FYST17 Exercises: QGP and heavy-ion physics

In order to pass one needs to solve at least 3 of the 4 exercises. To obtain the best grade one has to solve all 4 exercises.

Please note on the exercise if you worked together with anyone in solving it. The maximum group size is 4 people.

Each student always has to submit an individually formulated solution to the exercises.

1.1 Kinematics of heavy-ion collisions

- a) Show that if the maximum beam energy for pp collisions is $\sqrt{s_{pp}}$, the maximal Pb-Pb beam energy is $\sqrt{s_{Pb-Pb}} = Z\sqrt{s_{pp}}$ (so that the beam energy per nucleon pair is $\sqrt{s_{NN}} = \frac{Z}{A}\sqrt{s_{pp}}$).
- b) Consider each nuclei as a disk with radius $r_A = 1.2 \times A^{1/3}$ fm and assume that the two nuclei interact if the disks overlap, i.e., when the impact parameter (b) is less than $2r_A$. What range of impact parameters does the most central 0-5% Pb-Pb collisions correspond to?

(0-5% centrality is the 5\% fraction of the total cross section with smallest impact parameters)

Hint: use simple geometry to express the differential cross section $d\sigma/db$ and the total cross section

1.2 Azimuthal anisotropic flow

We assume in the following that the particles in an event is distributed according to:

$$f(\varphi) = \frac{1}{2\pi} \left(1 + \sum_{i=1}^{\infty} 2v_i \cos[i(\varphi - \Psi_i)] \right), \tag{1}$$

where φ is in the range from $0-2\pi$, v_i is the *i*-th order flow coefficient and Ψ_i is called the *i*-th order symmetry plane (really an angle in general). We only consider measurements of the elliptic flow, v_2 , in the following, so that:

$$f(\varphi) = \frac{1}{2\pi} \left(1 + 2v_2 \cos[2(\varphi - \Psi_2)] \right).$$
 (2)

- a) Make a drawing that illustrates the initial elliptic overlap region, indicating Ψ_2 .
- b) The event plane method. Show by explicit calculation that Ψ_2 can be estimated as:

$$\Psi_2 = \frac{1}{2} \tan^{-1} \left(\frac{\langle \sin(2\varphi) \rangle}{\langle \cos(2\varphi) \rangle} \right), \tag{3}$$

where the brackets indicate averages.

Hint: calculate the means using trigonometric identities.

c) Two particle correlations. The Event Plane method is criticized for the need to first determine the event plane and then measure v_2 since the statistical precision with which we can determine the event plane event-by-event will affect the result (even one typically corrects for this using a resolution function). One can avoid this by studying 2-particle correlations¹. Show that

$$\left\langle \cos[2(\varphi_1 - \varphi_2)] \right\rangle = v_2^2,\tag{4}$$

where the brackets indicate an average over all pairs, and the assumption is that there are no direct correlations between particle 1 and 2, but only indirect correlations through the common event plane Ψ_2 .

1.3 Jet quenching



FIG. 16. (Color online) Method of calculating average fractional momentum loss ($S_{\text{loss}} \equiv \delta p_T / p_T$). This plot is for illustration only; errors are not shown. In the order of procedure: (1) Scale the p + pdata by T_{AA} corresponding to centrality selection of Au + Au data, (2) shift the p + p points closest to Au + Au in yield, and (3) calculate momentum difference of p + p and Au + Au points.

PHENIX uses the relation: $dN_{pp}/dp_T(p_{T,initial}) =$ $R_{AA}(p_{T,final}) * dN_{pp}/dp_{T}(p_{T,final})$

Note for completeness that Ncoll cancels out in the equation because it both enters on the AA measurement side via: dN_{AA}/dp_{T} (measured) =

 $R_{AA}(p_T) * N_{coll} * dN_{pp}/dp_T$

and from the baseline: dN_{AA}/dp_{T} (baseline) = $N_{coll} * dN_{pp}/dp_T$

 $[R_{AA}(p_T) = dN_{AA}/dp_T \text{ (measured) }/$ dN_{AA}/dp_T (baseline)]

Figure from Phys. Rev. C 87, 034911 (2013)

- a) The PHENIX experiment at RHIC has developed a simple method for extracting the energy loss from the R_{AA} , see above Figure. If the initial spectrum is given by: $dN/dp_{\rm T} = a \times p_{\rm T}^b$ then they assume that the $p_{\rm T}$ is reduced, due to the jet quenching, by a factor $k = 1 - \Delta p_{\rm T}/p_{\rm T}$. The $R_{\rm AA}$ is then the ratio between the spectrum where the $p_{\rm T}$ is shifted down due to energy loss and the initial $p_{\rm t}$ -spectrum. Derive a relation between $R_{\rm AA}$, b, and k.
- b) At both RHIC and LHC the $R_{\rm AA}$ is ≈ 0.20 at $p_{\rm T} = 10$ GeV/c for the most central collisions, but the exponents of the pp spectra are b = -7.1 (RHIC) and b = -5.5 (LHC). What is the difference in energy loss?

¹The biggest gain is when we go to higher order correlations, but that is beyond the scope here, see e.g. Phys. Rev. C 83 (2011) 044913.

1.4 The medium temperature

- a) The charged particle multiplicity increases by a factor ≈ 2 from RHIC to LHC. Assuming this is true also for the energy density what is the estimated temperature increase of the medium from RHIC to LHC? Hint: what is the expected relation between energy density and temperature.
- b) Try to give an argument for why the increase is so small and reflect on what the gain would be for heavy-ion physics by increasing the beam energy by another factor 10.