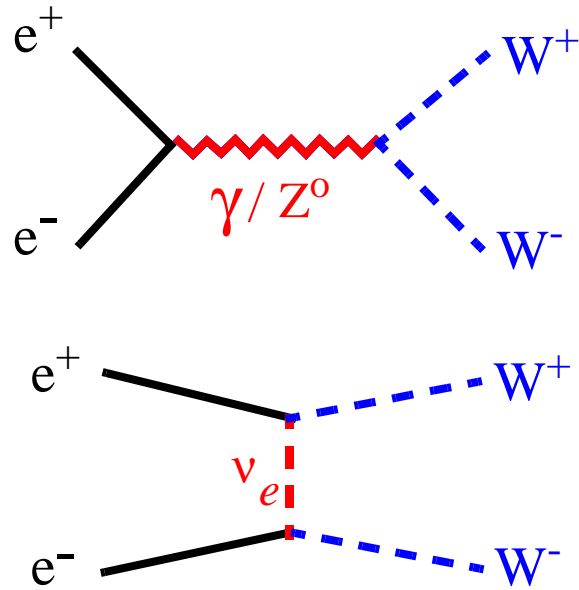
● No restrictions on N_{ν} in SM.

● LEP \Rightarrow three types of light neutrinos

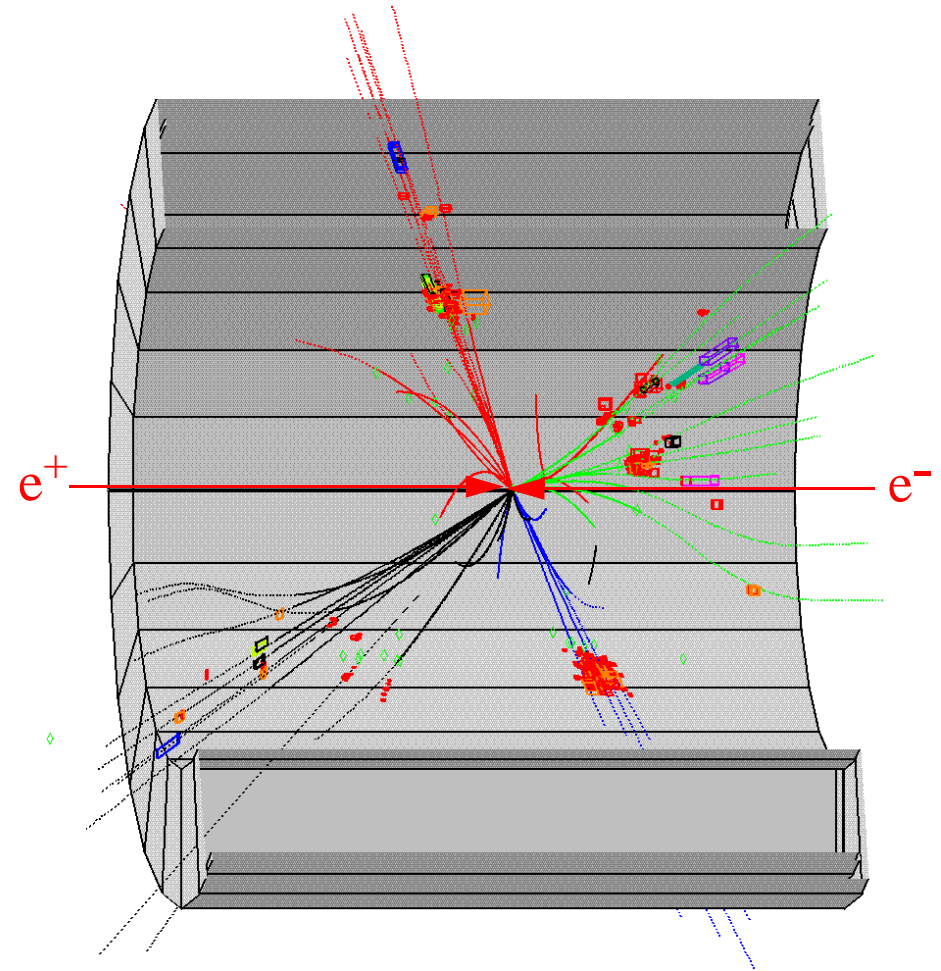
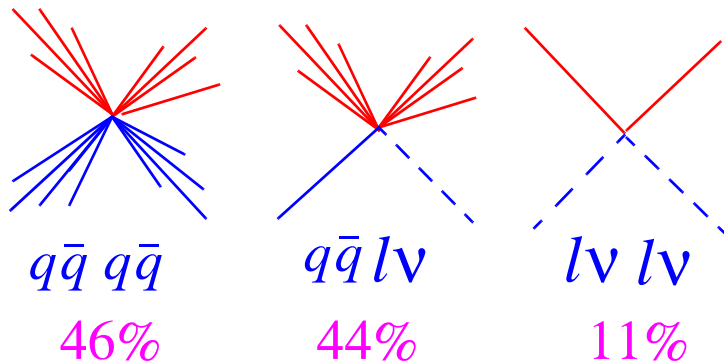
Precision studies of the W and Z bosons

➔ Studies of the W-boson

- W bosons were produced in pairs.



- The signature:



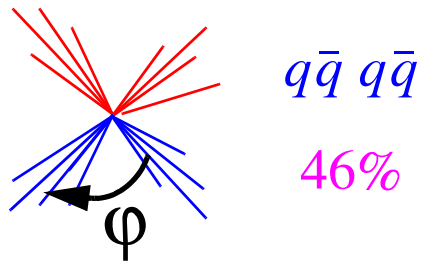
A WW-event with 4 jets

Precision studies of the W and Z bosons

→ Studies of the W-boson

- Step 1. Select WW-pair events

Step 2. Calculate the W-mass



$$M_W^2 = (\vec{P}_q + \vec{P}_{\bar{q}})^2 \quad (\text{4-vectors})$$

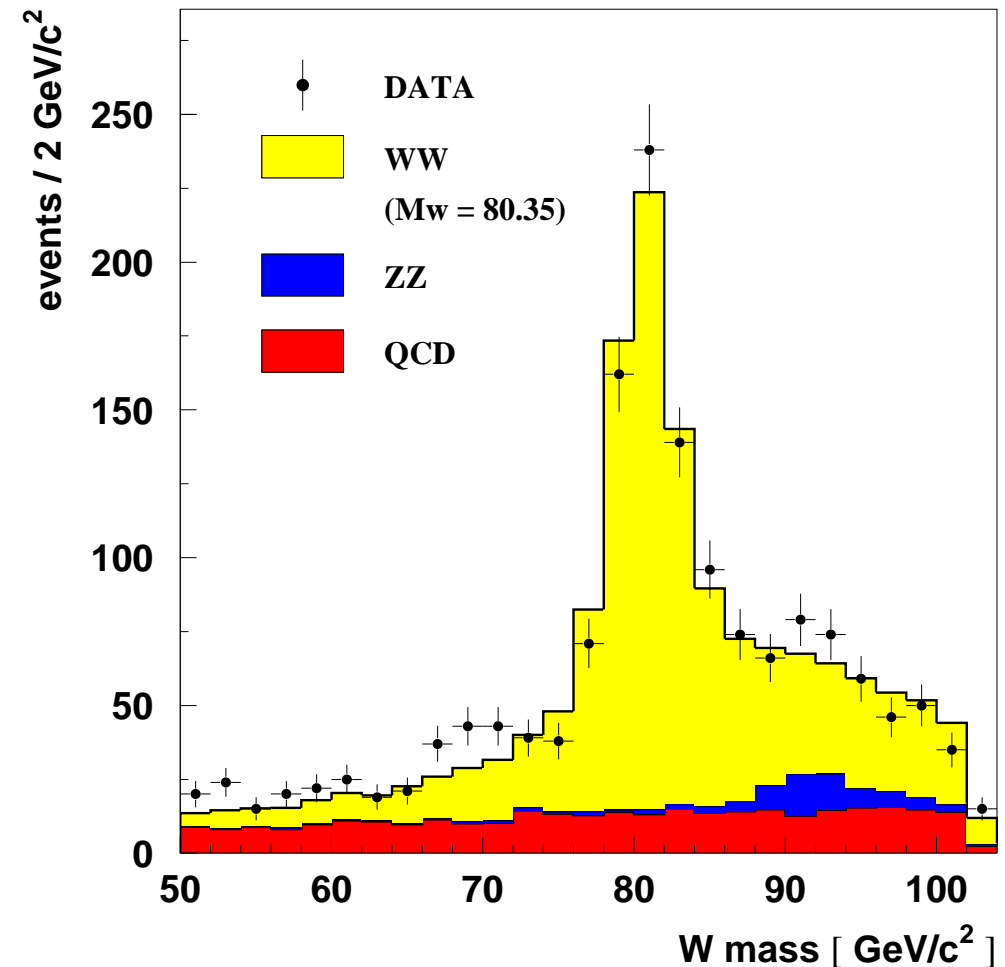
$$M_W^2 = 2 E_q E_{\bar{q}} (1 - \cos \varphi)$$

if $m_q = 0$

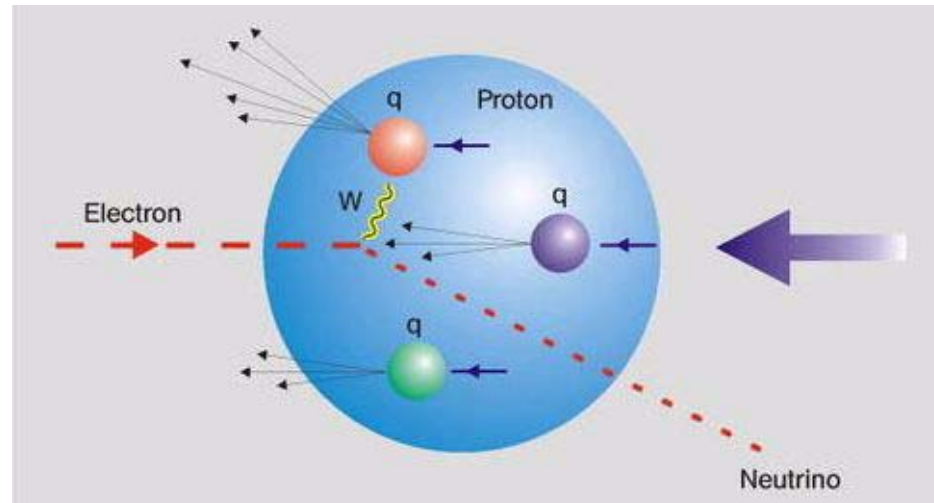
$$M_W = 80.376 \pm 0.033 \text{ GeV (LEP)}$$

$$M_W = 83.5 \pm 2.8 \text{ GeV (UA1)}$$

The mass distribution of jet-pairs



Charged current reactions



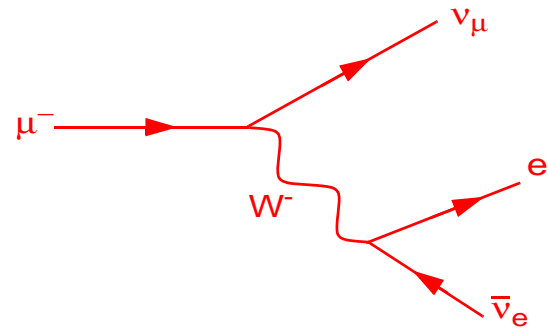
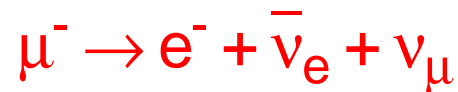
W and Z reactions

	Leptonic reactions	Hadronic reactions
Charge current reactions	<p>A Feynman diagram showing a neutrino ν_l and an antilepton l^+ meeting at a vertex. A wavy line representing a W^\pm boson is emitted from the vertex. The coupling constant is labeled g_w in red.</p>	<p>A Feynman diagram showing a quark q and an antiquark q' meeting at a vertex. A wavy line representing a W^\pm boson is emitted from the vertex. The coupling constant is labeled $V_{qq'} g_w$ in red.</p>
Neutral current reactions	<p>Two Feynman diagrams for leptonic neutral current reactions. The first shows a neutrino ν_l and an antineutrino $\bar{\nu}_l$ meeting at a vertex with a wavy line representing a Z^0 boson. The coupling constant is g_z. The second shows a lepton l^- and an antilepton l^+ meeting at a vertex with a wavy line representing a Z^0 boson. The coupling constant is g_z.</p>	<p>A Feynman diagram showing a quark q and an antiquark \bar{q} meeting at a vertex with a wavy line representing a Z^0 boson. The coupling constant is g_z.</p>

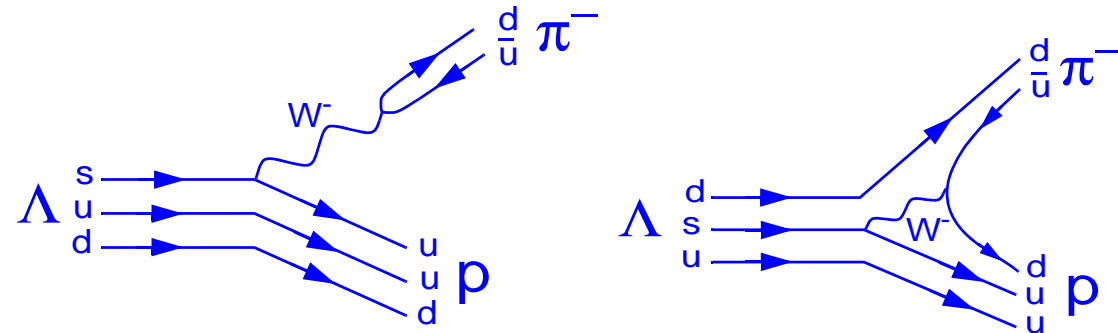
Charged current reactions

➔ Charged current reactions are mediated by a W .

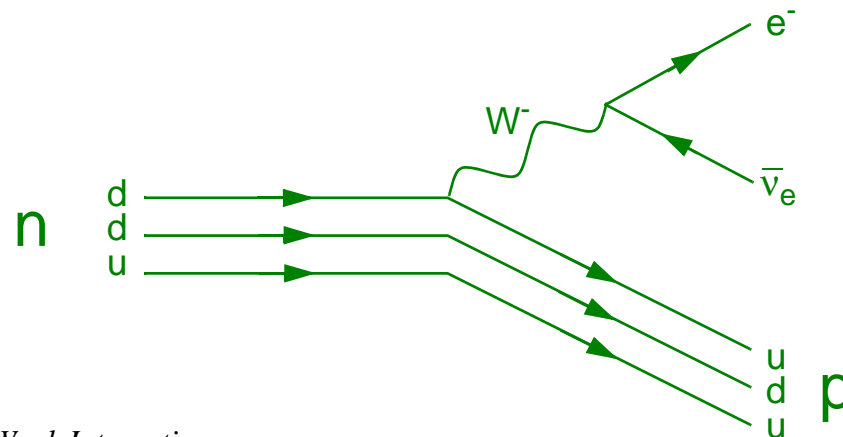
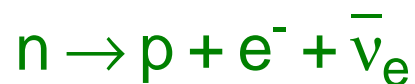
- Purely leptonic processes:



- Purely hadronic processes:

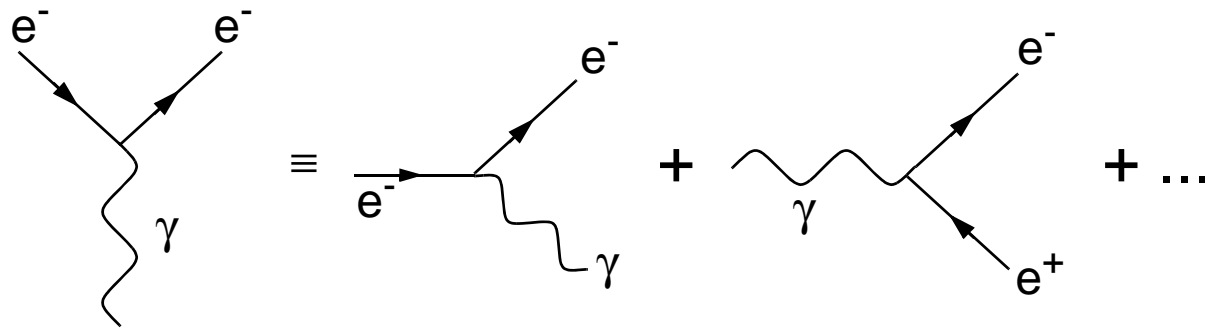


- Semileptonic reactions:



Leptonic charged current reactions

- All the **electromagnetic interactions** \Rightarrow eight basic interactions:



The basic vertex for electron-photon interactions.

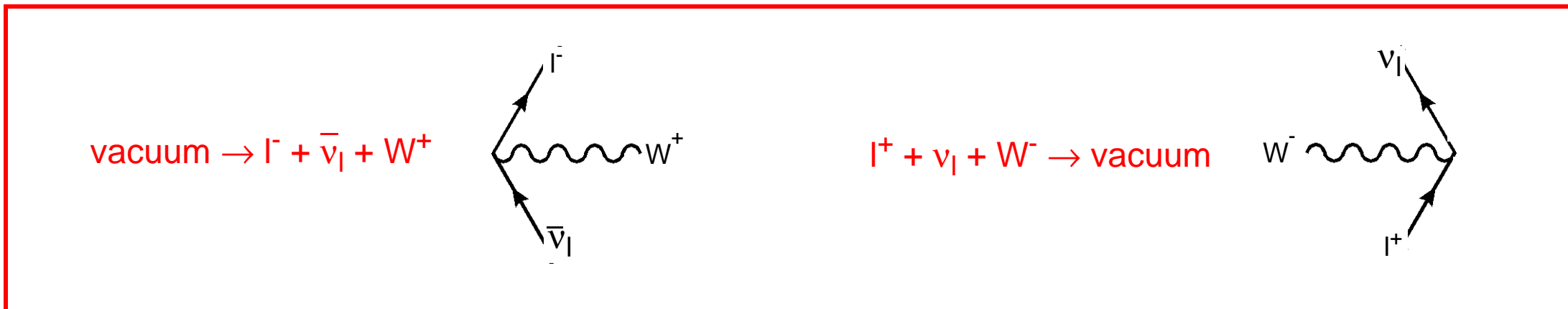
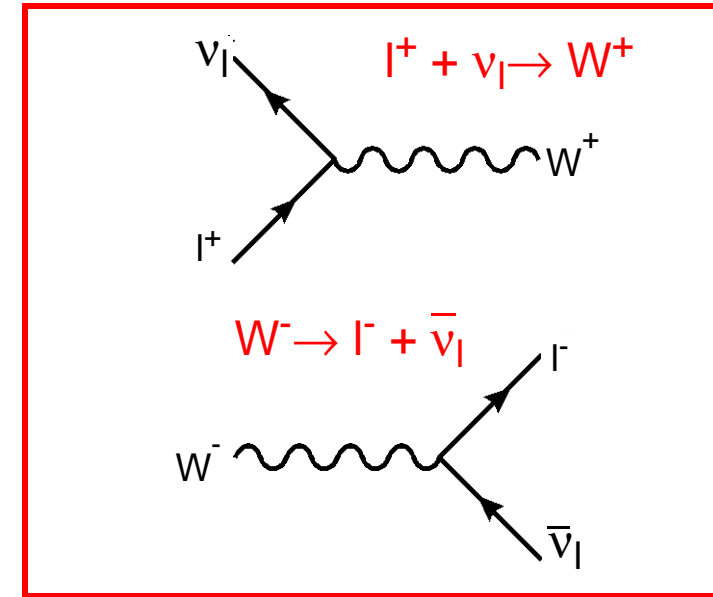
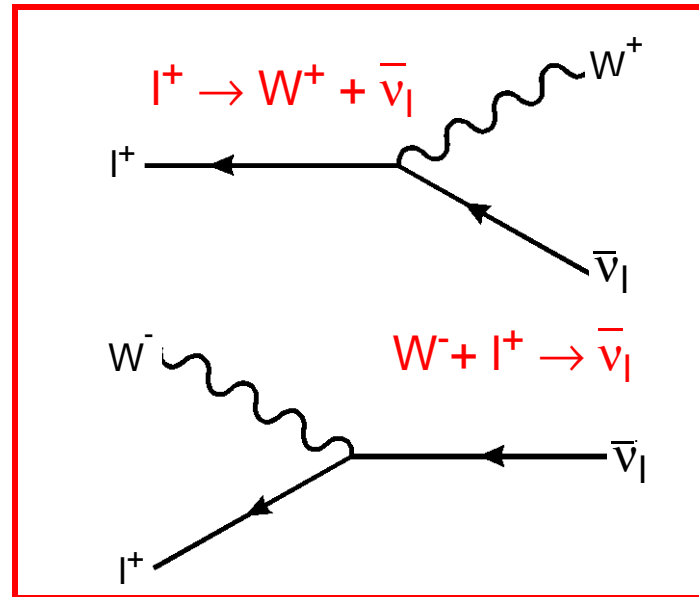
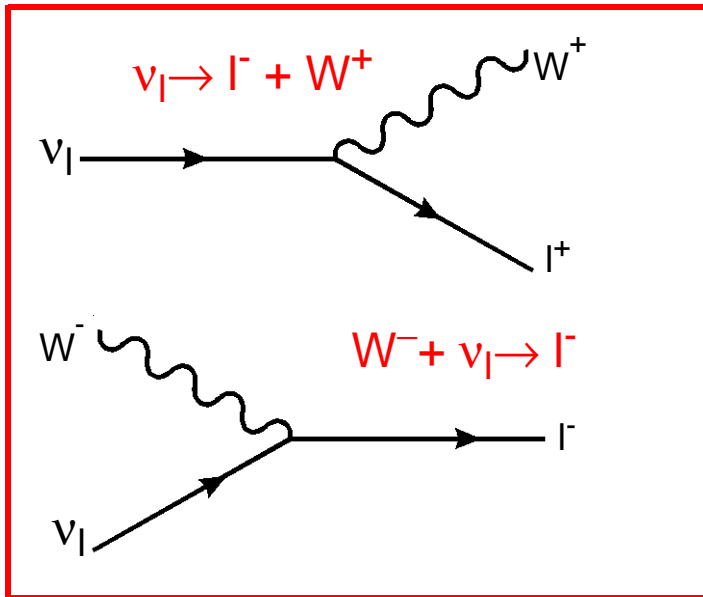
- **Leptonic weak interaction** \Rightarrow described by basic vertices:



The two basic vertices for W-lepton interactions..

Leptonic charged current reactions

- Eight basic charged current reactions from two basic W vertices:



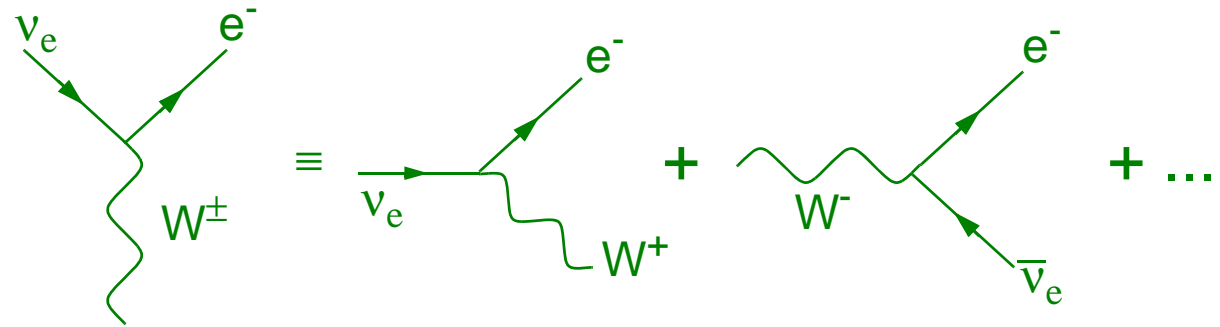
Leptonic charged current reactions

➔ Weak interactions conserve lepton numbers: L_e, L_μ, L_τ

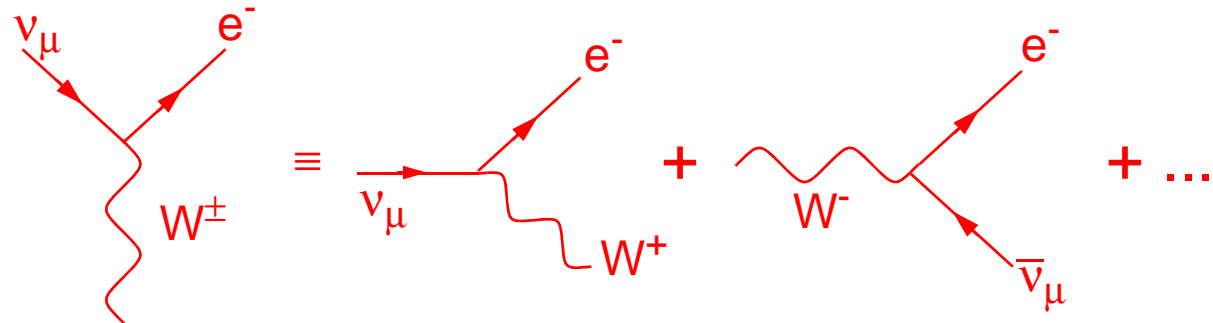
Feynman diagrams:

- 1) at each vertex, there is one arrow pointing in and one out
- 2) the lepton indices "l" are the same on both lines.

Allowed:



Forbidden:

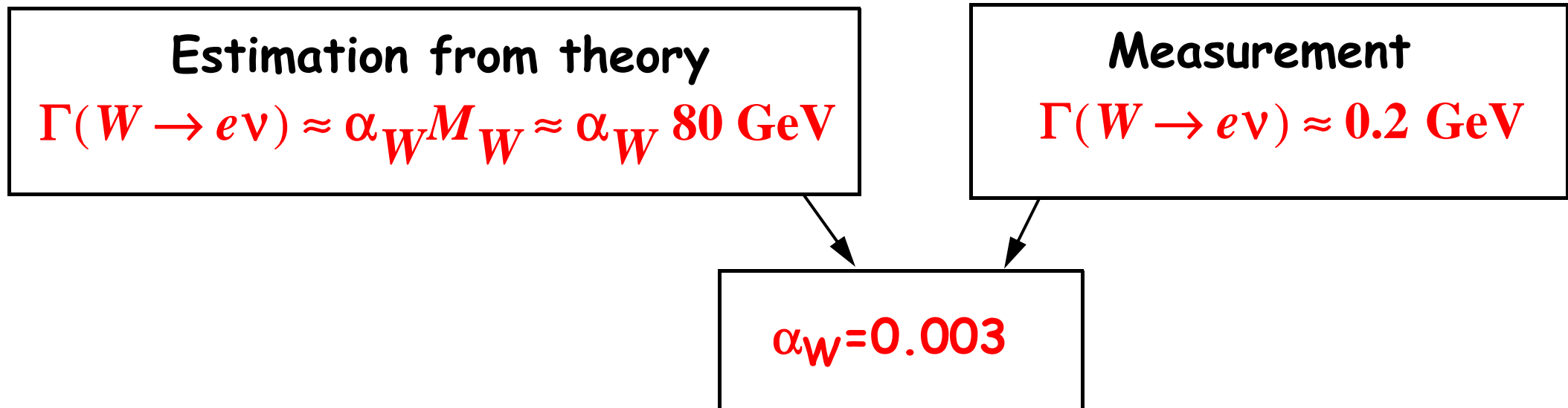


Leptonic charged current reactions

➔ The weak strength parameter: α_W

- α_W is the same at all leptonic vertices
(it does not depend on lepton type)

Example: The decay rate of W to $e+\nu$



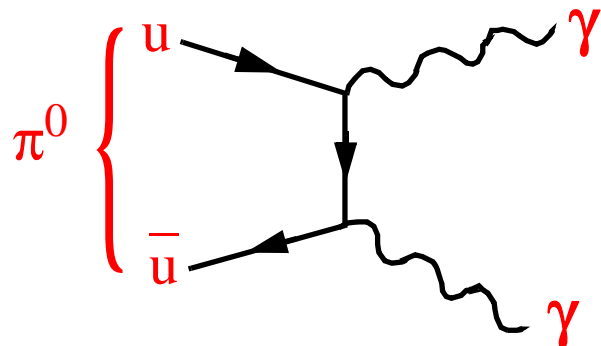
- Compare with the **electromagnetic** strength parameter: $\alpha_{em} = 0.007$

Leptonic charged current reactions

➔ Why is the weak interaction so weak if α_W and α_{em} is of a similar size ?

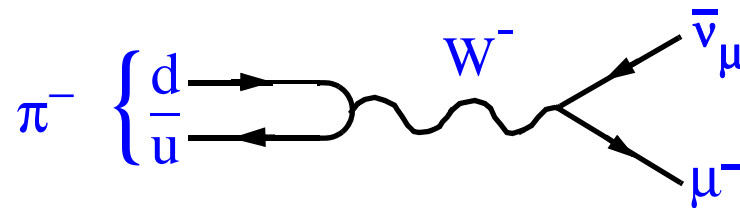
● Compare the decay of charged and neutral pions:

Electromagnetic decay



Lifetime = 8×10^{-17} s

Weak decay



Lifetime = $3000000000 \times 10^{-17}$ s

(Lifetime of a real W = $0.000000003 \times 10^{-17}$ s)

● **CONCLUSION:** Apparent weakness \Rightarrow large W and Z masses

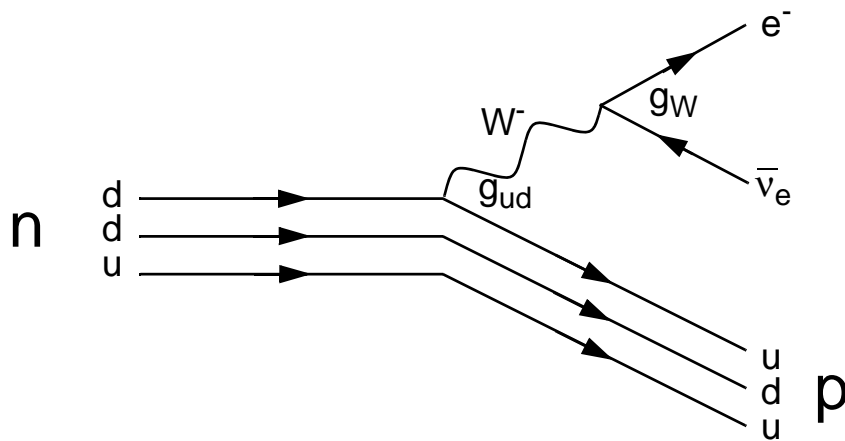
W and Z reactions

	Leptonic reactions	Hadronic reactions
Charge current reactions	<p>Diagram showing a neutrino ν_l and a lepton l^\pm interacting via a W^\pm boson. The vertex is labeled g_w.</p>	<p>Diagram showing a quark q and an antiquark q' interacting via a W^\pm boson. The vertex is labeled $V_{qq'} g_w$.</p>
Neutral current reactions	<p>Two diagrams showing neutrino and lepton interactions via a Z^0 boson. The first diagram shows ν_l and $\bar{\nu}_l$ interacting with vertex g_z. The second diagram shows l^- and l^+ interacting with vertex g_z.</p>	<p>Diagram showing a quark q and an antiquark \bar{q} interacting via a Z^0 boson. The vertex is labeled g_z.</p>

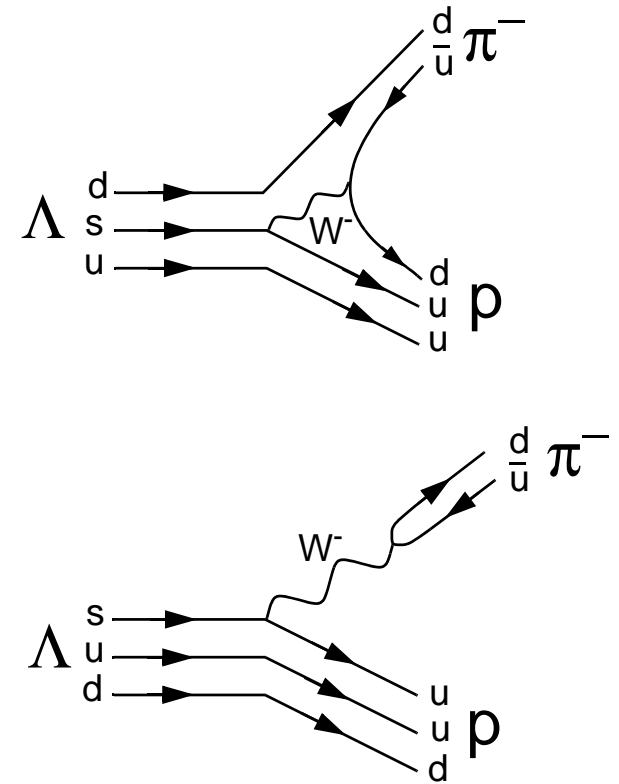
Hadronic charged current reactions

➔ In weak hadronic interactions, constituent quarks emit or absorb W or Z bosons.

Examples:



Neutron β -decay.



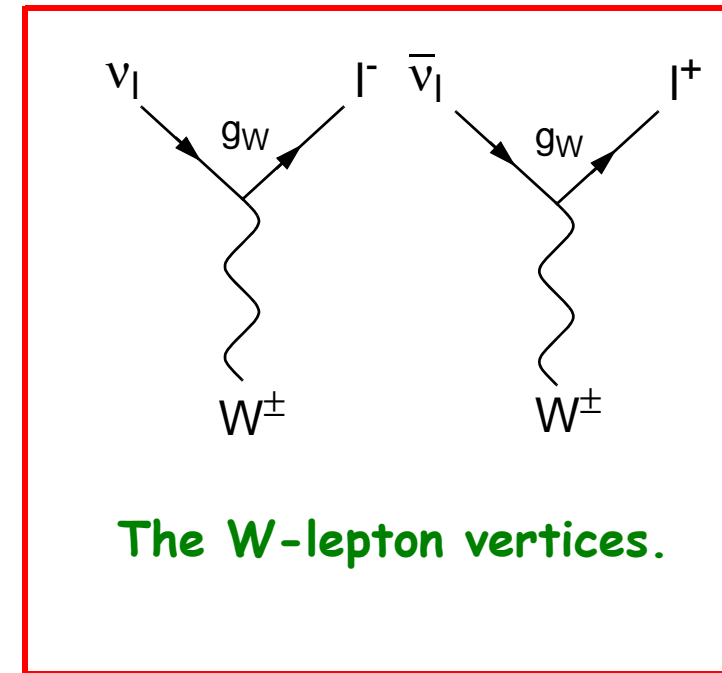
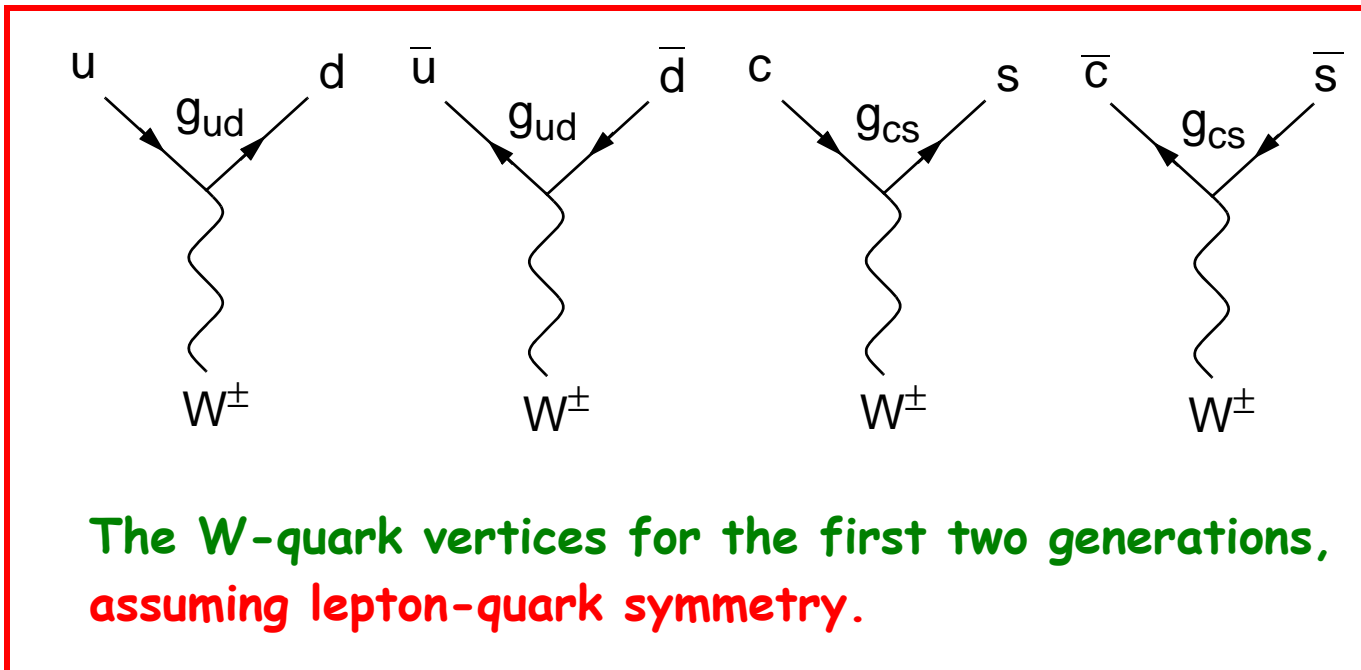
The dominant quark diagrams for Λ decay

Hadronic charged current reactions

➔ **ASSUMPTION:** Lepton-quark symmetry i.e. corresponding generations of quarks and leptons have identical weak interactions.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \leftrightarrow \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \leftrightarrow \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \leftrightarrow \begin{pmatrix} t \\ b \end{pmatrix}$$

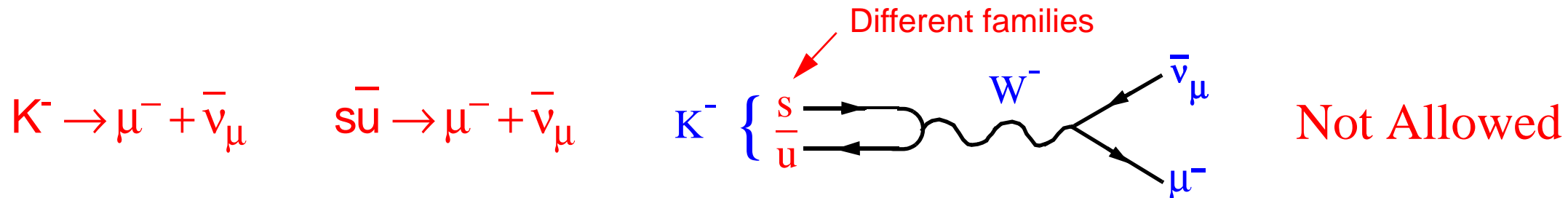
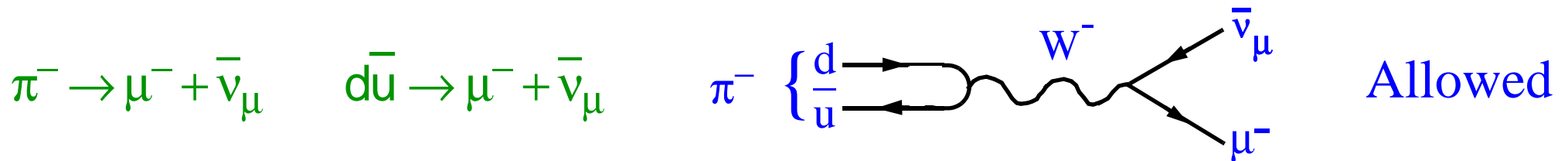
● Interactions will then only take place within a family !



Hadronic charged current reactions

➔ Experimental tests of the assumption of lepton-quark symmetry.

- Some weak reactions should be **allowed** and some should be **forbidden** if lepton-quark symmetry is true:



- Measurements of these decays give:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad \text{Branching ratio} = 0.9999 \quad \tau = 2.6 \times 10^{-8} \text{ s}$$

$$K^- \rightarrow \mu^- + \bar{\nu}_\mu \quad \text{Branching ratio} = 0.6343 \quad \tau = 1.2 \times 10^{-8} \text{ s}$$

- **CONCLUSION:** Quarks from **different generations** can **participate** in charged current interactions !

Hadronic charged current reactions

➔ Cabibbo \Rightarrow quark mixing in order to explain kaon decays.

- **Quark mixing scheme** \Rightarrow d- and s-quarks participate in weak interactions via the linear combinations:

$$d' = d \cos \theta_C + s \sin \theta_C \quad \text{where } \theta_C \text{ is called the Cabibbo angle.}$$

$$s' = -d \sin \theta_C + s \cos \theta_C$$

- The quark-lepton symmetry applies to doublets like

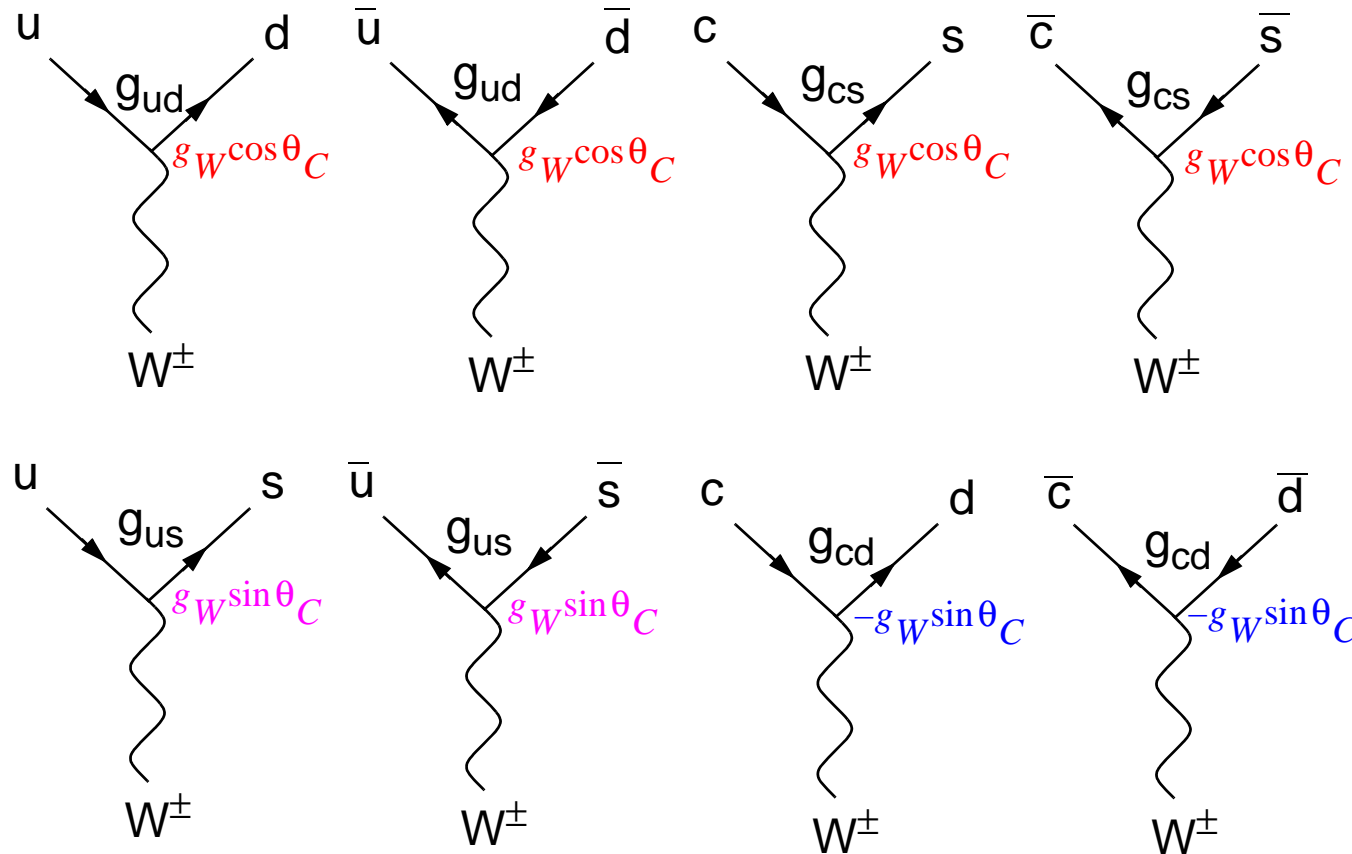
$$\begin{pmatrix} u \\ d' \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s' \end{pmatrix}$$

- The **ud'W vertex** \Rightarrow interpreted as a sum of the udW and usW vertices:

The diagram shows the decomposition of the $ud'W$ vertex into a sum of udW and usW vertices. The first diagram shows a vertex where a u quark and a d' quark meet at a W^\pm boson vertex, with a coupling constant g_W . This is shown to be equivalent to the sum of two diagrams: one where a u quark and a d quark meet at a W^\pm vertex with coupling g_{ud} , and another where a u quark and an s quark meet at a W^\pm vertex with coupling g_{us} . Below the diagrams, the couplings are defined as $g_{ud} = g_W \cos \theta_C$ and $g_{us} = g_W \sin \theta_C$.

Hadronic charged current reactions

➔ The quark mixing hypothesis \Rightarrow more W-quark vertices:



Within a generation: $g_{ud} = g_{cs} = g_W \cos \theta_C$

Between generations: $g_{us} = -g_{cd} = g_W \sin \theta_C$

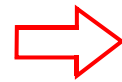
Hadronic charged current reactions

➔ Measurements of the Cabibbo angle.

● The **Cabibbo angle** has to be measured.

● Comparing the decay rates of $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ with $K^- \rightarrow \mu^- + \bar{\nu}_\mu$

$$\frac{\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2 \theta_C$$



$$\theta_C = 12.7^\circ \pm 0.1^\circ$$

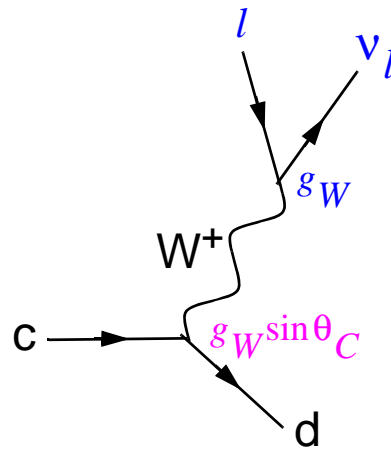
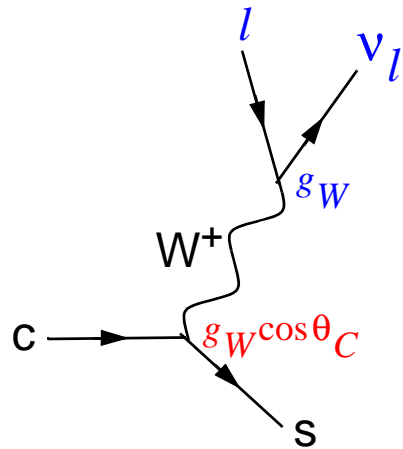
● The **coupling constants** within and between generations:

$$\begin{aligned} g_W \cos \theta_C &= 0,98 g_W \\ g_W \sin \theta_C &= 0,22 g_W \end{aligned}$$

Hadronic charged current reactions

➔ Charmed particle decays.

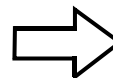
- Particles with **charm quarks** almost always give a **strange particle** in the final state because other decays are Cabibbo suppressed:



- The **suppression factor** is: $\frac{g_{cd}^2}{g_{cs}^2} = \tan^2 \theta_C = \frac{1}{20}$

$$\theta_C = 12,6^\circ$$

- Neutrino scattering experiments ➔
The **charmed quark couplings** g_{cd} and g_{cs}



$$\theta_C = 12^\circ \pm 1^\circ$$

Weak interactions and the third generation

➔ Two generation quark mixing can be written in matrix form:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \quad \text{with transitions within } \begin{pmatrix} u \\ d' \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s' \end{pmatrix}$$

● This means that the following weak transitions are **favoured**:

$$\begin{pmatrix} u \\ \updownarrow \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ \updownarrow \\ s \end{pmatrix}$$

● And that the following weak transitions are **suppressed**:

$$\begin{pmatrix} u \\ \swarrow \quad \searrow \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ \swarrow \quad \searrow \\ s \end{pmatrix}$$

● Charge conservation **forbids** the following charged current transitions:

$$\begin{pmatrix} u \\ \leftarrow \quad \rightarrow \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ \leftarrow \quad \rightarrow \\ s \end{pmatrix} \quad \begin{array}{l} \text{Charge} = 2/3 \\ \text{Charge} = -1/3 \end{array}$$

Weak interactions and the third generation

- The **c-quark** was **predicted** from lepton-quark symmetry \Rightarrow
Discovered in experiments in 1974.
- Discovery of the τ lepton and the b-quark \Rightarrow
The sixth quark was **predicted** to complete the symmetry \Rightarrow
Top quark was **discovered** in 1994
- The third generation gives rise to the **Cabibbo-Kobayashi-Maskawa (CKM) matrix** $V_{\alpha\beta}$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- $g_{\alpha\beta} = g_W V_{\alpha\beta}$
 $\alpha = u, c, t$ $\beta = d, s, b$

Weak interactions and the third generation

- Weak transitions can now take place between:

$$u \leftrightarrow d' = V_{ud}d + V_{us}s + V_{ub}b$$

$$c \leftrightarrow s' = V_{cd}d + V_{cs}s + V_{cb}b$$

$$t \leftrightarrow b' = V_{td}d + V_{ts}s + V_{tb}b$$

- If the mixing between the **b** and **t** quarks with lighter quarks can be **neglected** the CKM-matrix is reduced to:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- If V_{ub} , V_{cb} , V_{td} and V_{ts} are not small \Rightarrow
The two-generation mixing model would not work

Weak interactions and the third generation

→ b - quarks

$$V_{ub} = V_{cb} = V_{td} = V_{ts} = 0$$

t-quark decay only to b-quarks

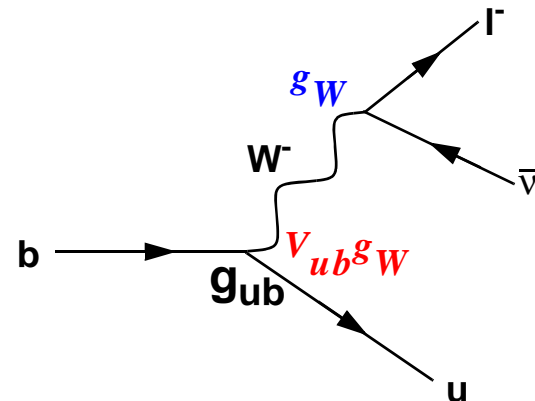
b-quark is stable
(since it cannot decay to u- or c-quarks)

↑ Not true

● **Semileptonic decays** of b-quarks to u- and c-quarks observed !

● The observed decay rate is proportional to the squared couplings:

$$|g_{ub}|^2 = |V_{ub}|^2 g_W^2$$



Weak interactions and the third generation

➔ b - quarks

- The most precise measurements at present

$$|V_{ub}| \approx 0,004 \quad \text{and} \quad |V_{cb}| \approx 0,04$$

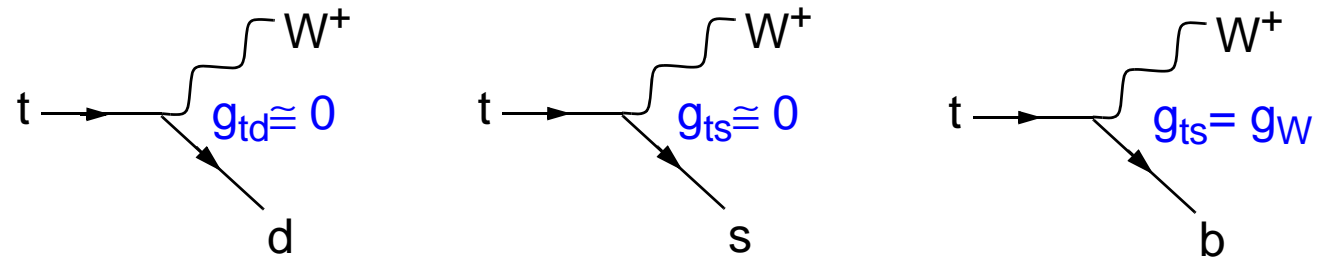
- The CKM-matrix becomes

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0,004 \\ -\sin\theta_C & \cos\theta_C & 0,04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix} \approx \begin{pmatrix} 0,98 & 0,22 & 0,004 \\ -0,22 & 0,98 & 0,04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix}$$

Weak interactions and the third generation

➔ The top quark

- The **top quark** is much heavier than even the W -bosons and it can **decay** by



- g_{td} and g_{ts} are close to zero \Rightarrow the only **significant decay mode**:

$$t \rightarrow W + b$$

rate proportional to $\alpha_W = g_W^2/4\pi \approx 0,0042$

- Estimation of the decay width of the top ($\Gamma \sim \alpha_W m_t \sim 1 \text{ GeV}$)

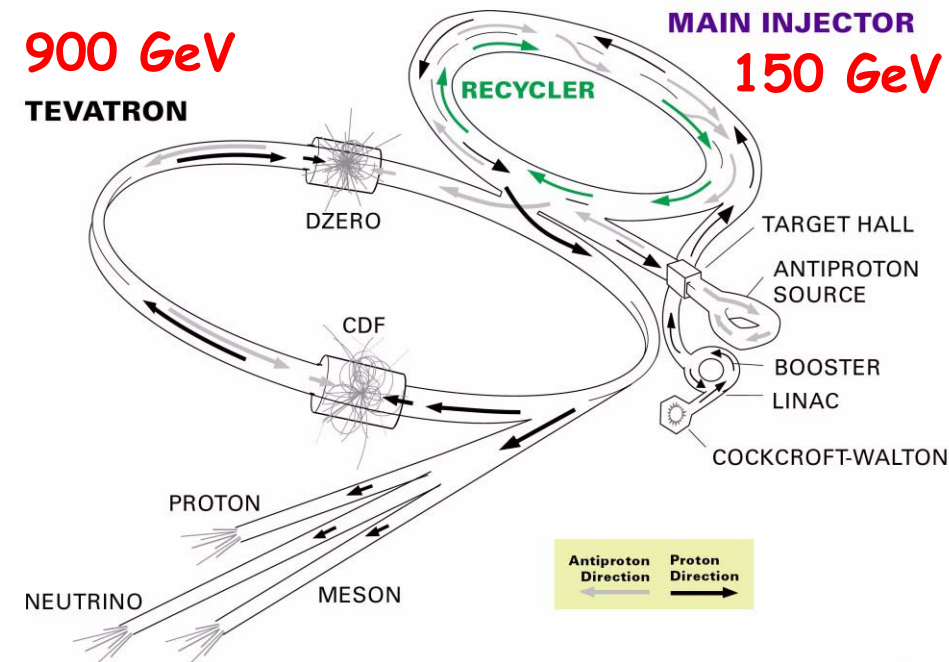
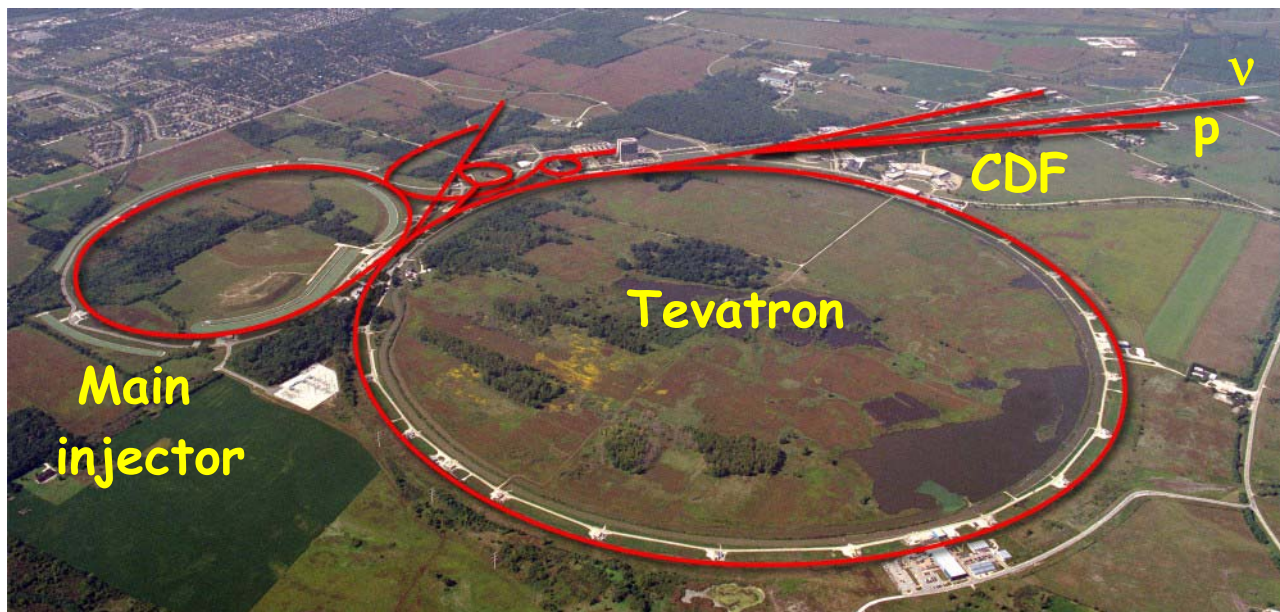
\Rightarrow **very short lifetime** $\Rightarrow \tau_t \approx 4 \times 10^{-25} \text{ s}$

The discovery of the top quark

➔ The accelerator: The Tevatron

	<u>Type</u>	<u>Bending field</u>	<u>Length</u>	<u>Collision energy</u>
Tevatron:	$p\bar{p}$ -collider	4.5 T	6.3 km	1960 GeV
SPS:	$p\bar{p}$ -collider	1.8 T	6.9 km	900 GeV
LHC:	pp-collider	8.4 T	27 km	14000 GeV

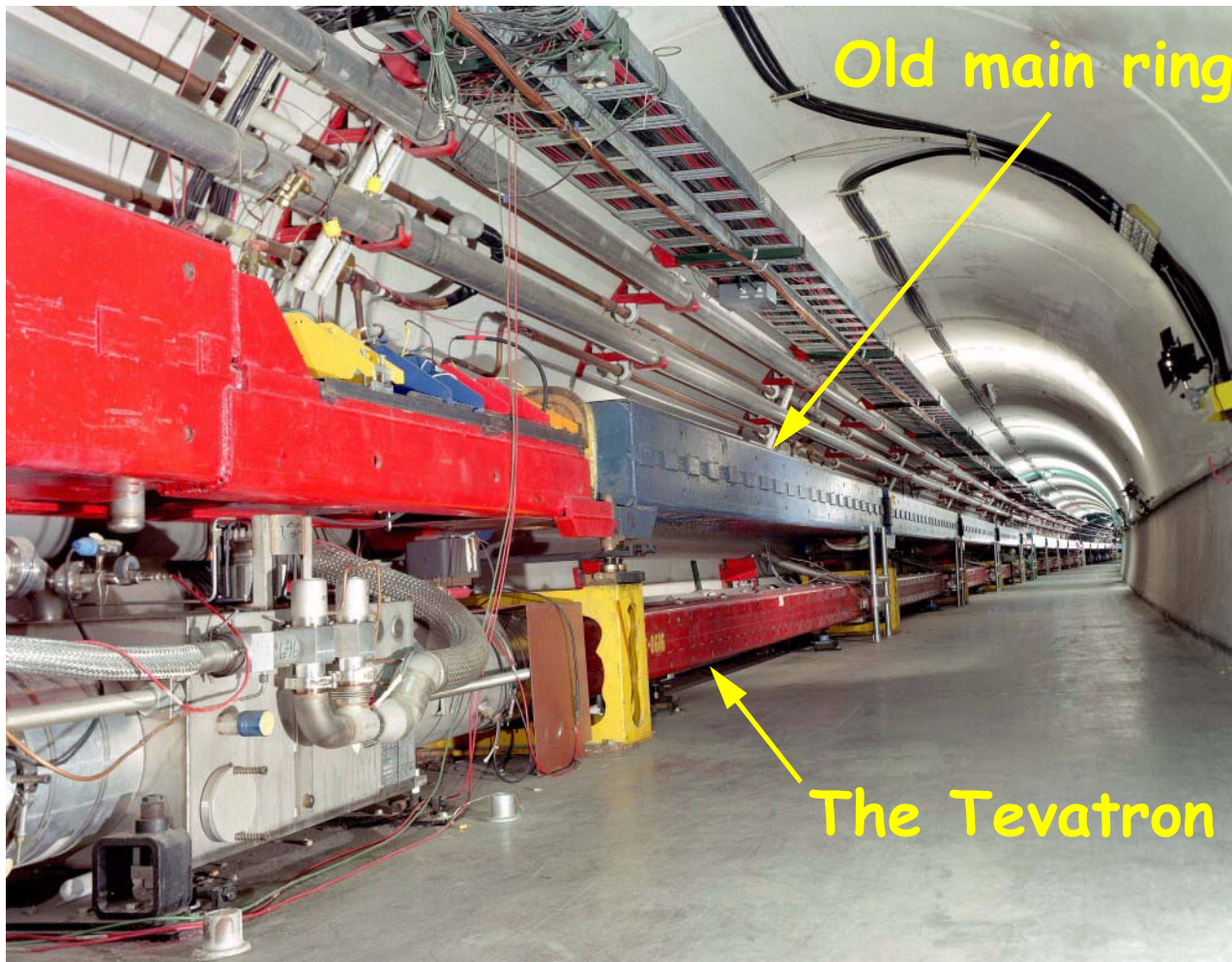
Superconducting magnets



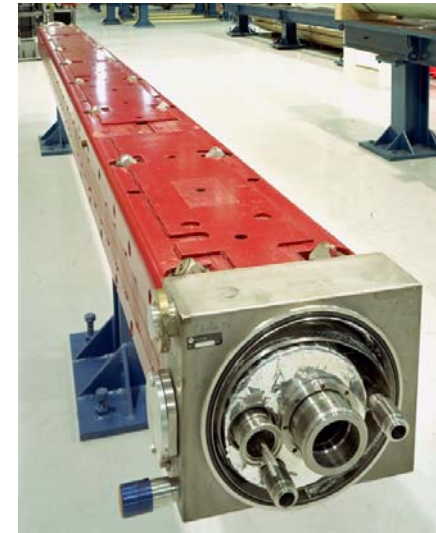
The discovery of the top quark

➔ The accelerator: The Tevatron

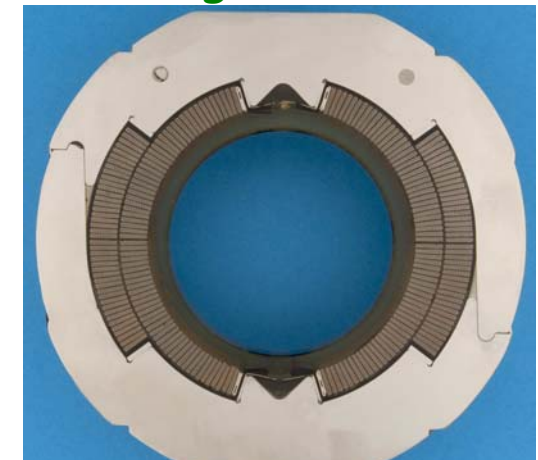
- The Tevatron accelerator was put under the **old main ring** which was used as a **pre-accelerator**.



Dipole magnet

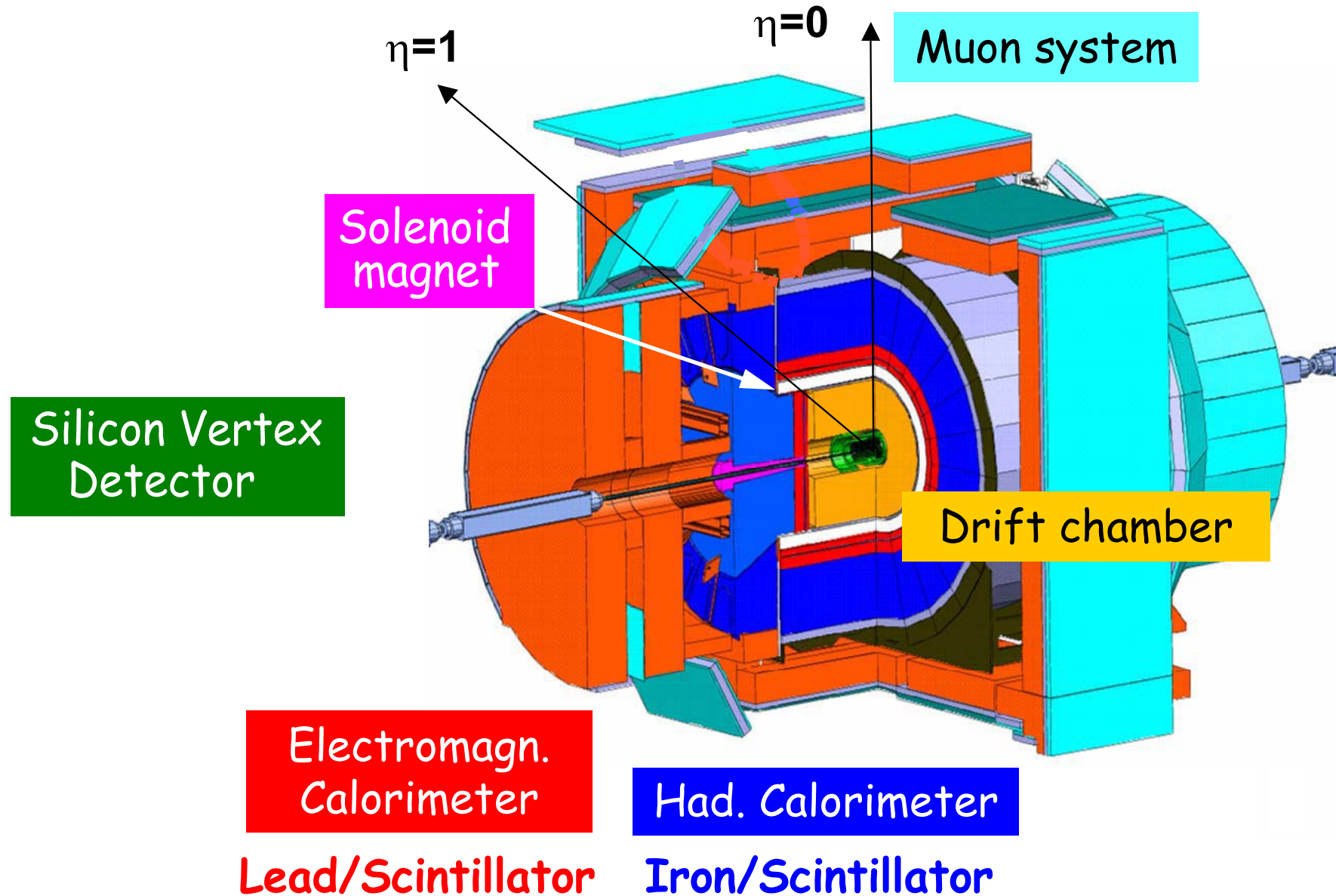


Magnet coil



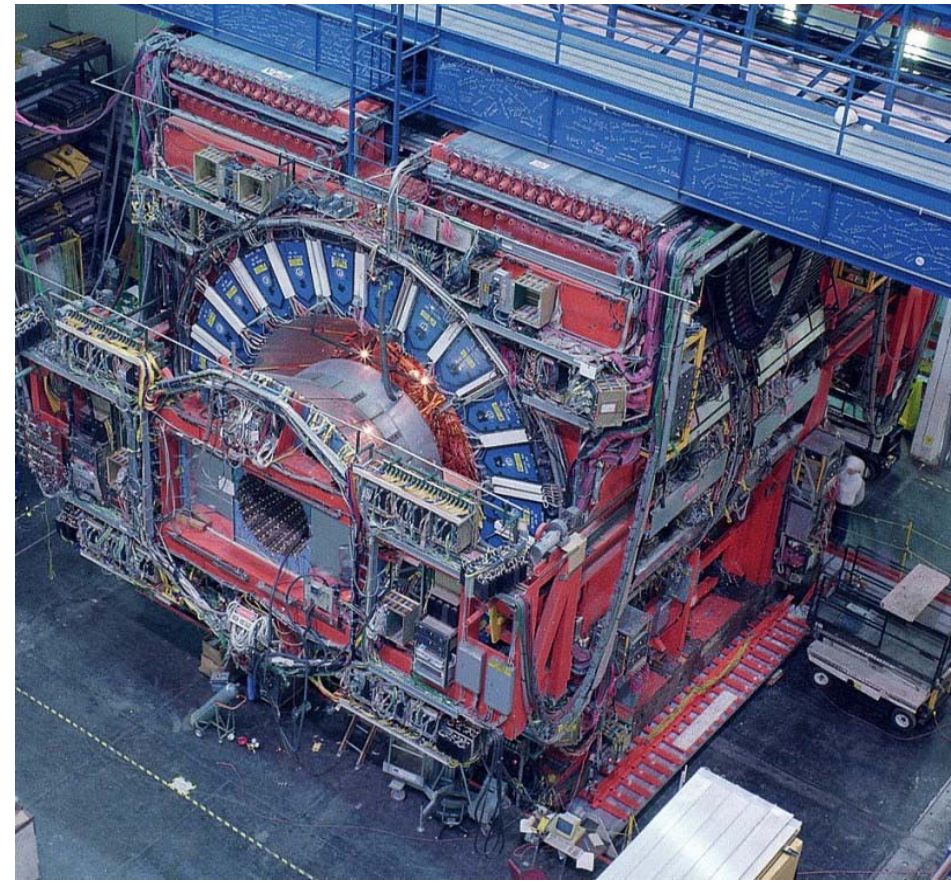
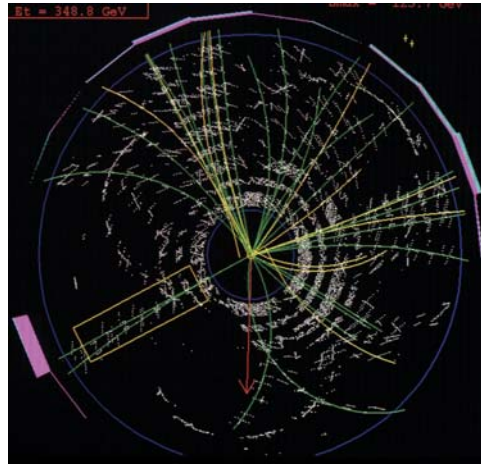
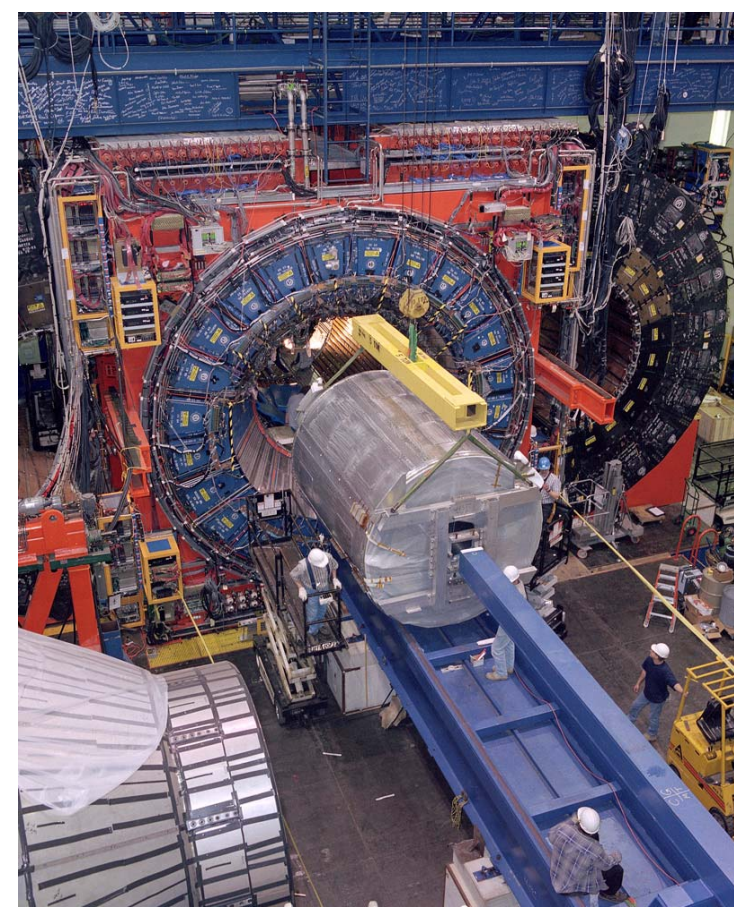
The discovery of the top quark

➔ The experiment: The Collider Detector at Fermilab



The discovery of the top quark

➔ The experiment: CDF



The discovery of the top quark

➔ Compare particles decay length

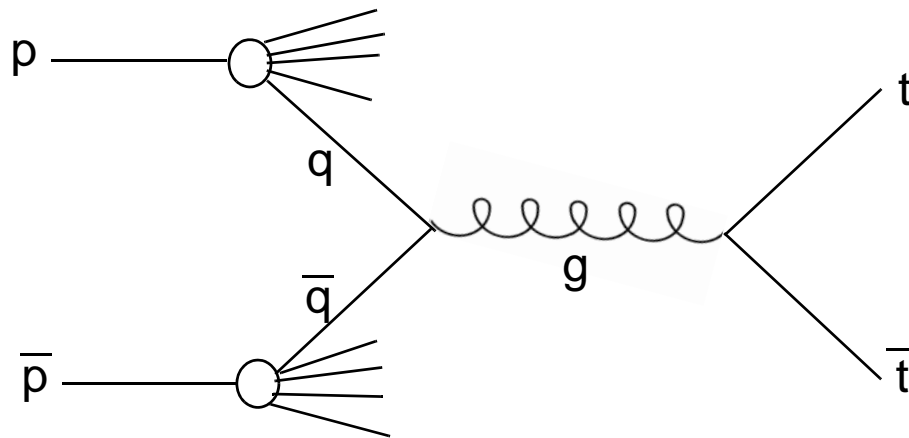
Particle	Lifetime	Decay Length	Decay Place	Decay Measurement
W, Z, top	$3-4 \times 10^{-13}$ ps	0	Beampipe	Not possible
π^0 ($\rightarrow \gamma\gamma$)	0.0008 ps	0.025 μm	Beampipe	Not possible
τ	0.3 ps	90 μm	Beampipe	Microvertex
Charm: $D^0/D^+/D_s$	0.4-1 ps	150-350 μm	Beampipe	Microvertex
Bottom: $B^0/B^+/B_s/$	1.5 ps	450 μm	Beampipe	Microvertex
K_S ($\rightarrow \pi\pi$)	80 ps	2.7 cm	Tracker	Tracker
K^\pm	10,000 ps	3.7 m	Tracker	Not possible
π^\pm	30,000 ps	7.8 m	No decay	Not possible
K_L ($\rightarrow \pi\pi\pi$)	50,000 ps	16 m	No decay	Not possible
μ ($\rightarrow e\bar{\nu}_e\nu_\mu$)	2,000,000 ps	659 m	No decay	Not possible

The discovery of the top quark

➔ Production of top-quarks

- In proton-antiproton colliders, pairs of top quarks are mostly produced by **quark-antiquark annihilation**:

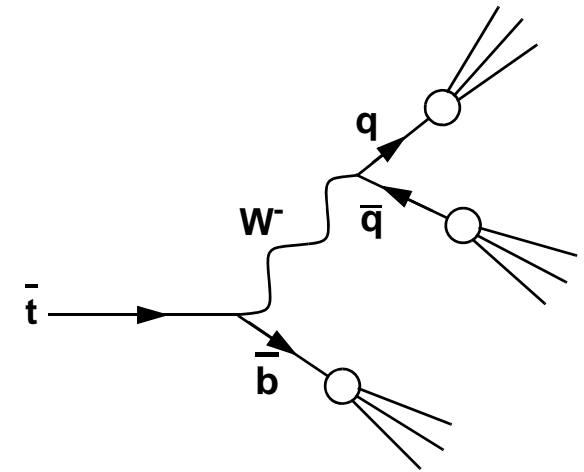
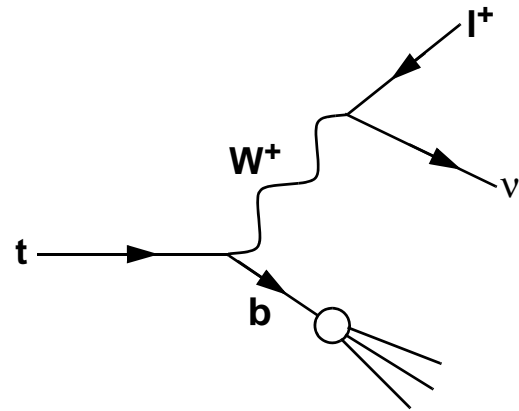
$$q + \bar{q} \rightarrow g \rightarrow t + \bar{t}$$



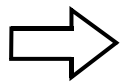
The discovery of the top quark

➔ The decay of top-quarks

- The most likely **decay** of a top quark is to a **b quark and to a W**.



- The **W** can decay to **leptons or hadrons**



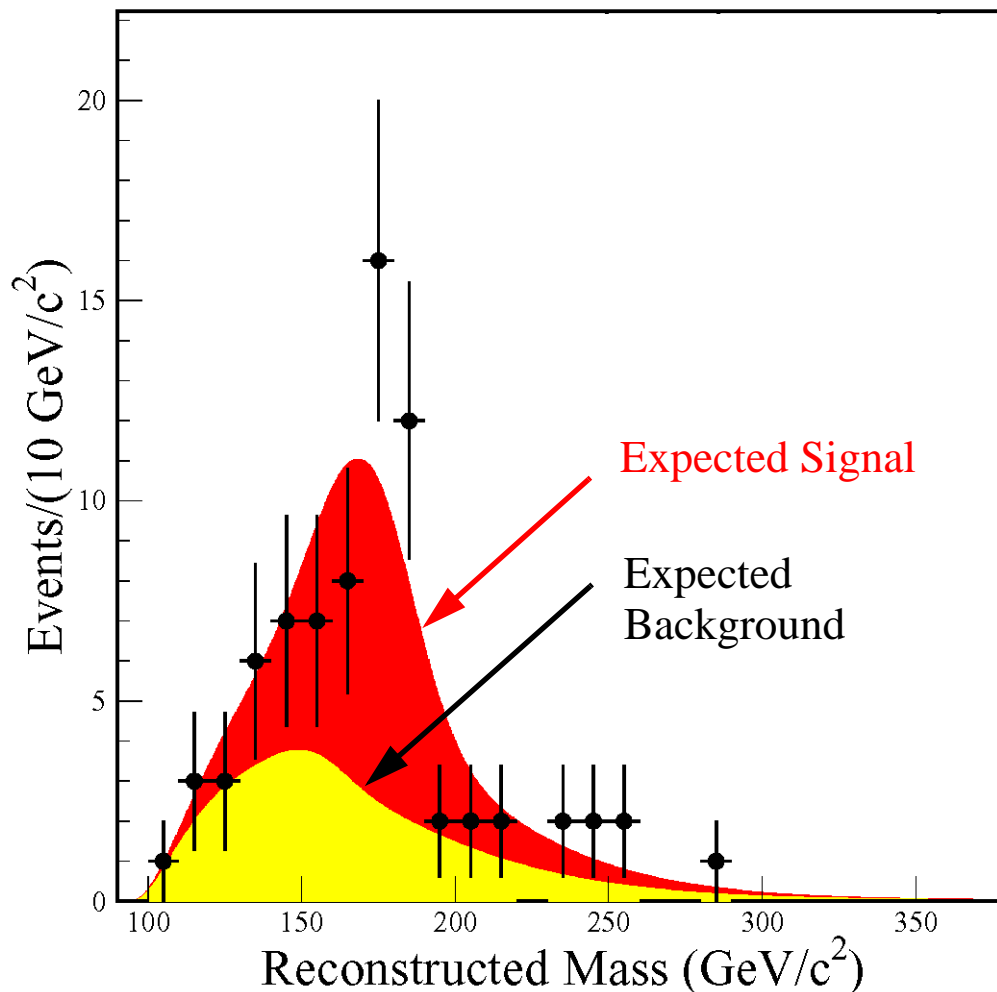
The final state is a complex mix of jets and leptons.

$$p + \bar{p} \longrightarrow t + \bar{t} + X$$

$\begin{cases} l^- + \bar{\nu}_1 \\ q + \bar{q} \end{cases}$
 $\begin{cases} W^- + \bar{b} \\ W^+ + b \end{cases}$
 $\begin{cases} l^+ + \nu_1 \\ q + \bar{q} \end{cases}$

The discovery of the top quark

- After a selection of likely top event \Rightarrow
plot mass distribution of the top-candidates.



- A large background component

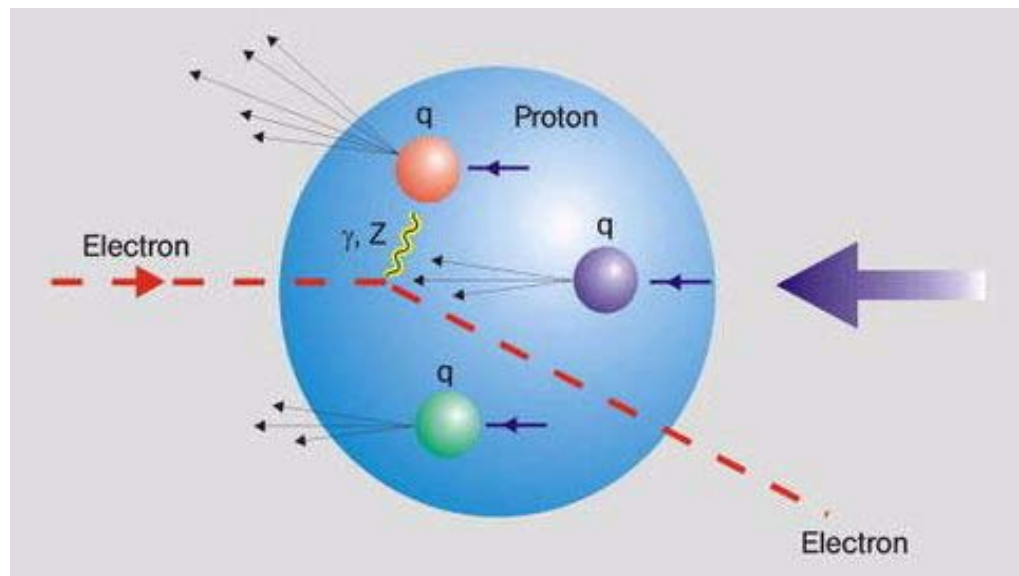
- Extract the **top mass**:

$$M_t = 176 \pm 5 \text{ GeV}$$

- Latest results:

$$M_t = 173 \pm 1 \text{ GeV}$$

Neutral current reactions

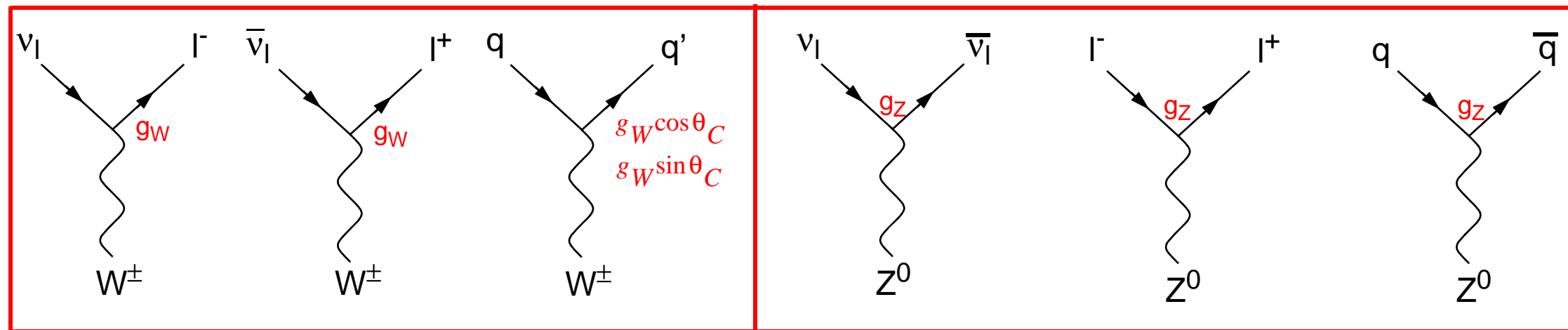


W and Z reactions

	Leptonic reactions	Hadronic reactions
Charge current reactions	<p>A Feynman diagram showing a neutrino ν_l and an antilepton l^+ meeting at a vertex. A wavy line representing a W^\pm boson is emitted from the vertex. The coupling constant is labeled g_w in red.</p>	<p>A Feynman diagram showing a quark q and an antiquark q' meeting at a vertex. A wavy line representing a W^\pm boson is emitted from the vertex. The coupling constant is labeled $V_{qq'} g_w$ in red.</p>
Neutral current reactions	<p>Two Feynman diagrams for leptonic neutral current reactions. The first shows a neutrino ν_l and an antineutrino $\bar{\nu}_l$ meeting at a vertex with a wavy line representing a Z^0 boson. The coupling constant is g_z. The second shows a lepton l^- and an antilepton l^+ meeting at a vertex with a wavy line representing a Z^0 boson. The coupling constant is g_z.</p>	<p>A Feynman diagram showing a quark q and an antiquark \bar{q} meeting at a vertex with a wavy line representing a Z^0 boson. The coupling constant is g_z.</p>

Neutral current reactions

- The basic vertices with W bosons:
 - Conserved lepton numbers
 - Not conserved quark flavour (quark mixing)
- The basic vertices with Z bosons:
 - Conserved lepton numbers
 - Conserved quark flavour (no quark mixing)

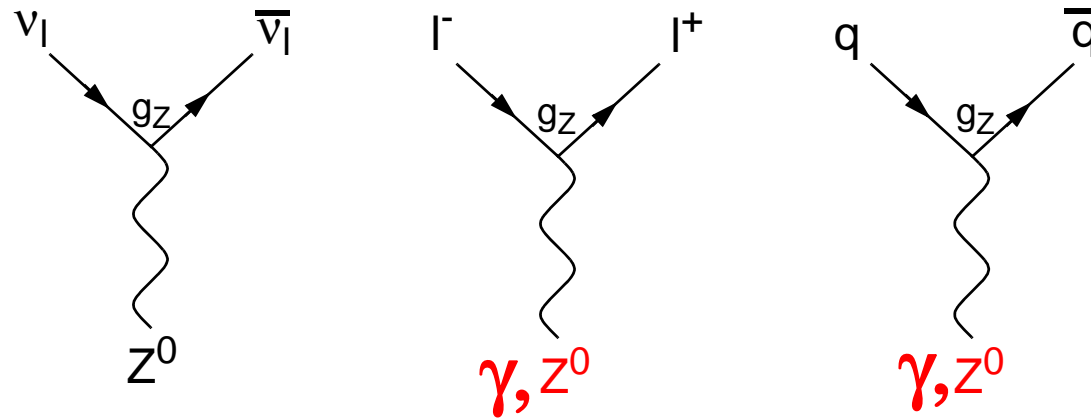


The basic W vertices

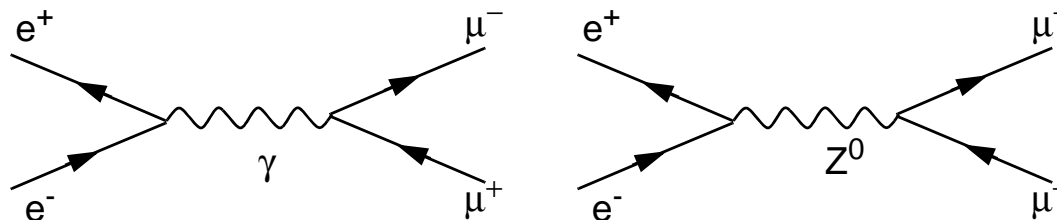
The basic Z vertices

Neutral current reactions

- In processes in which a **photon** can be exchanged, a **Z^0** boson can be exchanged **as well**:



- Example: $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions



Neutral current reactions

- Estimation of the cross-section for the photon- and Z-exchange process in e^+e^- collisions at low energy:

$$\sigma_\gamma \approx \frac{\alpha^2}{E_{CM}^2} \quad \sigma_Z \approx G_Z^2 E_{CM}^2$$

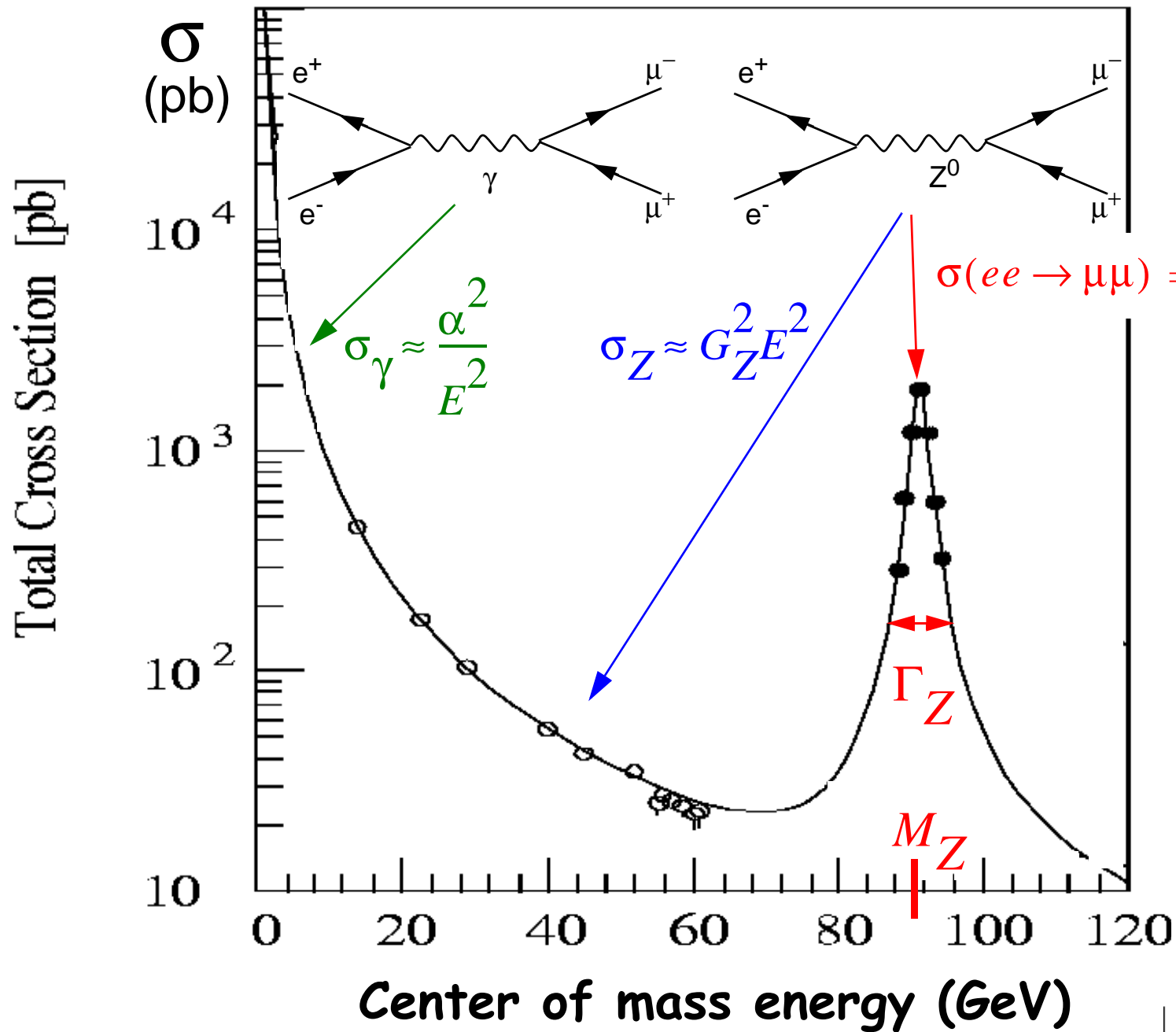
where E_{cm} is the collision energy.

- The **photon exchange** process will dominate **at low energies**.
- At $E_{cm}=M_Z$ this low-energy approximation fails and the **Z^0 peak** is described by the **Breit-Wigner formula**:

$$\sigma(E_{CM}) = \frac{M^2}{E_{CM}^2} \left[\frac{C}{(E_{CM}^2 - M^2)^2 + M^2 \Gamma^2} \right]$$

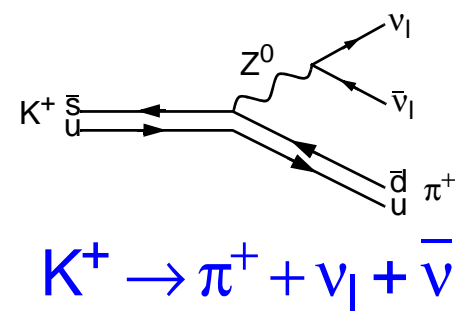
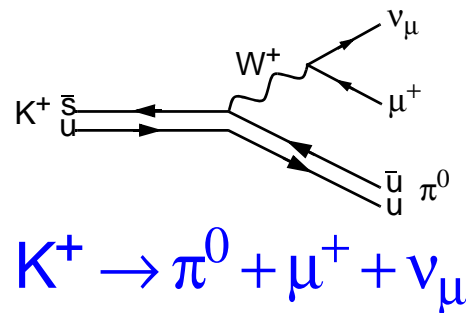
where M is the mass of the resonance and Γ its decay width.

Neutral current reactions



Test of flavour conservation

- That **flavour is conserved** at a Z^0 vertex can be verified by experiments.
- Example: measurement of the **decay rate of charged kaons**



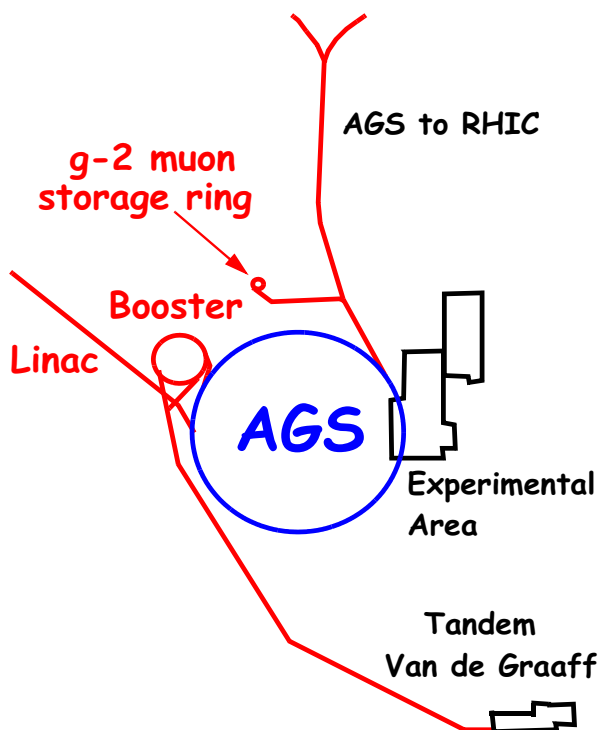
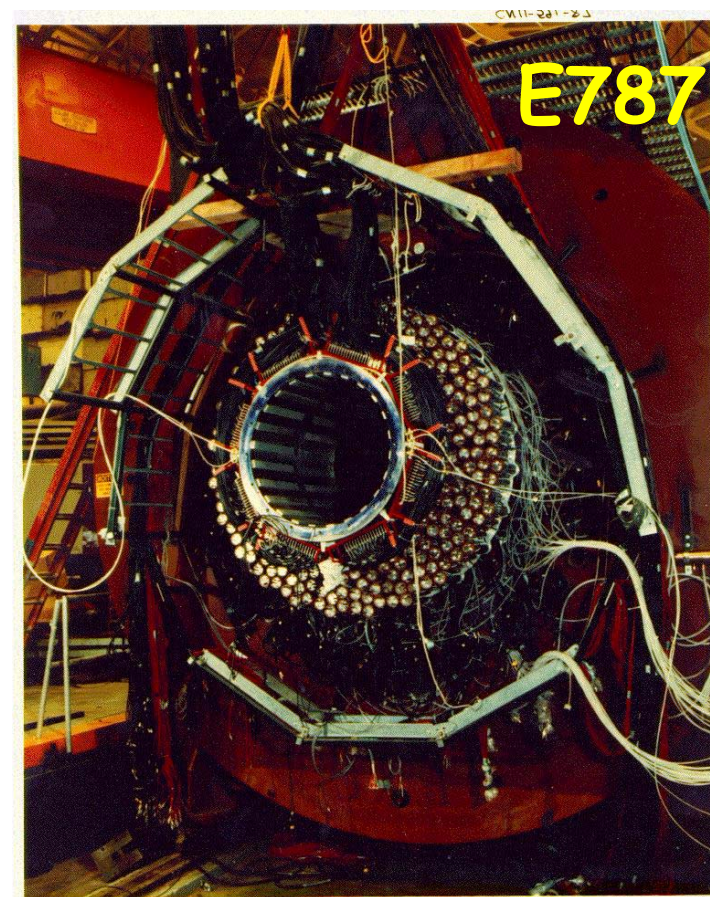
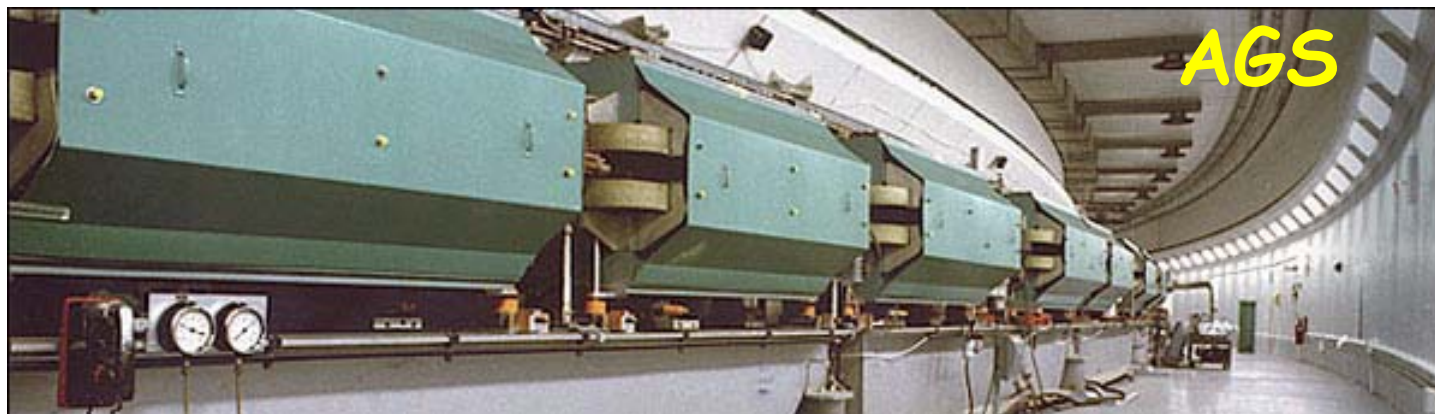
- The measured upper limit on the ratio of the decay rate of these two processes was

$$\frac{\sum_l \Gamma(K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

until experiment E787 came along

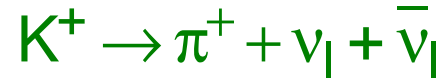
Test of flavour conservation

- The BNL experiment E787 was a **fixed target experiment**.
- It used a K^+ beam created by 24 GeV protons from the AGS.

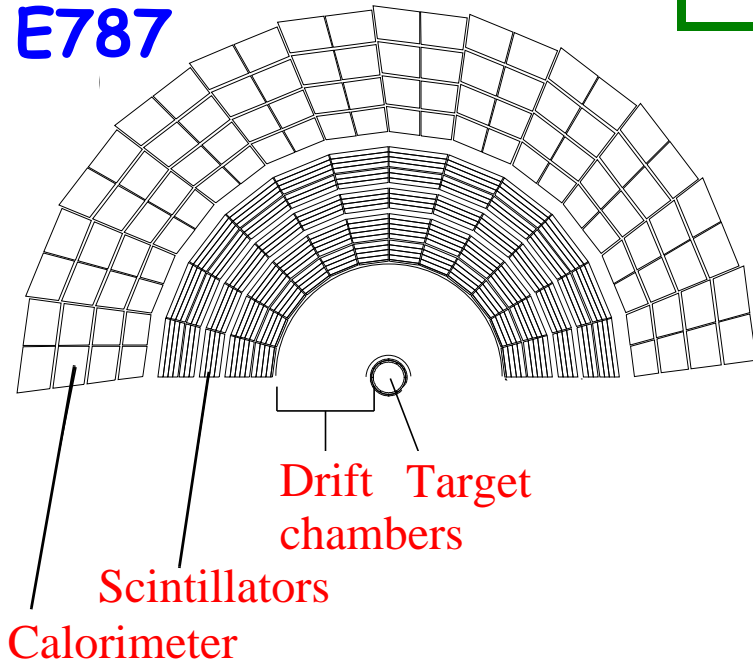


The 800 m long AGS has 240 magnets.

Test of flavour conservation



E787



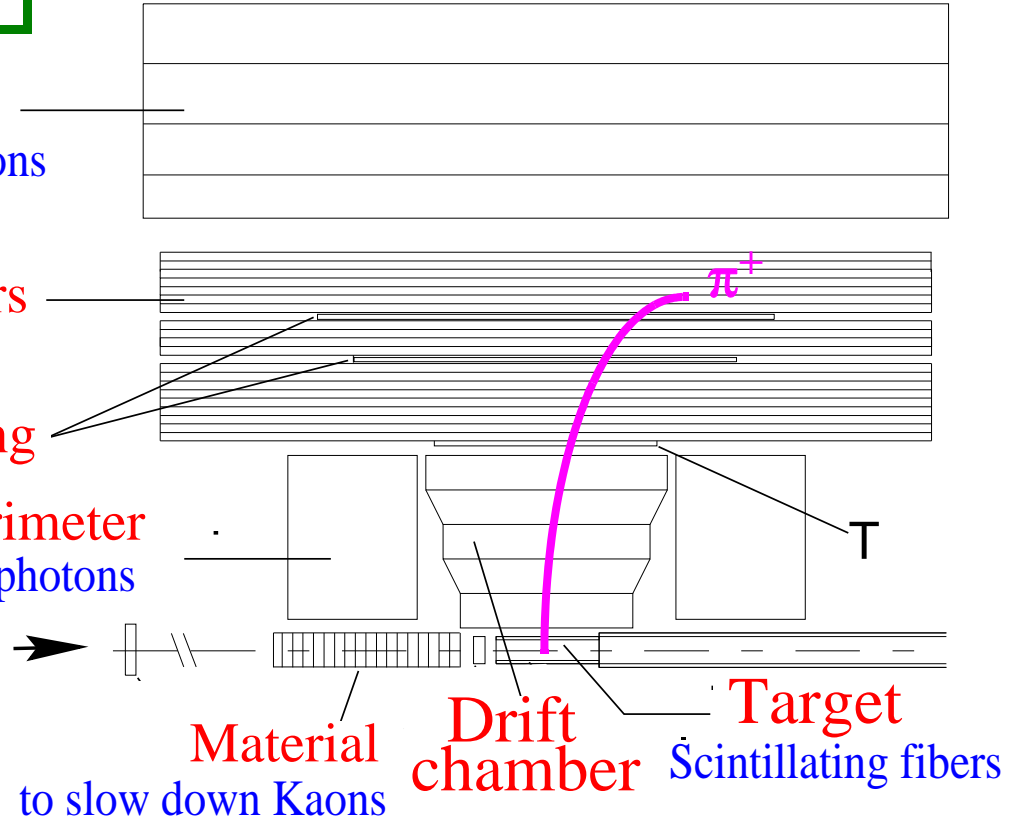
Calorimeter
to veto on photons

Scintillators

Tracking

CsI Calorimeter
to veto on photons

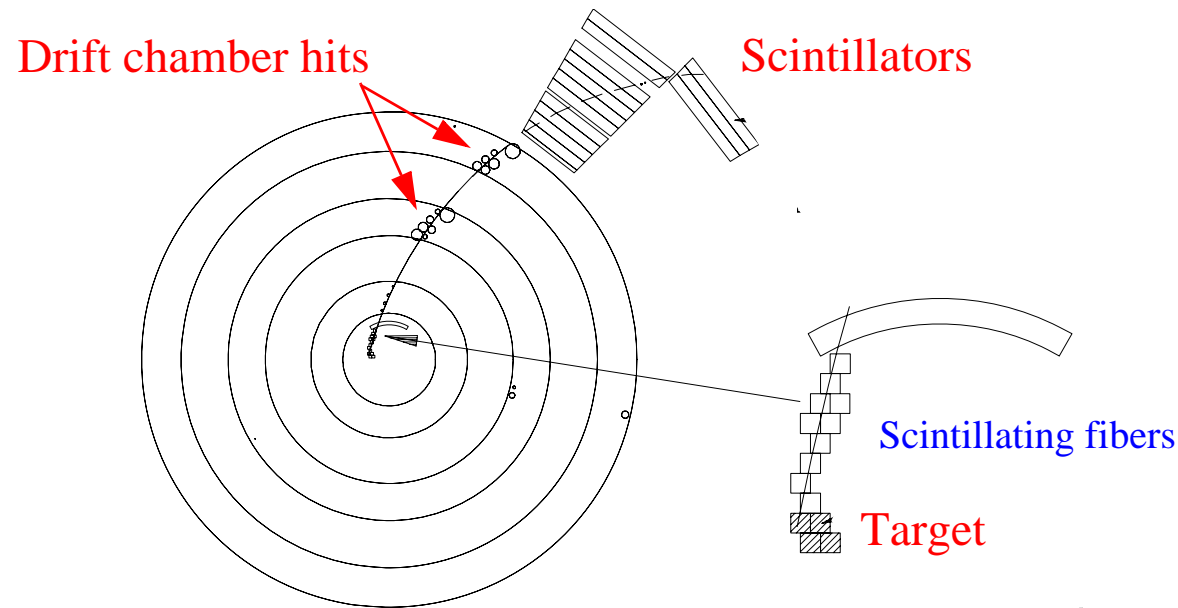
K^+ beam
0.8 GeV



- Kaons stopped in a **target** made of **scintillating fibers**
- The decay of the K^+ at rest was then studied.
- The **momentum, energy and range** of the particle from the decay was **measured**.

Test of flavour conservation

- After many years of running **two candidate events** for $K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l$ were found.

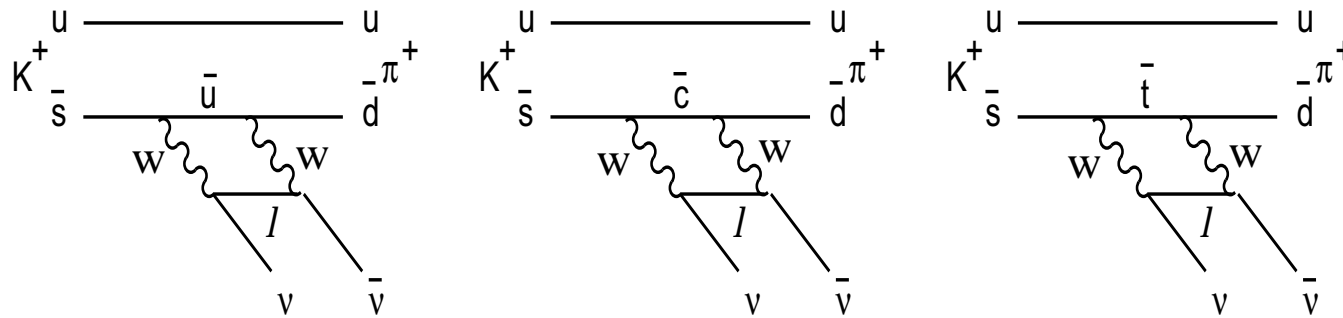


- The result from these two events were:

$$\frac{\sum_l \Gamma(K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu)} = \frac{1,6 \times 10^{-10}}{0,033} = 5 \times 10^{-9}$$

Test of flavour conservation

- Explanation: **second-order charged current reactions** and not neutral current processes:

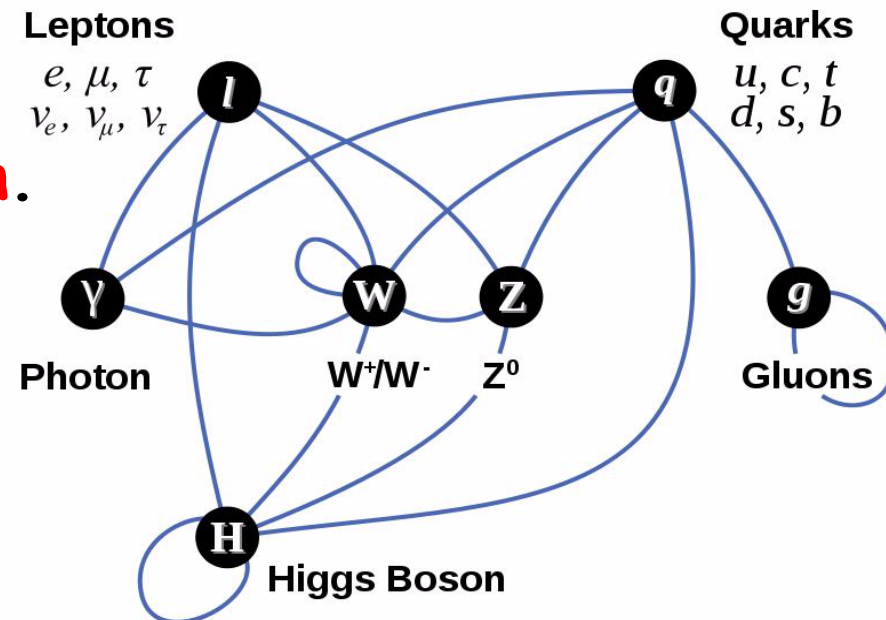


- The t - d vertex in the third diagram \Rightarrow **set limits** on the V_{td} element in the CKM matrix

$$0,007 < |V_{td}| < 0,030$$

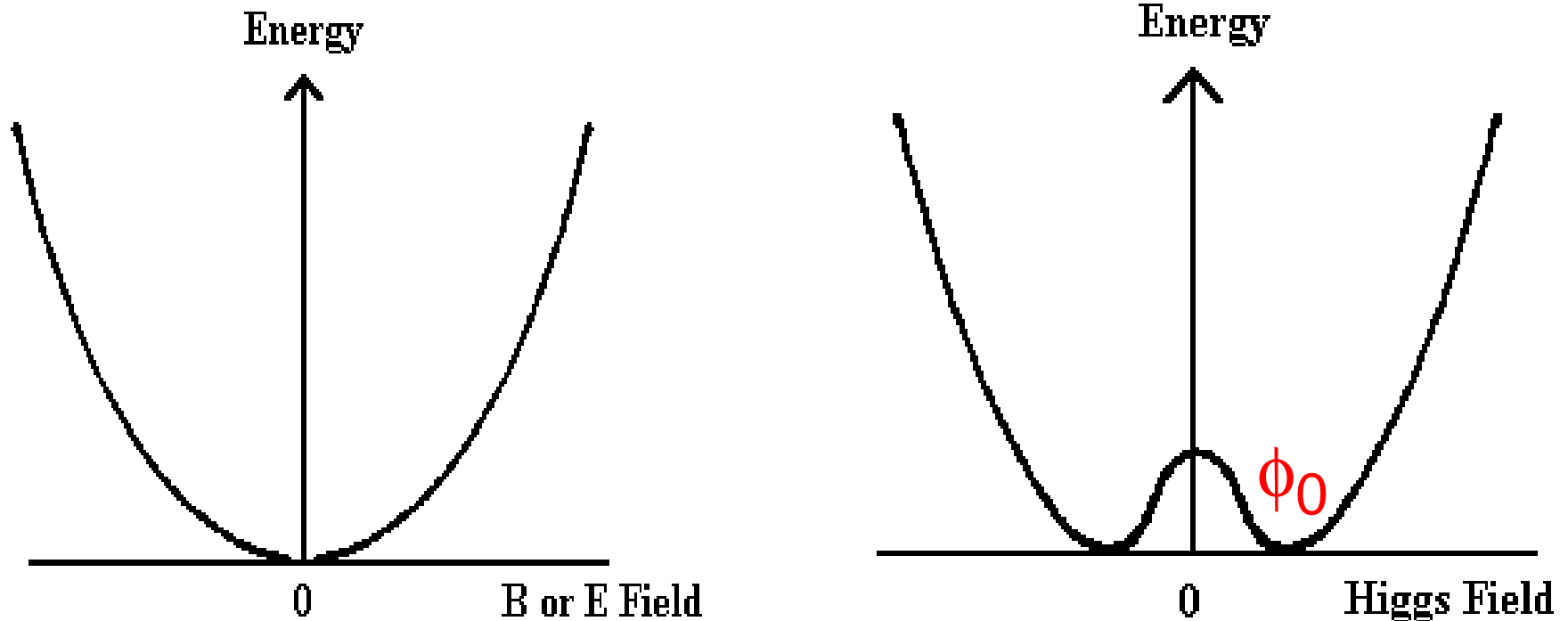
The Higgs boson

- Experimental **data agrees** extremely well with predictions of the gauge invariant **electroweak theory**.
- Gauge invariance \Rightarrow the **gauge bosons** have **zero mass**.
True for photons in QED and gluons in QCD but not for W and Z.
- A new scalar field called the **Higgs field** is introduced to **generate mass** to the W and Z bosons as well as fermion masses.
- Associated with the field is a new spin 0 particle called **the Higgs boson**.
- The theory predicts how the Higgs boson couples to other particles but **do not predict its mass**.



The Higgs boson

- Unusual characteristic of the Higgs field \Rightarrow
a non-zero value ϕ_0 in vacuum
(i.e. the field is not zero in its groundstate).



- Non-zero vacuum expectation value \Rightarrow

The vacuum is populated with massive Higgs bosons \Rightarrow

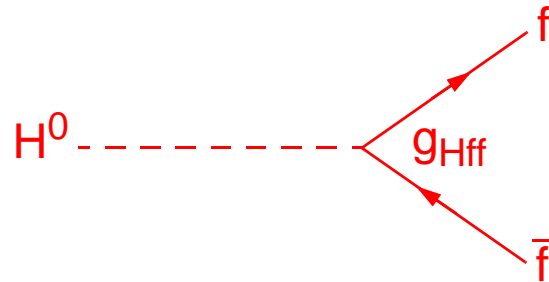
When a gauge field interacts with the Higgs field it acquires mass.

The Higgs boson

- The interaction with the Higgs field \Rightarrow
W and Z bosons obtain masses with the ratio given by

$$\cos\theta_W = \frac{M_W}{M_Z}$$

- Fermions acquire mass by interacting with the Higgs boson:



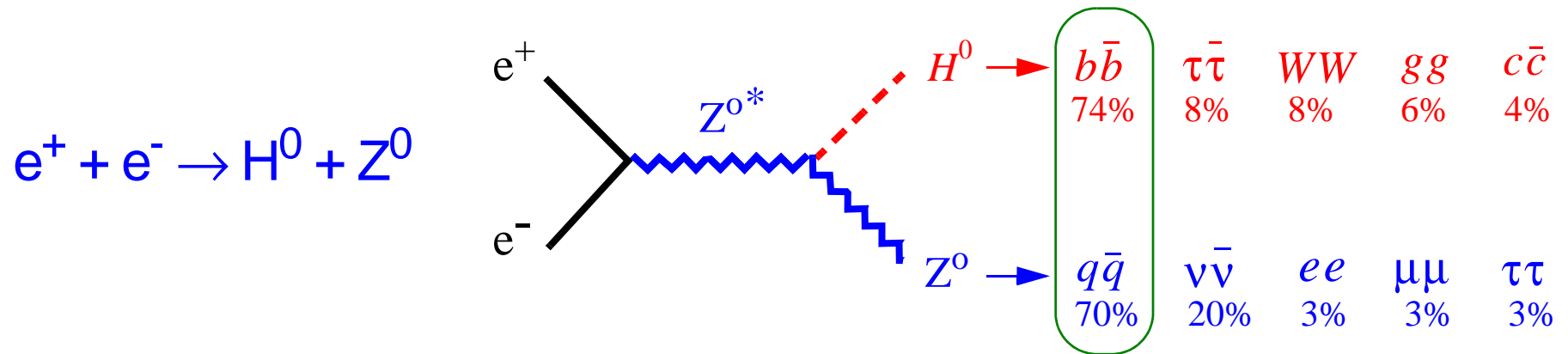
- The coupling constant depends on the fermion mass.

$$g_{Hff}^2 = \sqrt{2} G_F m_f^2$$

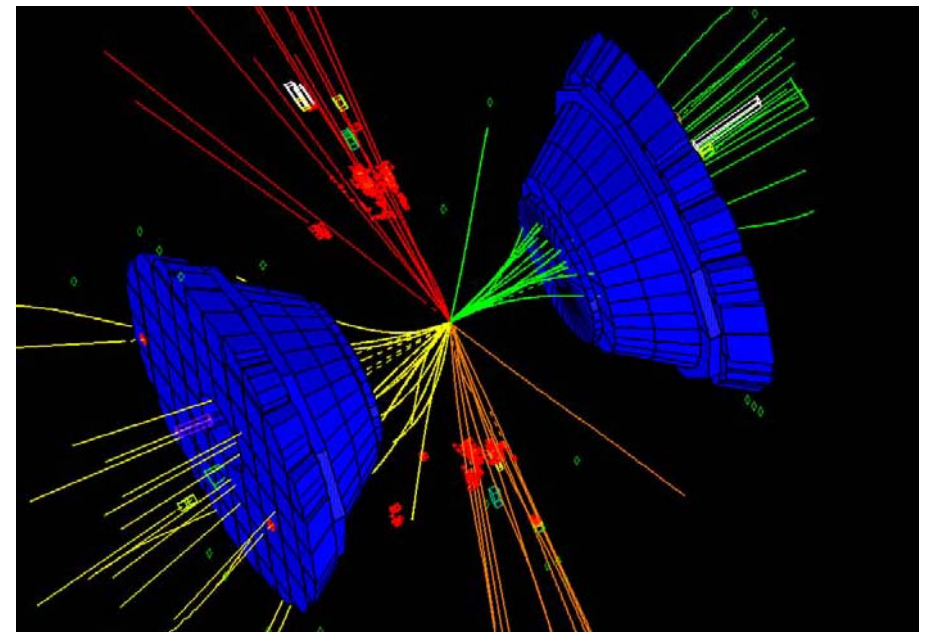
- Photons and gluons do not interact with the Higgs boson.

Search for the Higgs boson at LEP

- LEP 2: Higgs production by **Higgs strahlung**:



- Most of the Higgs events would have **4 jets** in the final state.
- Two of these should be coming from **b-quarks**.

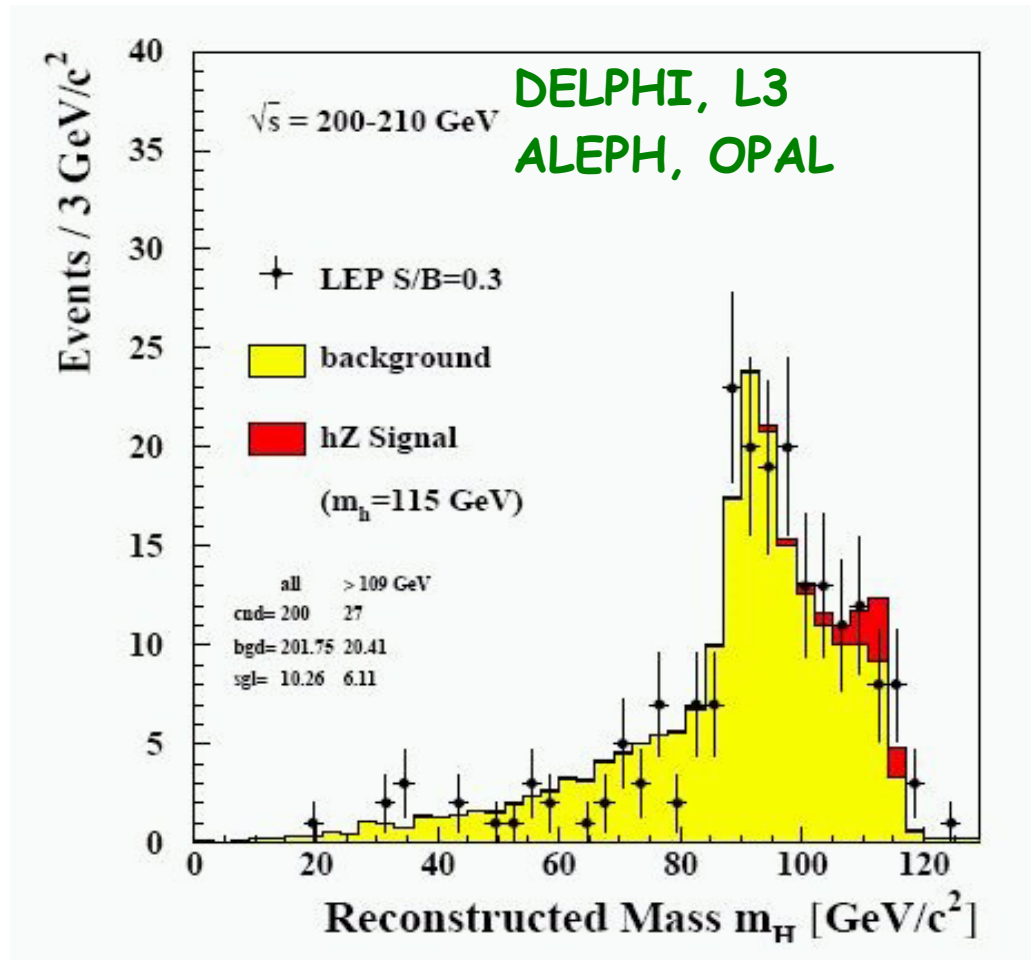


Search for the Higgs boson at LEP

- ALEPH reported a couple of Higgs candidates at 115 GeV
- DELPHI, L3 and OPAL had no signal.
- All data added together \Rightarrow no discovery.

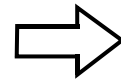
- LEP was turned off in 2000.
- The DELPHI experiment put a limit on the Higgs mass of:

$$M_H > 114 \text{ GeV}/c^2$$

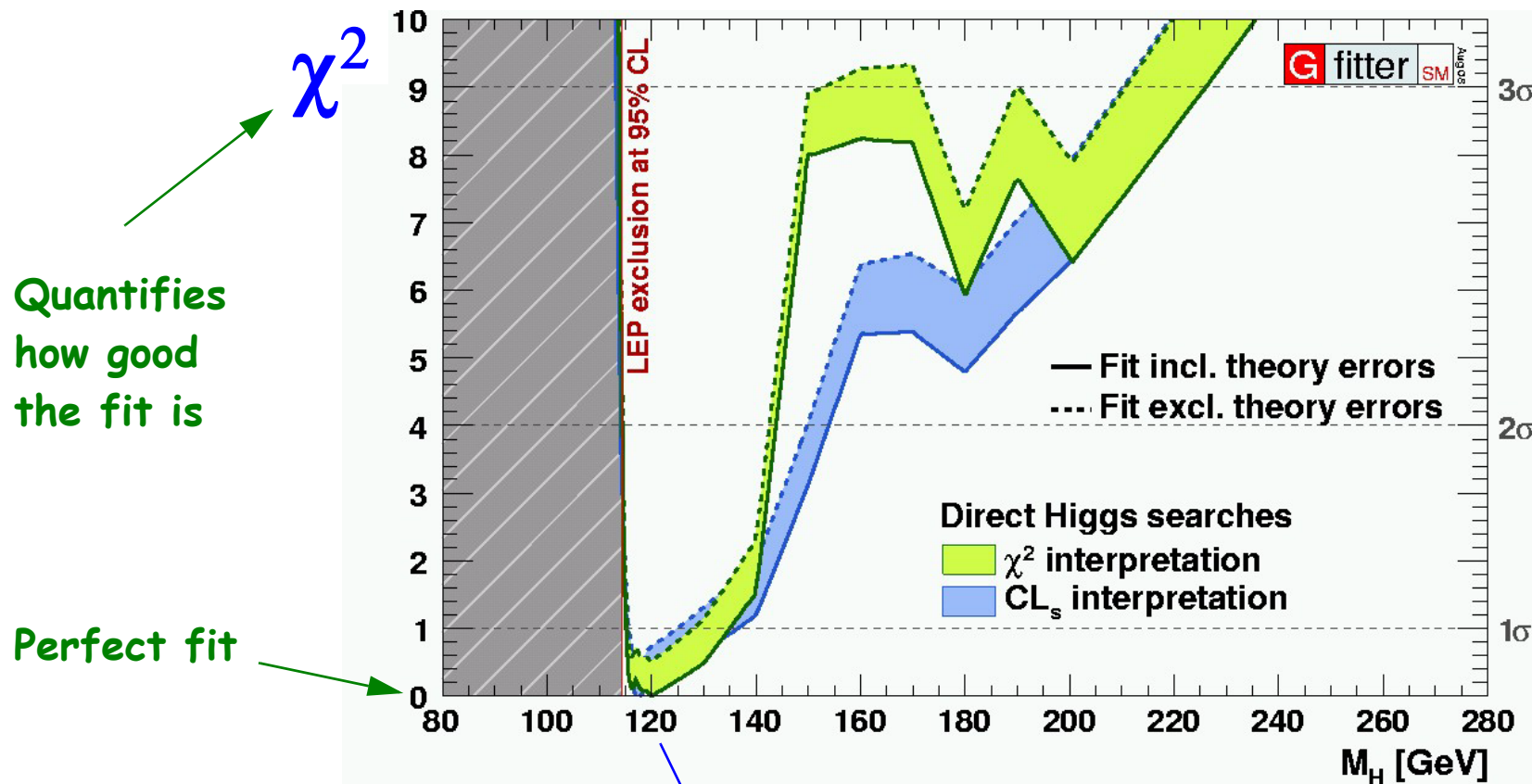


Search for the Higgs boson at LEP

- The measurement of many **electroweak parameters** at LEP



global fit with the Higgs mass as a free parameter.



$$m_H = 120^{+15}_{-5} \text{ GeV}$$