VI. Quark states and colours

- Forces acting between quarks in hadrons can be investigated by studying a simple quark-antiquark system
- Systems of heavy quarks, like $c\bar{c}$ (*charmonium*) and $b\bar{b}$ (*bottomonium*), are essentially non-relativistic (masses of quarks are much bigger than their kinetic energies)
  - Charmonium and bottomonium (*quarkonium*) are analogous to a hydrogen atom in a sense that they consist of many energy levels
  - While the hydrogen atom is governed by the electromagnetic force, the quarkonium system is dominated by the strong force

Like in a hydrogen atom, energy states of a quarkonium can be labelled by their principal quantum number n, and J, L, S, where $L \leq n$-1 and S can be either 0 or 1.

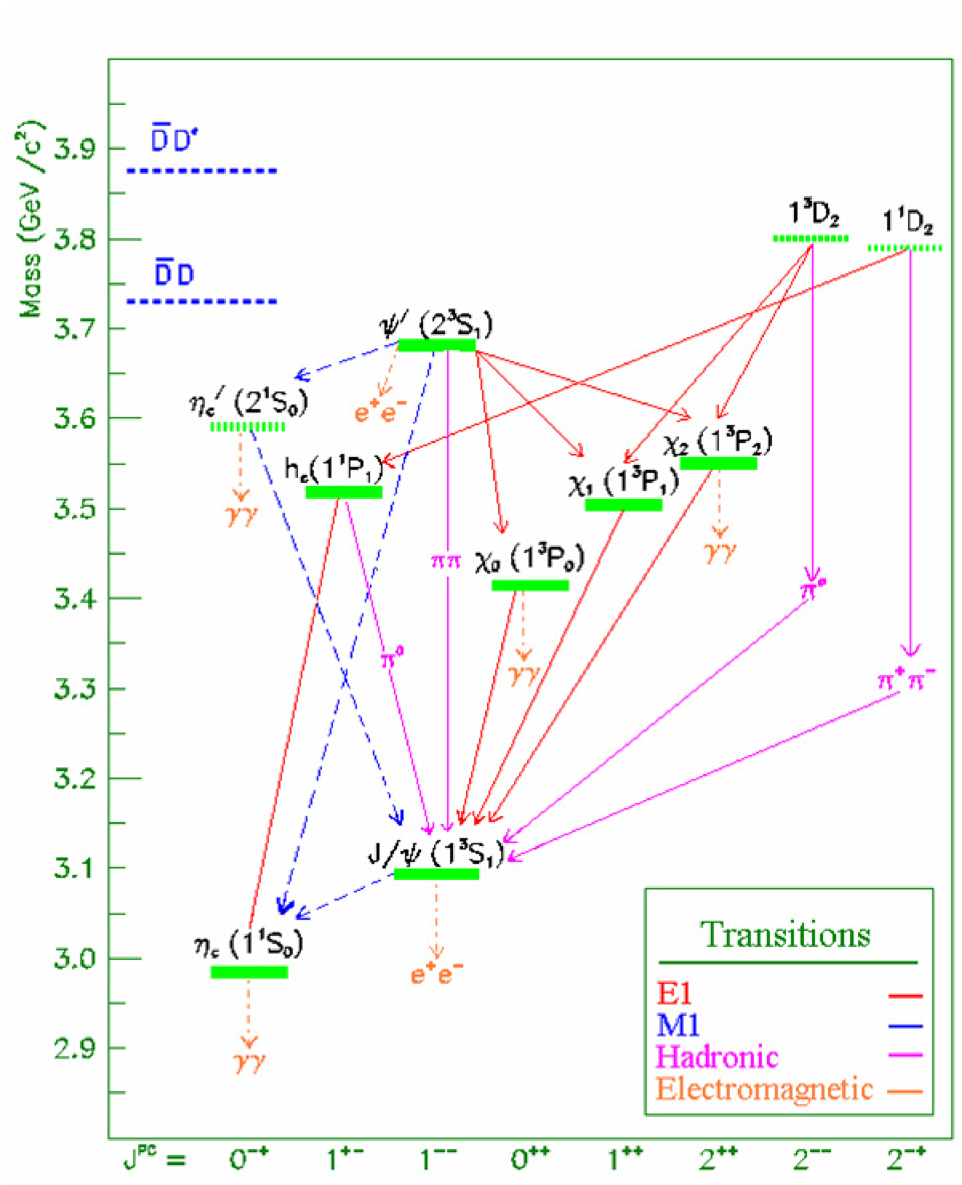

Figure 55: The charmonium spectrum

From Equations (66) and (77) , parity and C-parity of a quarkonium are:

$$P = P_q P_{\bar{q}}(-1)^L = (-1)^{L+1} \; ; \; C = (-1)^{L+S}$$

Predicted and observed charmonium and bottomonium states for n=1 and n=2:

		J^{PC}	$c\bar{c}$ state	$b\bar{b}$ state
n=1	$1S_0$	0^{-+}	$\eta_c(2980)$	–
n=1	$3S_1$	1^{--}	$J/\psi(3097)$	$\Upsilon(9460)$
n=2	$1S_0$	0^{-+}	–	–
n=2	$3S_1$	1^{--}	$\psi(3686)$	$\Upsilon(10023)$
n=2	$3P_0$	0^{++}	$\chi_{c0}(3415)$	$\chi_{b0}(9860)$
n=2	$3P_1$	1^{++}	$\chi_{c1}(3511)$	$\chi_{b1}(9892)$
n=2	$3P_2$	2^{++}	$\chi_{c2}(3556)$	$\chi_{b2}(9913)$
n=2	$1P_1$	1^{+-}	–	

- States J/ψ and ψ have the same J^{PC} quantum numbers as a photon: 1^{--}, and the most common way to form them is through e^+e^--annihilation, where virtual photon converts to a charmonium state

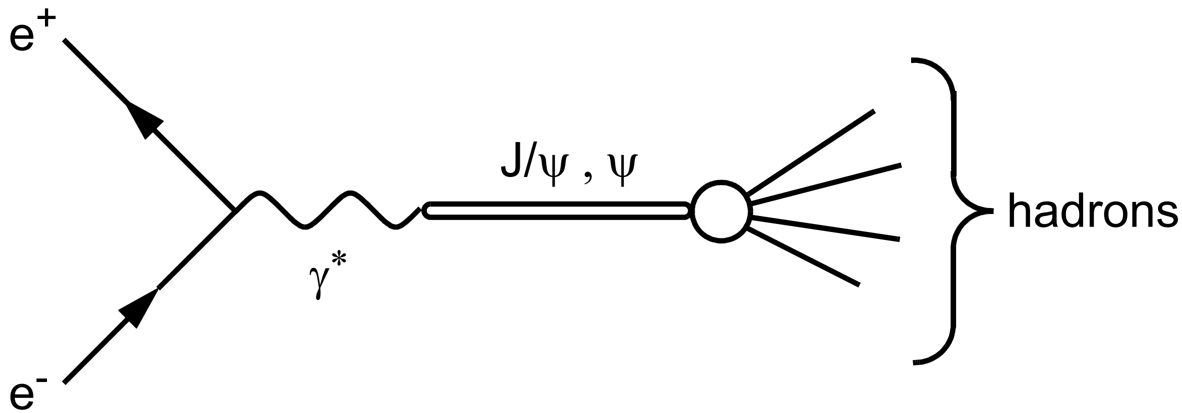

Figure 56: Formation and decay of J/ψ (ψ) mesons in e^+e^- annihilation

- If centre-of-mass energy of incident e^+ and e^- is equal to the quarkonium mass, formation of the latter is highly probable, which leads to the peak in the cross-section $\sigma(e^+e^-\to\text{hadrons})$.

- Cross-section σ in a collision is defined through

$$N = \sigma \times L \tag{80}$$

Here N is the count of reactions (*events*) in a time period, and L is the integrated *luminosity* – density of colliding particles integrated over this time period

- Cross-section is measured in barns:

$$[\sigma] = 1 \text{ barn } (1 \text{ b}) \equiv 10^{-24} \text{ cm}^2 \Rightarrow [L] = \text{cm}^{-2} \text{ or } 1 \text{ barn}^{-1} \ (1 \text{ b}^{-1})$$

An example:

- an LHC collider run will last 10^7 s, with instantaneous luminosity of 10^{34} cm^{-2}s^{-1} $\Rightarrow L = 10^{41}$ cm^{-2} = 100 fb^{-1}.
- The total production cross-section for $b\bar{b}$-pairs is about 500 μb $\Rightarrow$ in 10^7 s, the number of produced events will be N=500 μb ×100 fb^{-1} = 5 ×10^{13}

- Convenient way to represent cross-sections in e^+e^- annihilation:

$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \qquad (81)$$

Hadron production cross-section is normalized to muon cross-section

- Sharp peaks can be observed in R at E_{cm}=3.097 GeV (J/ψ) and 3.686 GeV (ψ)

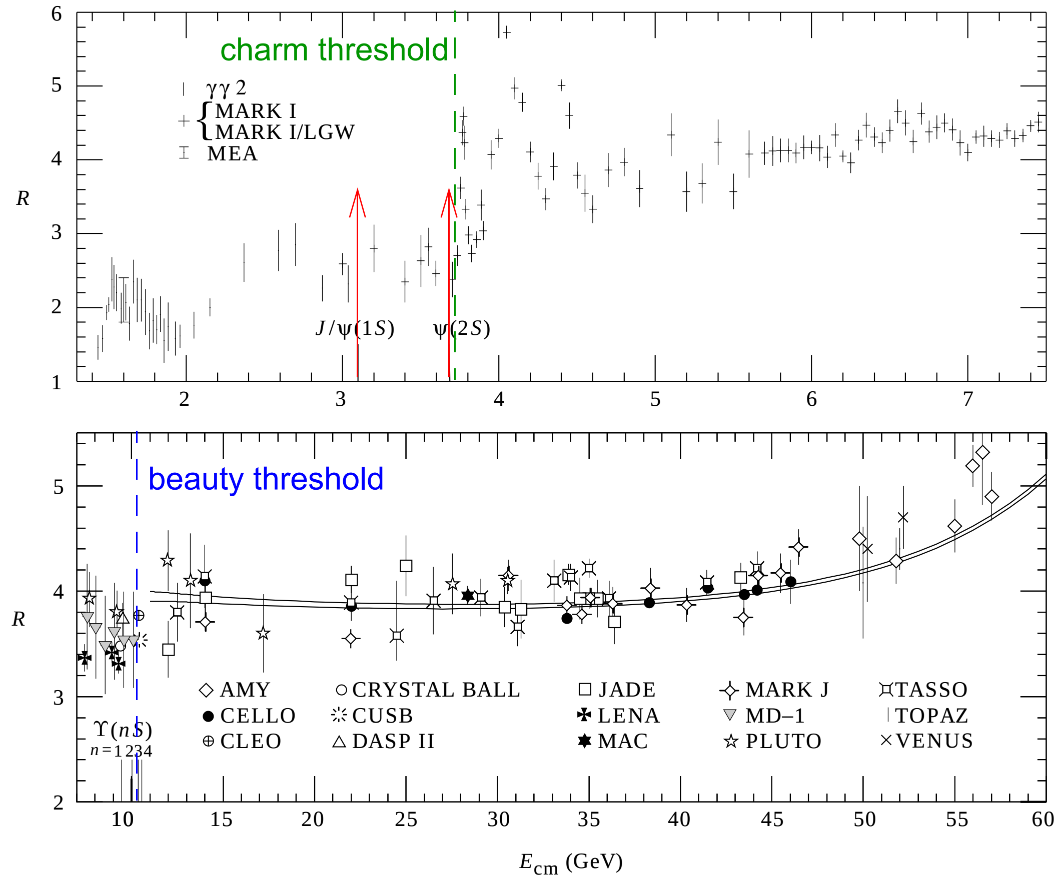

Figure 57: Cross-section ratio R in e^+e^- collision. Arrows indicate the peaks.

- Cross-section for a $\mu^+\mu^-$ final state depends only on E_{CM} and α:

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2} \quad (82)$$

- Charm *threshold* (3730 MeV): twice the mass of the lightest charmed meson, D
  - J/ψ, ψ are lighter ⇒ can not decay into charmed particles ⇒ long-living (narrow peaks below charm threshold)
  - Wide peaks above charm threshold: short-living resonances

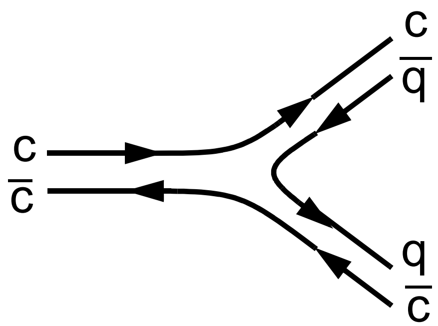

Figure 58: Charmonium resonance decay to charmed mesons

- J/ψ and ψ can only decay via annihilation of $c\bar{c}$ pair
  - Long lifetime since such annihilation is *suppressed* as opposed to light quarks
  - J/ψ and ψ can only decay to light hadrons (containing u, d, s), or to e^+e^-, or $\mu^+\mu^-$.

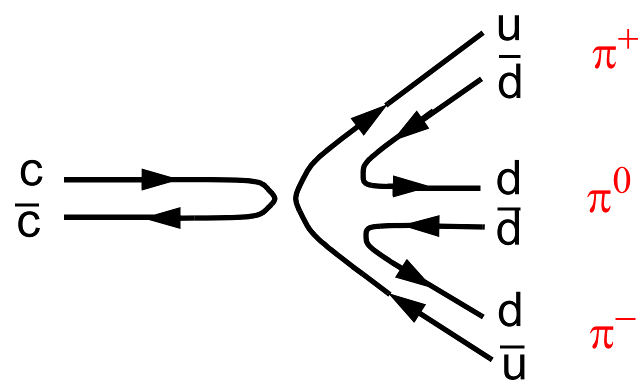

Figure 59: Charmonium decay to light non-charmed mesons

- Charmonium states with quantum numbers different of those of photon can not be produced in $c\bar{c}$ annihilation, but can be found in radiative decays of J/ψ or ψ:

$$\psi(3686) \to \chi_{ci} + \gamma \qquad (i=0,1,2) \tag{83}$$

$$\psi(3686) \to \eta_c(2980) + \gamma \tag{84}$$

$$J/\psi(3097) \to \eta_c(2980) + \gamma \tag{85}$$

- Bottomonium spectrum is observed in much the same way as charmonium
- Beauty threshold is at 10560 MeV/c^2 (twice the mass of the B meson)

- Similarities between spectra of bottomonium and charmonium suggest similarity of forces acting in two systems

The quark-antiquark potential

- Let's assume the $q\bar{q}$ potential being a central one, V(r), and the system to be non-relativistic

In the centre-of-mass frame of a $q\bar{q}$ pair, Schrödinger equation is

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{x}) + V(r)\psi(\vec{x}) = E\psi(\vec{x}) \tag{86}$$

Here $\mu = m_q/2$ is the *reduced mass* of a quark, and $r = |\vec{x}|$ is distance between the quarks.

Mass of a quarkonium state in this framework is

$$M(q\bar{q}) = 2m_q + E \tag{87}$$

- In the case of a Coulomb-like potential V(r)$\propto r^{-1}$, energy levels depend only on the *principal quantum number* n:

$$E_n = -\frac{\mu\alpha^2}{2n^2}$$

- In the case of a harmonic oscillator potential $V(r) \propto r^2$, the degeneracy of energy levels is broken: dependency on *L* arises

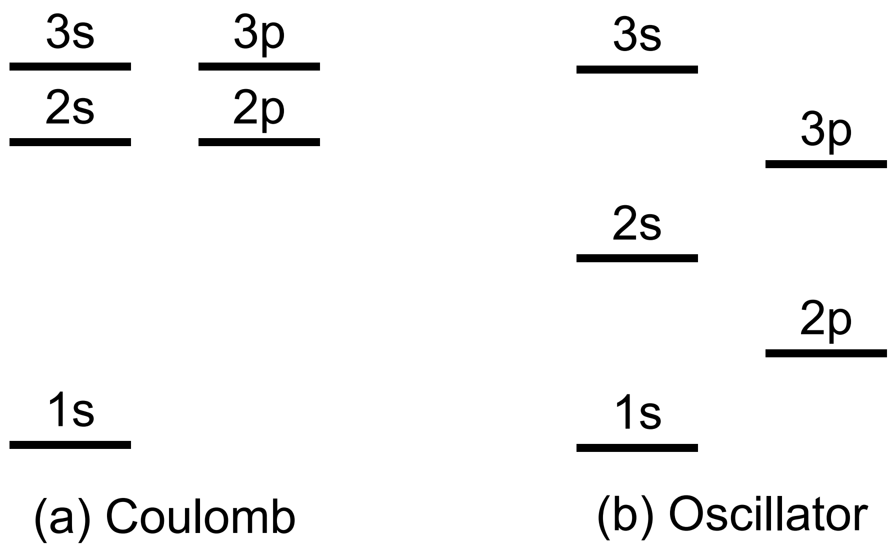

Figure 60: Energy levels arising from Coulomb and harmonic oscillator potentials for n=1,2,3

Cf Figure 55: one can see that heavy quarkonia spectra are inbetween the two approximations; the potential can be fitted by:

$$V(r) = -\frac{a}{r} + br \tag{88}$$

Coefficients a and b are determined by solving Equation (86) and fitting results to data:

$$a = 0.48 \qquad\qquad b = 0.18\ \mathrm{GeV}^2$$

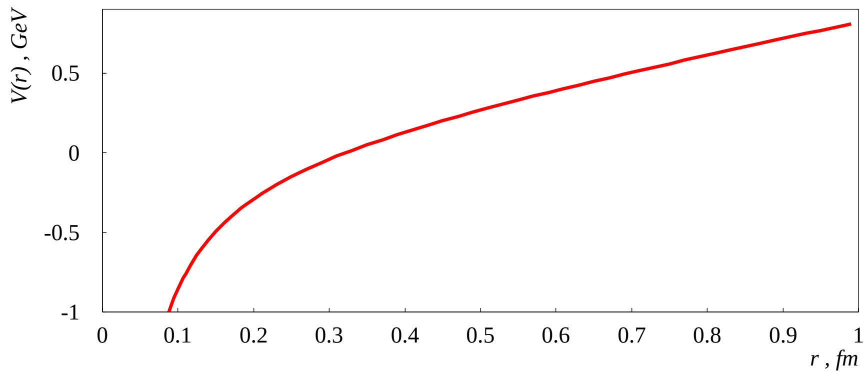

Figure 61: Modified Coulomb potential (88)

Other forms of the potential can give equally good fits, for example

$$V(r) = a\,ln(br) \tag{89}$$

where parameters appear to be

$$a = 0.7\ \mathrm{GeV} \qquad\qquad b = 0.5\ \mathrm{GeV}$$

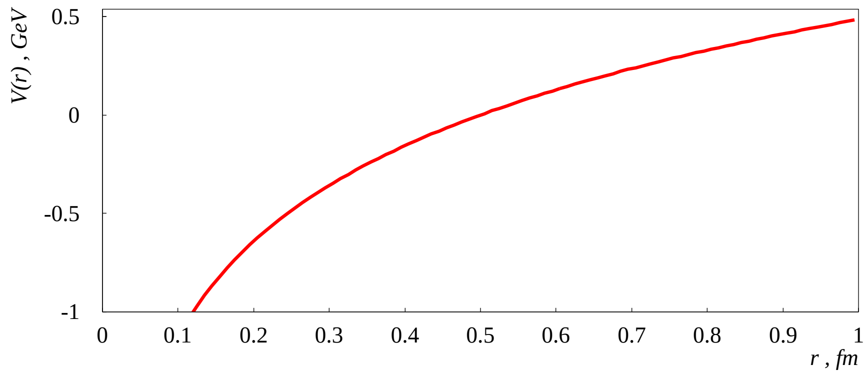

Figure 62: Logarithmic potential (89)

- In the range of $0.2 \le r \le 0.8$ fm potentials (88) and (89) are in good agreement $\Rightarrow$ in this region the quark-antiquark potential can be considered as well-defined

- Simple non-relativistic Schrödinger equation explains quite well existence of several energy states for a given heavy quark-antiquark system

Light mesons; nonets

- Spins of quarks are counter-directed $\Rightarrow J^P=0^-$, *pseudoscalar meson nonet* (9 possible $q\bar{q}$ combinations for u,d,s quarks)
- Spins of quarks are co-directed $\Rightarrow J^P=1^-$, *vector meson* nonet

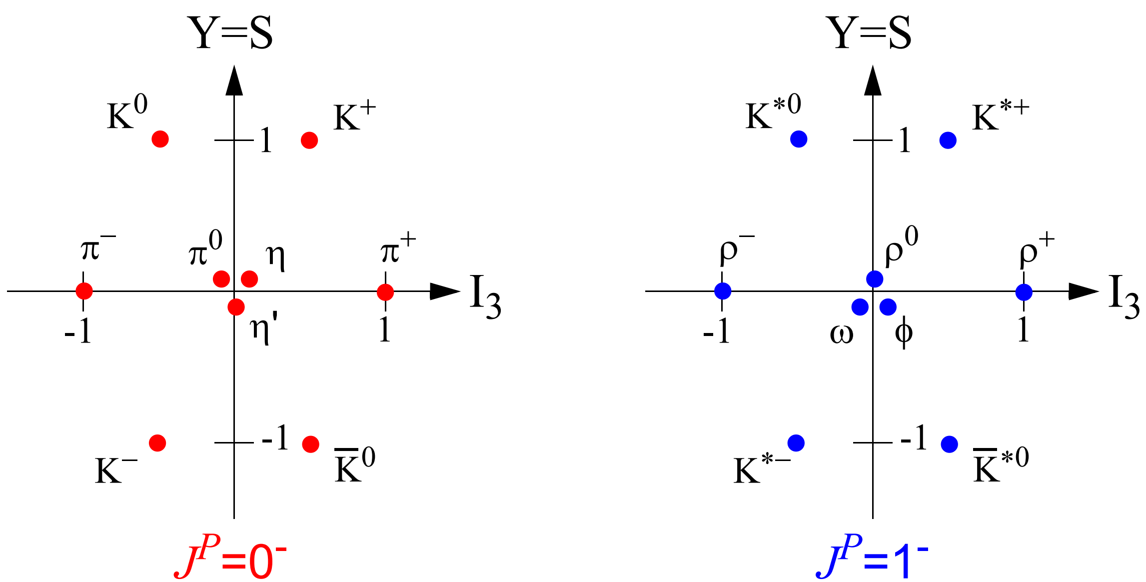

Figure 63: Light meson nonets in (I_3, Y) space ("weight diagrams")

- In each nonet, there are three particles with equal quantum numbers $Y=S=I_3=0$

  - They correspond to a $q\bar{q}$ pair like $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, or a *linear combination* of these states (follows from the isospin operator analysis):

$$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I = 1, I_3 = 0 \tag{90}$$

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad I = 0, I_3 = 0 \tag{91}$$

- π^0 and ρ^0 mesons are linear combinations of $u\bar{u}$ and $d\bar{d}$ states (90): $(u\bar{u} - d\bar{d})/(\sqrt{2})$

- ω meson is the linear combination (91): $(u\bar{u} + d\bar{d})/(\sqrt{2})$

Inclusion of an $s\bar{s}$ pair leads to further combinations:

$$\eta(547) = \frac{(d\bar{d} + u\bar{u} - 2s\bar{s})}{\sqrt{6}} \qquad I = 0, I_3 = 0 \tag{92}$$

$$\eta'(958) = \frac{(d\bar{d} + u\bar{u} + s\bar{s})}{\sqrt{3}} \qquad I = 0, I_3 = 0 \tag{93}$$

- There exists meson $\phi(1019)$, which is a quarkonium $s\bar{s}$, having I=0 and I_3=0

Light baryons

- Three-quark states of the lightest quarks (u,d,s) form baryons, which can be arranged in *supermultiplets* (*singlets, octets* and *decuplets*).

- The lightest baryon supermultiplets are *octet* of $J^P = \frac{1}{2}^+$ particles and *decuplet* of $J^P = \frac{3}{2}^+$ particles

  - Weight diagrams of baryons can be deduced from the quark model under assumption that the **combined** space-spin wavefunctions are *symmetric* under interchange of **like** quarks

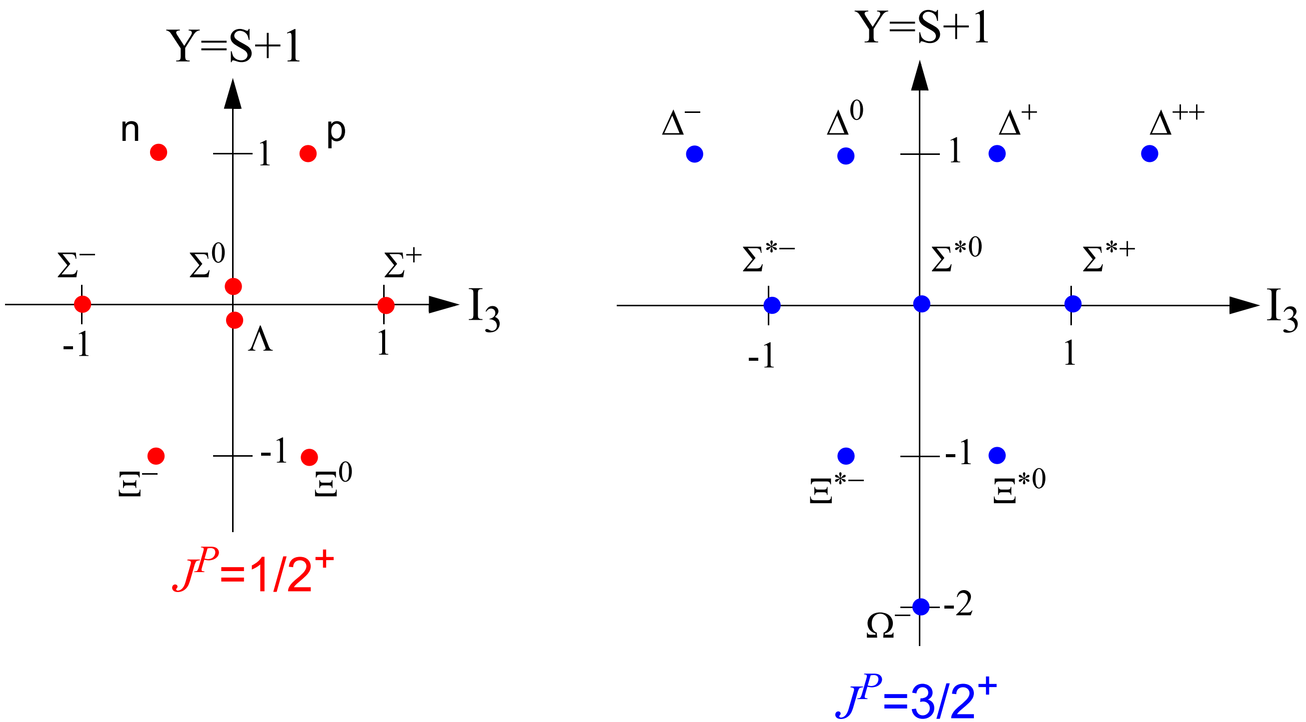

Figure 64: Weight diagrams for light baryons

– Parity of a 3-quark state $q_iq_jq_k$ is $P=P_iP_jP_k=1$

– Spin of such a state is sum of quark spins

– From presumption of symmetry under exchange of like quarks, any pair of like quarks qq must have spin-1

- There are six distinct combination of the form $q_i q_i q_j$:

uud, uus, ddu, dds, ssu, ssd

each of them can have spin J=1/2 or J=3/2

- three combinations of the form $q_i q_i q_i$ are possible:

uuu, ddd, sss

spins of all like-quarks have to be parallel (symmetry presumption), hence J=3/2 only

- The remaining combination is uds, with two distinct states having spin values J=1/2 and one state with J=3/2

- By adding up numbers, one gets 8 states with $J^P=1/2^+$ and 10 states with $J^P=3/2^+$, exactly what is shown by weight diagrams

- Measured masses of baryons show that mass difference between members of same isospin multiplets is much smaller than that between members of different isospin multiplets
  - In what follows, equal masses of isospin multiplet members are assumed, e.g.,

$$m_p = m_n \equiv m_N$$

Experimentally, more s-quarks contains a particle, heavier it is:

$$\Xi^0(1315)=(uss); \Sigma^+(1189)=(uus);\ p(938)=(uud)$$

$$\Omega^-(1672)=(sss);\ \Xi^{*0}(1532)=(uss);$$
$$\Sigma^{*+}(1383)=(uus);\ \Delta^{++}(1232)=(uuu)$$

- There is an evidence that the main contribution to big mass differences comes from the s-quark
  - Knowing masses of baryons, one can calculate 6 **simplistic** estimates of mass difference between s-quark and light quarks (u,d)

For the $3/2^+$ decuplet:

$$M_\Omega - M_\Xi = M_\Xi - M_\Sigma = M_\Sigma - M_\Delta = m_s - m_{u,d}$$

and for the $1/2^+$ octet:

$$M_\Xi - M_\Sigma = M_\Xi - M_\Lambda = M_\Lambda - M_N = m_s - m_{u,d}$$

Average value of those differences gives

$$m_s - m_{u,d} \approx 160\ MeV/c^2 \qquad (94)$$

- So far, so good – BUT quarks are spin-1/2 particles ⇒ fermions ⇒ their wavefunctions are *antisymmetric* and all the discussion above **contradicts Pauli principle!**

COLOUR

- Experimental data confirm predictions based on the assumption of symmetric wave functions
- That means that apart of space and spin degrees of freedom, quarks carry yet another attribute

In 1964-1965, Greenberg and Nambu with colleagues proposed the new property – the *colour* – with THREE possible states, and associated with the corresponding wavefunction χ^C:

$$\Psi = \psi(\vec{x})\chi\chi^C \qquad (95)$$

- Conserved quantum numbers associated with χ^C are *colour charges* – in strong interaction they play analogous role to the electric charge in e.m. interaction
- Hadrons can exist only in *colour singlet* states, with total colour charge of zero
- Quarks have to be *confined* within the hadrons, since non-zero colour states are forbidden

- Three independent colour wavefunctions are represented by *“colour spinors”*:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},\quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{96}$$

- They are acted on by eight independent *“colour operators”* which are represented by a set of 3-dimensional matrices (analogs of Pauli matrices)
- Colour charges I_3^C and Y^C are eigenvalues of corresponding operators

Values of I_3^C and Y^C for the colour states of quarks and antiquarks:

Quarks			**Antiquarks**		
	I_3^C	Y^C		I_3^C	Y^C
r (“red”)	1/2	1/3	$\bar{\mathbf{r}}$	-1/2	-1/3
g (“green”)	-1/2	1/3	$\bar{\mathbf{g}}$	1/2	-1/3
b (“blue”)	0	-2/3	$\bar{\mathbf{b}}$	0	2/3

- *Colour hypercharge* Y^C and *colour isospin charge* I_3^C are additive quantum numbers, having opposite sign for quark and antiquark

Confinement condition for the total colour charges of a hadron:

$$I_3^C = Y^C = 0 \tag{97}$$

The most general colour wavefunction for a baryon is a linear superposition of six possible combinations:

$$\begin{aligned}\chi_B^C = {} & \alpha_1 r_1 g_2 b_3 + \alpha_2 g_1 r_2 b_3 + \alpha_3 b_1 r_2 g_3 \\ & + \alpha_4 b_1 g_2 r_3 + \alpha_5 g_1 b_2 r_3 + \alpha_6 r_1 b_2 g_3\end{aligned} \tag{98}$$

where α_i are constants. Aparently, the color confinement demands the totally antisymmetric combination:

$$\begin{aligned}\chi_B^C = {} & \frac{1}{\sqrt{6}}(r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 \\ & - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3)\end{aligned} \tag{99}$$

Colour confinement principle (97) implies certain requirements for states containing both quarks and antiquarks:

– consider an arbitrary combination $q^m\bar{q}^n$ of *m* quarks and *n* antiquarks, $m \geq n$

– for a particle with α quarks in r-state, β quarks in g-state, γ quarks in b-state ($\alpha+\beta+\gamma=m$) and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ antiquarks in corresponding antistates ($\bar{\alpha}+\bar{\beta}+\bar{\gamma}=n$), the colour wavefunction is

$$r^{\alpha} g^{\beta} b^{\gamma} \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}} \tag{100}$$

Adding up colour charges (from the table above) and applying the confinement requirement,

$$I_3^C = (\alpha - \bar{\alpha})/2 - (\beta - \bar{\beta})/2 = 0$$

$$Y^C = (\alpha - \bar{\alpha})/3 + (\beta - \bar{\beta})/3 - 2(\gamma - \bar{\gamma})/3 = 0$$

$$\Downarrow$$

$$\alpha - \bar{\alpha} = \beta - \bar{\beta} = \gamma - \bar{\gamma} \equiv p$$

Here p is a non-negative integer $\Rightarrow$

- Numbers of quarks and antiquarks in a colorless state are related as: $m - n = 3p$

- The only combination $q^m\bar{q}^n$ allowed by the colour confinement principle is

$$(3q)^p(q\bar{q})^n\,,\qquad p,n \geq 0 \tag{101}$$

  - Form (101) forbids states with fractional electric charges
  - However, it allows exotic combinations like $qq\bar{q}\bar{q}$, $qqqq\bar{q}$ (like e.g. the pentaquark $\Theta^+ = uudd\bar{s}$)