

X. Charge conjugation and parity in weak interactions

REMINDER:

Parity

❖ The parity transformation is the transformation by reflection:

$$\vec{r} \rightarrow \vec{r}' = -\vec{r} \quad \text{or} \quad x, y, z \rightarrow x', y', z' = -x, -y, -z$$

it reverses the momentum (\vec{p}) but not $\vec{L} = \vec{r} \times \vec{p}$ or spin.

A parity operator \hat{P} is defined as

$$\rightarrow \hat{P}\psi(\vec{r}, t) = p\psi(-\vec{r}, t) \quad \text{where } p = \pm 1$$

Charge conjugation

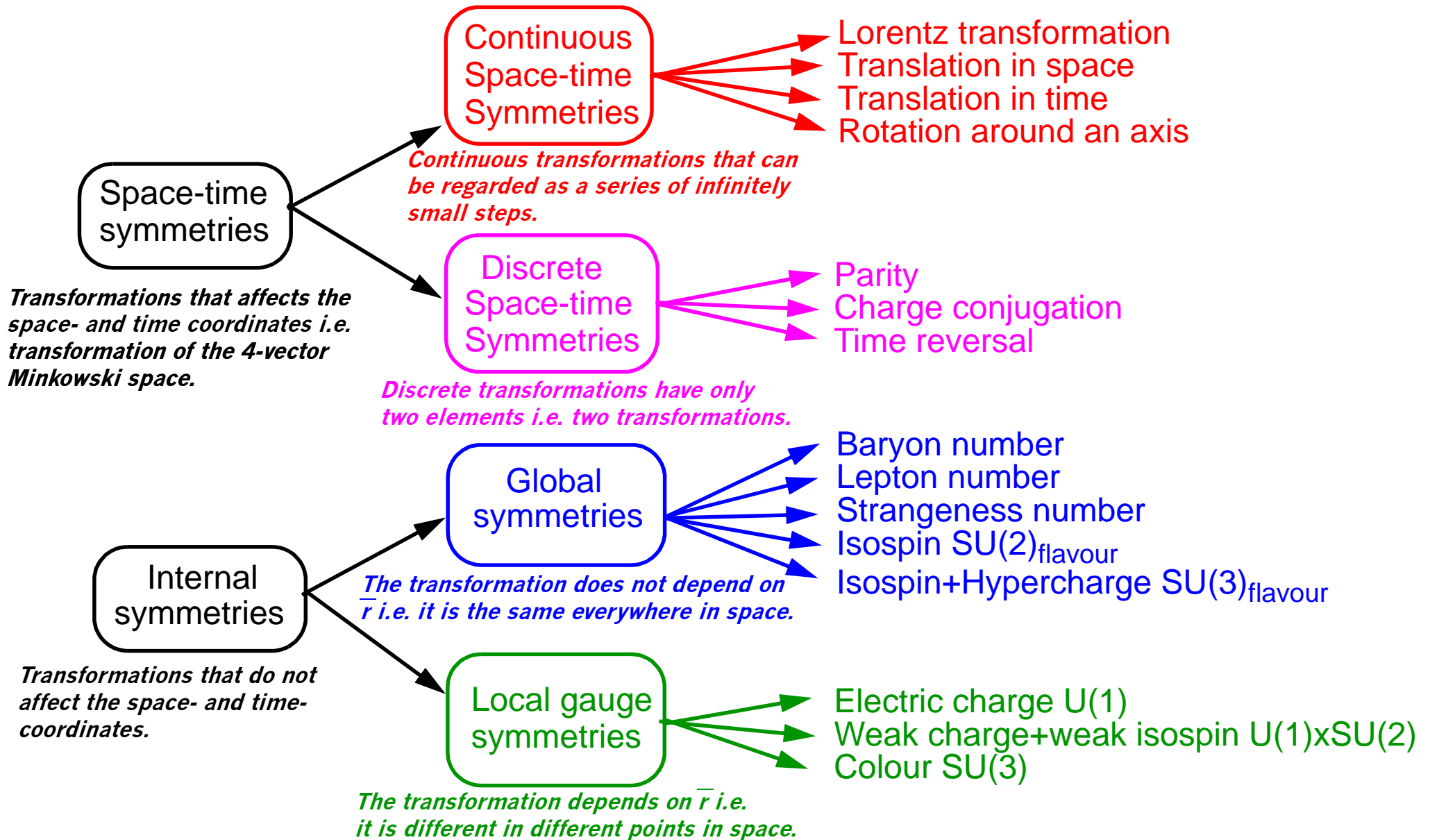
❖ The charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments

$$\rightarrow \hat{C}\Psi_a = c\Psi_{\bar{a}} \quad \text{where } c = \pm 1$$

meaning that from the particle in the initial state we go to the antiparticle in the final state.

Reminder

Symmetries



❖ While **parity** is conserved in strong and electromagnetic interactions, it is **violated** in weak processes:

– 1956: Based on the measurements of **Kaon decays**, Lee & Yang propose that parity is violated in weak processes:

Two known decays of the K^+ were:

$$K^+ \rightarrow \pi^0 + \pi^+ \quad \text{and} \quad K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

The intrinsic parity of a pion $P_\pi = -1$, and for the $\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ states the parities are

$$P_{\pi\pi} = P_\pi^2 (-1)^L = 1 \quad \text{since } L = L_{12} = 0$$

$$P_{\pi\pi\pi} = P_\pi^3 (-1)^L = -1 \quad \text{since } L = L_{12} + L_3 = 0$$

❖ Since the two final states have opposite parities, one of the K^+ decays must **violate parity!**

- 1957: Wu carries out studies of parity violation in β -decay. The ^{60}Co β -decay into $^{60}\text{Ni}^* + e^- + \bar{\nu}_e$ was studied.
- The ^{60}Co sample was cooled to 0.01 K to prevent thermal disorder.
- The sample was placed in a magnetic field \Rightarrow the nuclear spins were aligned along the field direction

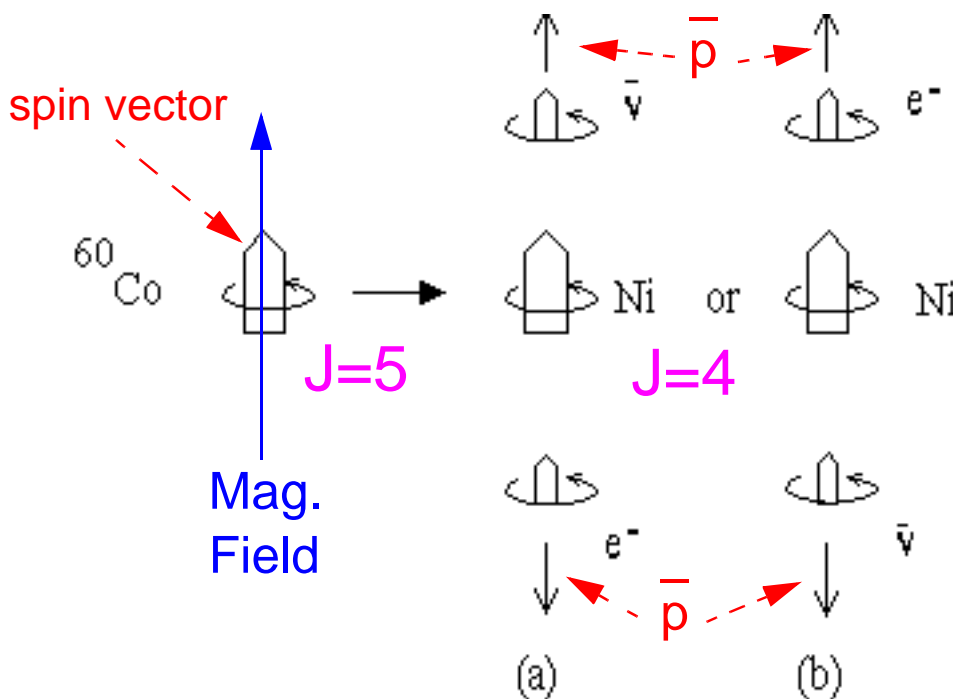


Figure 125: Possible β -decays of ^{60}Co : case (a) is preferred.

- If parity is conserved, processes (a) and (b) must have equal rates.

❖ *Electrons were emitted predominantly in the direction opposite the ^{60}Co spin*

→ Another case of both **parity and C-parity violation** was observed in **muon decays**:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

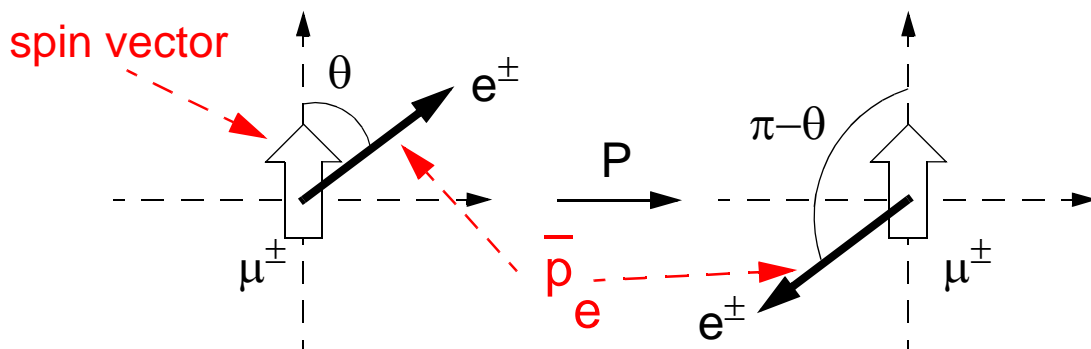


Figure 126: Effect of a parity transformation on the muon decays above

The **angular distribution** of the electrons (positrons) emitted in μ^- (μ^+) decay are given by

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm \left(1 - \frac{\xi_\pm}{3} \cos\theta \right) \tag{136}$$

here ξ_\pm are constants – “**asymmetry parameters**”, and Γ_\pm are total decay rates \Rightarrow **inverse lifetimes**

$$\Gamma_\pm = \int_{-1}^1 \Gamma_\pm(\cos\theta) d\cos\theta \equiv \frac{1}{\tau_\pm} \tag{137}$$

→ If the process is invariant under charge conjugation (**C-invariance**) \Rightarrow

$$\Gamma_+ = \Gamma_- \quad \xi_+ = \xi_- \quad (138)$$

(rates and angular distributions are the same for e^- and e^+)

→ If the process is **P-invariant**, then angular distributions in forward and backward directions are the same:

$$\Gamma_{\mu^\pm}(\cos\theta) = \Gamma_{\mu^\pm}(-\cos\theta) \quad \xi_+ = \xi_- = 0 \quad (139)$$

→ Experimental results:

$$\Gamma_+ = \Gamma_- \quad \xi_+ = -\xi_- = 1,00 \pm 0,04 \quad (140)$$



Both C- and P-invariance are violated!

→ However, the combined operation **CP** is **conserved** since that requires

$$\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(-\cos\theta) \tag{141}$$

$$\Gamma_+ = \Gamma_- \quad \xi_+ = -\xi_- \tag{142}$$

which is in agreement with the experiments.

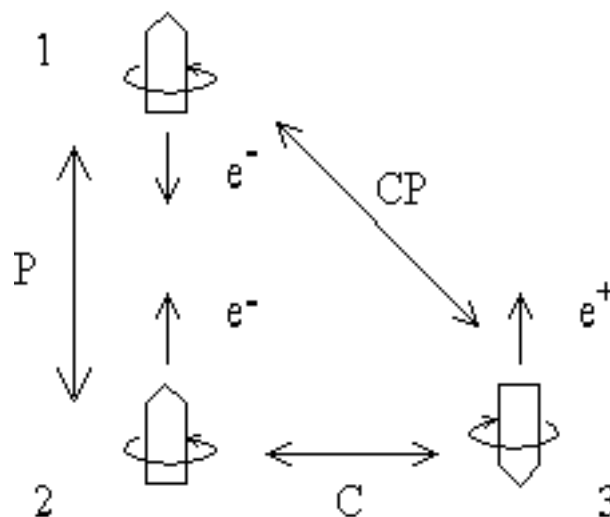


Figure 127: P-, C- and CP-transformation of an electron

❖ The combined transformation **CP** is a weaker requirement than the individual transformations P and C and it is **conserved**.

Helicity

helicity – the spin is quantized along the particle's direction of motion instead of along an arbitrary z-direction

$$\hat{\Lambda} = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \quad (143)$$

$$\hat{\Lambda}\psi = \lambda\psi$$

The eigenvalues of the helicity operator are $\lambda = -s, -s+1, \dots, +s$, \Rightarrow for spin-1/2 particle it can be either $-1/2$ or $1/2$

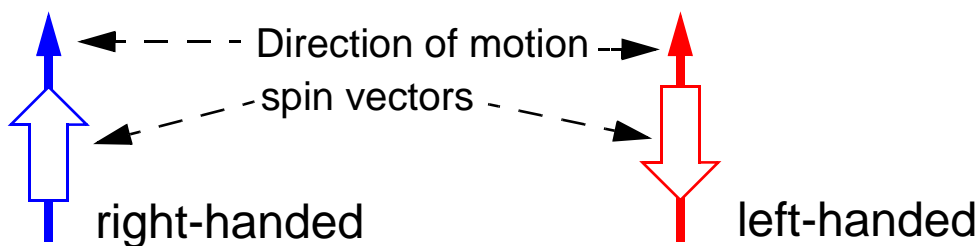


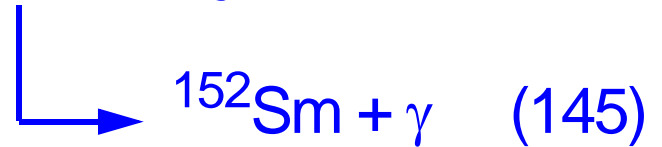
Figure 128: Helicity states of spin-1/2 particle

A particle with $\lambda = +1/2$ is called **right handed**.

A particle with $\lambda = -1/2$ is called **left handed**.

A **subscript R or L** is used to denote if a state is right or left handed e.g. e^-_R and ν_L

❖ 1958: Goldhaber et al. measured the **helicity of the neutrino** by studying electron capture in europium:



❖ In this reaction the initial state has zero momentum and ${}^{152}\text{Sm}^*$ and ν_e recoil in opposite directions.

❖ Events with the γ emitted in the direction of motion of the ${}^{152}\text{Sm}^*$ were selected so that the overall observed reaction was:



❖ The spin of the neutrino (+1/2 or -1/2) and the photon (+1 or -1) must add to give the spin of the electron (+1/2 or -1/2).

❖ The helicity (polarization) of the photons was determined by studying their absorption in magnetized iron.

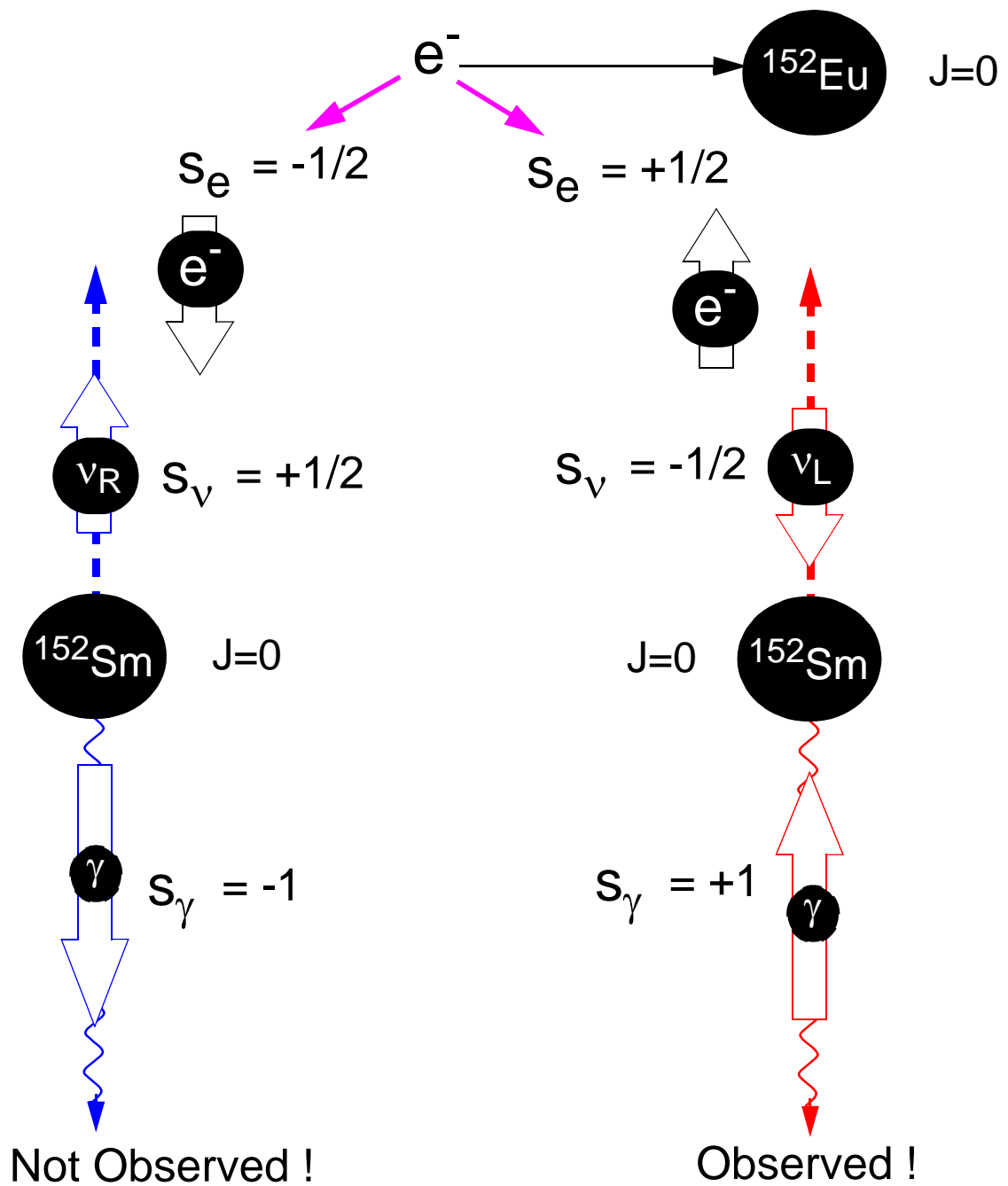


Figure 129: From the helicity of the photons it is possible to determine the helicity of the neutrinos.



From the measured photon helicity it was concluded that **neutrinos must be left-handed.**

V-A interaction

❖ **V-A interaction** theory was introduced by Fermi as an analytic description of spin dependence of charged current interactions.

❖ It denotes “polar Vector - Axial vector” interaction

– *A Polar vector* is one which direction is reversed by parity transformation e.g. momentum \vec{p}

– *An Axial vector* is one which direction is not changed by parity transformation e.g. spin \vec{s} or orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$

– The weak current has **both vector and axial** components, hence parity is not conserved in weak interactions

➔ Main conclusion: if $v \approx c$, only **left-handed** fermions ν_L, e_L^- etc. are emitted, and right-handed antifermions.

➔ *The very existence of preferred states violates both C- and P- invariance*

❖ **Neutrinos** (antineutrinos) are always **relativistic** and hence always **left(right)**-handed

❖ For other fermions, the **preferred states** are **left**-handed. Right-handed states are not completely forbidden but suppressed by the factor

$$\left(1 - \frac{v}{c}\right) \approx \frac{m^2}{2E^2} \tag{147}$$

Consider the two pion decay modes:

$$\pi^+ \rightarrow e^+ + \nu_e \tag{148}$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{149}$$

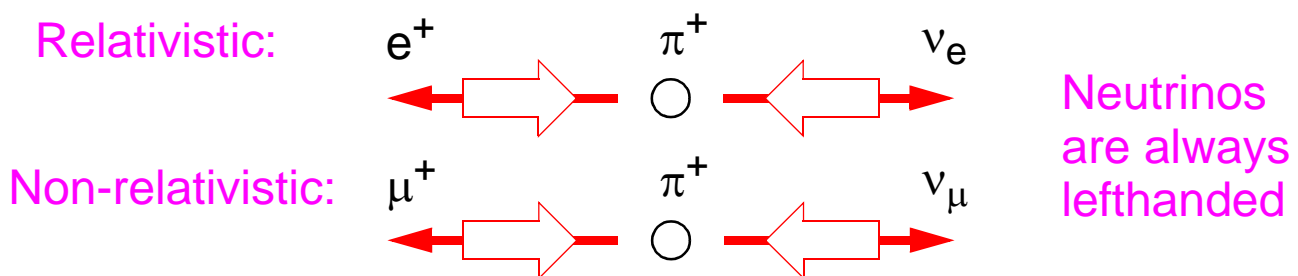


Figure 130: Helicities of leptons emitted in a pion decay

– The π^+ has spin-0 and it is at rest \Rightarrow the spins of the charged lepton and the neutrino must be opposite.

- The neutrinos are always left-handed \Rightarrow the charged leptons have to be left-handed as well.
- BUT: the e^+ and the μ^+ should be right-handed since they are anti-fermions.

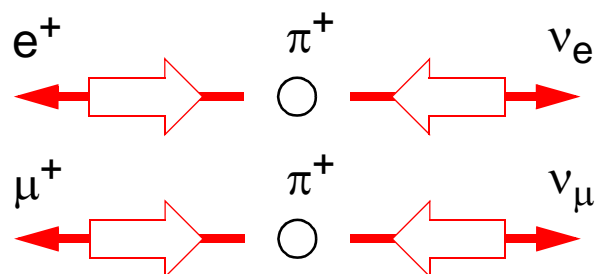
❖ In these decays the **electron** will be **relativistic** but **not the muon** (due to its large mass).

❖ It follows that the **pion to muon** decay should be **allowed** but the **pion to positron** decay should be **suppressed**.

Left-handed

Should be right-handed since it is relativistic

Can be left-handed since it is non-relativistic



- The suppression factor for positrons is expected to be of the order 10^{-5} .

The measured ratio:

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = (1,230 \pm 0,004) \times 10^{-4} \quad (150)$$

❖ **Muons** emitted in pion decays are always **polarized** and this can be used to measure muon decay symmetries by detecting the relativistic electrons in the following decays:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \tag{151}$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

The electrons are emitted in decays when both the ν_μ and the $\bar{\nu}_e$ are emitted in the direction opposite to the e^- :

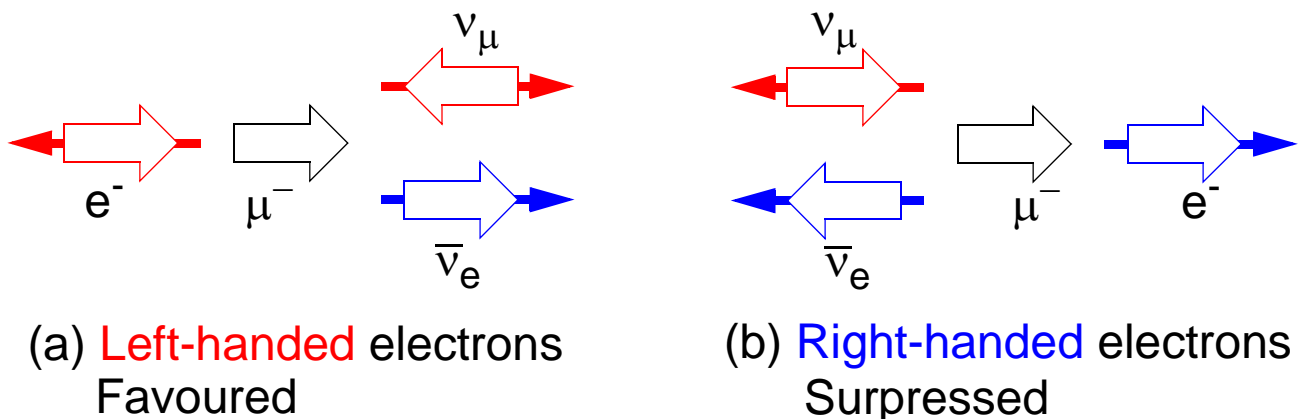


Figure 131: Muon decays with high energy electron emission.

The electron must have a spin parallel to the muon spin \Rightarrow configuration (a) with left-handed electrons is strongly preferred \Rightarrow this is observed experimentally as a forward-backward asymmetry.

Neutral kaons

❖ It is possible to produce the neutral kaons $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ in πp -collisions. This is a strong interaction process and strangeness has to be conserved:

$$\pi^- + p \rightarrow K^0 + \Lambda$$

s:	0	0	+1	-1
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$$\pi^+ + p \rightarrow \bar{K}^0 + p + K^+$$

s:	0	0	-1	0	+1
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❖ The kaons that are produced in this way are pure $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$ states.

❖ However, K^0 and \bar{K}^0 can be converted into each other since **strangeness is not conserved** in weak interactions:

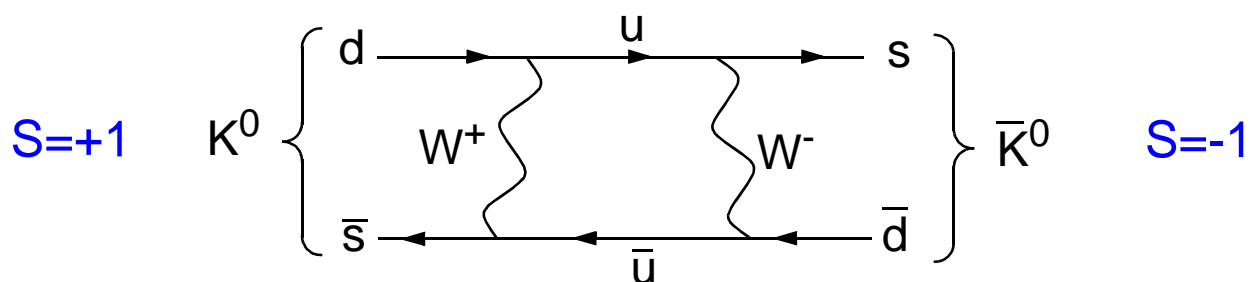


Figure 132: Example of a process converting K^0 to \bar{K}^0 .

❖ The observed physical particles are linear combinations of K^0 and \bar{K}^0 , since there is no conserved quantum number to distinguish them. The phenomenon is called *K^0 - \bar{K}^0 mixing*.

❖ We know that neither parity nor charge conjugation are conserved in weak decays. The combined operation **CP** is, however, **almost conserved**.

❖ In this case the CP operators eigenstates can be written as a mixture of K^0 and \bar{K}^0 :

$$K_1^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \quad (152)$$

$$K_2^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \quad (153)$$

so that

$$\hat{C}PK_1^0 = K_1^0 \text{ and } CPK_2^0 = -K_2^0 \quad (154)$$

i.e. the CP eigenvalues are $cp=+1$ for K_1^0 and $cp=-1$ for K_2^0 and the K_1^0 can therefore only decay to cp -even states while the K_2^0 only to cp -odd states.

Experimentally observed are two types of neutral kaons: K_S^0 (“S” for “short”, lifetime $\tau = 0,9 \times 10^{-10} s$) and K_L^0 (“long”, $\tau = 500 \times 10^{-10} s$).

❖ Can the K_S^0 be identified with the K_1^0 CP-eigenstate, and the K_L^0 with the K_2^0 ?

→ If CP-invariance holds for neutral kaons, K_S^0 should decay only into states with $cp=1$ such as 2π -states, and K_L^0 into states with $cp=-1$ such as 3π -states:

$$K_S^0 \rightarrow \pi^+ \pi^-, \quad K_S^0 \rightarrow \pi^0 \pi^0 \quad (155)$$

- The parity of a two-pion state is $P = P_\pi^2 (-1)^L = 1$
- The C-parity of a $\pi^0 \pi^0$ state is $C = (C_{\pi^0})^2 = 1$, and of a $\pi^+ \pi^-$ state: $C = (-1)^L = 1$
- i.e. $cp=1$ for the $\pi^+ \pi^-$ and $\pi^0 \pi^0$ states
- i.e. the assumption that $K_S^0 = K_1^0$ seem to be correct.

$$K_L^0 \rightarrow \pi^+ \pi^- \pi^0, \quad K_L^0 \rightarrow \pi^0 \pi^0 \pi^0 \quad (156)$$

- The parity of the 3- π states are -1
- The C-parity of $\pi^0 \pi^0 \pi^0$ is $C = (C_{\pi^0})^3 = 1$
- The C-parity of $\pi^+ \pi^- \pi^0$ is $C = C_{\pi^0} (-1)^{L_{\pi\pi}} = 1$
- i.e. the 3- π final states above have $CP = -1$
- i.e. the assumption that $K_L^0 = K_2^0$ seem to be correct.



Summary:

The neutral Kaon eigenstates in **strong interactions** are:

$$\begin{aligned} K^0 &= d\bar{s} \\ \bar{K}^0 &= s\bar{d} \end{aligned}$$

The neutral Kaon eigenstates in **weak interactions** (if CP is conserved) are:

$$\begin{aligned} K_s^0 = K_1^0 &= \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \\ K_L^0 = K_2^0 &= \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \end{aligned}$$

Strangeness Oscillations

$$\begin{cases} K_S^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \\ K_L^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \end{cases} \Rightarrow \begin{cases} K^0 = \frac{1}{\sqrt{2}}(K_S^0 + K_L^0) \\ \bar{K}^0 = \frac{1}{\sqrt{2}}(K_S^0 - K_L^0) \end{cases}$$

Kaons created at $t=0$ can be written as:

$$\begin{cases} K^0(0) = \frac{1}{\sqrt{2}}(K_S^0(0) + K_L^0(0)) \\ \bar{K}^0(0) = \frac{1}{\sqrt{2}}(K_S^0(0) - K_L^0(0)) \end{cases}$$

and the time-dependence of the kaon states as:

$$K^0(t) = \frac{1}{\sqrt{2}} \left(e^{-iE_S t} e^{-\Gamma_S t/2} K_S^0(0) + e^{-iE_L t} e^{-\Gamma_L t/2} K_L^0(0) \right)$$

$$\bar{K}^0(t) = \frac{1}{\sqrt{2}} \left(e^{-iE_S t} e^{-\Gamma_S t/2} K_S^0(0) - e^{-iE_L t} e^{-\Gamma_L t/2} K_L^0(0) \right)$$

if the time is measured in the restframe of the kaons then $E_S = m_S$ and $E_L = m_L$ are the masses of the K_S and K_L and Γ_S and Γ_L are the decay rates with $\Gamma = 1/\tau$ as usual.

Assume that a pure K^0 beam is produced by the reaction $\pi^- + p \rightarrow K^0 + \Lambda$ at $t=0$, then

$$K^0(t) = A(t)K^0(0) + B(t)\bar{K}^0(0) \quad \text{where}$$

$$\begin{cases} A(t) = \frac{1}{2} \left(e^{-(im_s + \Gamma_s/2)t} + e^{-(im_L + \Gamma_L/2)t} \right) \\ B(t) = \frac{1}{2} \left(e^{-(im_s + \Gamma_s/2)t} - e^{-(im_L + \Gamma_L/2)t} \right) \end{cases}$$

If the intensities of the K^0 and \bar{K}^0 is measured by strong interaction processes such as $\bar{K}^0 + p \rightarrow \pi^+ + \Lambda$ then one will find that they are

$$I(K^0) = |A(t)|^2 = \frac{1}{4} \left(e^{-t\Gamma_s} + e^{-t\Gamma_L} + 2e^{-t(\Gamma_s + \Gamma_L)/2} \cos(\Delta mt) \right)$$

$$I(\bar{K}^0) = |B(t)|^2 = \frac{1}{4} \left(e^{-t\Gamma_s} + e^{-t\Gamma_L} - 2e^{-t(\Gamma_s + \Gamma_L)/2} \cos(\Delta mt) \right)$$

$$\text{where } \Delta m = m_s - m_L = 3.5 \times 10^{-6} \text{ eV}$$

This phenomena when a K^0 beam partly turns into a \bar{K}^0 beam is called strangeness oscillations.

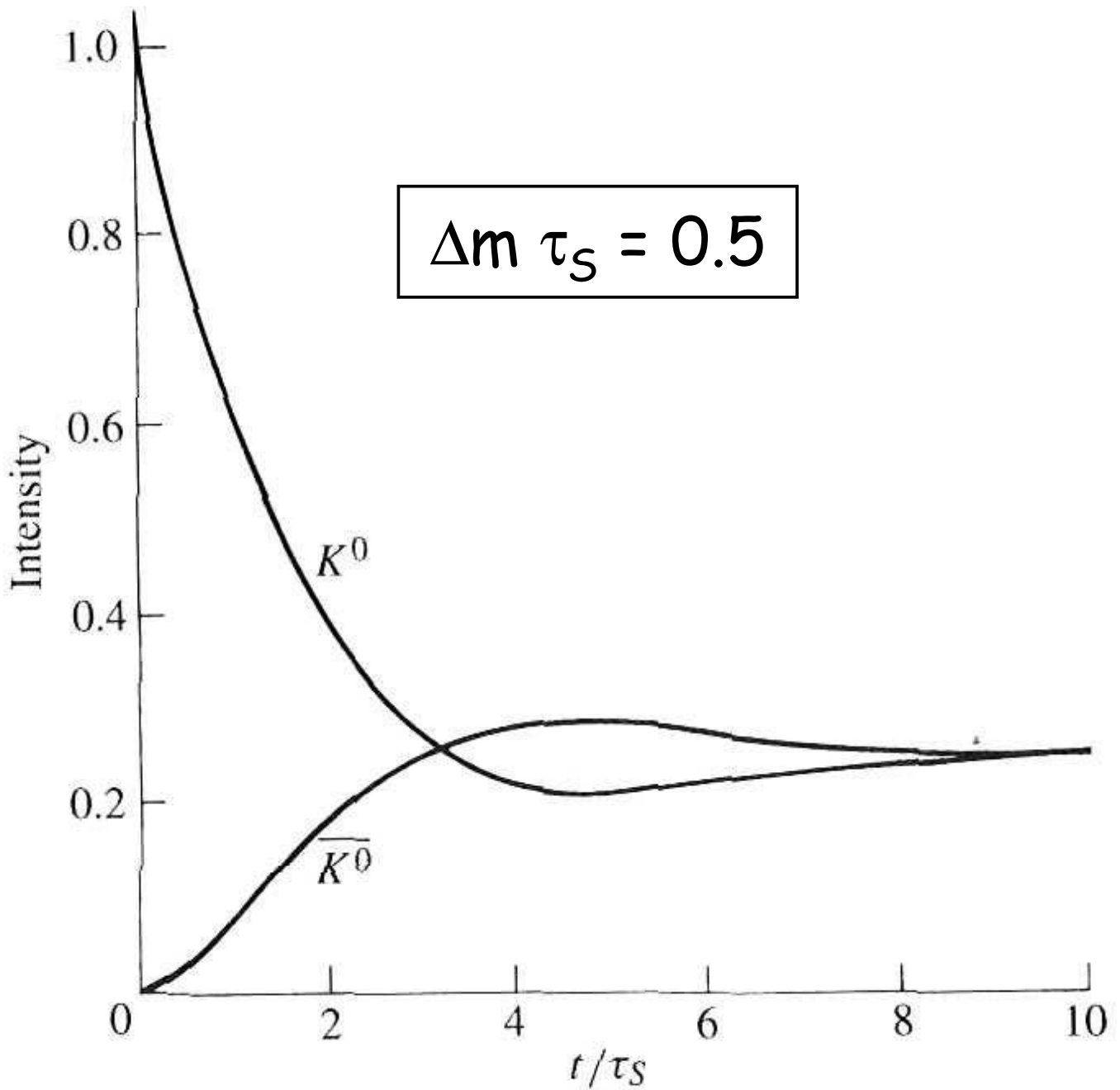


Figure 133: Strangeness oscillations: The intensity as a function of time of K^0 and \bar{K}^0 in a beam that was produced as pure K^0 .

CP-Violation

The *CP-violating* decay

$$K_L^0 \rightarrow \pi^+ \pi^- \tag{157}$$

was first observed in 1964, with a branching ratio of $B \approx 10^{-3}$.

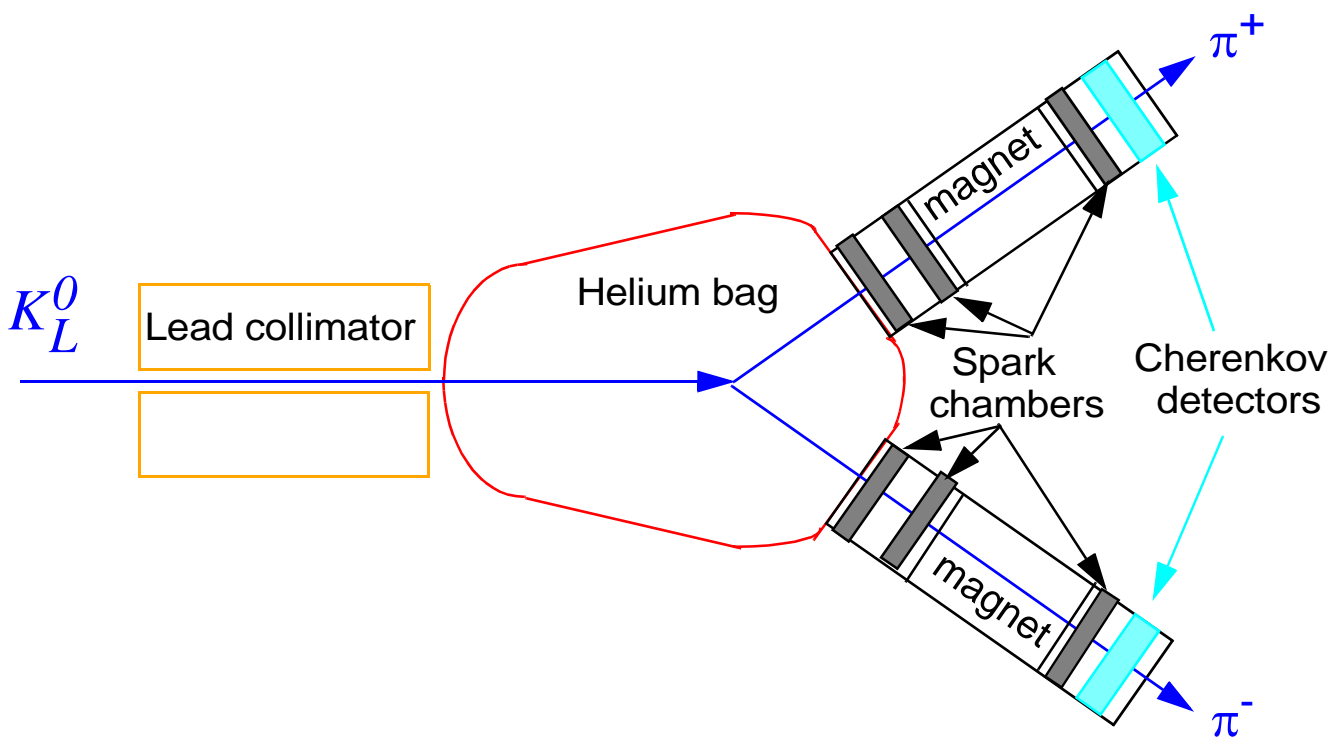


Figure 134: Sketch of the experiment that discovered CP-violation in weak decays.

❖ In general, the physical states K_S^0 and K_L^0 don't have to correspond to pure CP-eigenstates K_1^0 and K_2^0 . Instead

$$K_S^0 = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (K_1^0 + \varepsilon K_2^0)$$

$$K_L^0 = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (\varepsilon K_1^0 + K_2^0)$$

where ε is a small complex parameter: $|\varepsilon| = 2 \times 10^{-3}$

❖ K_S^0 contains **mostly** K_1^0 but has also a small K_2^0 component while K_L^0 consists mostly of K_2^0 with a small component of K_1^0 .

→ Mixing occur also for neutral **B-mesons** ($B^0 = \bar{d}b$, $\bar{B}^0 = bd$, $B_S^0 = \bar{s}b$ and $\bar{B}_S^0 = bs$) and for neutral D-mesons ($D^0 = c\bar{u}$ and $\bar{D}^0 = uc$).

→ There can be **different mechanisms** for CP-violation, especially in the B^0 - \bar{B}^0 systems. Several dedicated experiments have been built to study this system.

Summary

• Parity and charge conjugation

- a) Parity is violated in weak processes.
- b) Parity violation was first observed in ^{60}Co -decays.
- c) Muon decays can be used to show that both parity and charged conjugation is violated while the combined CP operation is conserved.

• Helicity

- d) Helicity is the spin quantized along the direction of motion.
- e) Neutrinos are left-handed and antineutrinos right-handed.
- f) This was first observed in reactions between electrons and ^{152}Eu atoms.

- **V-A interactions**

g) While neutrinos are always left-handed other fermions are exclusively left-handed only when they are relativistic.

- **Neutral kaons**

h) The neutral kaons decays that are observed experimentally (K^0_S and K^0_L) are due to K^0 - \bar{K}^0 mixing.

i) If a pure beam of K^0 is produced one can later detect \bar{K}^0 in the beam (by strong interaction hyperon production). The phenomena is called strangeness oscillations.

j) CP-violating decays of neutral kaons have been observed with a small branching ratios.