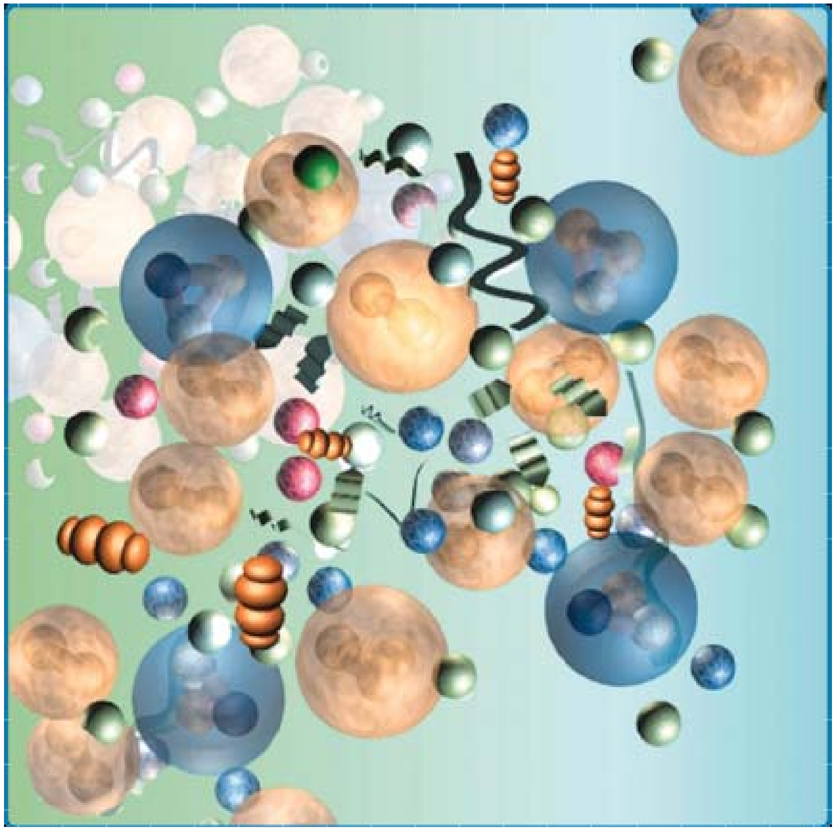
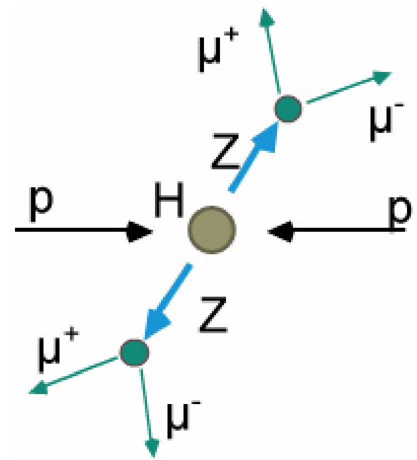
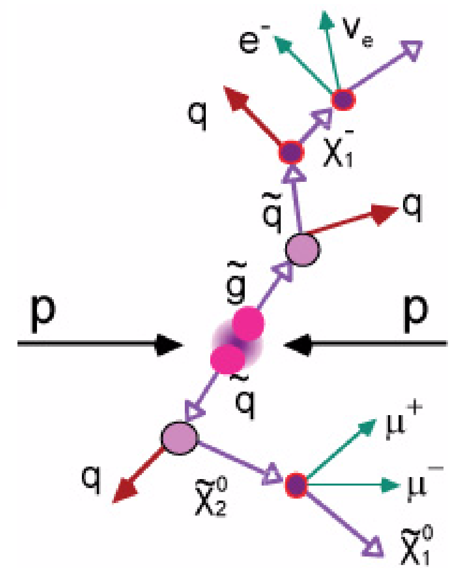
Modern Experimental Particle Physics







Oxana Smirnova
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Division of Experimental High Energy Physics
Lund University

I. Basic concepts

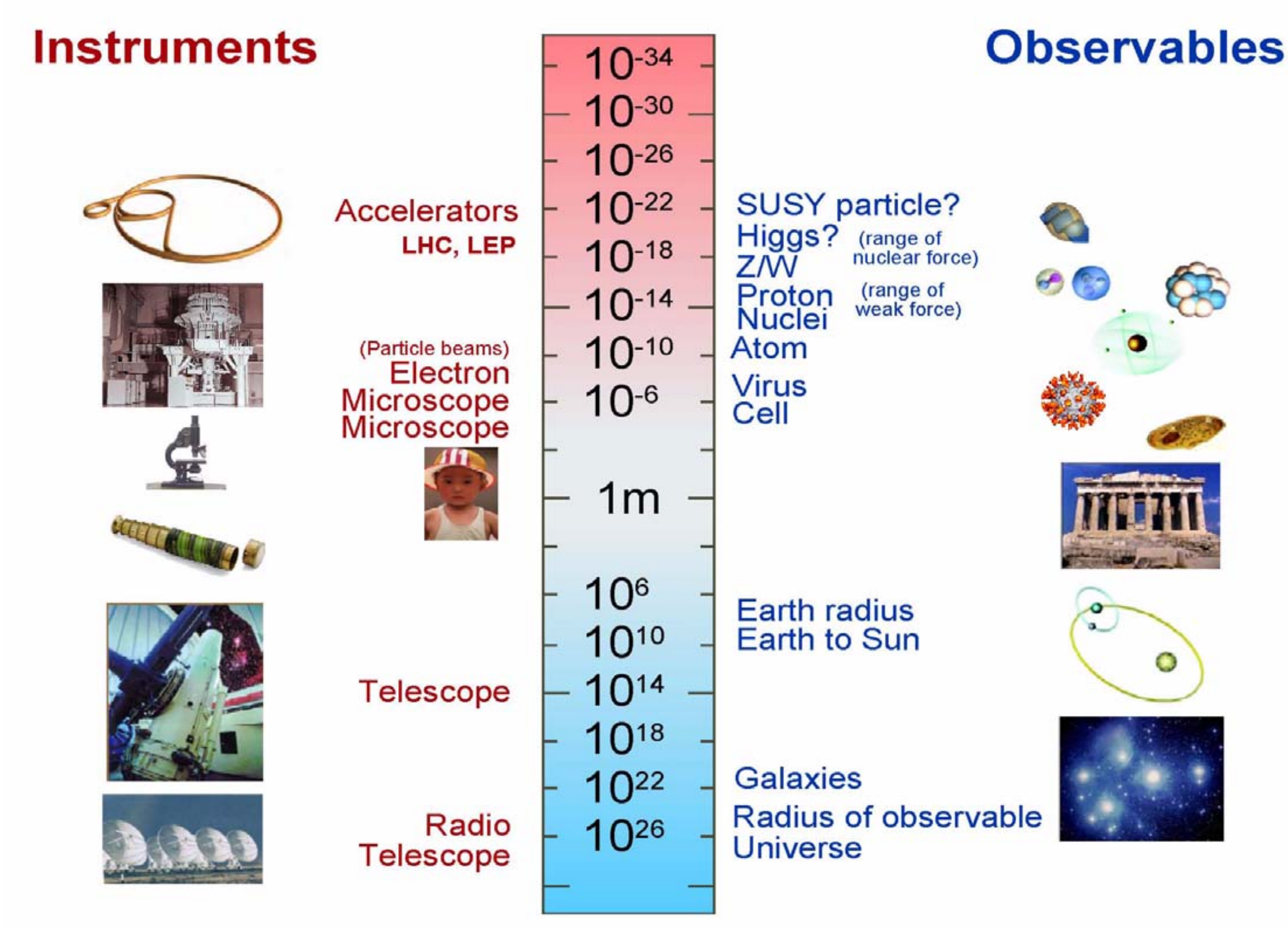

Figure 1: Object sizes and observation instruments

Main concepts

- Particle physics studies elementary “building blocks” of *matter* and *interactions* between them.
- Matter consists of *particles*.
  - Matter is built of particles called “fermions”: those that have half-integer spin, e.g. 1/2; obey *Fermi-Dirac statistics*.
- Particles interact via *forces*.
  - Interaction is an exchange of a force-carrying particle.
- Force-carrying particles are called *gauge bosons* (spin-1).

Forces of nature

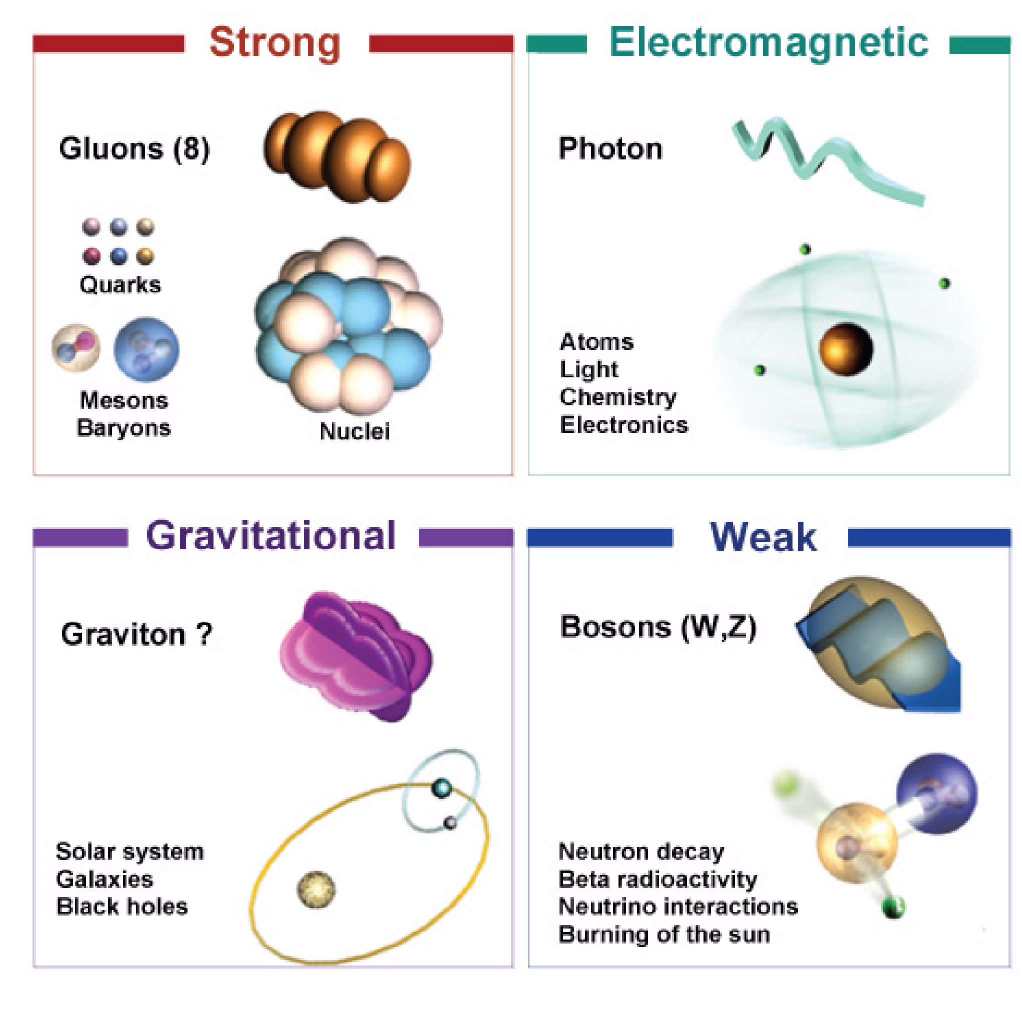

Figure 2: Forces and their carriers

Summary table of forces:

Force	Acts on/ *couples to:*	Carrier	Range	Strength	Stable systems	Induced reaction
Gravity	all particles *Mass/E-p tensor*	graviton G *(has not yet been observed)*	long $F \propto 1/r^2$	$\sim 10^{-39}$	Solar system	Object falling
Weak force	fermions *hypercharge*	bosons W^+,W^- and Z	$< 10^{-17}$ m	10^{-5}	None	β-decay
Electro-magnetism	charged particles *electric charge*	photon γ	long $F \propto 1/r^2$	$1/137$	Atoms, molecules	Chemical reactions
Strong force	quarks and gluons *colour charge*	gluons g (8 different)	10^{-15} m	1	Hadrons, nuclei	Nuclear reactions

The Standard Model

- Electromagnetic and weak forces can be described by a single theory $\Rightarrow$ the *"Electroweak Theory"* was developed in 1960s (Glashow, Weinberg, Salam).

- Theory of strong interactions appeared in 1970s: *"Quantum Chromodynamics"* (QCD).

- The *"Standard Model"* (SM) combines all the current knowledge.
  - Gravitation is VERY weak at particle scale, and it is not included in the SM. Moreover, quantum theory for gravitation does not exist yet.

Main postulates of SM:

1) Basic constituents of matter are *quarks* and *leptons* (spin 1/2)

2) They interact by exchanging *gauge bosons* (spin 1)

3) Quarks and leptons are subdivided into 3 *generations*

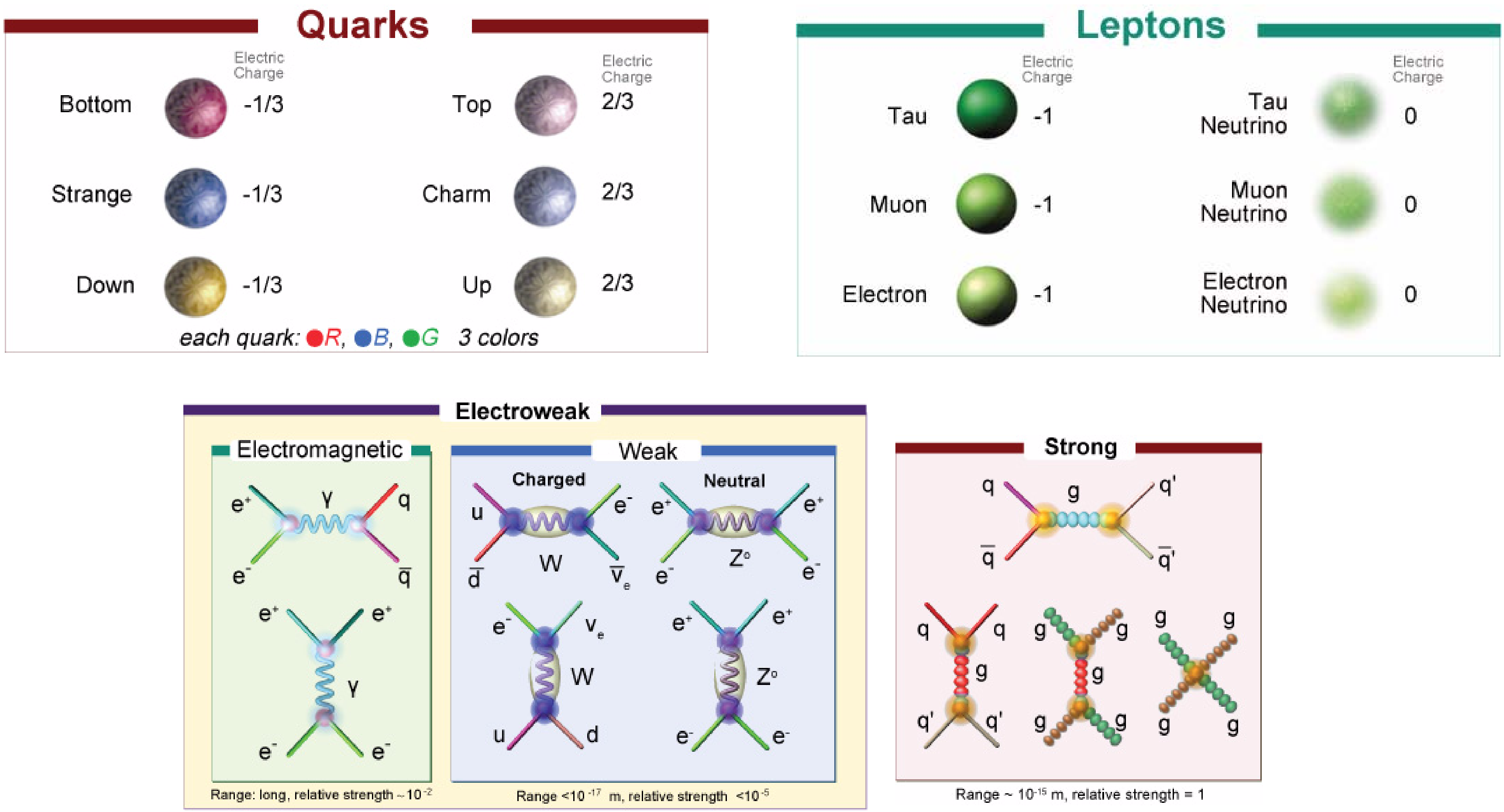

Figure 3: Standard Model: quarks, leptons and bosons

- SM does not explain neither appearance of the mass nor the reason for existence of the 3 generations.

History of the Universe as we know it

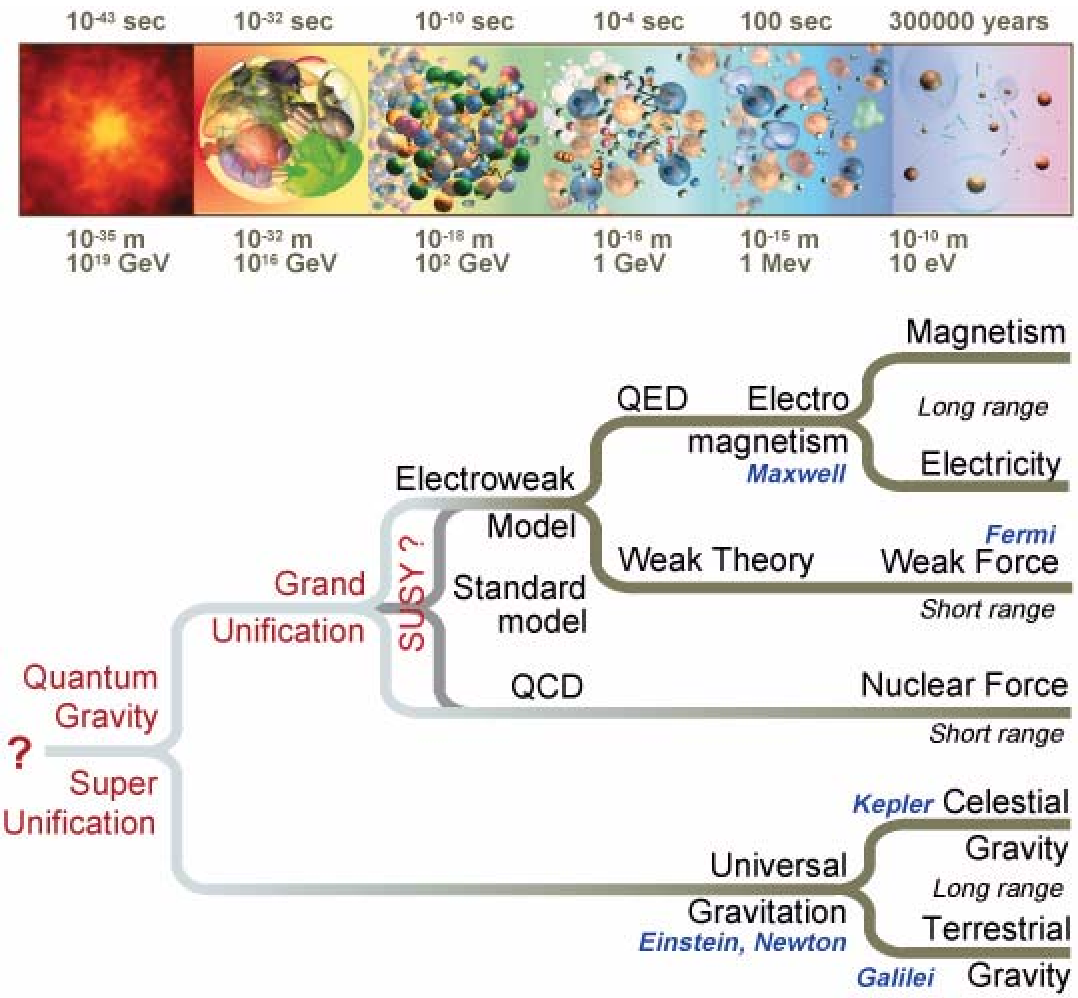

Figure 4: Summary of the current knowledge about our Universe

Units and dimensions

- Particle energy is measured in *electron-volts*:

$$1\ \text{eV} \approx 1.602 \times 10^{-19}\ \text{J} \tag{1}$$

  - 1 eV is energy of an electron upon passing a voltage of 1 Volt.
  - 1 keV = 10^3 eV; 1 MeV = 10^6 eV; 1 GeV = 10^9 eV

- The reduced *Planck constant* and the *speed of light*:

$$\hbar \equiv h/2\pi = 6.582 \times 10^{-22}\ \text{MeV s} \tag{2}$$

$$c = 2.9979 \times 10^{8}\ \text{m/s} \tag{3}$$

and the "*conversion constant*" is:

$$\hbar c = 197.327 \times 10^{-15}\ \text{MeV m} \tag{4}$$

- For simplicity, *natural units* are used:

$$\hbar = 1 \qquad \text{and} \qquad c = 1 \tag{5}$$

thus the unit of mass is *eV/c²*, and the unit of momentum is *eV/c*

Four-vector formalism

Relativistic kinematics is formulated with four-vectors:

- space-time four-vector: $x=(t,\bar{x})=(t,x,y,z)$, where t is time and $\bar{x}$ is a coordinate vector (c=1 notation is used)
- momentum four-vector: $p=(E,\bar{p})=(E,p_x,p_y,p_z)$, where E is particle energy and $\bar{p}$ is particle momentum vector

Calculus rules with four-vectors:

- 4-vectors are defined as
  - *contravariant*:

$$A^{\mu} = (A^0, \vec{A}), B^{\mu} = (B^0, \vec{B}), \tag{6}$$

  - and *covariant*:

$$A_{\mu} = (A^0, \overrightarrow{-A}), B_{\mu} = (B^0, \overrightarrow{-B}). \tag{7}$$

- *Scalar product* of two four-vectors is defined as:

$$A \cdot B = A^0 B^0 - (\vec{A} \cdot \vec{B}) = A_\mu B^\mu = A^\mu B_\mu . \quad (8)$$

- Scalar products of momentum and space-time four-vectors are thus:

$$x \cdot p = x^0 p^0 - (\vec{x} \cdot \vec{p}) = Et - (\vec{x} \cdot \vec{p}) \quad (9)$$

4-vector product of coordinate and momentum represents particle wavefunction

$$p \cdot p = p^2 = p^0 p^0 - (\vec{p} \cdot \vec{p}) = E^2 - \vec{p}^2 \equiv m^2 \quad (10)$$

4-momentum squared gives particle's invariant mass

For relativistic particles, we can see that

$$\boldsymbol{E^2 = p^2 + m^2} \quad (c=1) \quad (11)$$

Antiparticles

- Particles are described by wavefunctions:

$$\Psi(\vec{x}, t) = N e^{i(\vec{p}\vec{x} - Et)} \tag{12}$$

$\vec{x}$ is the coordinate vector, $\vec{p}$ - momentum vector, E and t are energy and time.

Particles obey the classical Schrödinger equation:

$$i\frac{\partial}{\partial t}\Psi(\vec{x}, t) = H\Psi(\vec{x}, t) = \frac{\vec{p}^2}{2m}\Psi(\vec{x}, t) = -\frac{1}{2m}\nabla^2\Psi(\vec{x}, t) \tag{13}$$

$$\text{here } \vec{p} = \frac{h}{2\pi i}\nabla \equiv \frac{\nabla}{i} \tag{14}$$

For relativistic particles, $\boldsymbol{E^2=p^2+m^2}$ (11), and (13) is replaced by the Klein-Gordon equation (15):

$$\Downarrow$$

$$-\frac{\partial^2}{\partial t^2}(\Psi) = H^2\Psi(\vec{x}, t) = -\nabla^2\Psi(\vec{x}, t) + m^2\Psi(\vec{x}, t) \tag{15}$$

- There exist *negative* energy solutions with $E_+ < 0$!

$$\Psi^*(\vec{x}, t) = N^* \cdot e^{i(-\vec{p}\vec{x} + E_+ t)}$$

- There is a problem with the Klein-Gordon equation: it is second order in derivatives. In 1928, Dirac found the first-order form having the same solutions:

$$i\frac{\partial\Psi}{\partial t} = -i\sum_i \alpha_i \frac{\partial\Psi}{\partial x_i} + \beta m\Psi \tag{16}$$

Here α_i and β are 4×4 matrices, and Ψ are four-component wavefunctions: *spinors* (for particles with spin 1/2).

$$\Psi(\vec{x}, t) = \begin{bmatrix} \Psi_1(\vec{x}, t) \\ \Psi_2(\vec{x}, t) \\ \Psi_3(\vec{x}, t) \\ \Psi_4(\vec{x}, t) \end{bmatrix}$$

Dirac-Pauli representation of matrices α_i and β:

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Here I is a 2×2 unit matrix, 0 is a 2×2 matrix of zeros, and σ_i are 2×2 *Pauli matrices*:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Also possible is *Weyl representation*:

$$\alpha_i = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Dirac's picture of vacuum

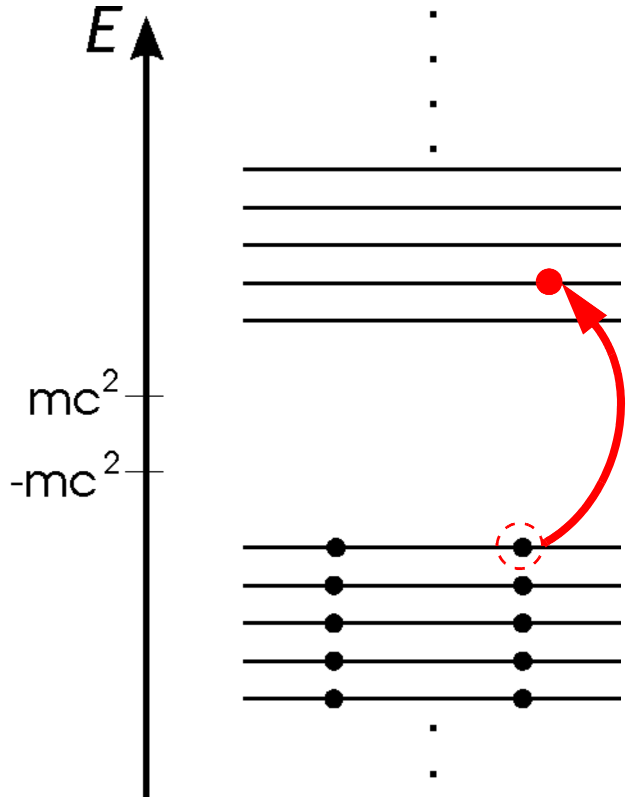

Figure 5: Fermions in Dirac's representation

- The "hole" created by appearance of an electron with "normal" energy is interpreted as the presence of electron's *antiparticle* with the opposite charge.
- Every charged particle must have an antiparticle of the same mass and opposite charge, to solve the mystery of "negative" energy.

Discovery of the positron

- 1933, C.D.Andersson, Univ. of California (Berkeley) observed with the Wilson cloud chamber 15 tracks in cosmic rays:

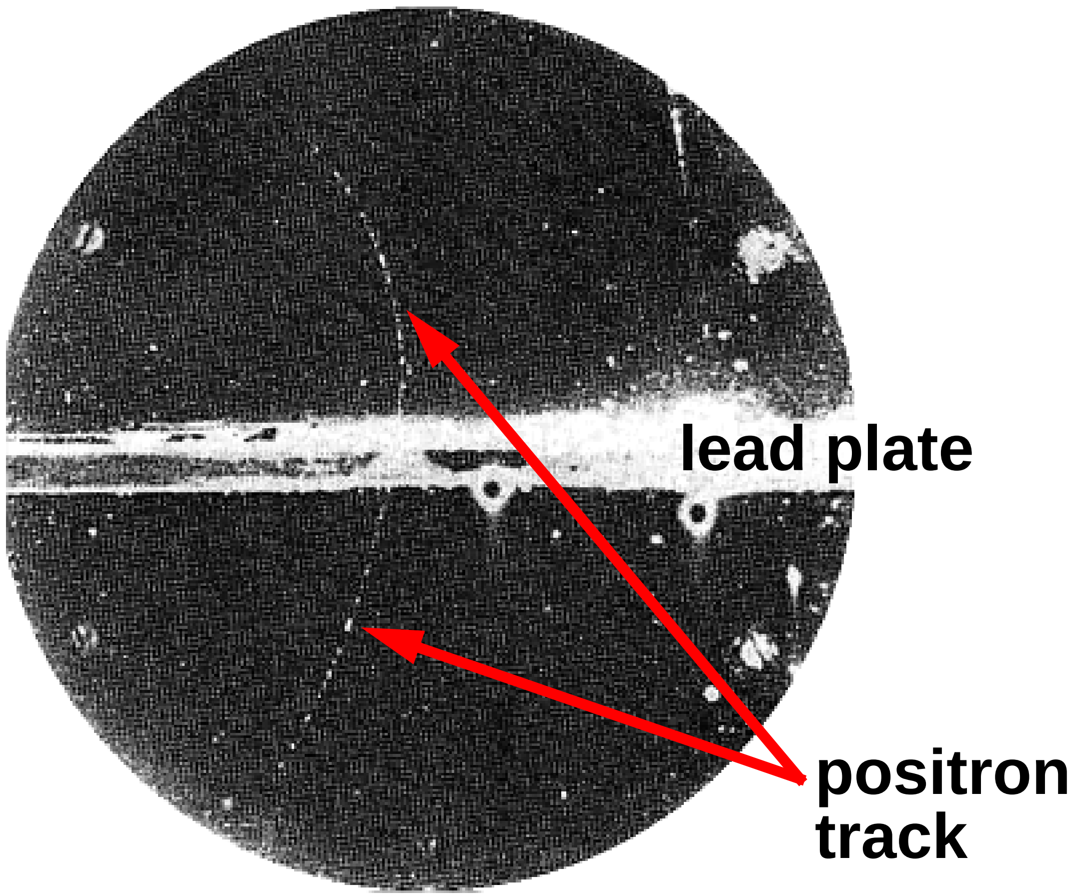

Figure 6: Photo of the track in the Wilson chamber

Feynman diagrams

In 1940s, R.Feynman developed a diagram technique for representing processes in particle physics.

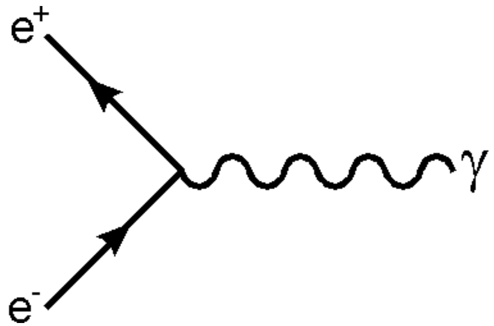

Figure 7: A Feynman diagram example: e+e- -> γ

Main assumptions and requirements:

- Time runs from left to right
- Arrow directed towards the right indicates a particle, and otherwise - antiparticle
- At every vertex, momentum, angular momentum and charge are conserved (but not necessarily energy)
- Particles are shown by solid lines, gauge bosons - by helices or dashed lines

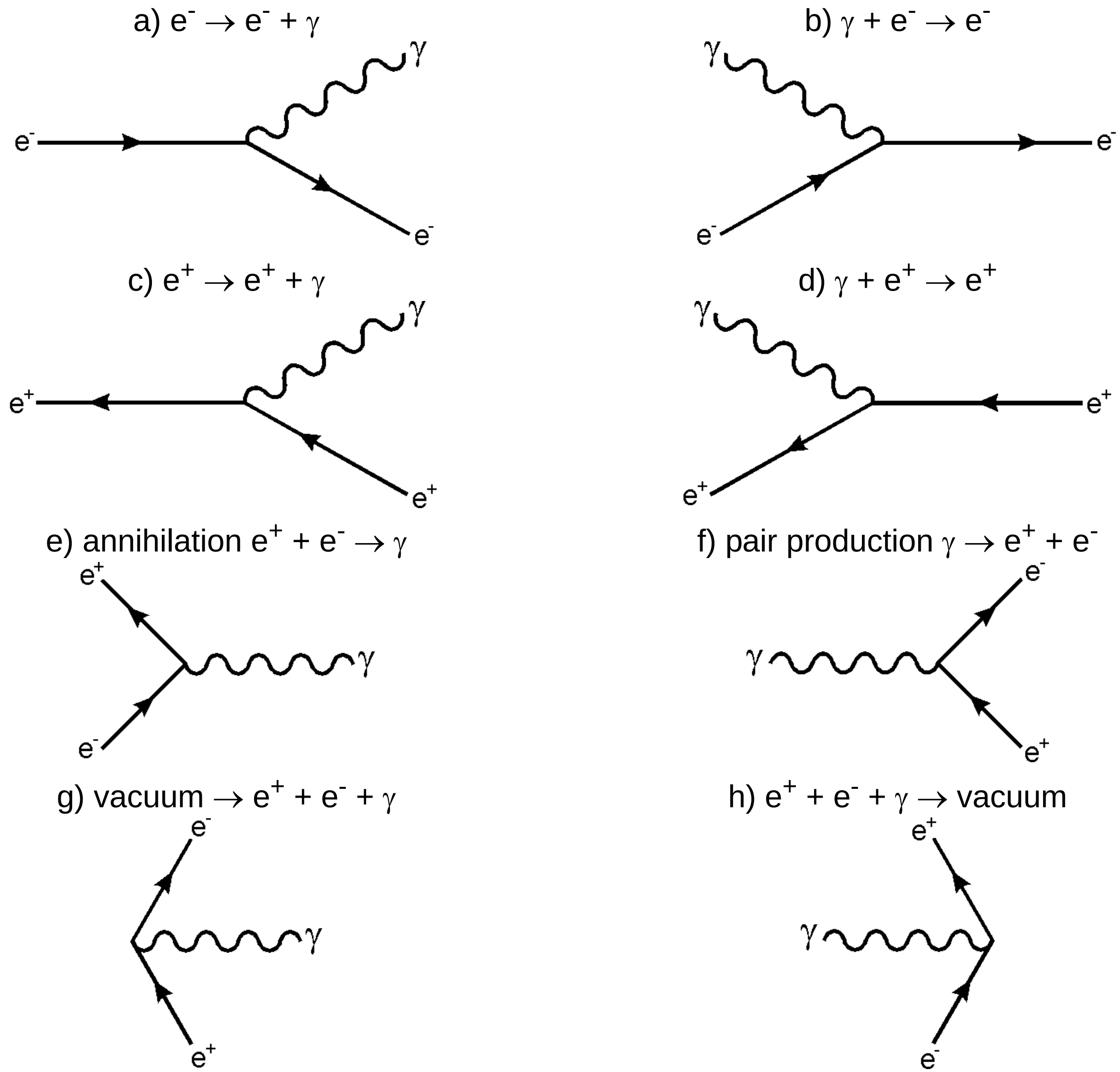

Figure 8: Feynman diagrams for **VIRTUAL** processes involving e^+, e^- and γ

- A virtual process does not require energy conservation

- A real process demands energy conservation, hence is a combination of virtual processes.

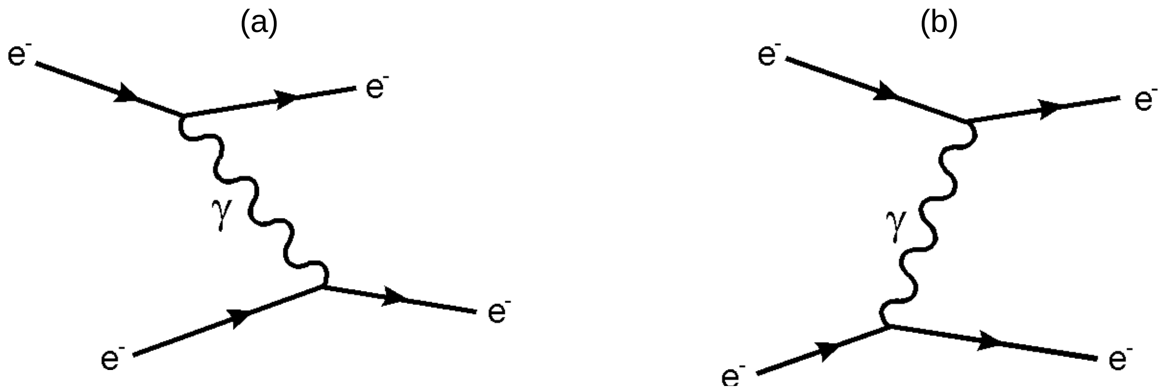

Figure 9: Electron-electron scattering, single photon exchange

- Any real process receives contributions from *all the possible* virtual processes.

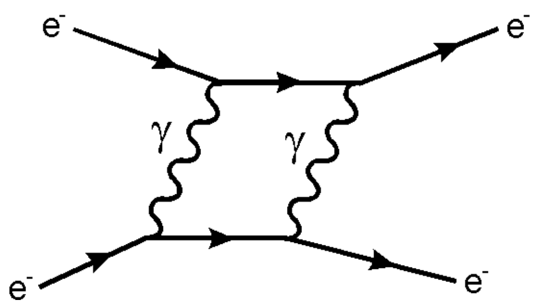

Figure 10: Two-photon exchange contribution

- Probability $P(e^-e^- \to e^-e^-) = |M(1\,\gamma \text{ exchange}) + M(2\,\gamma \text{ exchange}) + M(3\,\gamma \text{ exchange}) + \ldots|^2$ *(M stands for contribution, "Matrix element")*
  - Number of vertices in a diagram is called its *order*.
  - Each vertex has an associated probability proportional to a *coupling constant*, usually denoted as "α". In discussed processes this constant is

$$\alpha_{em} = \frac{e^2}{4\pi\varepsilon_0} \approx \frac{1}{137} \ll 1 \qquad (17)$$

  - Matrix element for a two-vertex process is proportional to $M = \sqrt{\alpha} \cdot \sqrt{\alpha}$, where each vertex has a factor $\sqrt{\alpha}$. Probability for a process is $P=|M|^2=\alpha^2$
  - For the real processes, a diagram of order n gives a contribution to probability of order α^n.

Provided sufficiently small α, high order contributions are smaller and smaller and the result is convergent: $P(\text{real}) = |M(\alpha)+M(\alpha^2)+M(\alpha^3)\ldots|^2$

Often lowest order calculation is precise enough.

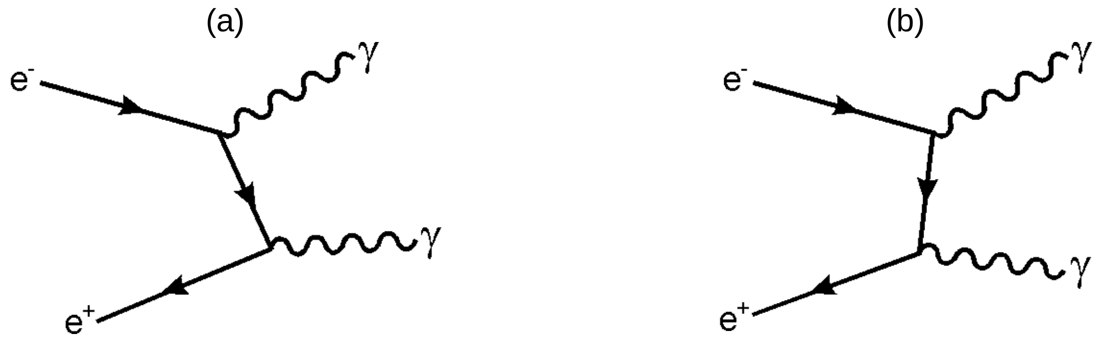

Figure 11: Lowest order contributions to $e^+e^- \rightarrow \gamma\gamma$. $M = \sqrt{\alpha} \cdot \sqrt{\alpha}$, $P=|M|^2=\alpha^2$

- Diagrams which differ only by time-ordering are usually implied by drawing only one of them

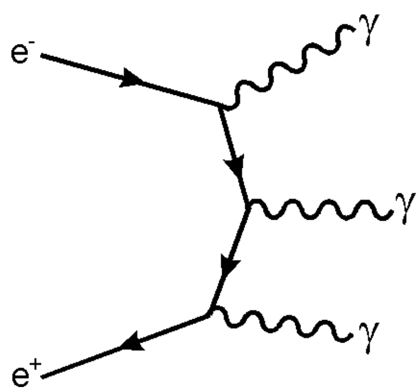

Figure 12: Lowest order of the process $e^+e^- \rightarrow \gamma\gamma\gamma$. $M = \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha}$, $P=|M|^2=\alpha^3$

This kind of process implies 3!=6 different time orderings

❖ Order of diagrams is sufficient to estimate the ratio of appearance rates of processes:

$$R \equiv \frac{Rate(e^+e^- \to \gamma\gamma\gamma)}{Rate(e^+e^- \to \gamma\gamma)} = \frac{O(\alpha^3)}{O(\alpha^2)} = O(\alpha)$$

This ratio can be measured experimentally; it appears to be $R = 0.9\times10^{-3}$, which is smaller than α_{em}, being only a first order prediction.

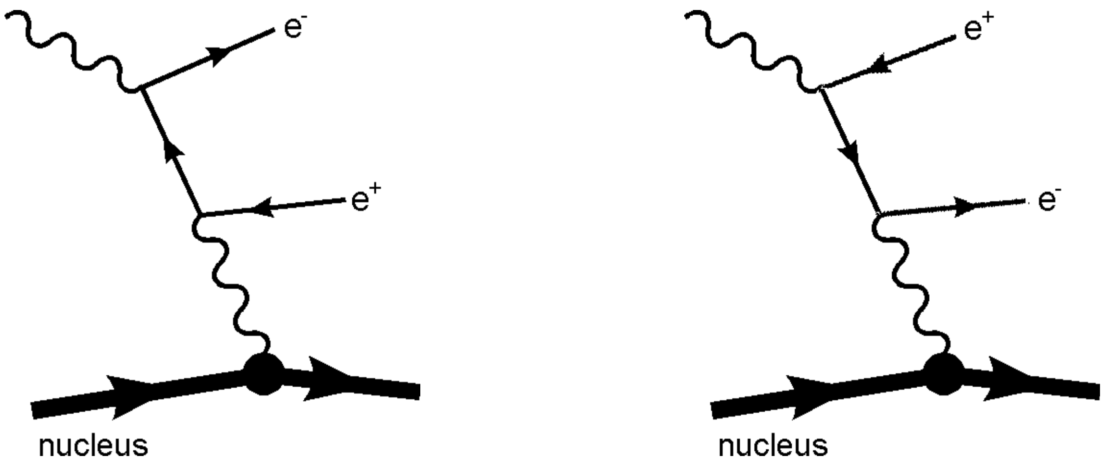

Figure 13: Diagrams that are ***not*** related by time ordering

- For nuclei, the coupling is proportional to $Z^2\alpha$, hence the rate of this process is of order $Z^2\alpha^3$

Exchange of a massive boson

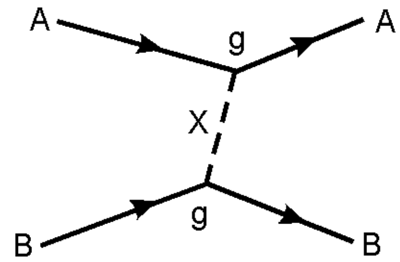

Figure 14: Exchange of a massive particle X

In the rest frame of particle A: $A(E_0, \vec{p}_0) \rightarrow A(E_A, \vec{p}) + X(E_X, -\vec{p})$

where $E_0 = M_A, \vec{p}_0 = (0, 0, 0), \ E_A = \sqrt{p^2 + M_A^2}, \ E_X = \sqrt{p^2 + M_X^2}$

From this one can estimate the maximum distance over which X can propagate before being absorbed: $\Delta E = E_X + E_A - M_A \geq M_X$, and this energy violation can exist only for a period of time $\Delta t \approx \hbar / \Delta E$ (Heisenberg's uncertainty relation), hence the *range of the interaction* is

r ≈ R $\equiv (\hbar / M_X)c = \Delta t\, c$

- For a massless exchanged particle, the interaction has an infinite range (e.g., electromagnetic)

- In case of a very heavy exchanged particle (e.g., a W boson in weak interaction), the interaction can be approximated by a *zero-range*, or *point interaction*:

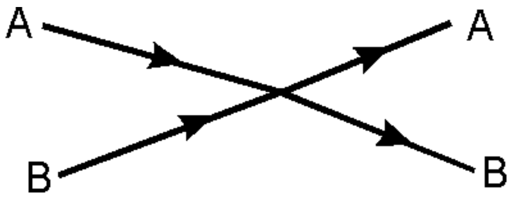

Figure 15: Point interaction as a result of $M_x \rightarrow \infty$

E.g., for a W boson: $R_W = \hbar / M_W = \hbar / (80.4\ \mathrm{GeV}/c^2) \approx 2 \times 10^{-18}$ m

Yukawa potential (1935)

- Considering particle X as an electrostatic spherically symmetric potential V(r), the Klein-Gordon equation (15) for it will look like

$$\nabla^2 V(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = M_X^2 V(r) \tag{18}$$

(In 3D polar coordinates system, $\frac{\partial^2}{\partial t^2}V(r) = (0)$, $\nabla^2 V(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}V(r)\right)$).

Integration of (18) gives the solution of

$$V(r) = -\frac{g^2}{4\pi r}e^{-r/R} \tag{19}$$

Here ***g*** is an integration constant, and it is interpreted as the *coupling strength* for particle X to the particles A and B.

❖ In Yukawa theory, g is analogous to the electric charge in QED, and the analogue of α_{em} is

$$\alpha_X = \frac{g^2}{4\pi}$$

α_X characterizes the strength of the interaction at distances $r \leq R$.

Consider a particle being scattered by the potential (19), thus receiving a momentum transfer $\vec{q}$

❖ Potential (19) has the corresponding amplitude, which is its Fourier-transform (like in optics):

$$f(\vec{q}) = \int V(\vec{x}) e^{i\vec{q}\vec{x}} d^3\vec{x} \tag{20}$$

Using polar coordinates, $d^3\vec{x} = r^2 \sin\theta d\theta dr d\phi$, and assuming $V(\vec{x}) = V(r)$, the amplitude is

$$f(\vec{q}) = 4\pi g \int_0^\infty V(r) \frac{\sin(qr)}{qr} r^2 dr = \frac{-g^2}{q^2 + M_X^2} \tag{21}$$

- For the point interaction, $M_X^2 \gg q^2$, hence $f(\vec{q})$ becomes a constant:

$$f(\vec{q}) = -G = \frac{-4\pi\alpha_X}{M_X^2}$$

That means that the point interaction is characterized not only by α_X, but by M_X as well

- Very useful approximation for weak interactions; in β-decays, this constant is called the "*Fermi coupling constant*", G_F