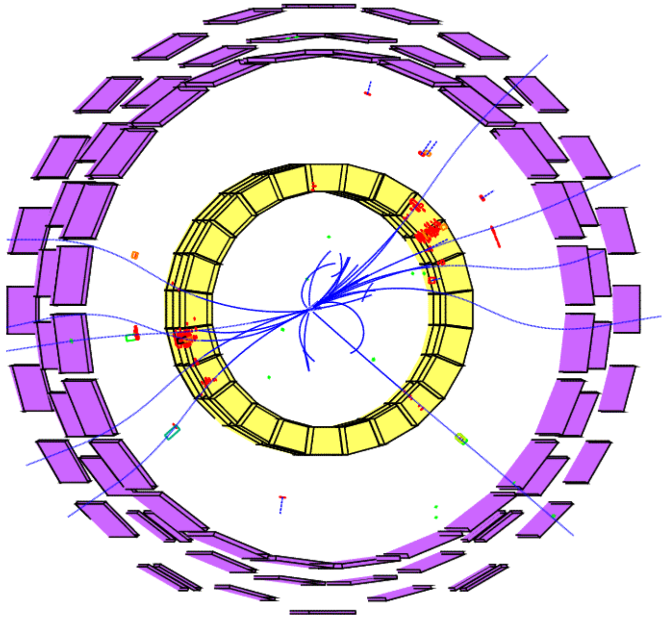
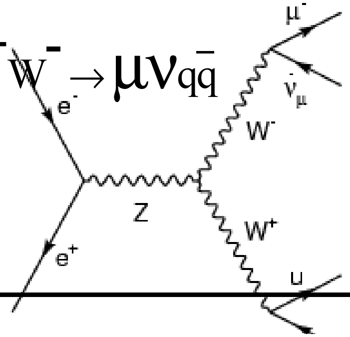
Particle Physics

experimental insight



$$e^+e^- \rightarrow W^+W^- \rightarrow \mu\nu q\bar{q}$$



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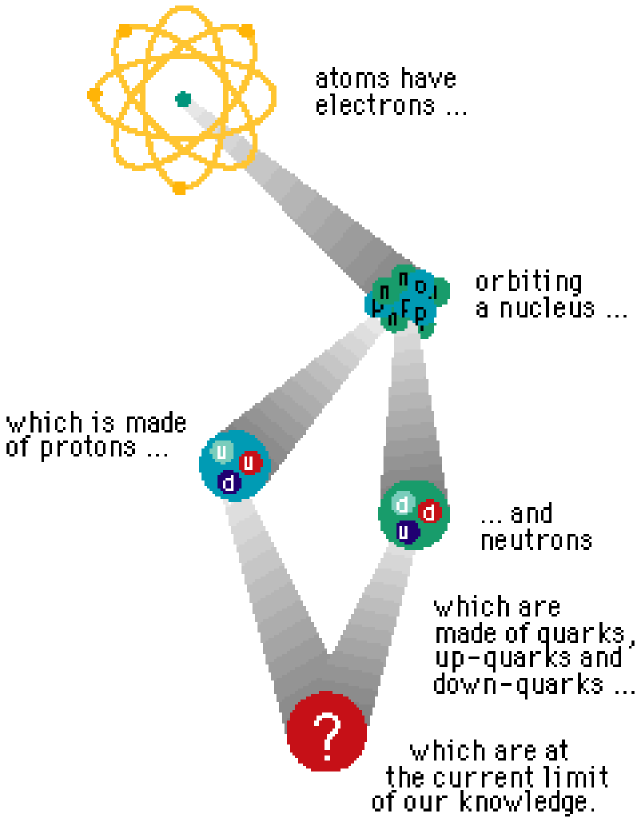
I. Basic concepts

→ Particle physics studies the elementary “building blocks” of *matter* and interactions between them.

→ Matter consists of *particles* = fermions (spin 1/2).

→ Particles interact via *forces*.
Interaction=exchange of force-carrying particle.

→ Force-carrying particles are called *gauge bosons* (spin-1).



Forces of nature:

1) gravitational

2) weak

3) electromagnetic

4) strong

Forces of nature

Name	Acts on/couples to:	Carrier	Range	Strength	Stable systems	Induced reaction
Gravity	all particles Mass/E-p tensor	graviton G *(has not yet been observed)*	long $F \propto 1/r^2$	$\sim 10^{-39}$	Solar system	Object falling
Weak force	fermions hypercharge	bosons W^+,W^- and Z	$< 10^{-17}$ m	10^{-5}	None	β-decay
Electromagnetism	charged particles electric charge	photon γ	long $F \propto 1/r^2$	$1/137$	Atoms, molecules	Chemical reactions
Strong force	quarks and gluons colour	8 gluons g	10^{-15} m	1	Hadrons, nuclei	Nuclear reactions

The Standard Model

→ Electromagnetic and weak forces can be described by a single theory ⇒ the *"Electroweak Theory"* was developed in 1960s (Glashow, Weinberg, Salam).

→ Theory of strong interactions appeared in 1970s: *"Quantum Chromodynamics"* (QCD).

→ The *"Standard Model"* (SM) combines both.

→ Gravitation is VERY weak at particle scale, and it is not included in the SM. Quantum theory for gravitation does not exist yet.

Main postulates of SM:

1) Basic constituents of matter are *quarks* and *leptons* (spin 1/2 particles = fermions).

2) They interact by exchanging gauge bosons (spin 1).

3) Quarks and leptons are subdivided into 3 *generations*.

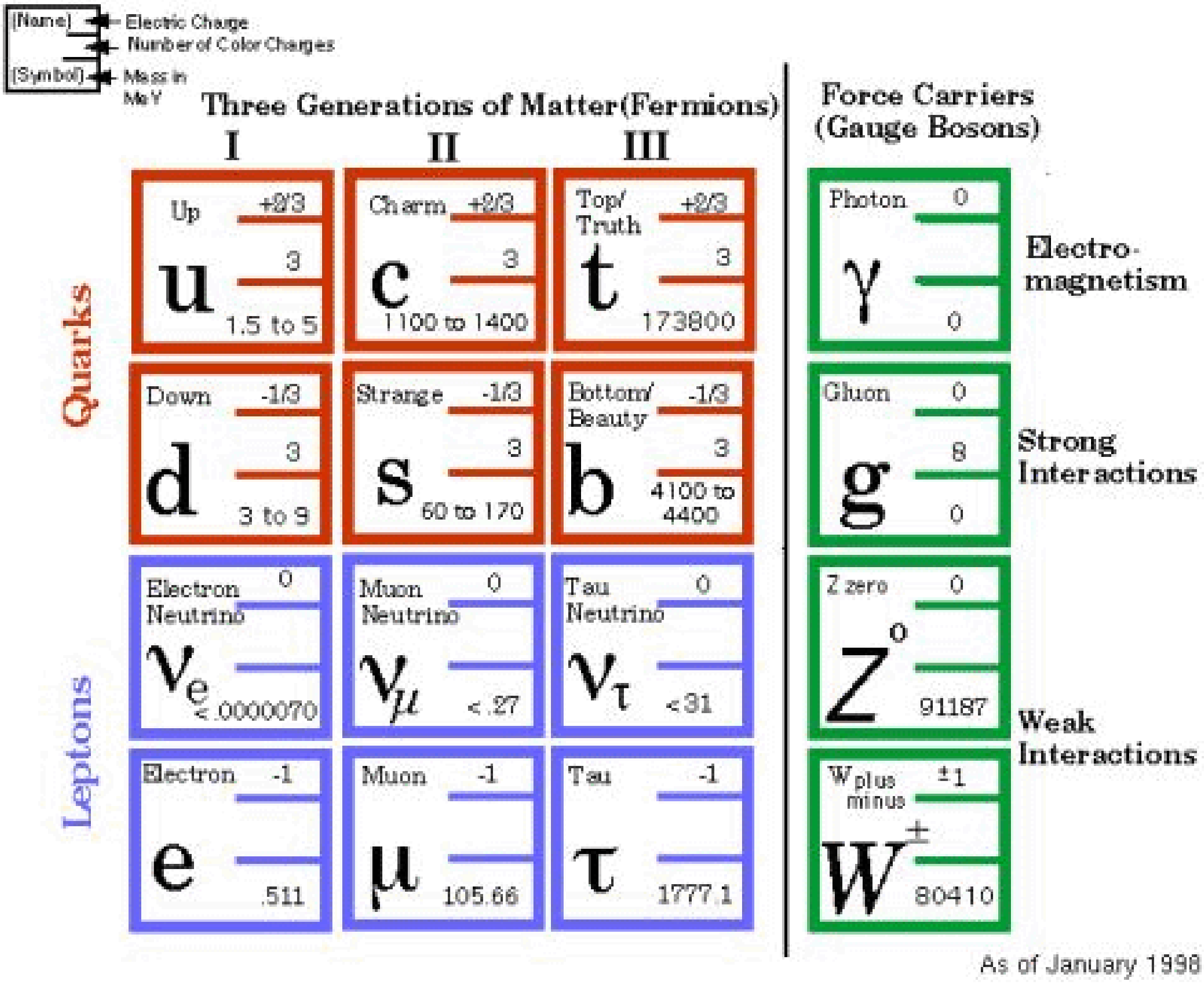


Figure 1: The Standard Model Chart

SM does not explain neither appearance of the mass nor the reason for existence of 3 generations.

Units and dimensions

→ The energy is measured in *electron-volts*:

$$1 \text{ eV} \approx 1.602 \times 10^{-19} \text{ J} \qquad (1)$$

1 eV = energy of e^- passing a voltage of 1 V.

1 keV = 10^3 eV; 1 MeV = 10^6 eV; 1 GeV = 10^9 eV

The reduced Planck constant and the speed of light:

$$\hbar \equiv h/2\pi = 6.582 \times 10^{-22} \text{ MeV s} \qquad (2)$$

$$c = 2.9979 \times 10^{8} \text{ m/s} \qquad (3)$$

and the "conversion constant" is:

$$\hbar c = 197.327 \times 10^{-15} \text{ MeV m} \qquad (4)$$

→ For simplicity, the *natural units* are used:

$$\hbar = 1 \quad \text{and} \quad c = 1 \qquad (5)$$

so the unit of mass is eV/c^2, and the unit of momentum is eV/c

Four-vector formalism

Remember normal three-vectors:

$$\overline{A} = (a_1,a_2,a_3) = (a_x,a_y,a_z) \text{ (cartesian coord.)}$$

Four-vectors are defined as

$$A = (A_0,\overline{A}) = (a_0,a_1,a_2,a_3) = (a_0,a_x,a_y,a_z) \text{ (cartesian c.)}$$

Four-vectors are actually of two kinds, kontravariant vectors, defined as

$$A^{\mu} = (A^0, \vec{A}) \qquad , \tag{6}$$

and covariant vectors

$$A_{\mu} = (A^0, -\vec{A}) \qquad . \tag{7}$$

The four-vectors which are needed in relativistic kinematics are momentum four-vectors p:

- $p = (p^0,\overline{p}) = (E,\overline{p}) = (E,p_x,p_y,p_z)$, where E = energy and $\underline{p}$ = three-momentum (c = 1)

and space-time four-vector x:

- $x = (x^0,\bar{x}) = (t,\bar{x}) = (t,x,y,z)$, where t = time and $\bar{x}$ = space-coordinate (normal three-vector) (c = 1)

Particle X with energy E and three-momentum $\bar{p}$ has thus a momentum four-vector $p=(E,\bar{p})=(E,p_x,p_y,p_z)$. The particle is also associated with a space-time four-vector $x=(t,\bar{x})=(t,x,y,z)$.

Calculation rules with four-vectors:

- Sum: $p_1 + p_2 = (E_1+E_2, \bar{p}_1+\bar{p}_2) = (E_1+E_2, p_{x1}+p_{x2}, p_{y1}+p_{y2}, p_{z1}+p_{z2})$
- Subtraction: similarly as sum

Scalar product of two four-vectors has a special rule.

Remember that the scalar product of two three-vectors $\bar{A}=(a_x,a_y,a_z)=(a_1,a_2,a_3)$ and $\bar{B}=(b_x,b_y,b_z)=(b_1,b_2,b_3)$ is defined as:

$$\vec{A} \cdot \vec{B} = (a_1 b_1 + a_2 b_2 + a_3 b_3) = (a_x b_x + a_y b_y + a_z b_z).$$

Scalar product of two four-vectors is defined as:

$$A \cdot B = A^0 B^0 - (\vec{A} \cdot \vec{B}) \equiv A_\mu B^\mu \equiv A^\mu B_\mu . \qquad (8)$$

The scalar product of a momentum and a space-time four-vector is:

$$x \cdot p = x^0 p^0 - (\vec{x} \cdot \vec{p}) = Et - (\vec{x} \cdot \vec{p}) \qquad (9)$$

This is the wavefunction of a particle with momentum four-vector *p* and space-time four-vector *x*!

The scalar product of two momentum four-vectors is:

$$p_1 \cdot p_2 = p_1^0 p_2^0 - (\vec{p}_1 \cdot \vec{p}_2) = E_1 E_2 - (\vec{p}_1 \cdot \vec{p}_2) \qquad (10)$$

For a particle with four-momentum *p*, the invariant mass is the scalar product of the momentum four-vector with itself .

$$m^2 \equiv p \cdot p = p^2 = p^0 p^0 - (\vec{p} \cdot \vec{p}) = E^2 - \vec{p}^2 \qquad (11)$$

For relativistic particles, we can thus see that

$$\boldsymbol{E^2 = \bar{p}^2 + m^2} \ (c=1) \qquad (12)$$

Antiparticles

→ Particles are described by a wavefunction:

$$\Psi(\vec{x}, t) = Ne^{i(\vec{p}\vec{x} - Et)} \qquad (13)$$

$\vec{x}$ is the coordinate vector, $\vec{p}$ - momentum vector, E and t are energy and time.

Particles obey the classical Schrödinger equation:

$$i\frac{\partial}{\partial t}\Psi(\vec{x}, t) = H\Psi(\vec{x}, t) = \frac{\vec{p}^2}{2m}\Psi(\vec{x}, t) = -\frac{1}{2m}\nabla^2\Psi(\vec{x}, t)$$

(14)

$$\text{where } \vec{p} = \frac{h}{2\pi i}\nabla \equiv \frac{\nabla}{i}. \qquad (15)$$

For relativistic particles, $\boldsymbol{E^2 = \bar{p}^2 + m^2}$(12), and (14) is replaced by the Klein-Gordon equation (16):

$$\Downarrow$$

$$-\frac{\partial^2}{\partial t^2}(\Psi) = H^2\Psi(\vec{x}, t) = -\nabla^2\Psi(\vec{x}, t) + m^2\Psi(\vec{x}, t) \qquad (16)$$

→ There exist *negative* energy solutions! $E_+ < 0$

$$\Psi^*(\vec{x}, t) = N^* \cdot e^{i(-\vec{p}\vec{x} + E_+ t)}$$

The problem with the Klein-Gordon equation: it is second order in derivatives. In 1928, Dirac found the first-order form having the same solutions:

$$i\frac{\partial\Psi}{\partial t} = -i\sum_i \alpha_i \frac{\partial\Psi}{\partial x_i} + \beta m\Psi \qquad (17)$$

where α_i and β are 4×4 matrices and Ψ are four-component wavefunctions: *spinors* (for particles with spin 1/2).

$$\Psi(\vec{x}, t) = \begin{bmatrix} \Psi_1(\vec{x}, t) \\ \Psi_2(\vec{x}, t) \\ \Psi_3(\vec{x}, t) \\ \Psi_4(\vec{x}, t) \end{bmatrix}$$

Dirac-Pauli representation of matrices α_i and β:

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Here I is 2x2 unit matrix, 0=2x2 matrix with zeros, and σ_i are 2x2 Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Also possible is Weyl representation:

$$\alpha_i = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Dirac's picture of vacuum

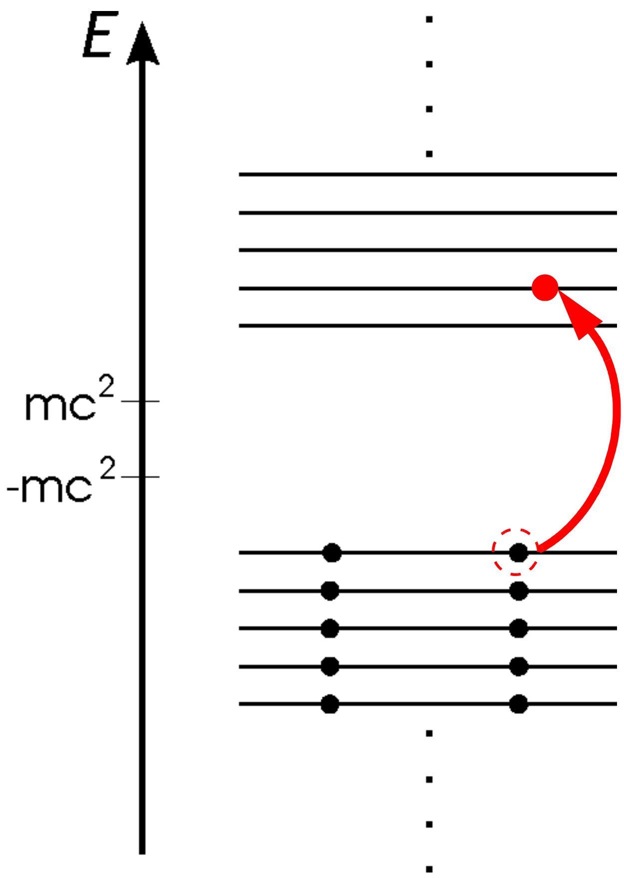


Figure 2: Fermions in Dirac's representation: a photon γ with an energy $E=2mc^2$ produces γ+vacuum -> e^+(hole) + e^-

Inserting a photon γ with an energy $E=2mc^2$ into the vacuum creates a "hole" in the vacuum since the electron jumps into a positive energy state. The "hole" is interpreted as the presence of electron's *antiparticle* with the opposite charge. Antiparticle e^+ has now a positive energy $E=mc^2$ (as well as the e^-).

→ Every charged particle has the antiparticle of the same mass and opposite charge.

Discovery of the positron

1933, C.D.Andersson, Univ. of California (Berkeley): observed with the Wilson cloud chamber 15 tracks in cosmic rays:

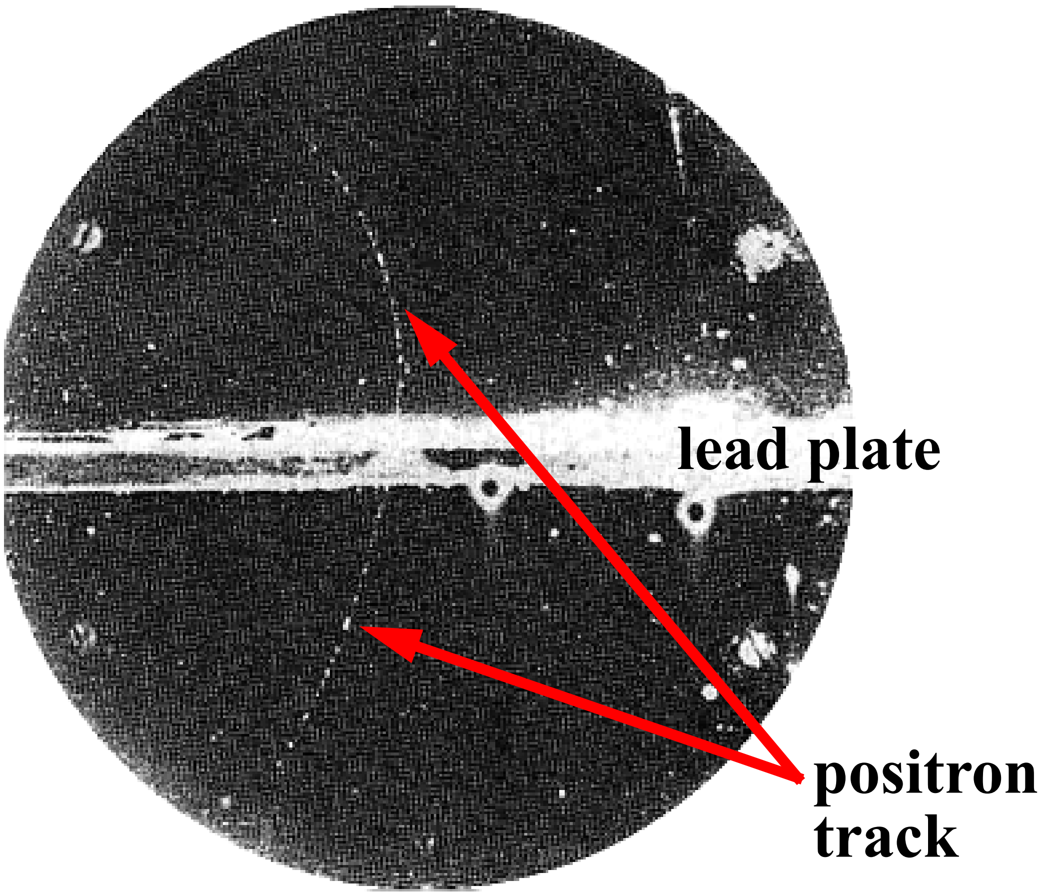


Figure 3: Photo of the track in the Wilson chamber

Feynman diagrams

In 1940s, R.Feynman developed a diagram technique for representing processes in particle physics.

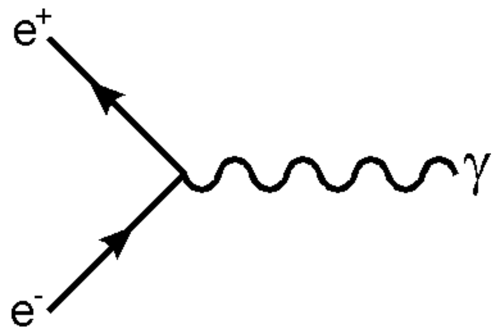


Figure 4: A Feynman diagram example: e+e- -> γ

Main assumptions and requirements:

- ❖ Time runs from left to right
- ❖ Arrow directed towards the right indicates a particle, and otherwise - antiparticle
- ❖ At every vertex, momentum, angular momentum and charge are conserved (but not energy)
- ❖ Particles are usually denoted with solid lines, and gauge bosons - with helices or dashed

lines

Virtual processes:

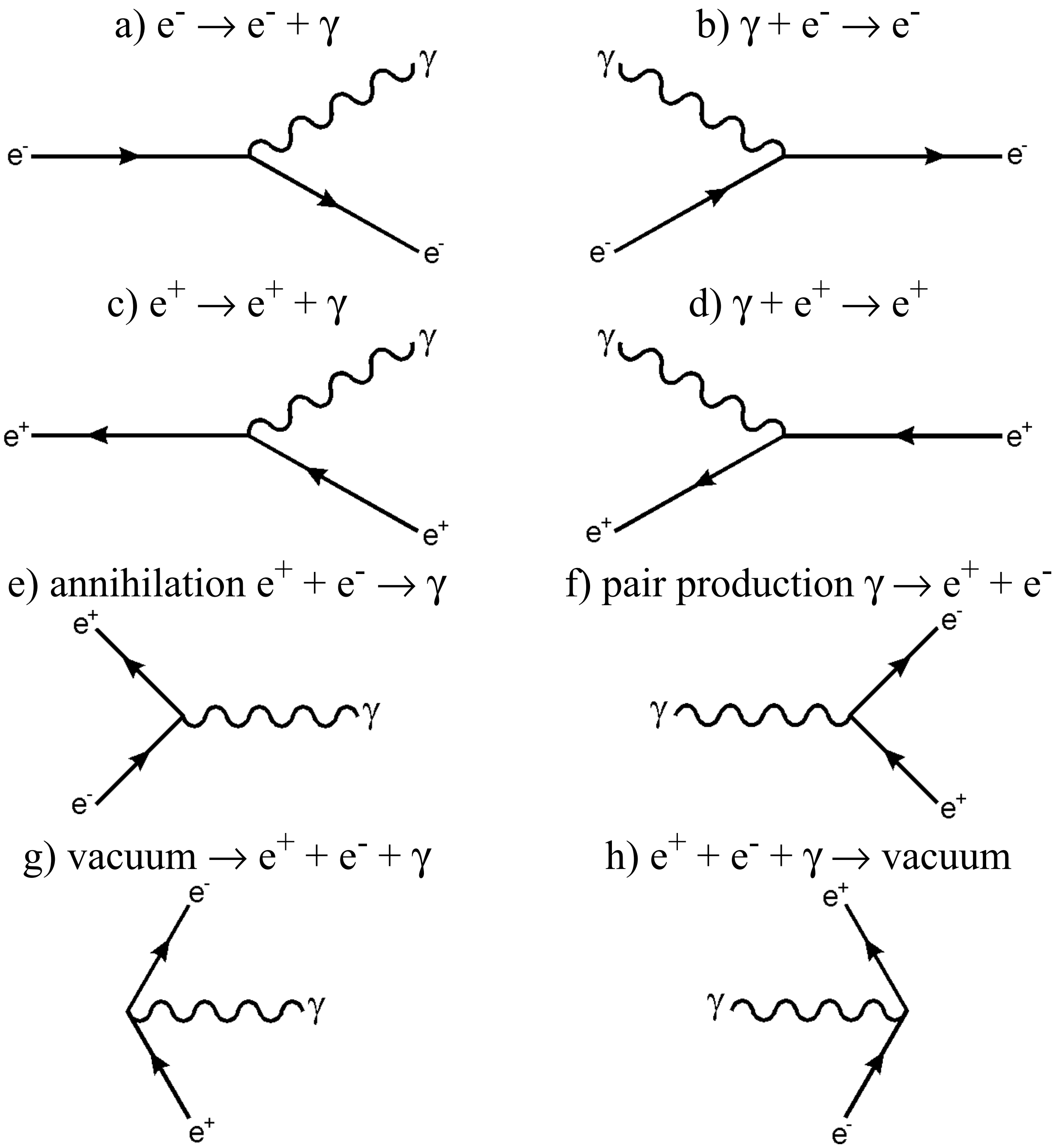


Figure 5: Feynman diagrams for basic processes involving electron, positron and photon

Real processes

→ A real process demands energy conservation, hence is a combination of virtual processes.

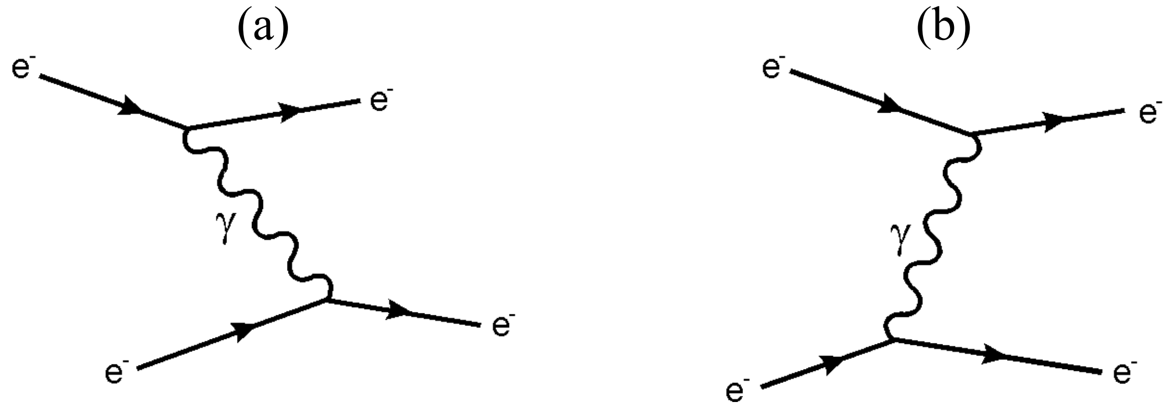


Figure 6: Electron-electron scattering, single photon exchange

→ Any real process receives contributions from all possible virtual processes.

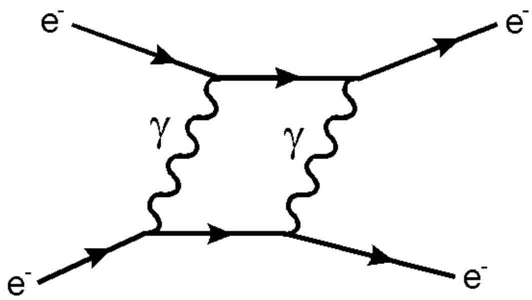


Figure 7: Two-photon exchange contribution

→ Probability $P(e^-e^- \rightarrow e^-e^-) = |M(1 \text{ photon exchange}) + M(2 \text{ photon exchange}) + M(3 \text{ photon exchange}) + \ldots|^2$ *(M=contribution=matrix element)*

❖ Number of vertices in a diagram is called its *order*.

❖ Each vertex has an associated probability proportional to a *coupling constant*, usually denoted as "α". In discussed processes this constant is

$$\alpha_{em} = \frac{e^2}{4\pi\varepsilon_0} \approx \frac{1}{137} \ll 1 \qquad (18)$$

❖ Matrix element for a two-vertex process is proportional to $M = \sqrt{\alpha} \cdot \sqrt{\alpha}$, where each vertex has a factor $\sqrt{\alpha}$. Probability for a process is $P = |M|^2 = \alpha^2$

❖ For the real processes, a diagram of order n gives a contribution to probability of order α^n.

Provided sufficiently small α, high order contributions are smaller and smaller and the result is convergent: $P(\text{real}) = |M(\alpha)+M(\alpha^2)+M(\alpha^3)...|^2$

Often lowest order calculation is precise enough.

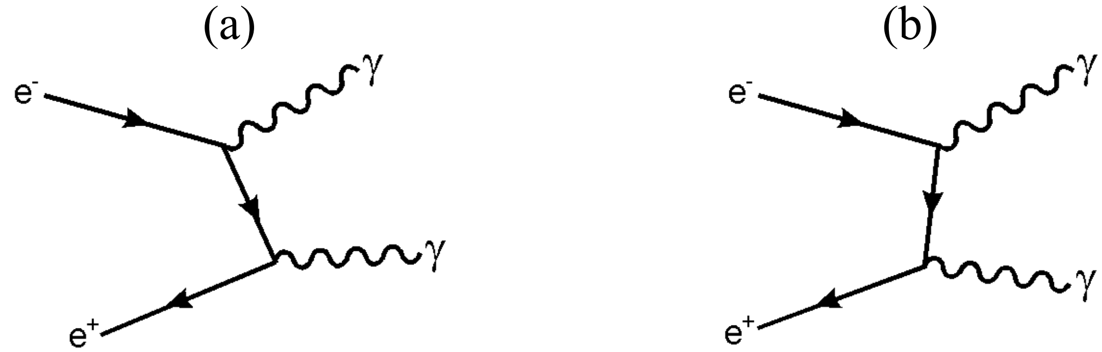


Figure 8: Lowest order contributions to $e^+e^- \to \gamma\gamma$. $M = \sqrt{\alpha} \cdot \sqrt{\alpha}$, $P=|M|^2=\alpha^2$

Diagrams which differ only by time-ordering are usually implied by drawing only one of them

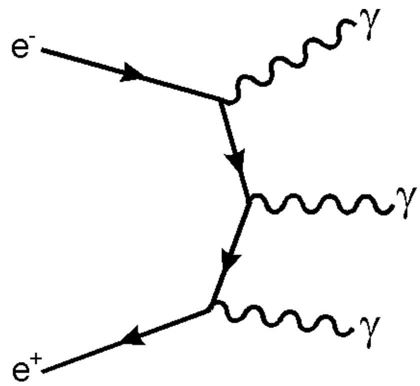


Figure 9: Lowest order of the process $e^+e^- \to \gamma\gamma\gamma$. $M = \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha}$, $P=|M|^2=\alpha^3$

This kind of process implies 3!=6 different time orderings

→ Only from the order of diagrams one can estimate the ratio of appearance rates of processes:

$$R \equiv \frac{Rate(e^+e^- \to \gamma\gamma\gamma)}{Rate(e^+e^- \to \gamma\gamma)} = \frac{O(\alpha^3)}{O(\alpha^2)} = O(\alpha)$$

This ratio can be measured experimentally; it appears to be $R = 0.9 \times 10^{-3}$, which is smaller than α_{em}, but the equation above is only a first order prediction.

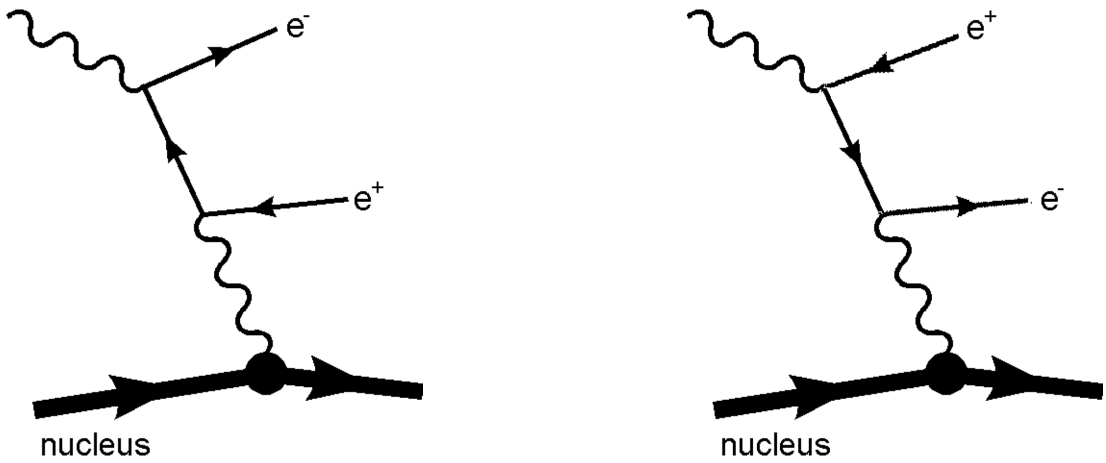


Figure 10: Diagrams are not related by time ordering

For nucleus, the coupling is proportional to $Z^2\alpha$, hence the rate of this process is of order $Z^2\alpha^3$

Exchange of a massive boson

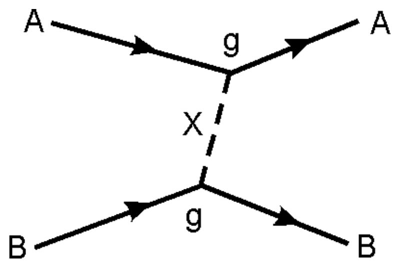


Figure 11: Exchange of a massive particle X

Consider the upper part of the diagram in Fig. 11,where particle A emits particle X: $A \rightarrow A + X$. The four-momentum of particle A is in the initial state $p = (E_0, \vec{p}_0)$.

In the rest frame of the particle A $\vec{p}_0 = (0, 0, 0)$, $E_0 = \sqrt{\vec{p}_0^{\,2} + M_A^2} = M_A$, so the four-vector is $p = (M_A, \vec{0})$. The reaction in the rest frame is thus:

$$A(M_A, \vec{0}) \rightarrow A(E_A, \vec{p}) + X(E_x, -\vec{p})$$

where

$$E_A = \sqrt{\vec{p}^{\,2} + M_A^2},\ E_X = \sqrt{\vec{p}^{\,2} + M_X^2}$$

From this one can estimate the maximum distance over which X can propagate before being absorbed. The energy violation is

$$\Delta E = E_{final} - E_{initial} = (E_X + E_A) - E_0$$

$$= (E_X + E_A) - M_A \geq M_X,$$

and this energy violation can exist only for a period of time $\Delta t \approx \hbar / \Delta E = \hbar / M_X$ (Heisenberg's uncertainty relation). Hence the *range of the interaction* is

$$r \approx R = \Delta t\, c \equiv (\hbar / M_X) c$$

→ For a massless exchanged particle, the interaction has an infinite range (e.g., electromagnetic)

→ In case of a very heavy exchanged particle (e.g., a W boson in weak interaction), the interaction can be approximated by a *zero-range*, or *point interaction*:

$$R_W = \hbar c / M_W = \hbar c / (80.4\ GeV/c^2) \approx 2 \times 10^{-18}\ \mathrm{m}$$

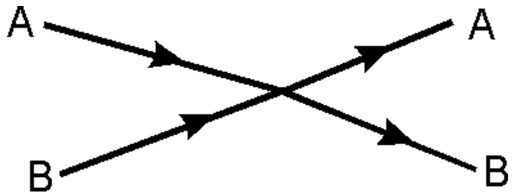


Figure 12: Point interaction as a result of $M_X \to \infty$

Consider particle X as an electrostatic potential V(r). Then the particle wavefunction $\Psi(\underline{x},t)$ can be replaced by V(r), assuming no time dependence. The Klein-Gordon equation (16) for particle X will then look like

$$\nabla^2 V(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = M_X^2 V(r) \tag{19}$$

(use spherical polar coordinate system, $\frac{\partial^2}{\partial t^2}V(r) = (0)$, $\nabla^2 V(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}V(r)\right)$).

Yukawa potential (1935)

Integration of the equation (18) gives the solution of

$$V(r) = -\frac{g^2}{4\pi r} e^{-r/R} \tag{20}$$

Here g is an integration constant, and it is interpreted as the coupling strength for particle X to the particles A and B.

→ In Yukawa theory, g is analogous to the electric charge in QED, and the analogue of α_{em} is

$$\alpha_X = \frac{g^2}{4\pi}$$

α_X characterizes the strength of the interaction at distances $r \le R$.

Consider a particle being scattered by the potential (20), thus receiving a momentum transfer $\vec{q}$

→ Potential (20) has the corresponding amplitude, which is its Fourier-transform (like in optics):

$$f(\vec{q}) = \int V(\vec{x}) e^{i\vec{q}\vec{x}} d^3\vec{x} \tag{21}$$

Using polar coordinates, $d^3\vec{x} = r^2 \sin\theta d\theta dr d\phi$, and assuming $V(\vec{x}) = V(r)$, the amplitude is

$$f(\vec{q}) = 4\pi g \int_0^{\infty} V(r) \frac{\sin(qr)}{qr} r^2 dr = \frac{-g^2}{q^2 + M_X^2} \qquad (22)$$

→ For the point interaction, $M_X^2 \gg q^2$, hence $f(\vec{q})$ becomes a constant:

$$f(\vec{q}) = -G = \frac{-4\pi\alpha_X}{M_X^2}$$

That means that the point interaction is characterized not only by α_X, but by M_X as well.

SUMMARY

❖ Matter consists of *particles = quarks and leptons* = fermions (spin 1/2). Particles interact via *4 forces*. Interaction = exchange of force-carrying particle.Force-carrying particles are called *gauge bosons* (spin-1).

❖ Particle kinematics is described by momentum- and space-time four-vectors p and x. Natural units are often used in calculations (c=hbar=1).

❖ Particles are described by a wavefunction $\Psi(x,t)$. Basic equations: the classical Schrödinger equation --> the Klein-Gordon equation for relativistic particles (but has negative E solutions) --> the Dirac equation. The Dirac equation describes correctly all the 4 particle states of spin-1/2 fermions and antifermions: two states for "spin-up" and "spin-down" particles, and two states for corresponding antiparticles.

❖ Antiparticles were first thought to be a weird consequence of quantum mechanics equations, but in 1933 the first antiparticle, positron, was actually found.

❖ Particle reactions can be described by Feynman diagrams. These are not just pictures, but they form a great aid in forming equations for particle reactions.

❖ Interactions: interactions are mediated by gauge bosons = particles. If these are massive, the interaction has a final distance. The strength of the interaction is characterized by a coupling constant α_X. If the gauge boson is “infinitely massive”, the interaction looks like a point interaction, characterized by both α_X. and M_X.