IV. Space-time symmetries

→ Conservation laws have their origin in the symmetries and invariance properties of the underlying interactions

→ Exact symmetry => conservation law => an observable whose absolute value cannot be defined ("non-observable")

Symmetries, conservation laws and "non-observables"

Symmetry transformation	Conservation law or selection rule	Non-observable
Space translation $\bar{x} \Rightarrow \bar{x} + \delta\bar{x}$	momentum	absolute spatial position
Rotation $\bar{x} \Rightarrow \bar{x}'$	angular momentum	absolute spatial direction
Time translation $t \Rightarrow t + \delta t$	energy	absolute time
Reflection $\bar{x} \Rightarrow \bar{x}' = -\bar{x}$	parity	"handedness" (absolute generalized right/left)

Symmetry transformation	Conservation law or selection rule	Non-observable
Charge conjugation $q \Rightarrow -q$	particle-antiparticle symmetry	absolute sign of electric charge
$\psi \Rightarrow e^{iq\theta}\psi$	charge q	rel. phase between states of different q
$\psi \Rightarrow e^{iL\theta}\psi$	lepton number L	rel. phase between states of different L
$\psi \Rightarrow e^{iB\theta}\psi$	baryon number B	rel. phase between states of different B

Translational invariance

❖ When a closed system of particles is moved from one position in space to another, its physical properties do not change

Consider an infinitesimal translation:

$$\vec{x}_i \rightarrow \vec{x}'_i = \vec{x}_i + \delta\vec{x}$$

the Hamiltonian of the system transforms as

$$H(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n) \rightarrow H(\vec{x}_1 + \delta\vec{x}, \vec{x}_2 + \delta\vec{x}, \ldots, \vec{x}_n + \delta\vec{x})$$

In the simplest case of a free particle,

$$H = -\frac{1}{2m}\nabla^2 = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \qquad (40)$$

From Equation (40) it is clear that

$$H(\vec{x}'_1, \vec{x}'_2, \ldots, \vec{x}'_n) = H(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n) \qquad (41)$$

which is true for any general closed system: the Hamiltonian is invariant under the translation operator $\hat{D}$, which is defined as an action onto an arbitrary wavefunction $\psi(\vec{x})$ such that

$$\hat{D}\psi(\vec{x}) \equiv \psi(\vec{x} + \delta\vec{x}) \tag{42}$$

For a single-particle state $\psi'(\vec{x}) = H(\vec{x})\psi(\vec{x})$. From eq. (42) one obtains:

$$\hat{D}\psi'(\vec{x}) = \psi'(\vec{x} + \delta\vec{x}) = H(\vec{x} + \delta\vec{x})\psi(\vec{x} + \delta\vec{x})$$

Since the Hamiltonian is invariant under translation, $\hat{D}\psi'(\vec{x}) = H(\vec{x})\psi(\vec{x} + \delta\vec{x})$ .Using (42) and ψ' definition

$$\hat{D}\psi'(\vec{x}) = \hat{D}H(\vec{x})\psi(\vec{x}) = H(\vec{x})\psi(\vec{x} + \delta\vec{x}) = H(\vec{x})\hat{D}\psi(\vec{x}) \tag{43}$$

This means that $\hat{D}$ *commutes* with Hamiltonian (a standard notation for this is $[\hat{D}, H] = \hat{D}H - H\hat{D} = 0$)

Since $\delta\vec{x}$ is an infinitely small quantity, translation (42) can be expanded as

$$\psi(\vec{x} + \delta\vec{x}) = \psi(\vec{x}) + \delta\vec{x} \cdot \nabla\psi(\vec{x}) \tag{44}$$

Form (44) includes explicitly the momentum operator $\hat{p} = -i\nabla$, hence the translation operator $\hat{D}$ can be rewritten as

$$\hat{D} = 1 + i\delta\vec{x} \cdot \hat{p} \tag{45}$$

Substituting (45) to (43), one obtains

$$[\hat{p}, H] = 0 \tag{46}$$

which is nothing but the *momentum conservation law* for a single-particle state whose Hamiltonian is invariant under translation.

Generalization of (45) and (46) for the case of multiparticle state leads to the general momentum conservation law for the total momentum $\vec{p} = \sum_{i=1}^{n} \vec{p}_i$

Rotational invariance

❖ When a closed system of particles is rotated about its centre-of-mass, its physical properties remain unchanged

Under the rotation about, for example, z-axis through an angle θ, coordinates x_i, y_i, z_i transform to new coordinates x'_i, y'_i, z'_i as following:

$$\begin{aligned} x'_i &= x_i cos\theta - y_i sin\theta \\ y'_i &= x_i sin\theta + y_i cos\theta \\ z'_i &= z \end{aligned} \qquad (47)$$

Correspondingly, the new Hamiltonian of the rotated system will be the same as the initial one,

$$H(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n) = H(\vec{x}'_1, \vec{x}'_2, \ldots, \vec{x}'_n)$$

Considering rotation through an infinitesimal angle $\delta\theta$, equations (47) transform to

$$x' = x - y\delta\theta \ , \ y' = y + x\delta\theta \ , \ z' = z$$

(θ small => cosθ = 1, sinθ = δθ)

A rotational operator is introduced by analogy with the translation operator $\hat{D}$:

$$\hat{R}_Z\psi(\vec{x}) \equiv \psi(\vec{x}') = \psi(x - y\delta\theta, y + x\delta\theta, z) \tag{48}$$

Expansion to first order in $\delta\theta$ gives

$$\psi(\vec{x}') = \psi(\vec{x}) - \delta\theta\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right)\psi(\vec{x}) = (1 + i\delta\theta\hat{L}_Z)\psi(\vec{x})$$

where $\hat{L}_z$ is the z-component of the orbital angular momentum operator $\hat{L}$:

$$\hat{L}_Z = -i\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \tag{49}$$

Remember: classical mechanics

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow L_z = (xp_y - yp_x)$$

→ For the general case of the rotation about an arbitrary direction specified by a unit vector $\vec{n}$, $\hat{L}_Z$ has to be replaced by the corresponding projection of $\hat{L}$: $\hat{L} \cdot \vec{n}$, hence

$$\hat{R}_n = 1 + i\delta\theta(\hat{L} \cdot \vec{n}) \tag{50}$$

Considering $\hat{R}_n$ acting on a single-particle state $\psi'(\vec{x}) = H(\vec{x})\psi(\vec{x})$ and repeating same steps as for the translation case, one gets:

$$[\hat{R}_n, H] = 0 \tag{51}$$

$$[\hat{L}, H] = 0 \tag{52}$$

=> conservation of angular momentum!

This applies for a spin-0 particle moving in a central potential, i.e., in a field which does not depend on a direction, but only on the absolute distance.

→ If a particle possesses a non-zero spin, the total angular momentum is the sum of the orbital and spin angular momenta:

$$\hat{J} = \hat{L} + \hat{S} \tag{53}$$

and the wavefunction is the product of the

[independent] space wavefuncion $\psi(\vec{x})$ and spin wavefunction χ:

$$\Psi = \psi(\vec{x})\chi \tag{54}$$

For the case of spin-1/2 particles, the spin operator is represented in terms of Pauli matrices σ:

$$\hat{S} = \frac{1}{2}\sigma \tag{55}$$

where σ has components :

(recall chapter 1 of these notes)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{56}$$

Let us denote now spin wavefunction for spin "up" state as $\chi = \alpha$ ($S_z = 1/2$) and for spin "down" state as $\chi = \beta$ ($S_z = -1/2$), so that

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{57}$$

Both α and β satisfy the eigenvalue equations for operator (55):

$$\hat{S}_z\alpha = \frac{1}{2}\alpha, \hat{S}_z\beta = -\frac{1}{2}\beta$$

Analogously to (50), the rotation operator for the spin-1/2 particle generalizes to

$$\hat{R}_n = 1 + i\delta\theta(\hat{J} \cdot \vec{n}) \tag{58}$$

When the rotation operator $\hat{R}_n$ acts onto the wave function $\Psi = \psi(\vec{x})\chi$, components $\hat{L}$ and $\hat{S}$ of $\hat{J}$ act independently on the corresponding wavefunctions:

$$\hat{J}\Psi = (\hat{L} + \hat{S})\psi(\vec{x})\chi = [\hat{L}\psi(\vec{x})]\chi + \psi(\vec{x})[\hat{S}\chi]$$

That means that although the total angular momentum has to be conserved, $[\hat{J}, H] = 0$, the rotational invariance does not in general lead to the conservation of $\hat{L}$ and $\hat{S}$ separately:

$$[\hat{L}, H] = -[\hat{S}, H] \neq 0$$

However, presuming that the forces can change only orientation of the spin, but not its absolute value $\Rightarrow$

$$[H, \hat{L}^2] = [H, \hat{S}^2] = 0$$

→ Good quantum numbers are those which are associated with conserved observables (operators commute with the Hamiltonian)

Spin is one of the quantum numbers which characterize any particle - elementary or composite.

❖ Spin $\vec{S}_P$ of a composite particle is the total angular momentum $\vec{J}$ of its constituents in their centre-of-mass frame

– Quarks are spin-1/2 particles ⇒ the spin quantum number $S_P=J$ can be either integer or half-integer for composite particles (hadrons)

– Its projections on the z-axis – J_z – can take any of $2J+1$ values, from $-J$ to J with the "step" of 1, depending on the particle's spin orientation

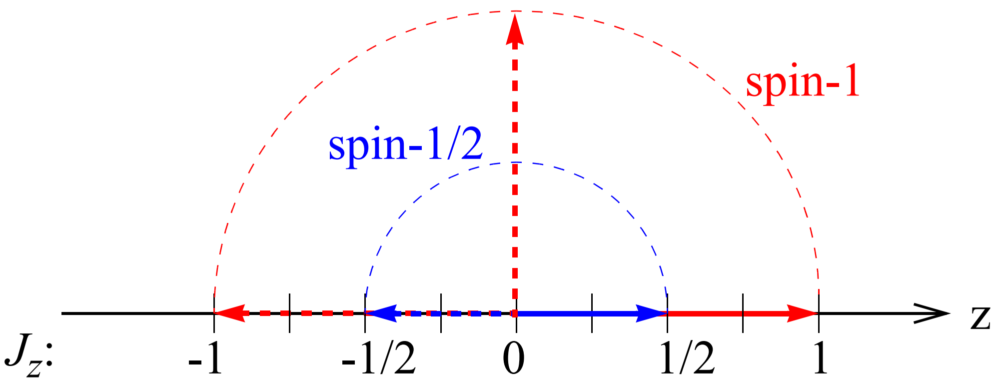

Figure 39: A naive illustration of possible J_z values for spin-1/2 and spin-1 particles

❖ Usually, it is assumed that L and S are "good" quantum numbers together with $J=S_P$, while J_z depends on the spin orientation.

Using “good” quantum numbers, one can refer to a particle via *spectroscopic notation*, like

$$^{2S+1}L_J \tag{59}$$

– Following chemistry traditions, instead of numerical values of $L=0,1,2,3...$, letters S,P,D,F... are used correspondingly

– In this notation, the lowest-lying ($L=0$) bound state of two particles of spin-1/2 will be $^1S_0$ or $^3S_1$

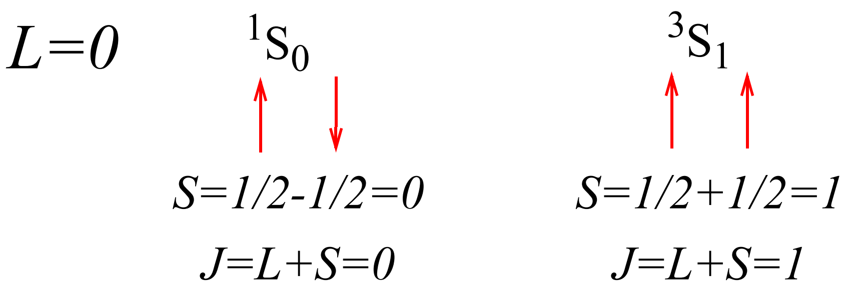

Figure 40: Quark-antiquark states for $L=0$

For mesons with $L \geq 1$, possible states are:

$$^1L_L\,,\ ^3L_{L+1}\,,\ ^3L_L\,,\ ^3L_{L-1}$$

→ Baryons are bound states of 3 quarks $\Rightarrow$ there are two orbital angular momenta connected to the relative motion of quarks.

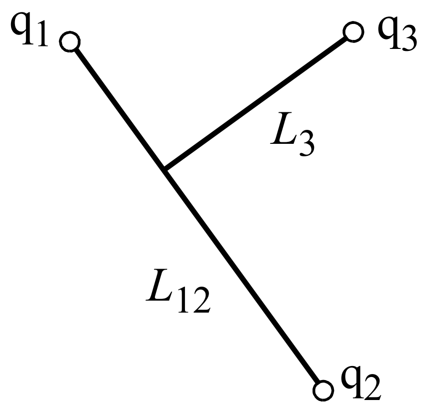

Figure 41: Internal orbital angular momenta of a three-quark state

– total orbital angular momentum is $L=L_{12}+L_3$.

– spin of a baryon $S=S_1+S_2+S_3 \Rightarrow S=1/2$ or $S=3/2$

Possible baryon states:

$$^2S_{1/2},\ ^4S_{3/2} \qquad (L=0)$$

$$^2P_{1/2},\ ^2P_{3/2},\ ^4P_{1/2},\ ^4P_{3/2},\ ^4P_{5/2} \qquad (L=1)$$

$$^2L_{L+1/2},\ ^2L_{L-1/2},\ ^4L_{L-3/2},\ ^4L_{L-1/2},\ ^4L_{L+1/2},\ ^4L_{L+3/2} \qquad (L\geq 2)$$

Parity

❖ Parity transformation is the transformation by reflection:

$$\vec{x}_i \rightarrow \vec{x}'_i = -\vec{x}_i \tag{60}$$

A system is invariant under parity transformation if

$$H(-\vec{x}_1, -\vec{x}_2, \ldots, -\vec{x}_n) = H(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n)$$

→ Parity is not an exact symmetry: it is violated in weak interaction => absolute "handedness" CAN be defined!

A parity operator $\hat{P}$ is defined as

$$\hat{P}\psi(\vec{x}, t) \equiv P_a \psi(-\vec{x}, t) \tag{61}$$

where P_a is the parity eigenvalue. Two consecutive reflections must give back the initial system:

$$\hat{P}^2 \psi(\vec{x}, t) = \psi(\vec{x}, t) \tag{62}$$

From equations (61) and (62), $P_a = +1, -1$

Consider a particle wavefunction which is a solution of the Dirac equation (17):

$$\psi_{\vec{p}}(\vec{x}, t) = u(\vec{p})e^{i(\vec{p}\vec{x} - Et)}, \tag{63}$$

where *u(p) is a four-component spinor (see p. 11) independent of x.* Parity operation on this wavefunction is:

$$\hat{P}\psi_{\vec{p}}(\vec{x}, t) = P_a u(-\vec{p})e^{i((-\vec{p})(-\vec{x}) - Et)} \tag{64}$$

When $\vec{p} = 0$ (the particle is at rest), the state ψ is an eigenstate of the parity operator:

$$\hat{P}\psi_0(\vec{x}, t) = P_a u(0)e^{-iEt} = P_a\psi_0(\vec{x}, t) \tag{65}$$

with eigenvalue P_a. P_a is called the intrinsic parity of a particle a: intrinsic parity= parity of a particle at rest.

For a system of *n* particles,

$$\hat{P}\psi(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n, t) \equiv P_1 P_2 \ldots P_n \psi(-\vec{x}_1, -\vec{x}_2, \ldots, -\vec{x}_n, t)$$

In polar coordinates, the parity transformation is:

$$r \to r' = r\ ,\ \theta \to \theta' = \pi - \theta\ ,\ \varphi \to \varphi' = \pi + \varphi$$

and a wavefunction can be written as

$$\psi_{nlm}(\vec{x}) = R_{nl}(r) Y_l^m(\theta, \varphi) \qquad (66)$$

In Equation (66), R_{nl} is a function of the radius only, and Y_l^m are *spherical harmonics*, which describe angular dependence.

Under the parity transformation, R_{nl} does not change, while spherical harmonics change as

$$Y_l^m(\theta, \varphi) \to Y_l^m(\pi - \theta, \pi + \varphi) = (-1)^l Y_l^m(\theta, \varphi)$$

$$\Downarrow$$

$$\hat{P}\psi_{nlm}(\vec{x}) = P_a \psi_{nlm}(-\vec{x}) = P_a (-1)^l \psi_{nlm}(\vec{x})$$

→ which means that a particle with a definite orbital angular momentum is also an eigenstate of parity with an eigenvalue $P_a(-1)^l$.

Considering only electromagnetic and strong interactions, and using the usual argumentation, one can prove that parity is conserved:

$$[\hat{P}, H] = 0$$

❖ Recall: the Dirac equation (17) (relativistic quantum mechanics) suggests a four-component wavefunction to describe both electrons and positrons: 2 components for electrons, 2 components for positrons. Note that in classical QM there would be no connection between parities of e^- and e^+.

→ Intrinsic parities of e^- and e^+ are related, namely:

$$P_{e^+} P_{e^-} = -1$$

This is true for all fermions (spin-1/2 particles), i.e.,

$$P_f P_{\bar{f}} = -1 \qquad (67)$$

Experimentally this can be confirmed by studying the reaction $e^+e^- \rightarrow \gamma\gamma$ where initial state has zero orbital momentum and parity of $P_{e^-} P_{e^+}$.

If the final state has relative orbital angular momentum l_γ, its parity is $P_\gamma^2 (-1)^{l_\gamma}$. Since $P_\gamma^2 = 1$, the parity conservation law requires that

$$P_{e^-} P_{e^+} = -1 = (-1)^{l_\gamma}$$

Experimental measurements of l_γ confirm (67).

While (67) can be proved in experiments, it is impossible to determine P_{e^-} or P_{e^+}, since these particles are created or destroyed only in pairs.

– Convention: define parities of leptons as:

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} \equiv 1 \qquad (68)$$

And consequently, parities of antileptons have opposite sign.

– Since quarks and antiquarks are also produced only in pairs, their parities are defined also by convention:

$$P_u = P_d = P_s = P_c = P_b = P_t = 1 \qquad (69)$$

with parities of antiquarks being -1.

For a meson M=($a\bar{b}$), parity is then calculated as

$$P_M = P_a P_{\bar{b}} (-1)^L = (-1)^{L+1} \qquad (70)$$

since $P_a P_{\bar{b}}$= -1. For the low-lying mesons ($L=0$) that means parity of -1, which is confirmed by observations.

For a baryon B=(abc), parity is given as

$$P_B = P_a P_b P_c (-1)^{L_{12}} (-1)^{L_3} = (-1)^{L_{12} + L_3} \qquad (71)$$

since $P_a P_b P_c$=1. For antibaryon $P_{\bar{B}} = -P_B$, similarly to the case of leptons.

For the low-lying baryons (L_{12}=L_3=0), Eq. (71) predicts positive parities, which is also confirmed by experiment.

Parity of the photon can be deduced from the classical field theory, considering the differential form of the Gauss's law:

$$\nabla \cdot \vec{E}(\vec{x}, t) = \frac{1}{\varepsilon_0}\rho(\vec{x}, t)$$

Under a parity transformation, charge density changes as $\rho(\vec{x}, t) \to \rho(-\vec{x}, t)$ and ∇ changes its sign, so that to keep the equation invariant, the electric field must transform as

$$\vec{E}(\vec{x}, t) \to -\vec{E}(-\vec{x}, t) \tag{72}$$

On the other hand, the electromagnetic field is described by the vector and scalar potentials:

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t} \tag{73}$$

For the photon, only the vector part corresponds to the wavefunction:

$$\vec{A}(\vec{x}, t) = N\vec{\varepsilon}(\vec{k})e^{i(\vec{k}\vec{x} - Et)}$$

Under the parity transformation,

$$\hat{P}\vec{A}(\vec{x}, t) \to P_\gamma \vec{A}(-\vec{x}, t)$$

and from (72) it is obtained that

$$\vec{E}(\vec{x}, t) \to P_\gamma \vec{E}(-\vec{x}, t)\,. \qquad (74)$$

Comparing (74) and (72), one concludes that parity of photon is $P_\gamma = -1$.

Charge conjugation

❖ Charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments

→ Charge conjugation is violated by the weak interaction => absolute sign of electric charge CAN be defined!

For the strong and electromagnetic interactions, charge conjugation is a symmetry:

$$[\hat{C}, H] = 0$$

– It is convenient now to denote a state in a compact notation, using Dirac's "ket" representation: $|\pi^+, \vec{p}\rangle$ denotes a pion having momentum $\vec{p}$, ($|\pi^+, \vec{p}\rangle = \psi_{\vec{p}}(\vec{x}, t) = u(\vec{p})e^{i(\vec{p}\vec{x} - Et)}$). In the general case,

$$|\pi^+\Psi_1;\pi^-\Psi_2\rangle \equiv |\pi^+\Psi_1\rangle|\pi^-\Psi_2\rangle \tag{75}$$

Next, we denote particles which have distinct antiparticles by "*a*" (*a* is the antiparticle of *a* and vice versa). Particles for which particle and antiparticle are the same are noted by "α".

In this notation, we describe the action of the charge conjugation operator to particles "α" as:

$$\hat{C}|\alpha, \Psi\rangle = C_\alpha|\alpha, \Psi\rangle \tag{76}$$

meaning that the final state acquires a phase factor C_α. The action of the charge conjugation operator to particles "*a*" is

$$\hat{C}|a, \Psi\rangle = |\bar{a}, \Psi\rangle \tag{77}$$

meaning that we transformed a particle in the initial state into an antiparticle in the final state.

Since a second transformation turns antiparticles back to particles, $\hat{C}^2 = 1$, and the eigenvalue is

$$C_\alpha = \pm 1 \tag{78}$$

For multiparticle states the transformation is:

$$\hat{C}|\alpha_1, \alpha_2, \ldots, a_1, a_2, \ldots;\Psi\rangle =$$

$$= C_{\alpha_1} C_{\alpha_2} \ldots |\alpha_1, \alpha_2, \ldots, \bar{a}_1, \bar{a}_2, \ldots;\Psi\rangle \tag{79}$$

– From (76) it is clear that particles $\alpha=\gamma,\pi^0$,... etc., are eigenstates of $\hat{C}$ with eigenvalues $C_\alpha=\pm 1$.

– Other eigenstates can be constructed from particle-antiparticle pairs:

$$\hat{C}|a, \Psi_1;\bar{a}, \Psi_2\rangle = |\bar{a}, \Psi_1;a\Psi_2\rangle = \pm|a, \Psi_1;\bar{a}, \Psi_2\rangle$$

For a state of definite orbital angular momentum, interchanging between particle and antiparticle reverses their relative position vector, for example:

$$\hat{C}|\pi^+\pi^-;L\rangle = (-1)^L|\pi^+\pi^-;L\rangle \tag{80}$$

For fermion-antifermion pairs theory predicts

$$\hat{C}|f\bar{f};J,L,S\rangle = (-1)^{L+S}|f\bar{f};J,L,S\rangle \tag{81}$$

This implies that π^0, being a $^1S_0$ state of $u\bar{u}$ and $d\bar{d}$, must have C-parity of 1.

Tests of C-invariance

Prediction of $C_{\pi^0} = 1$ can be confirmed

experimentally by studying the decay $\pi^0 \to \gamma\gamma$. The final state has *C=1,* and from the relations

$$\hat{C}|\pi^0\rangle = C_{\pi^0}|\pi^0\rangle$$

$$\hat{C}|\gamma\gamma\rangle = C_\gamma C_\gamma|\gamma\gamma\rangle = |\gamma\gamma\rangle$$

it stems that $C_{\pi^0} = 1$.

C_γ can be inferred from the classical field theory:

$$\vec{A}(\vec{x}, t) \to C_\gamma \vec{A}(\vec{x}, t)$$

under the charge conjugation, and since all electric charges swap, electric field and scalar potential also change sign:

$$\vec{E}(\vec{x}, t) \to -\vec{E}(\vec{x}, t) \ , \ \phi(\vec{x}, t) \to -\phi(\vec{x}, t) \ ,$$

which upon substitution into (73) gives $C_\gamma = -1$.

To check predictions of the C-invariance and of the value of C_γ, one can try to look for the decay

$$\pi^0 \to \gamma + \gamma + \gamma$$

If both predictions are true, this mode should be forbidden:

$$\hat{C}|\gamma\gamma\gamma\rangle = (C_\gamma)^3|\gamma\gamma\gamma\rangle = -|\gamma\gamma\gamma\rangle$$

which contradicts all previous observations.

Experimentally, this 3γ mode have never been observed.

Another confirmation of C-invariance comes from observation of η-meson decays:

$$\eta \to \gamma + \gamma$$
$$\eta \to \pi^0 + \pi^0 + \pi^0$$
$$\eta \to \pi^+ + \pi^- + \pi^0$$

They are electromagnetic decays, and first two clearly indicate that C_η=1. Identical charged pions momenta distribution in third confirm C-invariance.

SUMMARY

❖ Conservation laws stem from symmetries and invariance properties. Exact symmetry (invariance of the Hamiltonian H under an operation, i.e. the operator commutes with H) <=> conservation law <=> an observable whose absolute value cannot be defined.

❖ Invariance under spatial translation <=> *momentum conservation* <=> absolute spatial position undefined.

❖ Invariance under rotation <=> *angular momentum conservation* <=> absolute spatial direction undefined.

❖ Using "good" quantum numbers *L*, *S* and *J*=*S*$_P$, the *spectroscopic notation* of a particle is $^{2S+1}L_J$.

❖ Parity transformation is the transformation by reflection. Parity is violated in weak interaction => absolute "handedness" CAN be defined!

❖ A particle with a definite orbital angular momentum is an eigenstate of parity with an eigenvalue $P_a(-1)^l$.

❖ Intrinsic parities of a fermion and an antifermion are related, $P_f P_{\bar{f}} = -1$. Convention: parities of leptons/quarks are $P_l = P_q = 1$. Parities of antileptons/antiquarks have opposite sign.

❖ For a meson M=(a$\bar{\text{b}}$), parity is $P_M = P_a P_{\bar{b}}(-1)^L = (-1)^{L+1}$. For a baryon B=(abc), parity is $P_B = P_a P_b P_c (-1)^{L_{12}}(-1)^{L_3} = (-1)^{L_{12}+L_3}$.

❖ Charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments. Charge conjugation is violated by the weak interaction => absolute sign of electric charge CAN be defined!

❖ If particle=antiparticle = α (α=γ,π^0,... etc.), $\hat{C}|\alpha, \Psi\rangle = C_\alpha|\alpha, \Psi\rangle$. These particles are eigenstates of $\hat{C}$ with eigenvalues $C_\alpha = \pm 1$. Other eigenstates: particle-antiparticle pairs.

❖ For fermion-antifermion pairs

$$\hat{C}|f\bar{f}; J, L, S\rangle = (-1)^{L+S}|f\bar{f}; J, L, S\rangle \quad .$$