

# VII. Quark states and colours

- ❖ Forces acting between quarks in hadrons can be investigated by studying a simple quark-antiquark system
- ❖ Systems of heavy quarks, like  $c\bar{c}$  (*charmonium*) and  $b\bar{b}$  (*bottomonium*), are essentially non-relativistic (masses of quarks are much bigger than their kinetic energies)
  - ☉ Charmonium and bottomonium (*quarkonium*) are analogous to a hydrogen atom in a sense that they manifest many energy levels
  - ☉ While the hydrogen atom is governed by the electromagnetic force, the quarkonium system is dominated by the strong force

Like in a hydrogen atom, energy states of a quarkonium can be labelled by their principal (radial) quantum number  $n$ , and  $J, L, S$ , where  $L \leq n-1$  and  $S$  can be either 0 or 1.

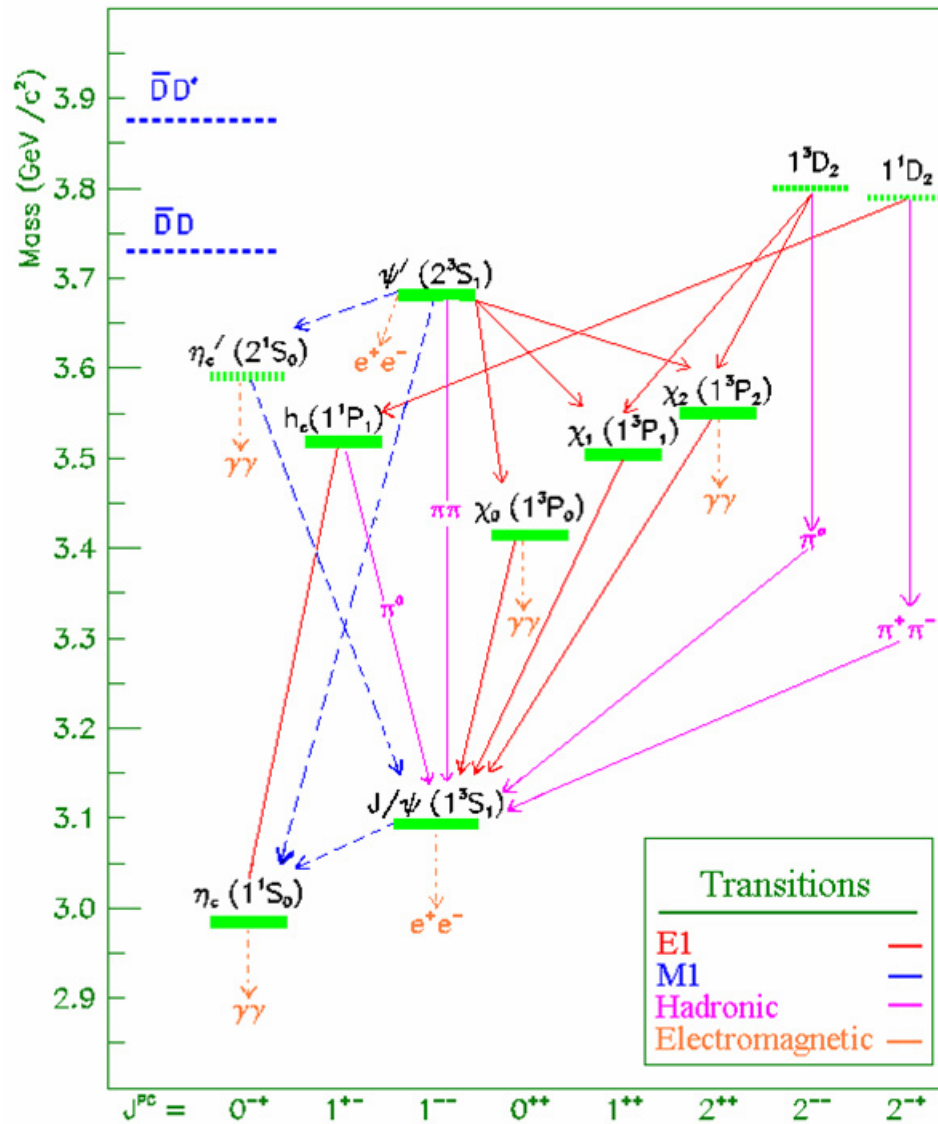


Figure 101: The charmonium spectrum

From Equations (63) and (74) , parity and C-parity of a quarkonium are:

$$P = P_q P_{\bar{q}} (-1)^L = (-1)^{L+1} \quad ; \quad C = (-1)^{L+S}$$

Some predicted and observed charmonium and bottomonium states for  $n=1$  and  $n=2$ :

		$J^{PC}$	$c\bar{c}$ state	$b\bar{b}$ state
n=1	$^1S_0$	$0^{-+}$	$\eta_c(2984)$	$\eta_b(9398)$
n=1	$^3S_1$	$1^{--}$	$J/\psi(3097)$	$Y(9460)$
n=2	$^1S_0$	$0^{-+}$	$\eta_c(3639)$	$\eta_b(9999) (?)$
n=2	$^3S_1$	$1^{--}$	$\psi(3686)$	$Y(10023)$
n=2	$^3P_0$	$0^{++}$	$\chi_{c0}(3415)$	$\chi_{b0}(9859)$
n=2	$^3P_1$	$1^{++}$	$\chi_{c1}(3511)$	$\chi_{b1}(9893)$
n=2	$^3P_2$	$2^{++}$	$\chi_{c2}(3556)$	$\chi_{b2}(9912)$
n=2	$^1P_1$	$1^{+-}$	$h_c(3525) (?)$	$h_b(9899) (?)$

☉ States  $J/\psi$  and  $\psi$  have the same  $J^{PC}$  quantum numbers as a photon:  $1^{--}$ , and the most common way to form them is through  $e^+e^-$ -annihilation, where virtual photon converts to a charmonium state

## Electron-positron collisions, cross-section

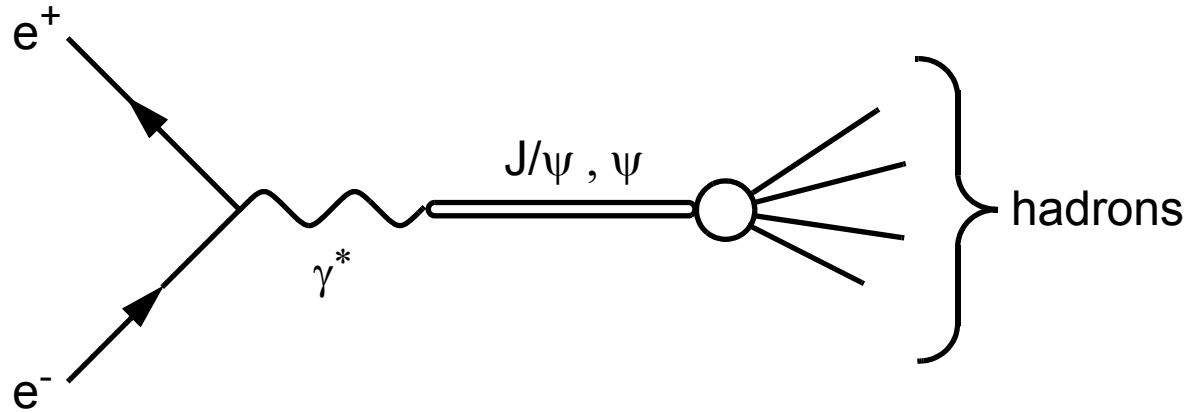


Figure 102: Formation and decay of  $J/\psi$  ( $\psi$ ) mesons in  $e^+e^-$  annihilation

☉ If centre-of-mass energy of incident  $e^+$  and  $e^-$  is equal to the quarkonium mass, formation of the latter is highly probable, which leads to the peak in the cross-section  $\sigma(e^+e^- \rightarrow \text{hadrons})$ .

❖ Cross-section  $\sigma$  in a collision is defined through

$$N = \sigma \times L \quad (104)$$

Here  $N$  is the count of reactions (*events*) in a time period, and  $L$  is the integrated *luminosity* – density of colliding particles integrated over this time period

⊙ Cross-section is measured in barns:

$$[\sigma] = 1 \text{ barn (1 b)} \equiv 10^{-24} \text{ cm}^2 \Rightarrow [L] = \text{cm}^{-2} \text{ or } 1 \text{ barn}^{-1} (1 \text{ b}^{-1})$$

An example:

⊙ an LHC collider run will last  $10^7$  s, with instantaneous luminosity of  $10^{34} \text{ cm}^{-2}\text{s}^{-1}$   
 $\Rightarrow L = 10^{41} \text{ cm}^{-2} = 100 \text{ fb}^{-1}$ .

⊙ The total production cross-section for  $b\bar{b}$ -pairs is about  $500 \mu\text{b}$   $\Rightarrow$  in  $10^7$  s, the number of produced events will be  $N = 500 \mu\text{b} \times 100 \text{ fb}^{-1} = 5 \times 10^{13}$

❖  $e^+e^-$  collisions provide clean study environment; it is convenient to normalize the cross-sections to that of muon production:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (105)$$

❖ Sharp peaks can be observed in  $R$  at  $E_{cm} = 3.097 \text{ GeV}$  ( $J/\psi$ ) and  $3.686 \text{ GeV}$  ( $\psi$ )

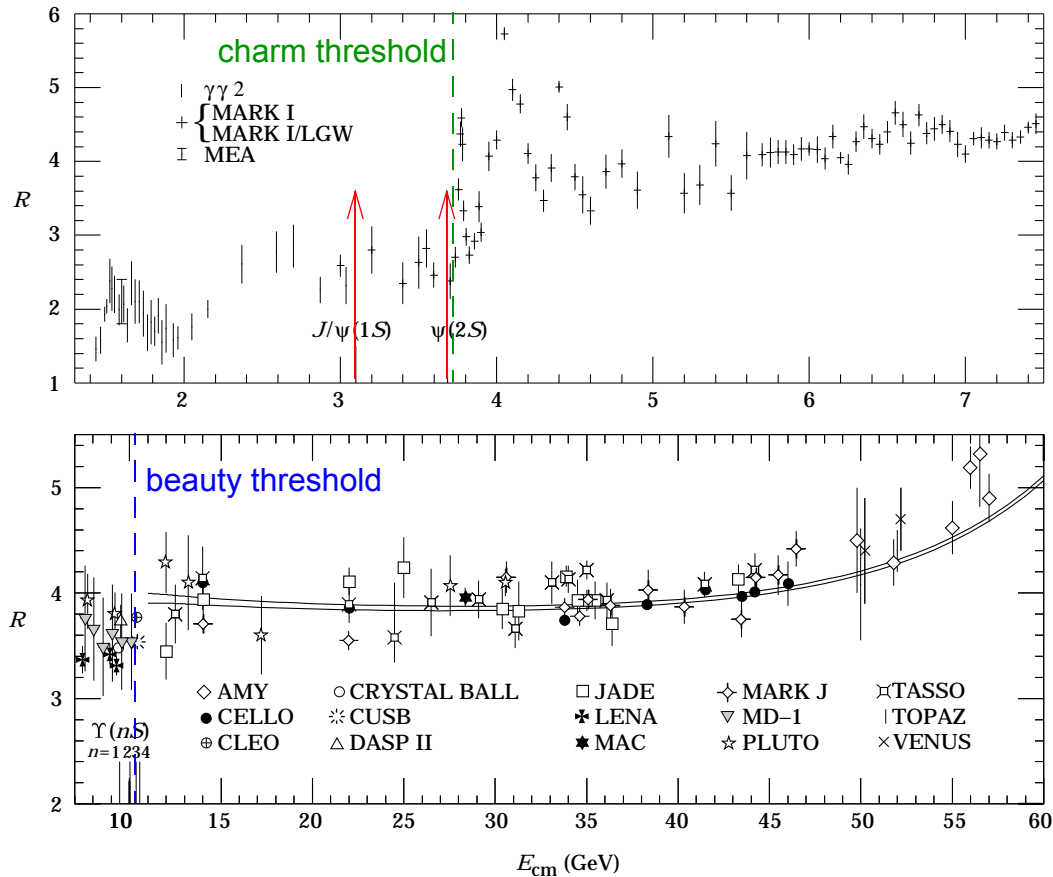


Figure 103: Cross-section ratio  $R$  in  $e^+e^-$  collision. Arrows indicate the peaks.

☉ Cross-section for a  $\mu^+\mu^-$  final state is known and depends only on  $E_{CM}$  and  $\alpha$ :

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2} \quad (106)$$

❖ Charm *threshold* (3730 MeV): twice the mass of the lightest charmed meson, D

- ⊙  $J/\psi$ ,  $\psi$  are lighter  $\Rightarrow$  can not decay into charmed particles  $\Rightarrow$  long-living (narrow peaks below charm threshold)
- ⊙ Wide peaks above charm threshold: short-living resonances

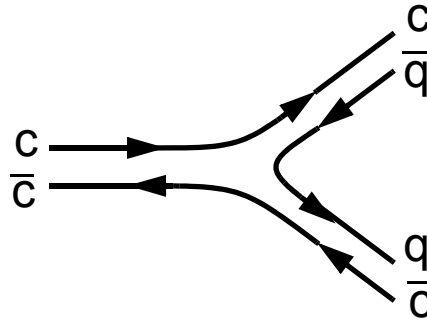


Figure 104: Charmonium resonance decay to charmed mesons

❖  $J/\psi$  and  $\psi$  can only decay via annihilation of  $c\bar{c}$  pair

- ⊙ Long lifetime since such annihilation is *suppressed* as opposed to light quarks (e.g. in  $\pi_0$ )
- ⊙  $J/\psi$  and  $\psi$  can only decay to light hadrons (containing u, d, s), or to  $e^+e^-$ , or  $\mu^+\mu^-$ .

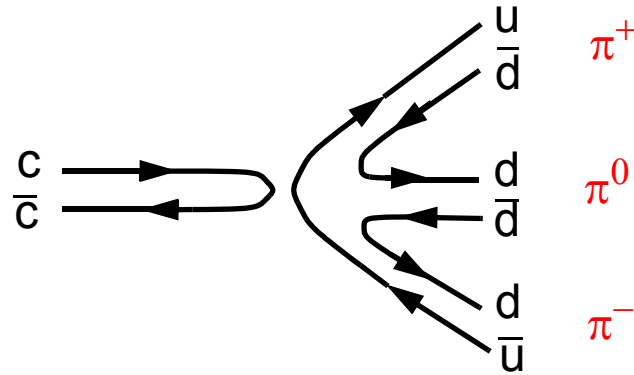


Figure 105: Charmonium decay to light non-charmed mesons

- Charmonium states with quantum numbers different of those of photon can not be produced in  $c\bar{c}$  annihilation, but can be found in radiative decays of  $J/\psi$  or  $\psi$ :

$$\psi(3686) \rightarrow \chi_{ci} + \gamma \quad (i=0,1,2) \quad (107)$$

$$\psi(3686) \rightarrow \eta_c(2980) + \gamma \quad (108)$$

$$J/\psi(3097) \rightarrow \eta_c(2980) + \gamma \quad (109)$$

- Bottomonium spectrum is observed in much the same way as charmonium

- Beauty threshold is at  $10560 \text{ MeV}/c^2$  (twice the mass of the B meson)

- Similarities between spectra of bottomonium and charmonium suggest similarity of forces acting in two systems



## The quark-antiquark potential

- ❖ Let's assume the  $q\bar{q}$  potential being a central one,  $V(r)$ , and the system to be non-relativistic

In the centre-of-mass frame of a  $q\bar{q}$  pair, Schrödinger equation is

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{x}) + V(r)\psi(\vec{x}) = E\psi(\vec{x}) \quad (110)$$

Here  $\mu = m_q/2$  is the *reduced mass* of a quark, and  $r = |\vec{x}|$  is distance between the quarks.

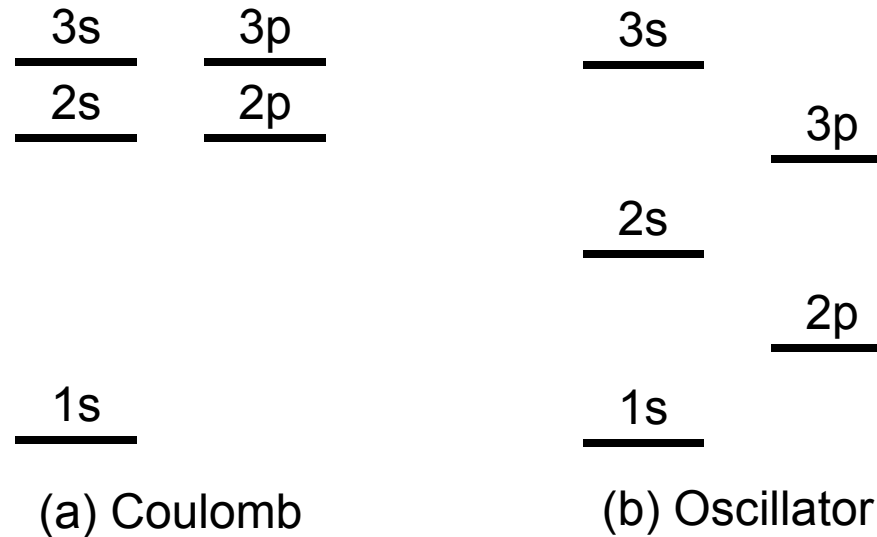
Mass of a quarkonium state in this framework is

$$M(q\bar{q}) = 2m_q + E \quad (111)$$

- ☉ In the case of a Coulomb-like approximation  $V(r) \propto r^{-1}$ , energy levels are quantized, depending only on the *principal quantum number*  $n$ :

$$E_n = -\frac{\mu\alpha^2}{2n^2}$$

- ⊙ In the case of a harmonic oscillator potential  $V(r) \propto r^2$ , the degeneracy of energy levels is broken: dependency on  $L$  arises



(a) Coulomb

(b) Oscillator

Figure 106: Energy levels arising from Coulomb and harmonic oscillator potentials for  $n=1,2,3$

Cf Figure 101: one can see that heavy quarkonia spectra are inbetween the two approximations; the actual potential can be described by:

$$V(r) = -\frac{a}{r} + br \quad (112)$$

Coefficients  $a$  and  $b$  are determined by solving Equation (110) and fitting results to data:

$$a = 0.48$$

$$b = 0.18 \text{ GeV}^2$$

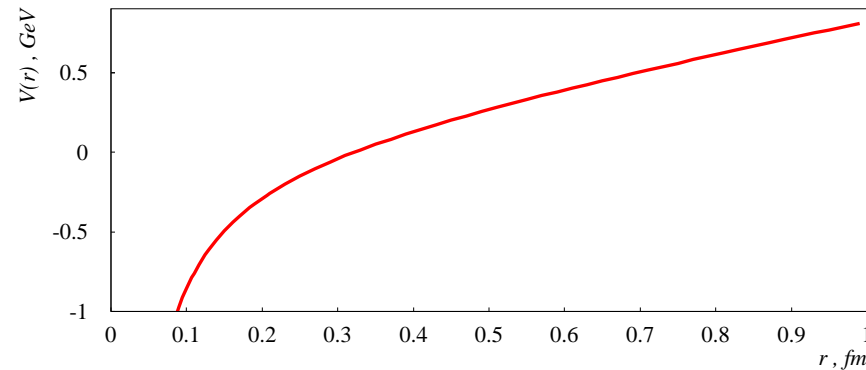


Figure 107: Modified Coulomb potential (112)

Other forms of the potential can give equally good results, for example

$$V(r) = a \ln(br) \quad (113)$$

where parameters appear to be

$$a = 0.7 \text{ GeV}$$

$$b = 0.5 \text{ GeV}$$

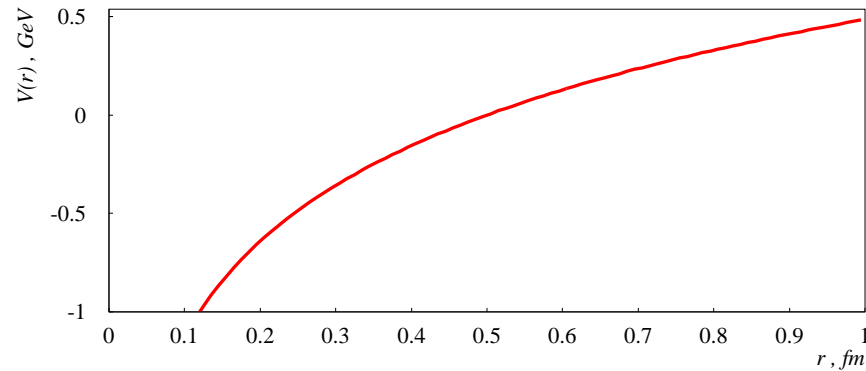


Figure 108: Logarithmic potential (113)

🎯 In the range of  $0.2 \leq r \leq 0.8$  fm potentials (112) and (113) are in good agreement  
 $\Rightarrow$  in this region the quark-antiquark potential can be considered as well-defined

❖ Simple non-relativistic Schrödinger equation explains quite well existence of several energy states for a given heavy quark-antiquark system

## Light mesons; nonets

- ❖ Spins of quarks are counter-directed  $\Rightarrow J^P=0^-$ , *pseudoscalar meson nonet* (9 possible  $q\bar{q}$  combinations for u,d,s quarks)
- ❖ Spins of quarks are co-directed  $\Rightarrow J^P=1^-$ , *vector meson nonet*

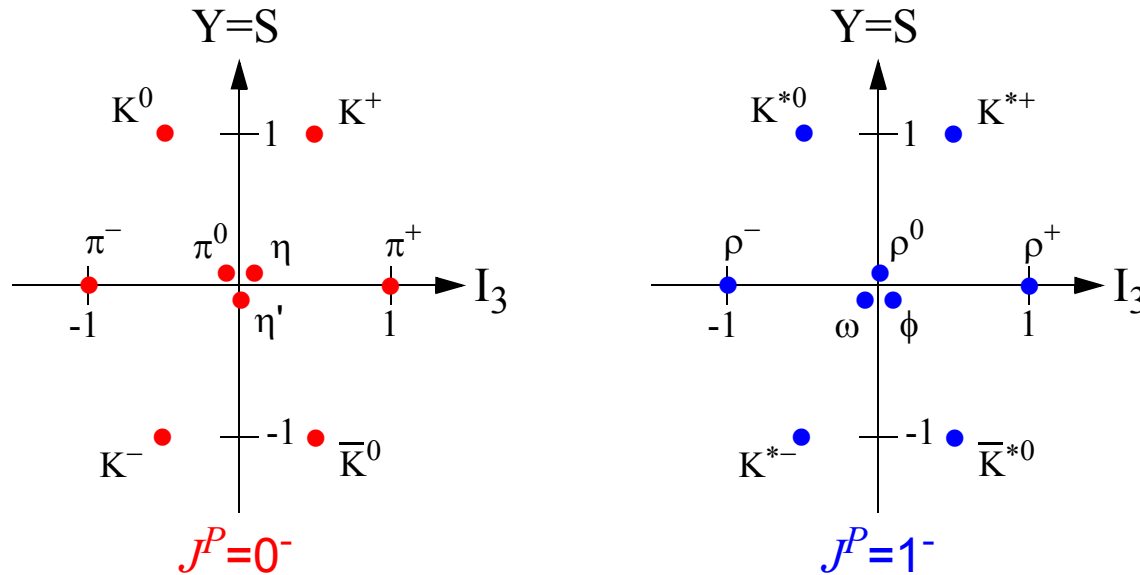


Figure 109: Light meson nonets in  $(I_3, Y)$  space (“weight diagrams”)

❖ In each nonet, there are three particles with equal quantum numbers  $Y=S=I_3=0$

☉ They correspond to a  $q\bar{q}$  pair like  $u\bar{u}$ ,  $d\bar{d}$  or a *linear combination* of these states (follows from the isospin operator analysis):

$$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad I = 1, I_3 = 0 \quad (114)$$

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \quad I = 0, I_3 = 0 \quad (115)$$

❖  $\pi^0$  and  $\rho^0$  mesons are linear combinations of  $u\bar{u}$  and  $d\bar{d}$  states (114):  
 $(u\bar{u} - d\bar{d})/(\sqrt{2})$

❖  $\omega$  meson is the linear combination (115):  $(u\bar{u} + d\bar{d})/(\sqrt{2})$

Inclusion of an  $s\bar{s}$  pair leads to further combinations:

$$\eta(547) = \frac{(d\bar{d} + u\bar{u} - 2s\bar{s})}{\sqrt{6}} \quad I = 0, I_3 = 0 \quad (116)$$

$$\eta'(958) = \frac{(d\bar{d} + u\bar{u} + s\bar{s})}{\sqrt{3}} \quad I = 0, I_3 = 0 \quad (117)$$

- ❖ There exists meson  $\phi(1019)$ , which is a quarkonium  $s\bar{s}$ , having  $I=0$  and  $I_3=0$

### *Light baryons*

- ❖ Three-quark states of the lightest quarks (u,d,s) form baryons, which can be arranged in *supermultiplets* (*singlets*, *octets* and *decuplets*).
- ❖ The lightest baryon supermultiplets are *octet* of  $J^P = \frac{1}{2}^+$  particles and *decuplet* of  $J^P = \frac{3}{2}^+$  particles
- ☉ Weight diagrams of baryons can be deduced from the quark model under assumption that the **combined** space-spin wavefunctions are *symmetric* under interchange of **like** quarks

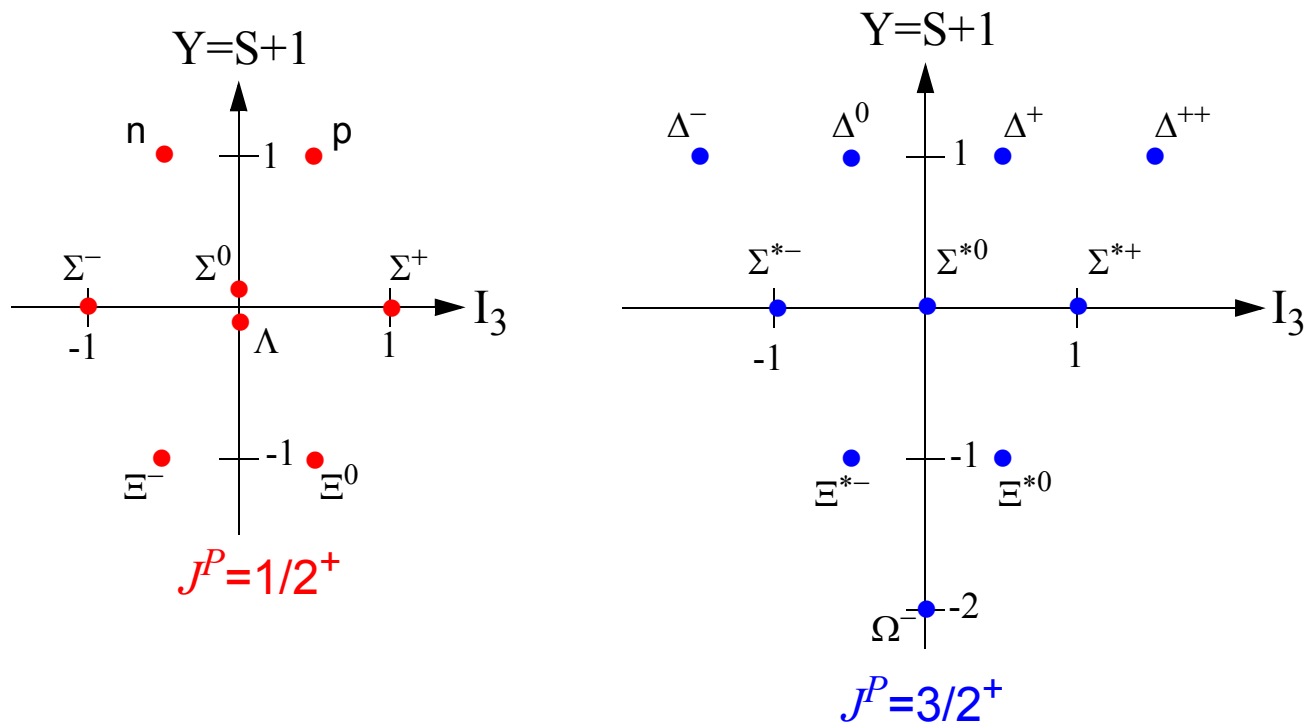


Figure 110: Weight diagrams for light baryons

- Parity of a 3-quark state  $q_i q_j q_k$  is  $P = P_i P_j P_k = 1$
- Spin of such a state is sum of quark spins
- From presumption of symmetry under exchange of like quarks, any pair of like quarks  $qq$  must have total spin-1 (quark spins co-directed)



❖ There are six distinct combinations of the form  $q_i q_j q_k$ :

$uud, uus, ddu, dds, ssu, ssd$

each of them can have spin  $J=1/2$  or  $J=3/2$

❖ three combinations of the form  $q_i q_i q_i$  are possible:

$uuu, ddd, sss$

spins of all like-quarks have to be parallel (symmetry presumption), hence  $J=3/2$  only

❖ The remaining combination is  $uds$ , with two distinct states having spin values  $J=1/2$  and one state with  $J=3/2$

❖ By adding up numbers, one gets 8 states with  $J^P=1/2^+$  and 10 states with  $J^P=3/2^+$ , exactly what is shown by weight diagrams

❖ Measured masses of baryons show that mass difference between members of same isospin multiplets is much smaller than that between members of different isospin multiplets

☉ In what follows, equal masses of isospin multiplet members are assumed, e.g.,

$$m_p = m_n \equiv m_N$$

Experimentally, more s-quarks contains a particle, heavier it is:

$$\Xi^0(1315) = (uss); \Sigma^+(1189) = (uus); p(938) = (uud)$$

$$\Omega^-(1672) = (sss); \Xi^{*0}(1532) = (uss);$$

$$\Sigma^{*+}(1383) = (uus); \Delta^{++}(1232) = (uuu)$$

❖ There is an evidence that the main contribution to big mass differences comes from the s-quark

☉ Knowing masses of baryons, one can calculate 6 **simplistic** estimates of mass difference between s-quark and light quarks (u,d)

For the  $3/2^+$  decuplet:

$$M_{\Omega} - M_{\Xi} = M_{\Xi} - M_{\Sigma} = M_{\Sigma} - M_{\Delta} = m_s - m_{u,d}$$

and for the  $1/2^+$  octet:

$$M_{\Xi} - M_{\Sigma} = M_{\Xi} - M_{\Lambda} = M_{\Lambda} - M_N = m_s - m_{u,d}$$

Average value of those differences gives

$$m_s - m_{u,d} \approx 160 \text{ MeV}/c^2 \quad (118)$$

❖ So far, so good – BUT quarks are spin-1/2 particles  $\Rightarrow$  fermions  $\Rightarrow$  their wavefunctions are *antisymmetric* and all the discussion above **contradicts Pauli principle!**

# COLOUR

- ❖ Experimental data confirm predictions based on the assumption of symmetric wave functions
- ❖ That means that apart of space and spin degrees of freedom, quarks carry yet another attribute

In 1964-1965, Greenberg and Nambu with colleagues proposed the new property – the *colour* – with THREE possible states, and associated with the corresponding wavefunction  $\chi^C$ :

$$\Psi = \psi(\vec{x})\chi\chi^C \quad (119)$$

- ⊙ Conserved quantum numbers associated with  $\chi^C$  are *colour charges* – in strong interaction they play analogous role to the electric charge in e.m. interaction
- ⊙ Hadrons are presumed to exist only in *colour singlet* states, with total colour charge of zero
- ⊙ Quarks have to be *confined* within the hadrons, since non-zero colour states are forbidden

☉ Three independent colour wavefunctions are represented by “*colour spinors*”:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (120)$$

- ☉ They are acted upon by eight independent “*colour operators*” which are represented by a set of 3-dimensional matrices (analogs of Pauli matrices)
- ☉ Colour charges  $I_3^C$  and  $Y^C$  are eigenvalues of corresponding operators

In this formalism, colours can be quantified. Values of  $I_3^C$  and  $Y^C$  for the colour states of quarks and antiquarks are:

	Quarks		Antiquarks		
	$I_3^C$	$Y^C$	$I_3^C$	$Y^C$	
<b>r</b> (“red”)	1/2	1/3	$\bar{r}$	-1/2	-1/3
<b>g</b> (“green”)	-1/2	1/3	$\bar{g}$	1/2	-1/3
<b>b</b> (“blue”)	0	-2/3	$\bar{b}$	0	2/3

⊙ *Colour hypercharge*  $Y^C$  and *colour isospin charge*  $I_3^C$  are additive quantum numbers, having opposite sign for quark and antiquark

*Confinement condition* for the total colour charges of a hadron:

$$I_3^C = Y^C = 0 \quad (121)$$

The most general colour wavefunction for a baryon is a linear superposition of six possible combinations:

$$\begin{aligned} \chi_B^C = & \alpha_1 r_1 g_2 b_3 + \alpha_2 g_1 r_2 b_3 + \alpha_3 b_1 r_2 g_3 \\ & + \alpha_4 b_1 g_2 r_3 + \alpha_5 g_1 b_2 r_3 + \alpha_6 r_1 b_2 g_3 \end{aligned} \quad (122)$$

where  $\alpha_i$  are constants. It can be shown that the color confinement requires the totally antisymmetric combination:

$$\begin{aligned} \chi_B^C = & \frac{1}{\sqrt{6}} (r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 \\ & - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3) \end{aligned} \quad (123)$$

Colour confinement principle (121) implies certain requirements for states containing both quarks and antiquarks:

- consider an arbitrary combination  $q^m \bar{q}^n$  of  $m$  quarks and  $n$  antiquarks,  $m \geq n$
- for a particle with  $\alpha$  quarks in r-state,  $\beta$  quarks in g-state,  $\gamma$  quarks in b-state ( $\alpha + \beta + \gamma = m$ ) and  $\bar{\alpha}$ ,  $\bar{\beta}$ ,  $\bar{\gamma}$  antiquarks in corresponding antistates ( $\bar{\alpha} + \bar{\beta} + \bar{\gamma} = n$ ), the colour wavefunction is

$$r^\alpha g^\beta b^\gamma \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}} \quad (124)$$

Adding up colour charges (from the table above) and applying the confinement requirement,

$$I_3^C = (\alpha - \bar{\alpha})/2 - (\beta - \bar{\beta})/2 = 0$$

$$Y^C = (\alpha - \bar{\alpha})/3 + (\beta - \bar{\beta})/3 - 2(\gamma - \bar{\gamma})/3 = 0$$

⇓

$$\alpha - \bar{\alpha} = \beta - \bar{\beta} = \gamma - \bar{\gamma} \equiv p$$

Here  $p$  is a non-negative integer  $\Rightarrow$

❖ Numbers of quarks and antiquarks in a colorless state are related as:  
 $m - n = 3p$

❖ The only combination  $q^m \bar{q}^n$  allowed by the colour confinement principle is

$$(3q)^p (q\bar{q})^n, \quad p, n \geq 0 \quad (125)$$

⊙ Form (125) forbids states with fractional electric charges

⊙ However, it allows exotic combinations like  $qqq\bar{q}$ ,  $qqqq\bar{q}$  (like e.g. the pentaquark  $\Theta^+ = uud\bar{s}$ )

### Unconventional states

A number of unforeseen states in  $c\bar{c}$  and  $b\bar{b}$  regions are being observed

$X(3872)$  is one of the most intriguing: it is well studied, and yet does not fit the quark model:

⊙ A possible candidate was charmonium  $\eta_{c2}(1^1D_2)$ , but in 2013 LHCb measured its  $J^{PC}=1^{++}$



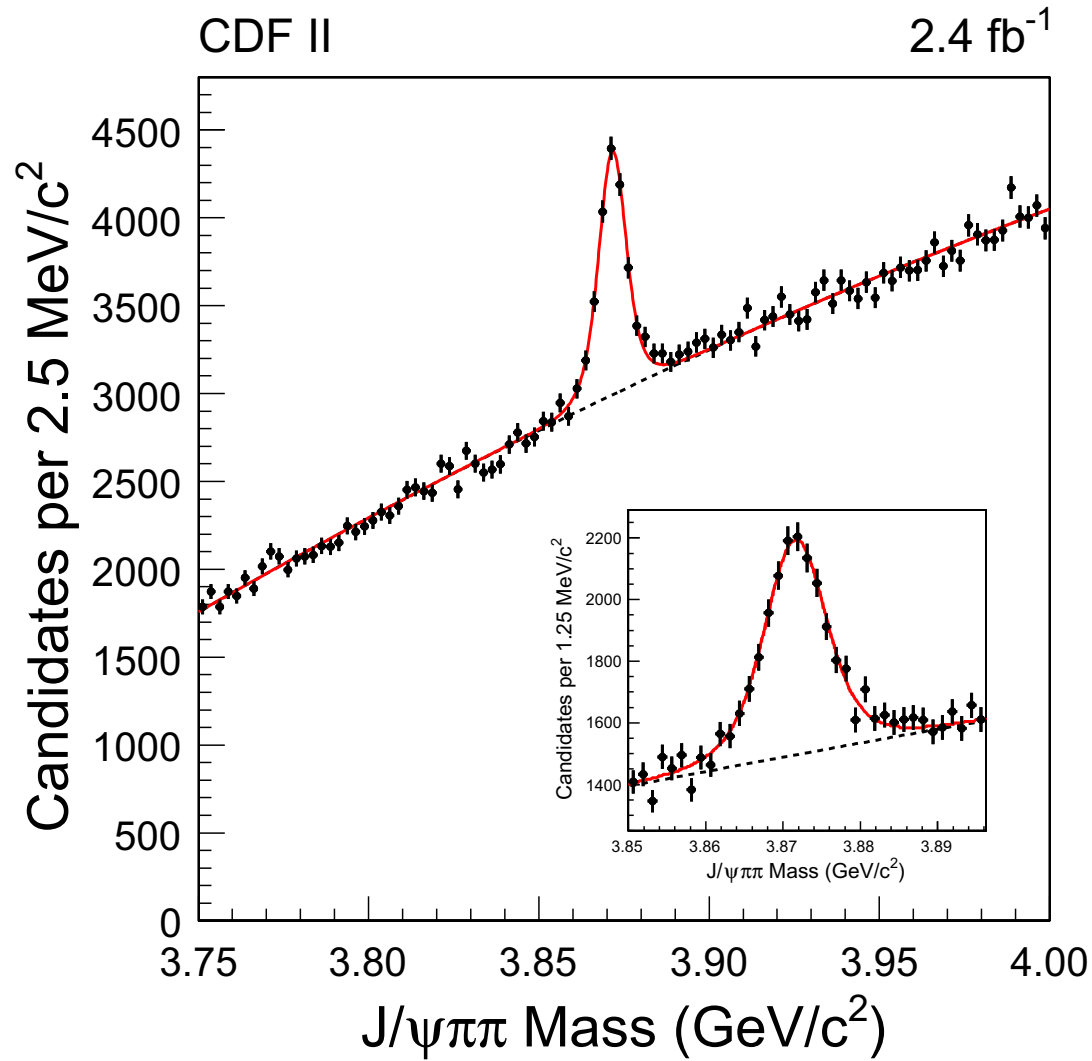


Figure 111: X(3872) as measured by the CDF experiment