FYST17 Lecture 2

Symmetries and CP violation

Thanks to A Hocker and M. Bona

Today's topics

- Symmetries
  - Broken symmetries
- Neutral kaon mixing
- CP violation
  - Matter / anti-matter asymmetry
- The CKM matrix

What do we mean by conservation/violation of a symmetry?

- Define a quantum mechanical operator O.
- If O describes a good symmetry:

Physics 'looks the' same before and after applying the symmetry i.e. the observed quantity associated with O is conserved (same before and after the operator is applied). e.g. conservation of energy-momentum etc.

e.g. probabilities are the same for matter and antimatter doing something.

- If this condition is not met – the symmetry is broken.
  - That is, the symmetry is not respected by nature. So O is (at best) a mathematical tool used to help our understanding of nature.
  - Slightly broken symmetries (like isospin in EW interactions) can be very useful)!

e.g. Isospin symmetry assumes that $m_u=m_d$. In doing so we can estimate branching fractions where the final state differs by a π^0 vs a $\pi^\pm$ etc. The difference comes from a Clebsch-Gordan coefficient.

Continuous Symmetries and Conservation Laws

- In classical mechanics we have learned that to each continuous symmetry transformation, which leaves the scalar Lagrange density invariant, can be attributed a conservation law and a constant of movement (E. Noether, 1915)

- Continuous symmetry transformations lead to additive conservation laws

Symmetry	Invariance under movement in time	Homogeneity of space	Isotropy of space
Transformation	Translation in time	Translation in space	Rotation in space
Conserved quantity	Energy	Linear momentum	Angular momentum

→ No evidence for violation of these symmetries seen so far

Continuous Symmetries and Conservation Laws

In general, if U is a symmetry of the Hamiltonian H, one has: $[H,U]=0 \Rightarrow H=U^\dagger H U$

$$\langle f'|H|i'\rangle = \langle Uf|H|Ui\rangle = \langle f|U^\dagger H U|i\rangle = \langle f|H|i\rangle$$

- Accordingly, the Standard Model Lagrangian satisfies local gauge symmetries (the physics must not depend on local (and global) phases that cannot be observed):

U(1) gauge transformation	➔	**Electromagnetic interaction**
SU(2) gauge transformation	➔	**Weak interaction**
$SU(3)_C$ gauge transformation	➔	**Strong interaction (QCD)**

- Conserved additive quantum numbers:
  - Electric charge (processes can move charge between quantum fields, but the sum of all charges is constant)
  - Similar: color charge of quarks and gluons, and the weak charge
  - Quark (baryon) and lepton numbers (however, no theory for these, therefore believed to be only approximate asymmetries) → evidence for lepton flavor violation in "neutrino oscillation"

Discrete Symmetries

Discrete symmetry transformations lead to multiplicative conservation laws

The following discrete transformations are fundamental in particle physics:

- **Parity *P* ("handedness"):**

  Reflection of space around an arbitrary center;
  P invariance ⍰ cannot know whether we live in *this* world, or in *its mirror* world

- **Particle-antiparticle transformation *C* :**

  Change of all additive quantum numbers (for example the electrical charge) in its opposite ("charge conjugation")

- **Time reversal *T* :**

  The time arrow is reversed in the equations;
  T invariance ⇒ if a movement is allowed by a the physics law, the movement in the opposite direction is also allowed

In particle physics:

$$P\left|e_L^-\right\rangle = \left|e_R^-\right\rangle$$

$$P\left|\pi^0\right\rangle = -\left|\pi^0\right\rangle$$

$$P\left|n\right\rangle = +\left|n\right\rangle$$

$$C\left|e_L^-\right\rangle = \left|e_L^+\right\rangle$$

$$C\left|u\right\rangle = \left|\bar{u}\right\rangle$$

$$C\left|d\right\rangle = \left|\bar{d}\right\rangle$$

$$C\left|\pi^0\right\rangle = +\left|\pi^0\right\rangle$$

Time reversal symmetry (invariance under change of time direction) does certainly not correspond to our daily experience. The macroscopic violation of *T* symmetry follows from maximising thermodynamic entropy (leaving a parking spot has a larger solution space than entering it). In the microscopic world of single particle reactions thermodynamic effects can be neglected, and *T* invariance is realised.

Discrete Symmetries

Discrete symmetry transformations lead to multiplicative conservation laws

The following discrete transformations are fundamental in particle physics:

- Parity ***P*** ("handedness"):

In particle physics:

These are interesting because it is not obvious whether the laws of nature should look the same for any of these transformations, and the answer was surprising when these symmetries were first tested !

$$C|u\rangle = |u\rangle$$

$$C|d\rangle = |\bar{d}\rangle$$

$$C|\pi^0\rangle = +|\pi^0\rangle$$

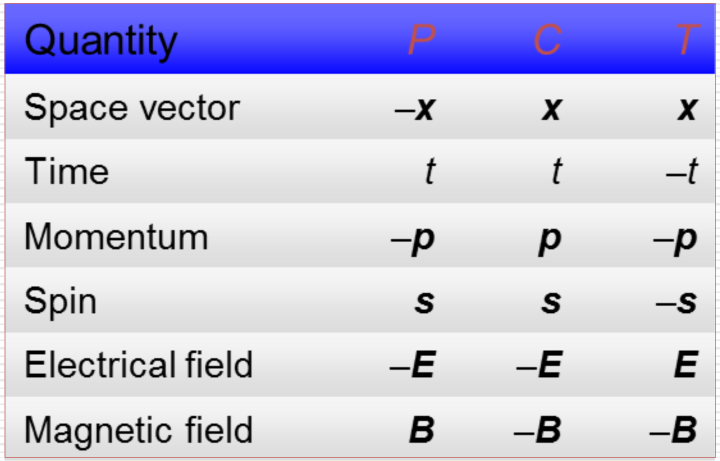
- Time reversal ***T*** :

The time arrow is reversed in the equations;

T invariance ▯ if a movement is allowed by a the physics law, the movement in the opposite direction is also allowed

Time reversal symmetry (invariance under change of time direction) does certainly not correspond to our daily experience. The macroscopic violation of *T* symmetry follows from maximising thermodynamic entropy (leaving a parking spot has a larger solution space than entering it). In the microscopic world of single particle reactions thermodynamic effects can be neglected, and *T* invariance is realised.

C, *P*, *T* Transformations and the *CPT* Theorem



Quantity	*P*	*C*	*T*
Space vector	$-\boldsymbol{x}$	$\boldsymbol{x}$	$\boldsymbol{x}$
Time	t	t	$-t$
Momentum	$-\boldsymbol{p}$	$\boldsymbol{p}$	$-\boldsymbol{p}$
Spin	$\boldsymbol{s}$	$\boldsymbol{s}$	$-\boldsymbol{s}$
Electrical field	$-\boldsymbol{E}$	$-\boldsymbol{E}$	$\boldsymbol{E}$
Magnetic field	$\boldsymbol{B}$	$-\boldsymbol{B}$	$-\boldsymbol{B}$

The *CPT* theorem (1954): **"Any Lorentz-invariant local quantum field theory is invariant under the successive application of *C*, *P* and *T*"**

proofs: G. Lüders, W. Pauli; J. Schwinger

- Fundamental consequences:
  - Relation between spin and statistics: fields with integer spin ("bosons") commute and fields with half-numbered spin ("fermions") anticommute ⍰ Pauli exclusion principle
  - Particles and antiparticles have **equal mass and lifetime**, equal magnetic moments with opposite sign, and **opposite quantum numbers**

- Best experimental test: $\left|\left(m_{K^0} - m_{\bar{K}^0}\right)/m_{K^0}\right| < 10^{-18}$

Not so in the microscopic World ?

Electromagnetic and strong interactions are (so far) *C*, *P* and *T* invariant

- Example: neutral pion decays via electromagnetic (EM) interaction : $\pi^0 \to \gamma\gamma$ but not $\pi^0 \to \gamma\gamma\gamma$

$$\pi^0 = \frac{1}{\sqrt{2}}\left[u\bar{u} - d\bar{d}\right]_{L=0,S=0} \quad \Rightarrow \quad C\left|\pi^0\right\rangle = +\left|\pi^0\right\rangle$$

$$C \cdot \vec{B},\vec{E} = -\vec{B},-\vec{E} \quad \Rightarrow \quad C\left|\gamma\right\rangle = -\left|\gamma\right\rangle$$

the initial (π^0) and final states ($\gamma\gamma$) are *C* even: hence, *C* is conserved !

- Generalization: $P\left|q\bar{q}'\right\rangle = (-1)^{L+1}\left|q\bar{q}'\right\rangle$, $C\left|q\bar{q}\right\rangle = (-1)^{L+S}\left|q\bar{q}\right\rangle$, $G\left|u\bar{u}(\bar{d})\right\rangle = (-1)^{L+S+I}\left|u\bar{u}(\bar{d})\right\rangle$

- Experimental tests of *P* and *C* invariance of the EM interaction:

$$C \text{ invariance: BR}\left(\pi^0 \to 3\gamma\right) < 3.1\times10^{-8}$$

$$P \text{ invariance: BR}\left(\eta \to 4\pi^0\right) < 6.9\times10^{-7}$$

- Experimental tests of *C* invariance of strong interaction: compare rates of positive and negative particles in reactions like: $p\bar{p} \to \pi^+\pi^- X,\ K^+K^- X$

And … the Surprise in Weak Interaction !

T.D. Lee and C.N. Yang pointed out in 1956 (to explain the observation of the decays $K \rightarrow 2\pi$ *and* 3π - the cosmic-ray θ/τ puzzle) that *P* invariance had not been tested in weak interaction → C.S. Wu performed in 1957 the experiment they suggested and observed parity violation

- Angular distribution of electron intensity:

$$I(\theta) = 1 + \alpha \frac{\vec{\sigma} \cdot \vec{P}_e}{E_e} = 1 + \boxed{\alpha \frac{v}{c}} \cos\theta$$

helicity

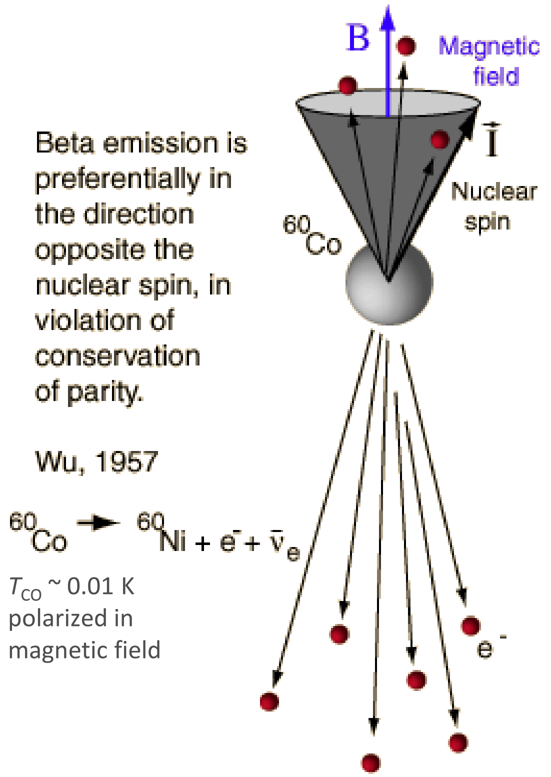
where:

$\vec{\sigma}$ - spin vector of electron

$\vec{P}_e$ - electron momentum

E_e - electron energy

$$\alpha = \begin{cases} -1 & \text{for electron} \\ +1 & \text{for positron} \end{cases}$$



Beta emission is preferentially in the direction opposite the nuclear spin, in violation of conservation of parity.

Wu, 1957

$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e$

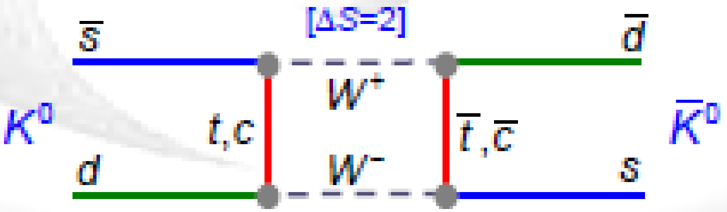
T_{CO} ~ 0.01 K polarized in magnetic field

- It was found that parity is even *maximally* violated in weak interactions !

Neutral Kaon Mixing

- Neutral kaons "mix" through the charged weak current, which does not conserve strangeness, neither *P* nor *C*. Weak interaction *cannot* distinguish K^0 from $\bar{K}^0$
- Simple picture: they mix through common virtual states:

- Because $\Delta m(K) = m(K_L) - m(K_S) = 3.5 \times 10^{-12}$ MeV > 0, a K^0 will change with time into a $\bar{K}^0$ and vice versa
- These oscillations are described in QCD by $\Delta S = 2$ Feynman "box" diagrams:



Neutral Kaon Mixing

- An initially pure K^0 state, will evolve into a superposition of states:

$$|K(t)\rangle = g(t)|K^0\rangle + h(t)|\bar{K}^0\rangle$$

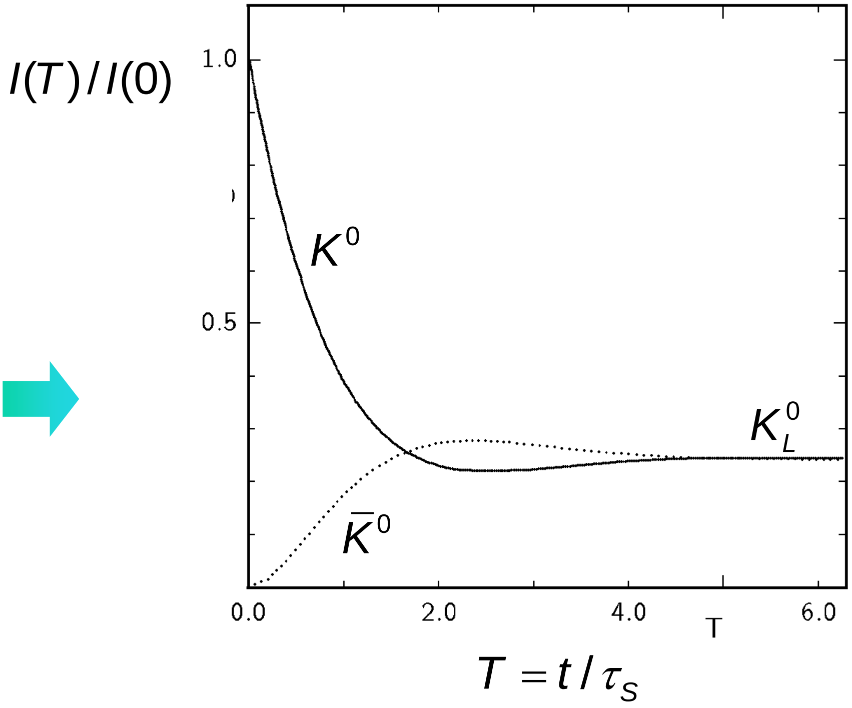
- The time dependence is obtained by solving the time-dependent Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = \left(M - \tfrac{i}{2}\Gamma\right)\begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix}$$

with 2x2 matrices M, Γ, of which the off-diagonals $\propto \Delta m$, $\Delta\Gamma$ govern the mixing

- The respective time-dependent intensities are found to be (neglecting *CP* violation):

$$I_{K^0}(t) \propto e^{-\Gamma_L t} + 2e^{-\Gamma_L t/2}\cos(\Delta m \cdot t)$$
$$I_{\bar{K}^0}(t) \propto e^{-\Gamma_L t} - 2e^{-\Gamma_L t/2}\cos(\Delta m \cdot t)$$



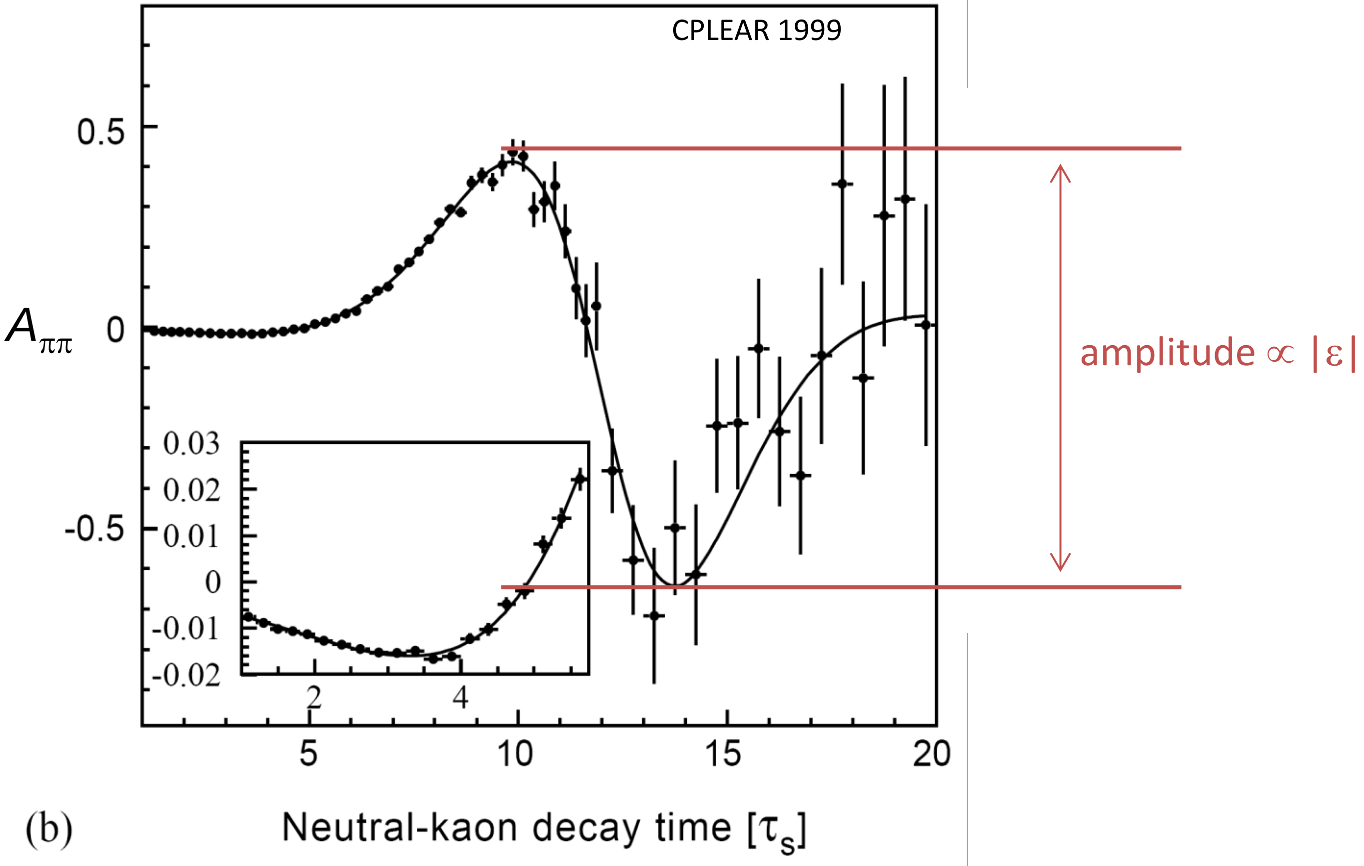
- After several K_S lifetimes, only K_L are left

Neutral Kaon Mixing and *CP* Violation

- Since K_S and K_L are not *CP* eigenstates, the time dependence has to be slightly modified by the size of ε, giving rise to an additional sine term.

Asymmetry: $$A_{\pi\pi} = \frac{\Gamma\left(\bar{K}^0 \to \pi^+\pi^-\right) - \Gamma\left(K^0 \to \pi^+\pi^-\right)}{\Gamma\left(\bar{K}^0 \to \pi^+\pi^-\right) + \Gamma\left(K^0 \to \pi^+\pi^-\right)} \propto |\varepsilon| \cos(\Delta m \cdot t - \varphi)$$

Neglecting other sources of *CP* violation & assuming arg(ε) = π/4.

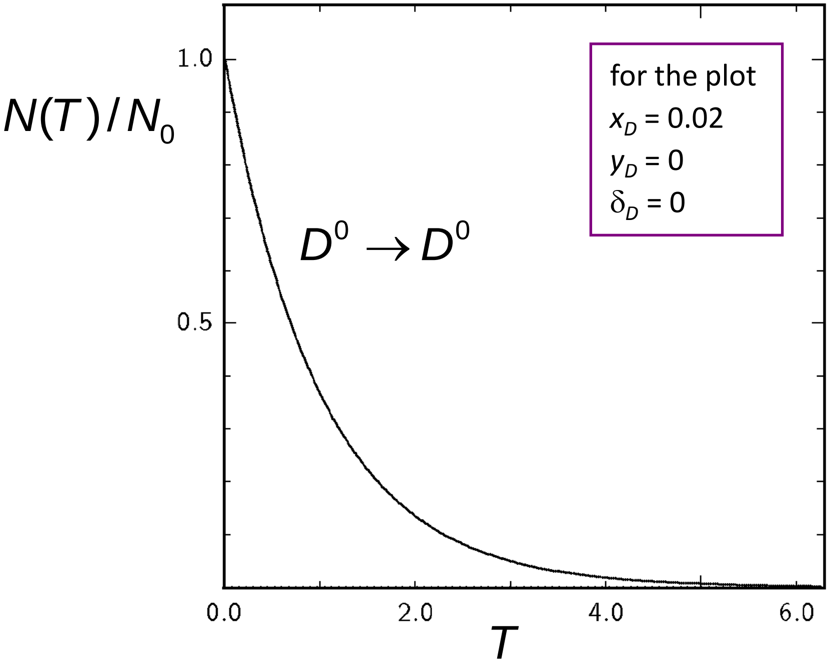


There are in Fact Four Meson Systems with Mixing

- Pairs of self-conjugate mesons that can be transformed to each other via flavor changing weak interaction transitions are:

$$|K^0\rangle = |\bar{s}d\rangle \qquad |D^0\rangle = |c\bar{u}\rangle \qquad |B_d^0\rangle = |\bar{b}d\rangle \qquad |B_s^0\rangle = |\bar{b}s\rangle$$

- They have very different oscillation properties that can be understood from the “CKM couplings” (see later in this lecture) occurring in the box diagrams



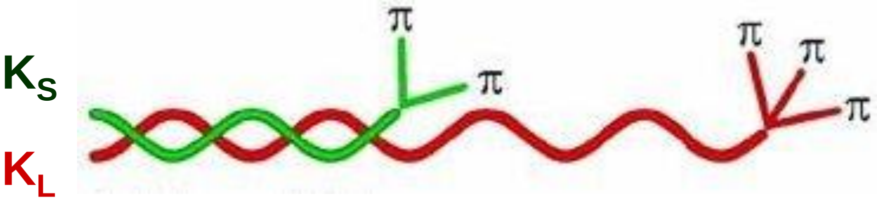
for the plot
$x_D = 0.02$
$y_D = 0$
$\delta_D = 0$

mixing probability:
$\chi \sim 2\text{x}10^{-6}$

CP violation

From Schrödinger eqn:

$$|K_{S,L}(t)\rangle = e^{-im_{S,L}t}\, e^{-\Gamma_{S,L}t/2}\, |K_{S,L}(0)\rangle$$



3 types of CP violation:

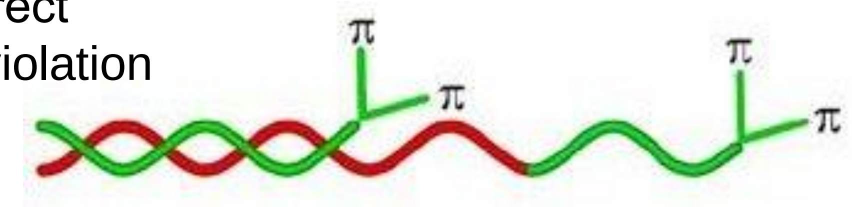
violation in mixing

$$\mathrm{Prob}(K^0 \to \bar{K}^0) \neq \mathrm{Prob}(\bar{K}^0 \to K^0)$$

violation in interference

$$\mathrm{Prob}(K^0(t) \to \pi^+\pi^-) \neq \mathrm{Prob}(\bar{K}^0(t) \to \pi^+\pi^-)$$

Parameter ε
“indirect”
CP violation

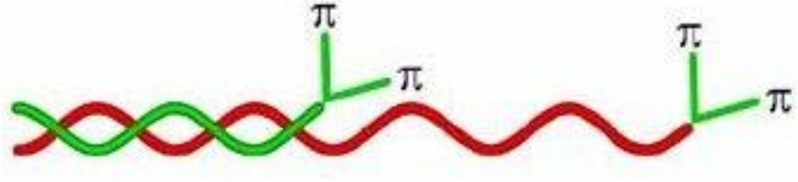


violation in decays

$$\mathrm{Prob}(K \to f) \neq \mathrm{Prob}(\bar{K} \to \bar{f})$$

“direct”
CP violation

Parameter ε’



“Direct” *CP* Violation = *CP* Violation in Decay

- General signature: rate differences between *CP*-conjugated processes:

$$\Gamma\left(|i\rangle \to |f\rangle\right) \neq \bar{\Gamma}\left(|\bar{i}\rangle \to |\bar{f}\rangle\right)$$

- It *necessarily* involves interference of amplitudes contributing to the processes.
  - To obtain interference, we need phases that change sign under *CP*
  - Example: if the decay amplitudes are given by: $\{a_{1,2}, \phi_{1,2} \in \Re$

$$A\left(|i\rangle \to |f\rangle\right) = a_1 e^{i\theta_1} e^{i\phi_1} + a_2 e^{i\theta_2} e^{i\phi_2}$$

$$\bar{A}\left(|\bar{i}\rangle \to |\bar{f}\rangle\right) = \left(a_1 e^{i\theta_1} e^{-i\phi_1} + a_2 e^{i\theta_2} e^{-i\phi_2}\right) \underbrace{e^{-2i(\xi_i - \xi_f)}}_{\text{unphysical phase}}$$

$$\begin{cases} \phi_j & \text{alters sign under } CP \text{ (“weak phase”)} \\ \theta_j & CP \text{ invariant (“strong phase”)} \end{cases}$$

where: $\Gamma\left(|i\rangle \to |f\rangle\right) \propto \left|A\left(|i\rangle \to |f\rangle\right)\right|^2$ and $\bar{\Gamma}\left(|\bar{i}\rangle \to |\bar{f}\rangle\right) \propto \left|\bar{A}\left(|\underline{i}\rangle \to |\bar{f}\rangle\right)\right|^2$

We can define the following *CP* asymmetry A_{CP}:

$$A_{CP} = \frac{\bar{\Gamma}\left(|\bar{i}\rangle \to |\bar{f}\rangle\right) - \Gamma\left(|i\rangle \to |f\rangle\right)}{\bar{\Gamma}\left(|\bar{i}\rangle \to |\bar{f}\rangle\right) + \Gamma\left(|i\rangle \to |f\rangle\right)} = \frac{2a_1 a_2 \sin(\theta_1 - \theta_2)\sin(\phi_1 - \phi_2)}{a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2)\cos(\phi_1 - \phi_2)}$$

CP Violation in the Kaon Decay

- We have seen that at least two amplitudes with different *CP*-violating (weak) *and* conserving (strong) phases have to contribute to the decay for direct CPV. This suppresses this type of CPV, so that the observable effect should be small compared to ε.

- To allow for (*small*) direct CPV, we need to slightly modify our previous definitions:

$$|\varepsilon + \varepsilon'|^2 = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} \quad \text{and use also:} \quad |\varepsilon - \textcircled{2}\,\varepsilon'|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)}$$

"Clebsch-Gordon isospin" factor when passing from charged to neutral pions

- If the observed *CP* violation is different in the two decay modes, we have a prove for a contribution from direct *CP* violation. From the measurement of the *ratio of these decay-rate ratios* we can determine ε'

The observable

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \Big/ \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = \left|\frac{\varepsilon - 2\varepsilon'}{\varepsilon + \varepsilon'}\right|^2 \overset{\varepsilon' \ll \varepsilon}{\cong} 1 - 6 \times \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)$$

First order Taylor expansion

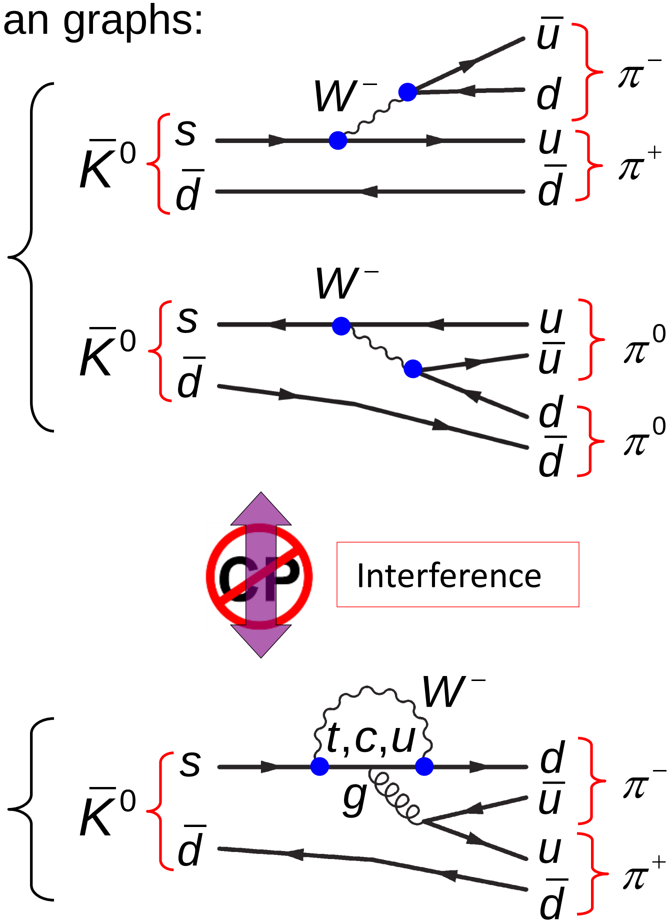
The Discovery of *CP* Violation in the Decay

- Due to the smallness of the effect, it took several experiments and over 30 years of effort to establish the existence of direct CPV

- Feynman graphs:

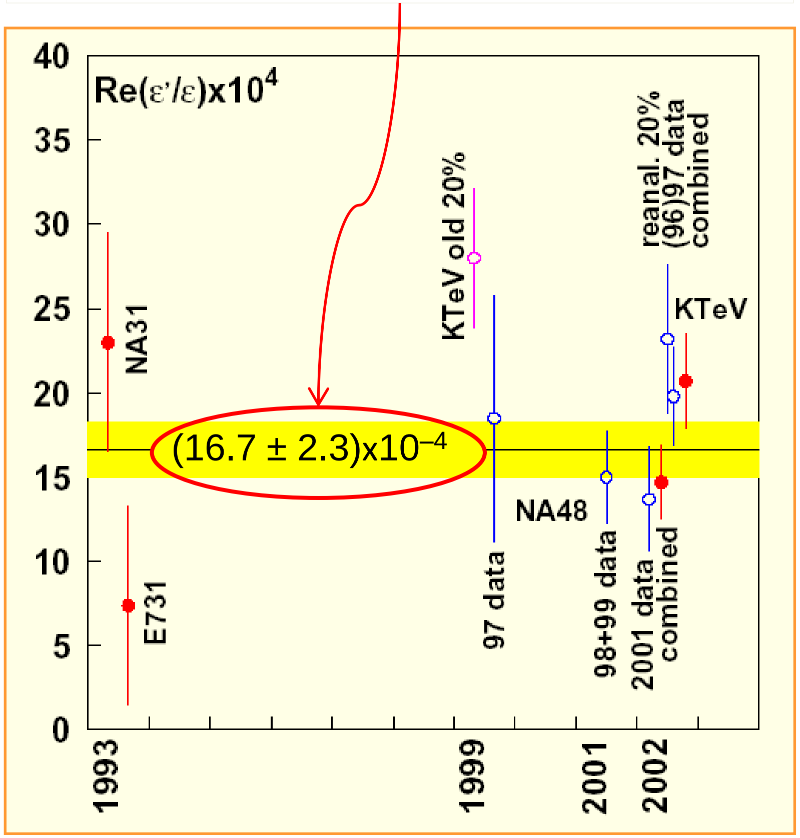
"Tree"
(born-level)
amplitudes

"Penguin"
(loop-level)
amplitude

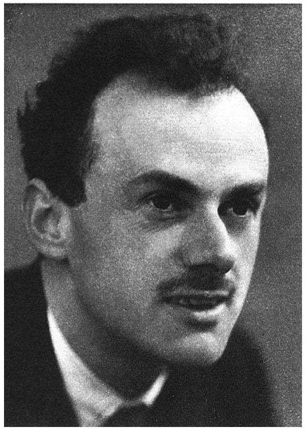


Experimental average

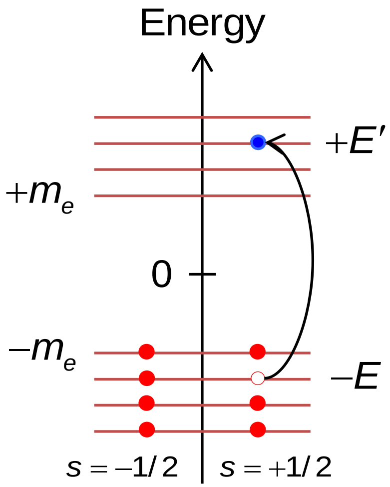
Indeed, a *very* small CPV effect !



Anti-particles



Dirac, imagining holes and seas in 1928



This picture fails for bosons !

- Combining quantum mechanics with special relativity, and the wish to linearize δ/δ*t,* leads Dirac to the equation

$$i\gamma^{\mu}\partial_{\mu}\psi(x,t)-m\psi(x,t)=0$$

(1928)

for which **solutions with negative energy** appear

- Vacuum represents a “sea” of such negative-energy particles (fully filled according to Pauli’s principle)

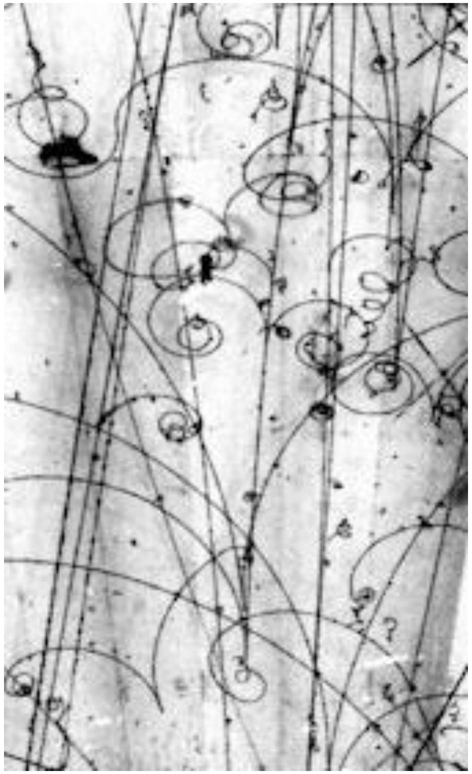
- Dirac identified holes in this sea as “antiparticles” with opposite charge to particles … (however, he conjectured that these holes were protons, despite their large difference in mass, because he thought “positrons” would have been discovered already)

- An electron with energy *E* can fill this hole, emitting an energy 2*E* and leaving the vacuum (hence, the hole has effectively the charge +*e* and positive energy).

Particles and Antiparticles Annihilate

What happens if we bring particles and antiparticles together ?

- A particle can annihilate with its antiparticle to form gamma rays
- An example whereby matter is converted into pure energy by Einstein's formula $E = mc^2$
- Conversely, gamma rays with sufficiently high energy can turn into a particle-antiparticle pair

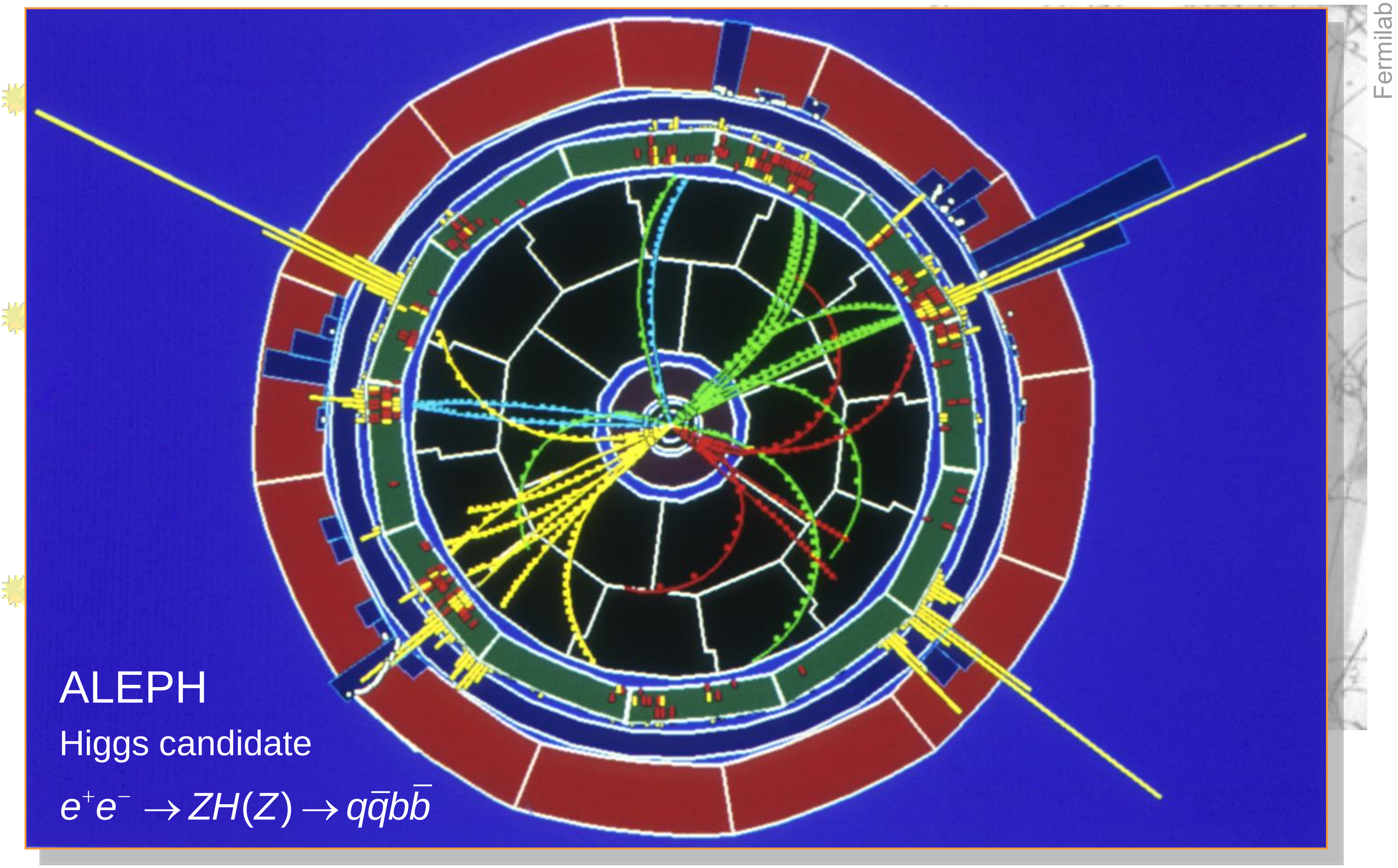


Fermilab

Particle-antiparticle tracks in a bubble chamber

Particles and Antiparticles Annihilate

What happens if we bring particles and antiparticles together ?



ALEPH

Higgs candidate

$e^+e^- \to ZH(Z) \to q\bar{q}b\bar{b}$

So the Standard Model can handle both particles and anti-particles

in most cases same couplings

What about anti-matter in our Universe?

Antimatter in the Universe ?

- Does stable antimatter exist in the universe ?

Balloon-borne Superconducting Solenoidal (BESS) spectrometer

  - No antinuclei (e.g., Antihelium) seen in cosmic rays (relative limit from BESS: $< 10^{-6}$)
  - No significant (diffuse) cosmic γ rays from nucleon-antinucleon annihilation in the boundary between matter & antimatter regions

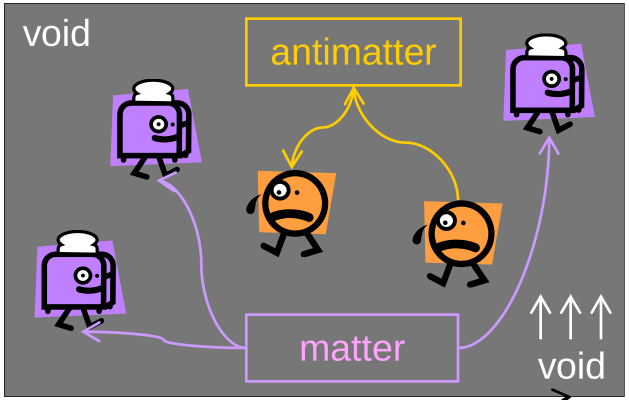
➡ No evidence of antimatter in our domain of the universe (~20 Mpc = 0.6×10^8 light years)

- Could our universe be like inverse Swiss cheese, with distant matter or antimatter regions(*) ?

➡ Difficult within the current limits

- Likely: no antimatter in our universe

(apart from the antimatter created dynamically in particle collisions)



The *voids* would create anisotropy in CMB spectrum, which is not seen

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(*) “If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of nature,we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons. In fact there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them from present astronomical methods." P. A. M. Dirac, Nobel Lecture (1933)

CP violation and the baryon asymmetry

We can estimate the magnitude of the baryon asymmetry of the Universe caused by KM CP violation

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \sim \frac{J \times P_u \times P_d}{M^{12}}$$

$$J = \cos(\theta_{12})\cos(\theta_{23})\cos^2(\theta_{13})\sin(\theta_{12})\sin(\theta_{23})\sin(\theta_{13})\sin(\delta)$$
$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$
$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

- The Jarlskog parameter J is a parametrization invariant measure of CP violation in the quark sector: J ~ O(10^{-5})
- The mass scale M can be taken to be the electroweak scale O(100 GeV)
- This gives an asymmetry O(10^{-17}):

much much below the observed value of O(10^{-10})

What causes the difference between matter and antimatter?

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks:
  - It is a 3x3 complex unitary matrix described by 9 (real) parameters:
    - 5 can be absorbed as phase differences between the quark fields
    - 3 can be expressed as (Euler) mixing angles
    - the fourth makes the CKM matrix complex (i.e. gives it a phase)
      - weak interaction couplings differ for quarks and antiquarks
      - CP violation

We need more CP violation

To create a larger asymmetry, require:

- new sources of CP violation
  - that occur at high energy scales

Where might we find it?

- lepton sector: CP violation in neutrino oscillations
- quark sector: discrepancies with KM predictions
- gauge sector, extra dimensions, other new physics:
  - precision measurements of flavour observables are generically sensitive to additions to the Standard Model

- Cp violation in the SM

Parameters of the Standard Model

- 3 gauge couplings
- 2 Higgs parameters
- 6 quark masses
- 3 quark mixing angles + 1 phase
- 3 (+3) lepton masses
- (3 lepton mixing angles + 1 phase)

Cabibbo–Kobayashi–Maskawa

CKM matrix

PMNS matrix

Pontecorvo–Maki–Nakagawa–Sakata

flavour parameters

( ) = with Dirac neutrino masses

What breaks the flavour symmetries?

- In the Standard Model, the vacuum expectation value of the Higgs field breaks the electroweak symmetry
- Fermion masses arise from the Yukawa couplings of the quarks and charged leptons to the Higgs field (taking $m_\nu=0$)
- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks
- Consequently, the only flavour-changing interactions are the charged current weak interactions
    - no flavour-changing neutral currents (GIM mechanism)
    - not generically true in most extensions of the SM
    - flavour-changing processes provide sensitive tests

Flavour for new physics discoveries

A lesson from history:

- New physics shows up at precision frontier before energy frontier
  - GIM mechanism before discovery of charm
  - CP violation / CKM before discovery of bottom & top
  - Neutral currents before discovery of Z

FCNC suppressed
$\Delta S=2$ suppressed
wrt $\Delta S=1$

- Particularly sensitive – loop processes
  - Standard Model contributions suppressed / absent
  - flavour changing neutral currents (rare decays)
  - CP violation
  - lepton flavour / number violation / lepton universality

More on the CKM matrix

CKM matrix in the Standard Model

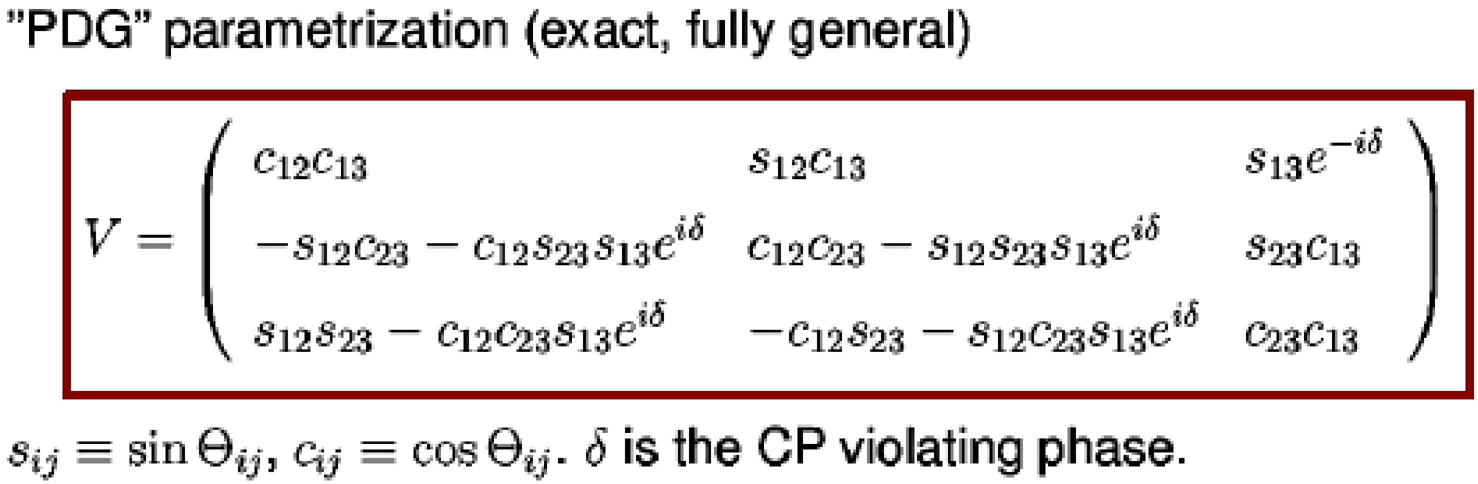
The charged current interaction gets a flavor structure, encoded in the Cabibbo Kobayashi Maskawa (CKM) matrix V.

$$\boxed{\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(\bar{\tilde{U}}_L \gamma^\mu W_\mu^+ V \tilde{D}_L + \bar{\tilde{D}}_L \gamma^\mu W_\mu^- V^\dagger \tilde{U}_L \right) .}$$

V_{ij} connects left-handed up-type quark of the ith gen. to left-handed down-type quark of jth gen. Intuitive labelling by flavor:

$$\boxed{V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}} \qquad V_{13} = V_{ub} \;\; etc$$

Via W exchange is the only way to change flavor in the SM.



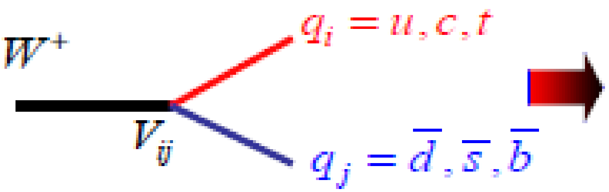
"PDG" parametrization (exact, fully general)

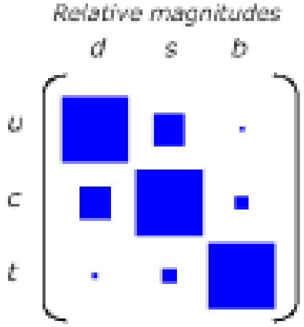
$$\boxed{V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}}$$

$s_{ij} \equiv \sin\Theta_{ij}$, $c_{ij} \equiv \cos\Theta_{ij}$. δ is the CP violating phase.

CKM matrix in the Standard Model

- Quarks change type in weak interactions:





$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- We parameterise the couplings V_{ij} in the CKM matrix:

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

where here I use
the Buras correction
to the Wolfestein
parameterisation

$$\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{\mathrm{ud}}V_{\mathrm{ub}}^*}{V_{\mathrm{cd}}V_{\mathrm{cb}}^*}$$

$$\bar{\rho} = \rho\,(1-\lambda^2/2)$$
$$\bar{\eta} = \eta\,(1-\lambda^2/2)$$

Unitarity relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

multiply with its conjugate transpose
$VV^\dagger = V^\dagger V = 1$

$$\Sigma_i V_{ij} V^*_{ik} = \delta_{jk}$$

column orthogonality

$$\Sigma_j V_{ij} V^*_{kj} = \delta_{ik}$$

row orthogonality

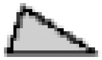
Unitarity relations

column orthogonality

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* \simeq \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$

Areas have to be the same
→ Jarlskog parameter

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* \simeq \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0$$

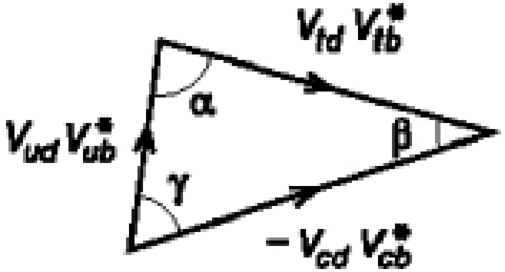


$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

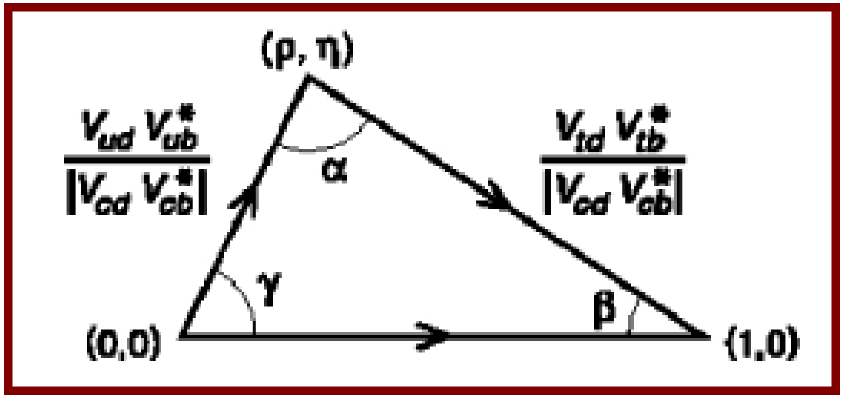
Third unitarity relation

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}\simeq\mathcal{O}(\lambda^{3})+\mathcal{O}(\lambda^{3})+\mathcal{O}(\lambda^{3})=0$$

$V_{id}V^{*}_{ib}=0$ represents the orthogonality condition between the first and the third column of the CKM matrix (the orientation depends on the phase convention)



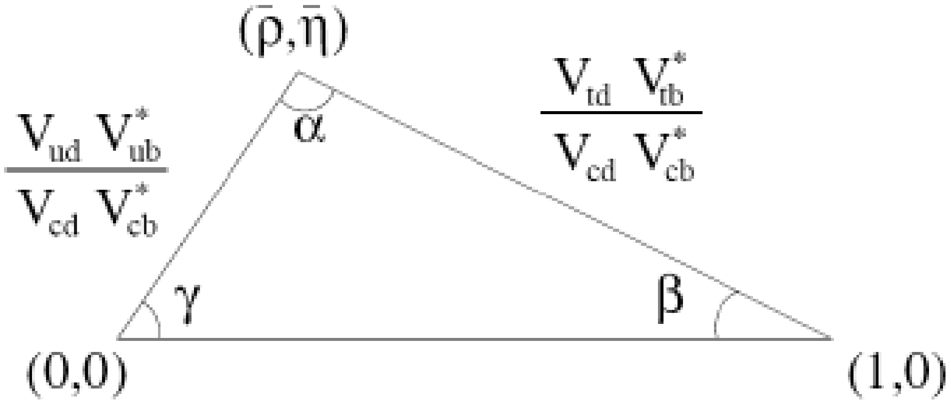
re-scaled version where sides have been divided by $|V_{cd}V^{*}_{cb}|$



In terms of the Wolfenstein parameterization, the coordinates of this triangle are (0, 0), (1, 0) and (ρ, η):
the two sides are $(\rho - i\eta)$ and $(1 - \rho + i\eta)$.

Probing the structure of the CKM mechanism

$$V_{ud}V_{ub}^* + V_{td}V_{tb}^* + V_{cd}V_{cb}^* = 0$$



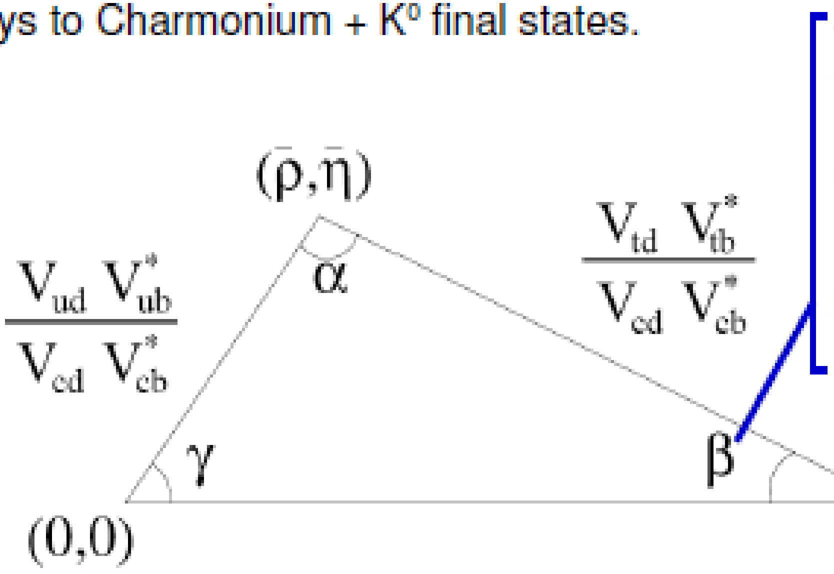
The angles can be written in terms of CKM matrix elements as:

$$\alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$$
$$\beta \equiv \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$$
$$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$$

- We need to measure the angles and sides to over-constrain this triangle, and test that it closes.
- Need experiments to measure these quantities

Constraining the angles

Theoretically clean (SM uncertainties ~10^{-2} to 10^{-3}) tree dominated decays to Charmonium + K^0 final states.



$b \to c\bar{c}s$

$B^0 \to J/\psi K_L^0$

$B^0 \to J/\psi K_S^0$

$B^0 \to \psi(2S) K_S^0$

$B^0 \to \chi_{1c} K_S^0$

$B^0 \to \eta_c K_S^0$

$B^0 \to J/\psi K^{*0}$

$B \to J/\psi \pi^0$

$B \to D^{(*)+} D^{(*)-}$

$B \to \eta' K^0$

$B \to \rho K^0$

$B \to \omega K^0$

$B \to \pi^0 K^0$

$B \to \phi K^{(*)0}$

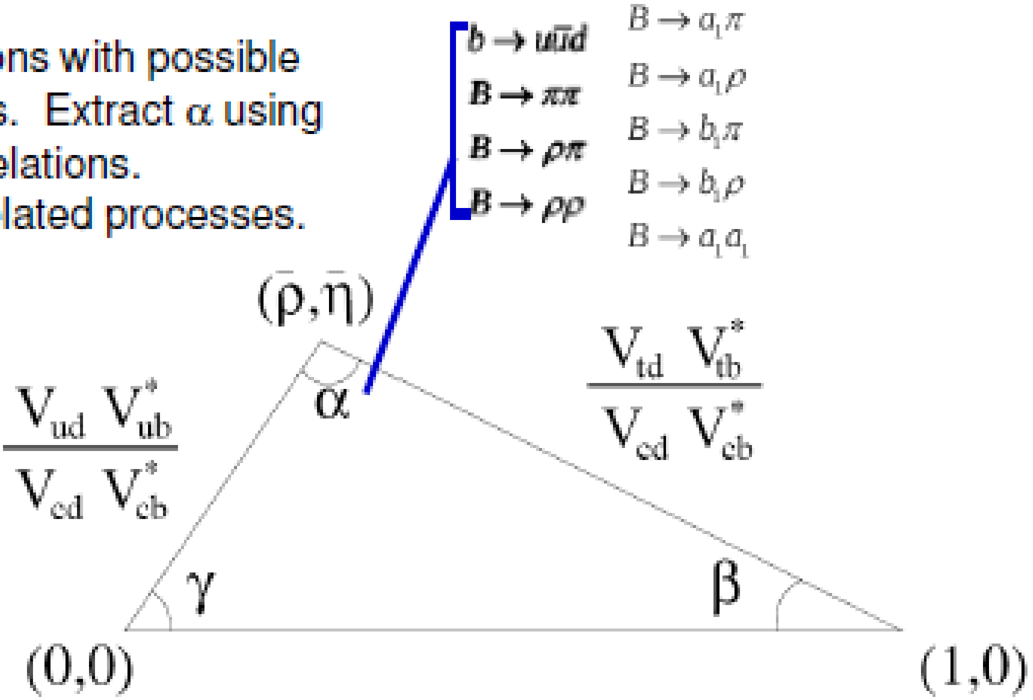
$B \to KKK^0$

$B \to f^0(980) K^0$

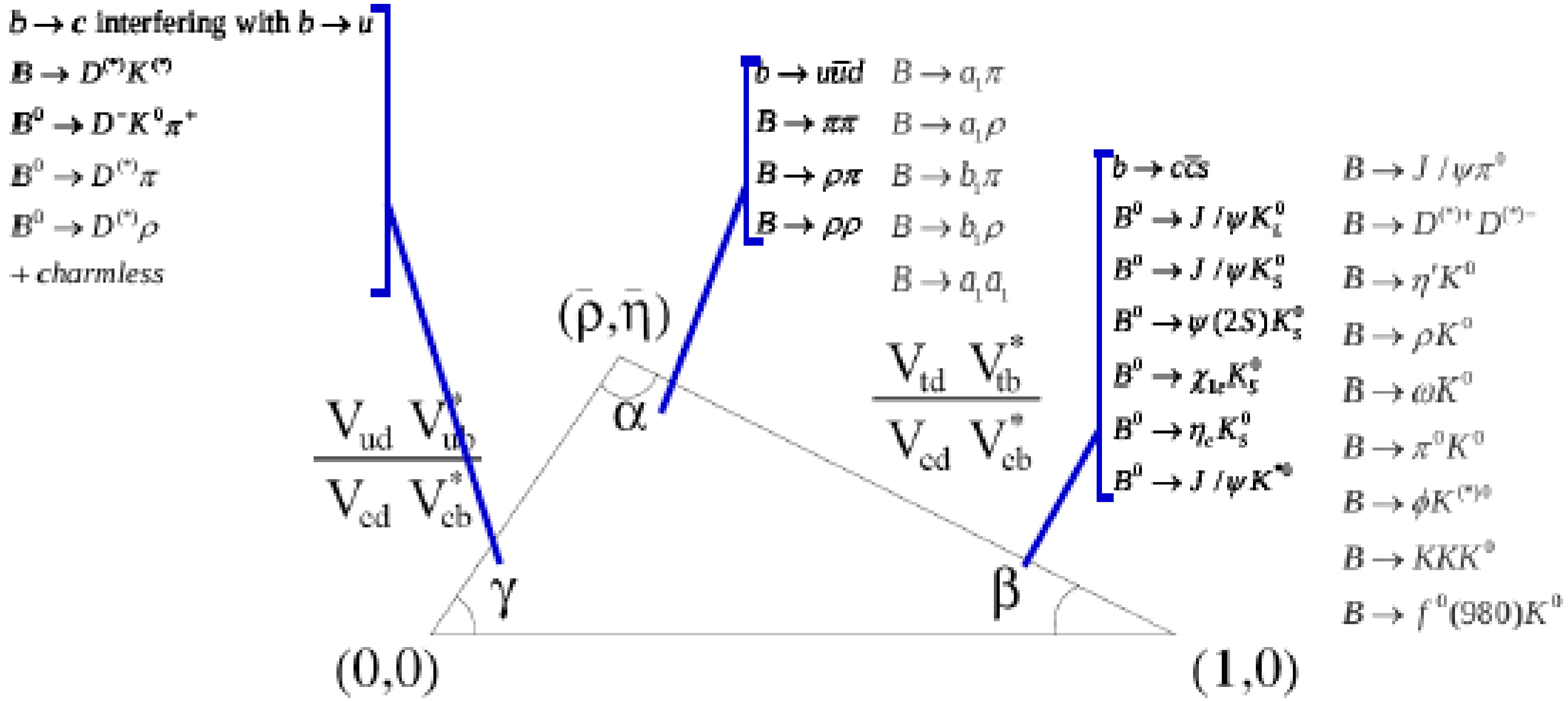
Constraining the angles

b → $\bar{u}ud$ transitions with possible loop contributions. Extract α using

- SU(2) Isospin relations.
- SU(3) flavour related processes.



Constraining the angles



$b \to c$ interfering with $b \to u$
$B \to D^{(*)}K^{(*)}$
$B^0 \to D^- K^0 \pi^+$
$B^0 \to D^{(*)}\pi$
$B^0 \to D^{(*)}\rho$
$+$ *charmless*

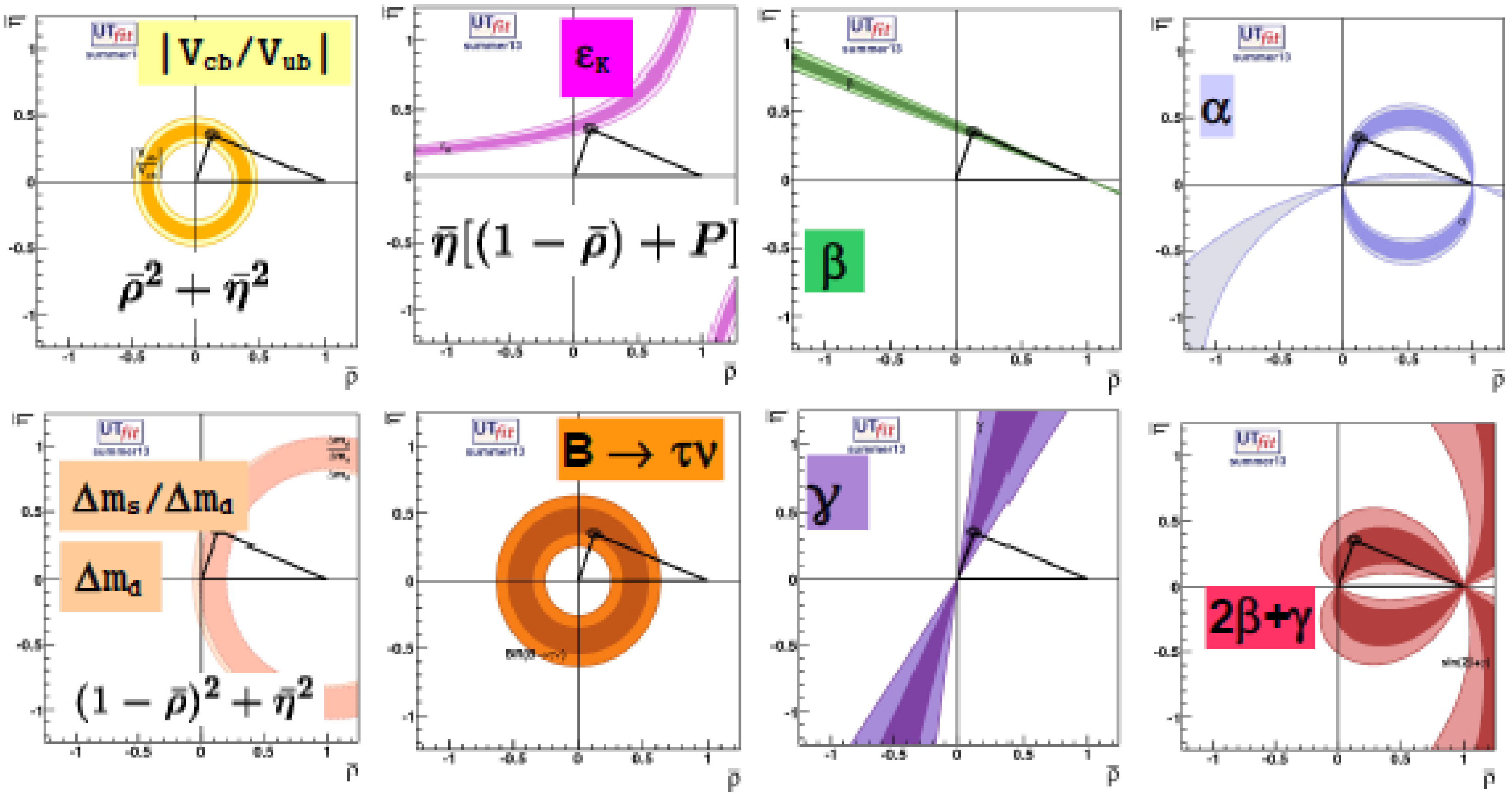
$b \to u\bar{u}d$
$B \to \pi\pi$
$B \to \rho\pi$
$B \to \rho\rho$

$B \to a_1\pi$
$B \to a_1\rho$
$B \to b_1\pi$
$B \to b_1\rho$
$B \to a_1a_1$

$b \to c\bar{c}s$
$B^0 \to J/\psi K_L^0$
$B^0 \to J/\psi K_S^0$
$B^0 \to \psi(2S)K_S^*$
$B^0 \to \chi_{1c}K_S^*$
$B^0 \to \eta_c K_S^0$
$B^0 \to J/\psi K^{*0}$

$B \to J/\psi\pi^0$
$B \to D^{(*)+}D^{(*)-}$
$B \to \eta' K^0$
$B \to \rho K^0$
$B \to \omega K^0$
$B \to \pi^0 K^0$
$B \to \phi K^{(*)0}$
$B \to KKK^0$
$B \to f^0(980)K^0$

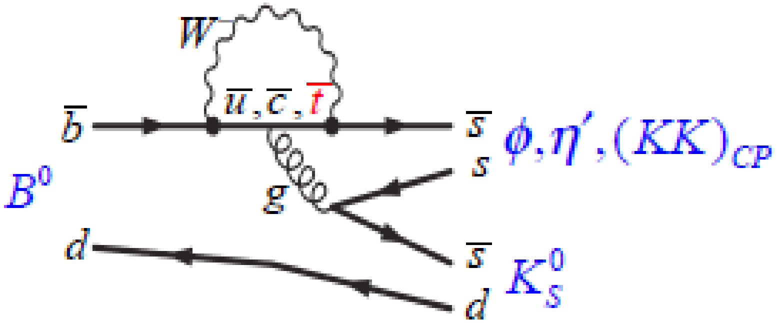
Unitarity Triangle analysis in the SM



So far it closes – all measurements consistent

CP violation: Searching for new physics

- sin2β has been measured to O(1°) accuracy in b → $\bar{c}cs$ decays.
- Can use this to search for signs of New Physics (NP) if:
  - Identify a rare decay sensitive to sin2β (loop dominated process).
  - Measure S precisely in that mode (S_{eff}).
  - Control the theoretical uncertainty on the Standard Model 'pollution' (ΔS_{SM}).
  - Compute $\Delta S_{\mathrm{NP}} = S_{eff} - S_{c\bar{c}s} - \Delta S_{\mathrm{SM}}$
- In the presence of NP: $\Delta S_{NP} \neq 0$



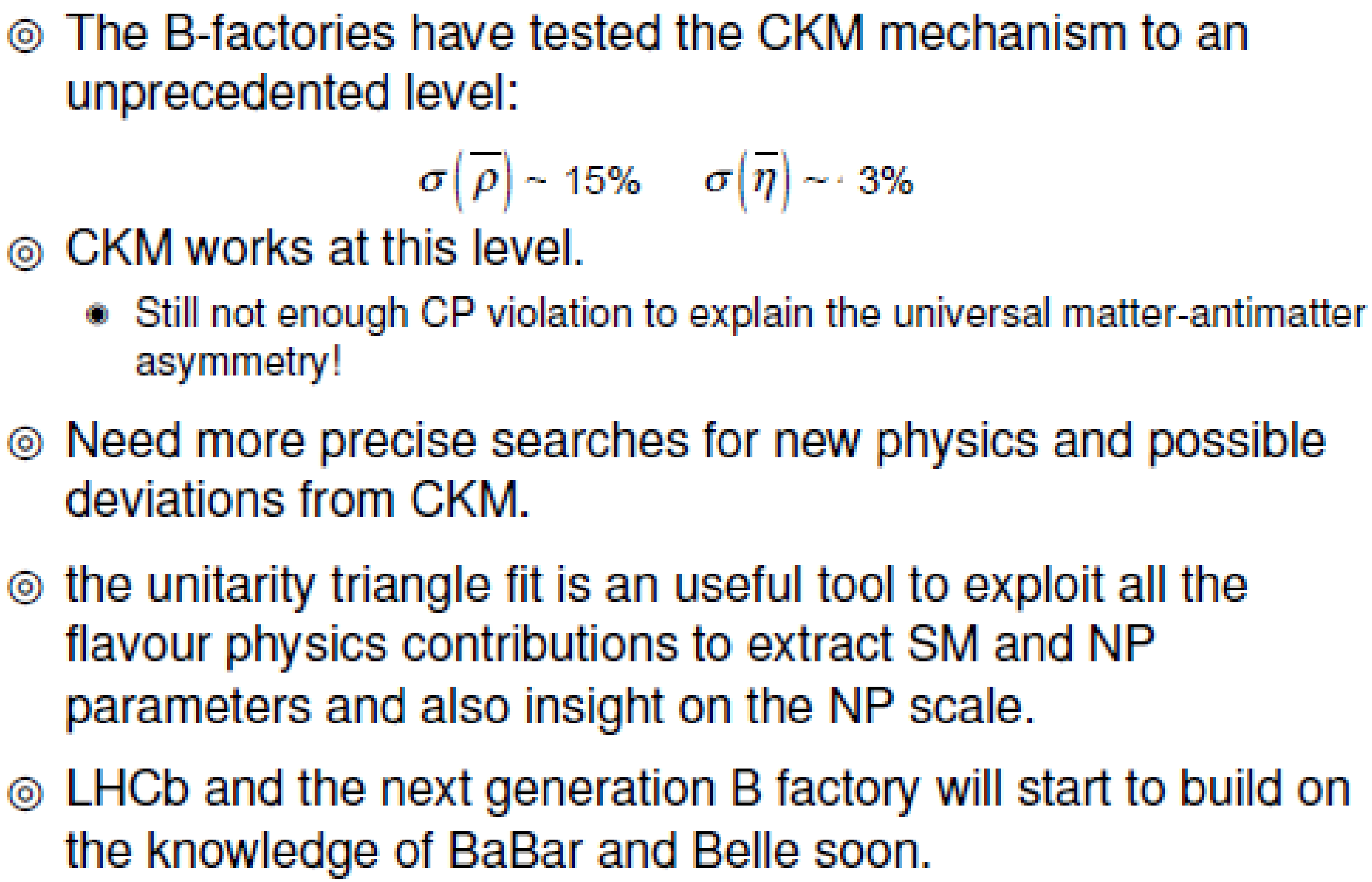
- Many tests have been performed in:
  - B→d processes.
  - B→s processes.

► Unknown heavy particles can introduce new amplitudes that can affect physical observables of loop dominated processes.

► Observables that might be affected include branching fractions, CP asymmetries, forward backward asymmetries … and so on.

► A successful search requires that we understand Standard Model contributions well!

Summary



- The B-factories have tested the CKM mechanism to an unprecedented level:

$$\sigma\left(\bar{\rho}\right) \sim 15\% \qquad \sigma\left(\bar{\eta}\right) \sim 3\%$$

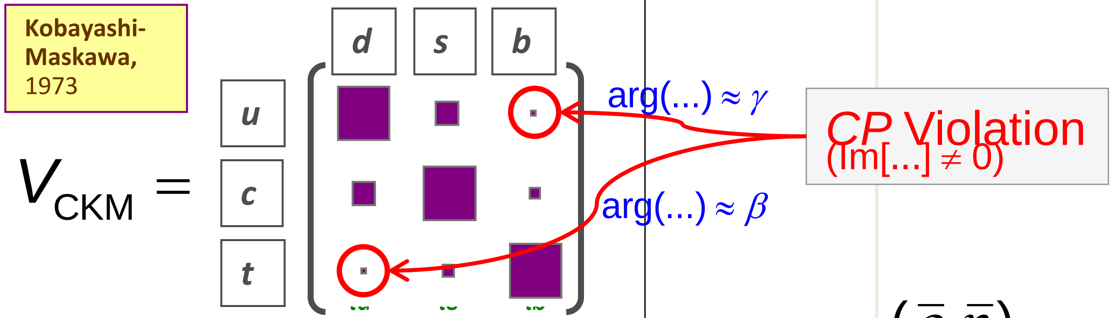
- CKM works at this level.
  - Still not enough CP violation to explain the universal matter-antimatter asymmetry!
- Need more precise searches for new physics and possible deviations from CKM.
- the unitarity triangle fit is an useful tool to exploit all the flavour physics contributions to extract SM and NP parameters and also insight on the NP scale.
- LHCb and the next generation B factory will start to build on the knowledge of BaBar and Belle soon.

Summary

- The study of CP violation is a fundamental part of particle physics, and cosmology.
  - It revolves around EPR experiments with correlated B, D, K, mesons, and quantum interference studies.
- We don't really understand it.
  - The CKM mechanism works well but it is incomplete. It is only a small part of the story. We don't know if CP violation in leptons, or some new physics scenario really explains the matter-antimatter asymmetry questions arising from the Big Bang.
  - Eventually we hope to understand the reason behind this conundrum, and in doing so we will either find new particle physics, or new cosmological effects.
    - Given that its very hard to have an asymmetry in the big bang that doesn't get washed out by inflation – the money is on new physics effects/particles to be discovered!

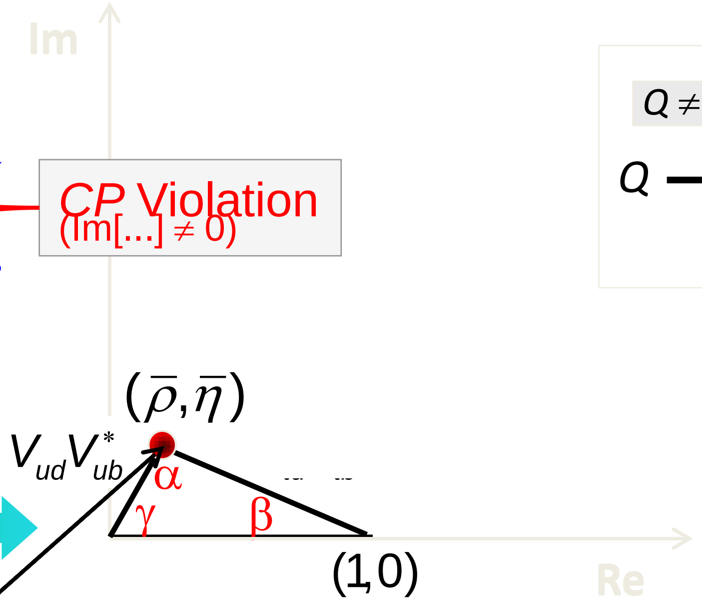
One slide: The CKM Matrix and the Unitarity Triangle

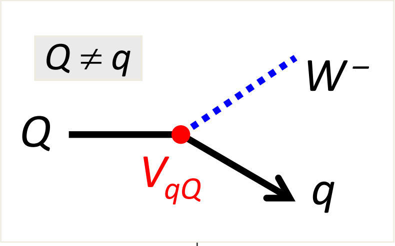
Kobayashi-Maskawa,
1973



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\left(\propto A\lambda^3 \ \propto -A\lambda^3 \ \propto A\lambda^3\right)$$

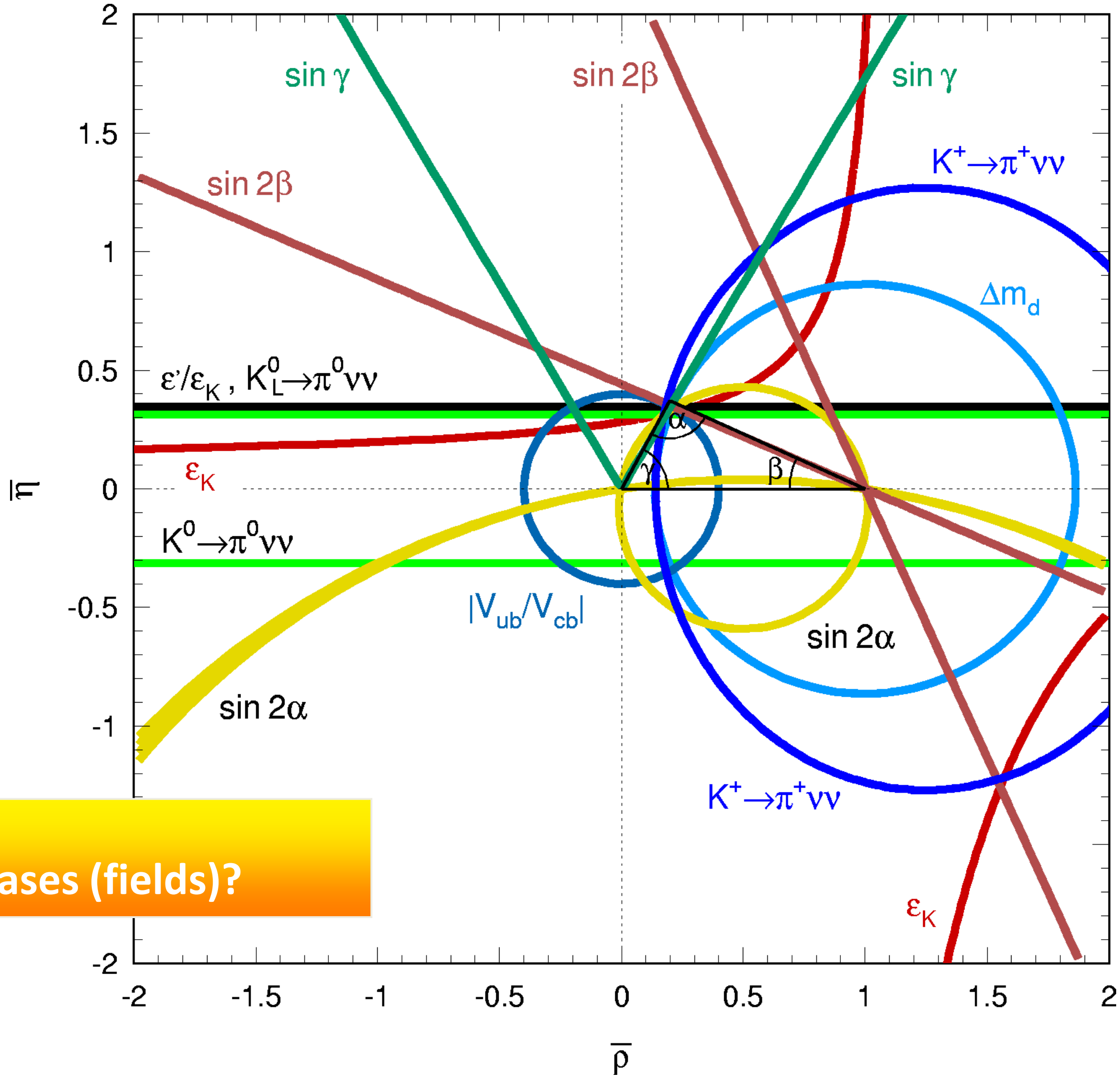




$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \underbrace{\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}}_{} = 0$$

phase invariant : $\bar{\rho} + i\bar{\eta}$

One slide: The CKM Matrix and the Unitarity Triangle



Culminating Point

SM or new phases (fields)?

Observables for direct ~~CP~~

CPV effect small, direct CPV expected to be even smaller or zero

If no direct CPV then the observable ratios of $K_{L,S}$ to $\pi^+\pi^-$ and $\pi^0\pi^0$ should both equal ε:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon' \qquad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon'$$

The ratio between the rates related to the ratio of direct to indirect CPV:

$$\boxed{\mathrm{Re}(\varepsilon'/\varepsilon) \cong \frac{1}{6}\left[\left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 - 1\right] \cong \frac{1}{6}\left[\frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)} - 1\right]}$$

Rare decays **"normal" decays**

From theory:

- Standard Model: Re(ε'/ε) ~ 0 – 30 × 10^{-4}
- Superweak theory: Re(ε'/ε) = 0

A=amplitude
Γ=decay rate