

VII. (VI) Quark states and colours

- ❖ Forces acting between quarks in hadrons can be investigated by studying a simple quark-antiquark system
- ❖ Systems of heavy quarks, like $c\bar{c}$ (*charmonium*) and $b\bar{b}$ (*bottomonium*), are essentially non-relativistic (masses of quarks are much bigger than their kinetic energies)
 - ☉ Charmonium and bottomonium (*quarkonium*) are analogous to a hydrogen atom in a sense that they consist of many energy levels
 - ☉ While the hydrogen atom is governed by the electromagnetic force, the quarkonium system is dominated by the strong force

Like in a hydrogen atom, energy states of a quarkonium can be labelled by their principal quantum number n , and J, L, S , where $L \leq n-1$ and S can be either 0 or 1.

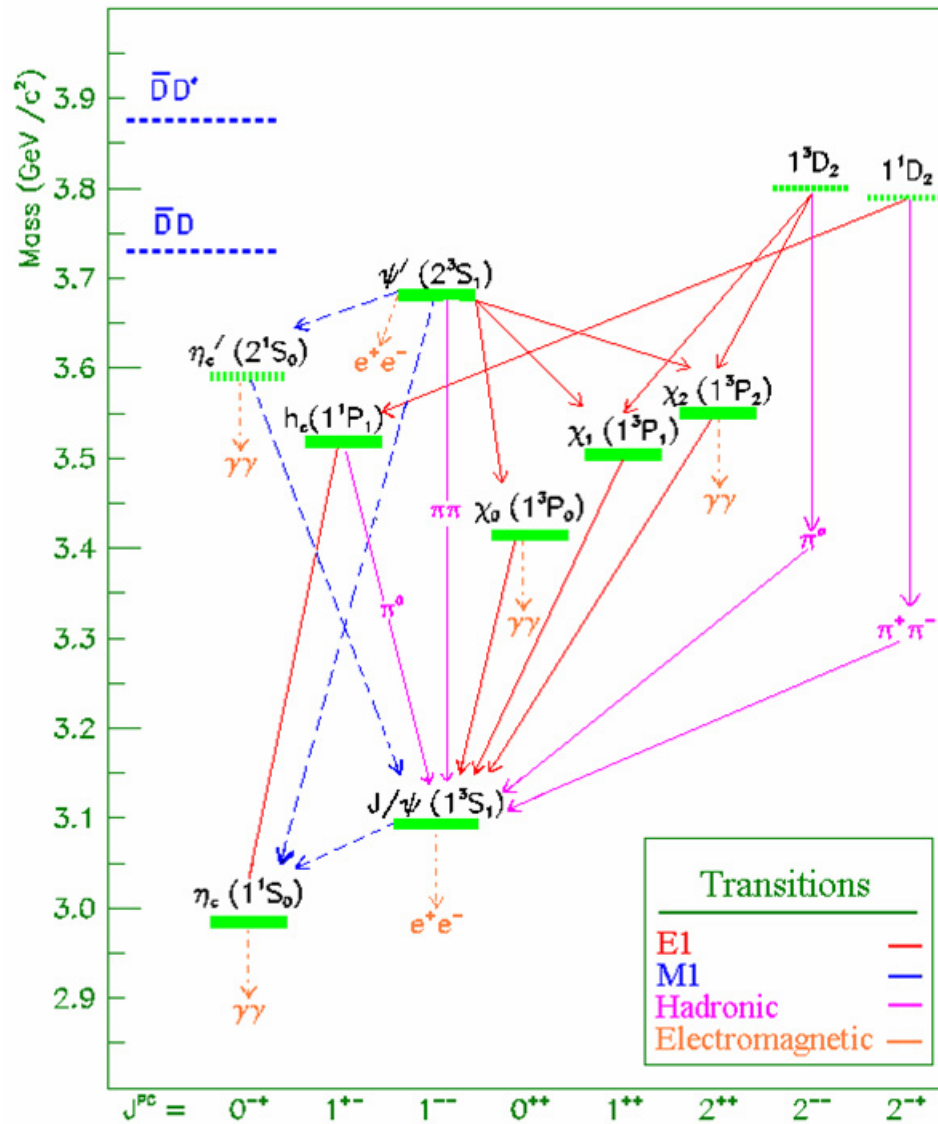


Figure 99: The charmonium spectrum

From Equations (67) and (78) , parity and C-parity of a quarkonium are:

$$P = P_q P_{\bar{q}} (-1)^L = (-1)^{L+1} ; C = (-1)^{L+S}$$

Predicted and observed charmonium and bottomonium states for $n=1$ and $n=2$:

		J^{PC}	$c\bar{c}$ state	$b\bar{b}$ state
n=1	1S ₀	0 ⁻⁺	$\eta_c(2980)$	–
n=1	3S ₁	1 ⁻⁻	J/ $\psi(3097)$	Y(9460)
n=2	1S ₀	0 ⁻⁺	–	–
n=2	3S ₁	1 ⁻⁻	$\psi(3686)$	Y(10023)
n=2	3P ₀	0 ⁺⁺	$\chi_{c0}(3415)$	$\chi_{b0}(9860)$
n=2	3P ₁	1 ⁺⁺	$\chi_{c1}(3511)$	$\chi_{b1}(9892)$
n=2	3P ₂	2 ⁺⁺	$\chi_{c2}(3556)$	$\chi_{b2}(9913)$
n=2	1P ₁	1 ^{+–}	–	

☉ States J/ ψ and ψ have the same J^{PC} quantum numbers as a photon: 1⁻⁻, and the most common way to form them is through e⁺e⁻-annihilation, where virtual photon converts to a charmonium state

Electron-positron collisions, cross-section

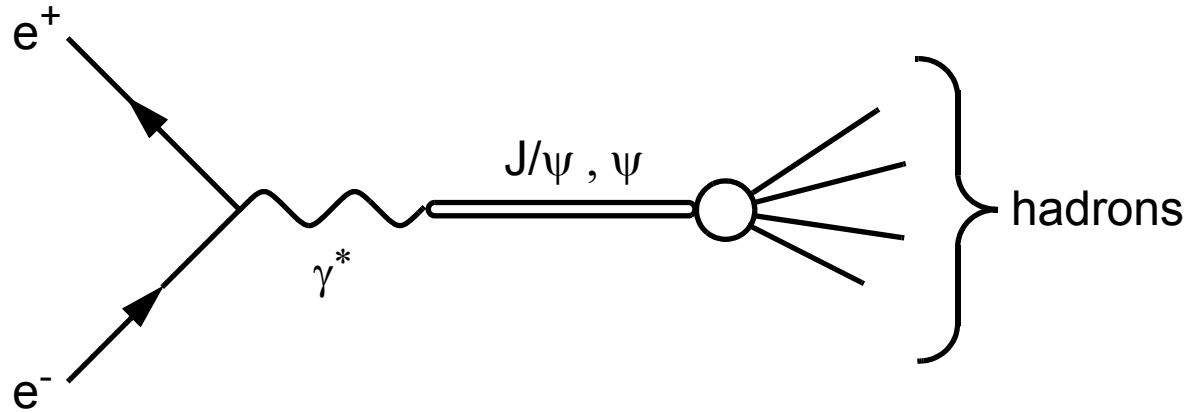


Figure 100: Formation and decay of J/ψ (ψ) mesons in e^+e^- annihilation

☉ If centre-of-mass energy of incident e^+ and e^- is equal to the quarkonium mass, formation of the latter is highly probable, which leads to the peak in the cross-section $\sigma(e^+e^- \rightarrow \text{hadrons})$.

❖ Cross-section σ in a collision is defined through

$$N = \sigma \times L \quad (108)$$

Here N is the count of reactions (*events*) in a time period, and L is the integrated *luminosity* – density of colliding particles integrated over this time period

⊙ Cross-section is measured in barns:

$$[\sigma] = 1 \text{ barn (1 b)} \equiv 10^{-24} \text{ cm}^2 \Rightarrow [L] = \text{cm}^{-2} \text{ or } 1 \text{ barn}^{-1} (1 \text{ b}^{-1})$$

An example:

⊙ an LHC collider run will last 10^7 s, with instantaneous luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$
 $\Rightarrow L = 10^{41} \text{ cm}^{-2} = 100 \text{ fb}^{-1}$.

⊙ The total production cross-section for $b\bar{b}$ -pairs is about $500 \mu\text{b}$ \Rightarrow in 10^7 s, the number of produced events will be $N = 500 \mu\text{b} \times 100 \text{ fb}^{-1} = 5 \times 10^{13}$

❖ e^+e^- collisions provide clean study environment; it is convenient to normalize the cross-sections to that of muon production:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \quad (109)$$

❖ Sharp peaks can be observed in R at $E_{cm} = 3.097 \text{ GeV}$ (J/ψ) and 3.686 GeV (ψ)

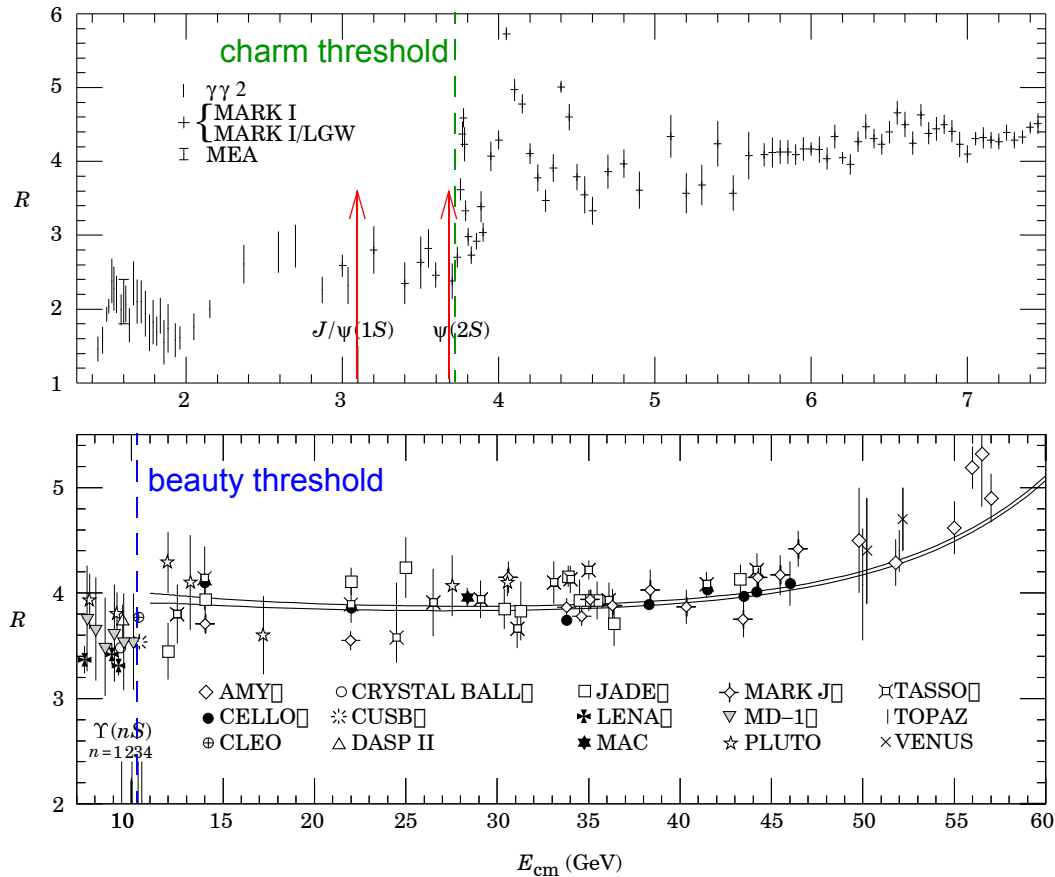


Figure 101: Cross-section ratio R in e^+e^- collision. Arrows indicate the peaks.

☉ Cross-section for a $\mu^+\mu^-$ final state is known and depends only on E_{CM} and α :

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2} \quad (110)$$

❖ Charm *threshold* (3730 MeV): twice the mass of the lightest charmed meson, D

- ⊙ J/ψ , ψ are lighter \Rightarrow can not decay into charmed particles \Rightarrow long-living (narrow peaks below charm threshold)
- ⊙ Wide peaks above charm threshold: short-living resonances

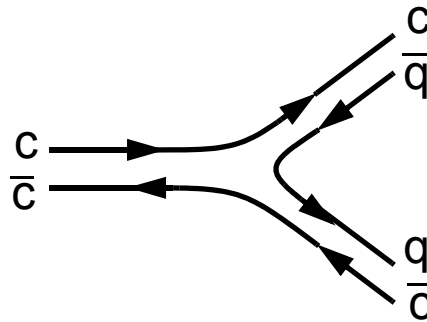


Figure 102: Charmonium resonance decay to charmed mesons

❖ J/ψ and ψ can only decay via annihilation of $c\bar{c}$ pair

- ⊙ Long lifetime since such annihilation is *suppressed* as opposed to light quarks
- ⊙ J/ψ and ψ can only decay to light hadrons (containing u, d, s), or to e^+e^- , or $\mu^+\mu^-$.

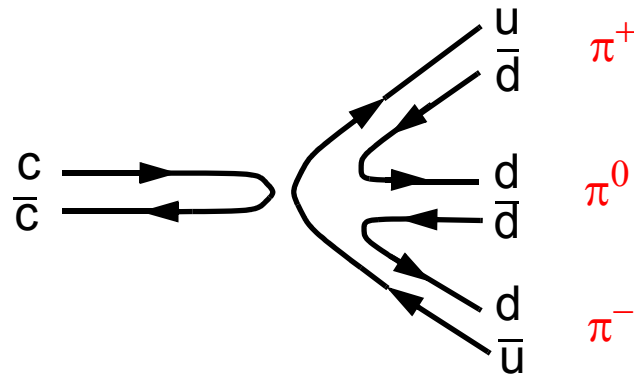


Figure 103: Charmonium decay to light non-charmed mesons

- Charmonium states with quantum numbers different of those of photon can not be produced in $c\bar{c}$ annihilation, but can be found in radiative decays of J/ψ or ψ :

$$\psi(3686) \rightarrow \chi_{ci} + \gamma \quad (i=0,1,2) \quad (111)$$

$$\psi(3686) \rightarrow \eta_c(2980) + \gamma \quad (112)$$

$$J/\psi(3097) \rightarrow \eta_c(2980) + \gamma \quad (113)$$

- Bottomonium spectrum is observed in much the same way as charmonium

- Beauty threshold is at $10560 \text{ MeV}/c^2$ (twice the mass of the B meson)

- Similarities between spectra of bottomonium and charmonium suggest similarity of forces acting in two systems

The quark-antiquark potential

- ❖ Let's assume the $q\bar{q}$ potential being a central one, $V(r)$, and the system to be non-relativistic

In the centre-of-mass frame of a $q\bar{q}$ pair, Schrödinger equation is

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{x}) + V(r)\psi(\vec{x}) = E\psi(\vec{x}) \quad (114)$$

Here $\mu = m_q/2$ is the *reduced mass* of a quark, and $r = |\vec{x}|$ is distance between the quarks.

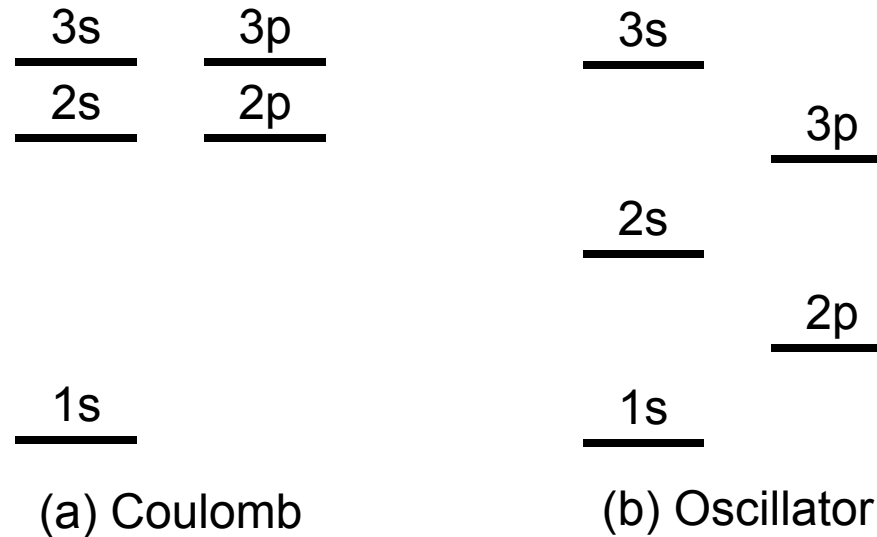
Mass of a quarkonium state in this framework is

$$M(q\bar{q}) = 2m_q + E \quad (115)$$

- ☉ In the case of a Coulomb-like approximation $V(r) \propto r^{-1}$, energy levels are quantized, depending only on the *principal quantum number* n :

$$E_n = -\frac{\mu\alpha^2}{2n^2}$$

- ⊙ In the case of a harmonic oscillator potential $V(r) \propto r^2$, the degeneracy of energy levels is broken: dependency on L arises



(a) Coulomb

(b) Oscillator

Figure 104: Energy levels arising from Coulomb and harmonic oscillator potentials for $n=1,2,3$

Cf Figure 99: one can see that heavy quarkonia spectra are inbetween the two approximations; the actual potential can be described by:

$$V(r) = -\frac{a}{r} + br \quad (116)$$

Coefficients a and b are determined by solving Equation (114) and fitting results to data:

$$a = 0.48$$

$$b = 0.18 \text{ GeV}^2$$

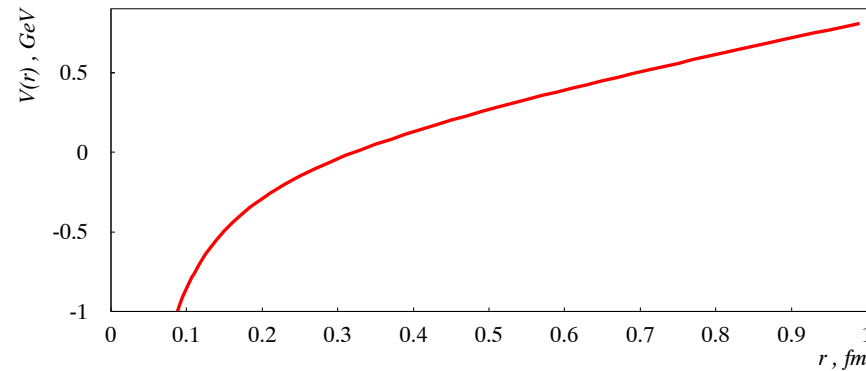


Figure 105: Modified Coulomb potential (116)

Other forms of the potential can give equally good results, for example

$$V(r) = a \ln(br) \quad (117)$$

where parameters appear to be

$$a = 0.7 \text{ GeV}$$

$$b = 0.5 \text{ GeV}$$

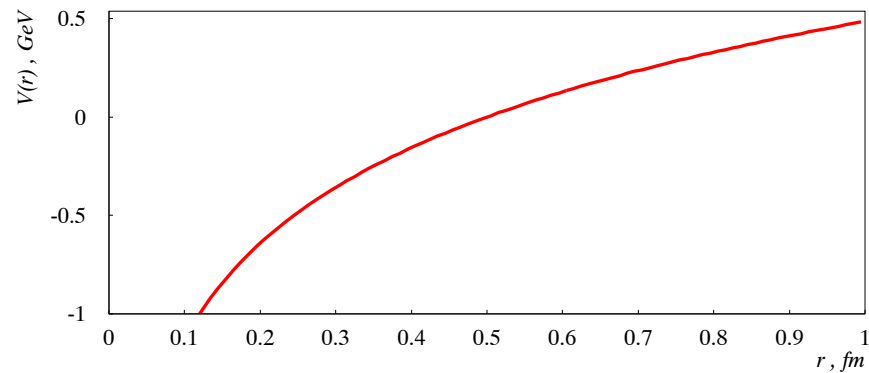


Figure 106: Logarithmic potential (117)

🎯 In the range of $0.2 \leq r \leq 0.8$ fm potentials (116) and (117) are in good agreement
 \Rightarrow in this region the quark-antiquark potential can be considered as well-defined

❖ Simple non-relativistic Schrödinger equation explains quite well existence of several energy states for a given heavy quark-antiquark system

Light mesons; nonets

- ❖ Spins of quarks are counter-directed $\Rightarrow J^P=0^-$, *pseudoscalar meson nonet* (9 possible $q\bar{q}$ combinations for u,d,s quarks)
- ❖ Spins of quarks are co-directed $\Rightarrow J^P=1^-$, *vector meson nonet*

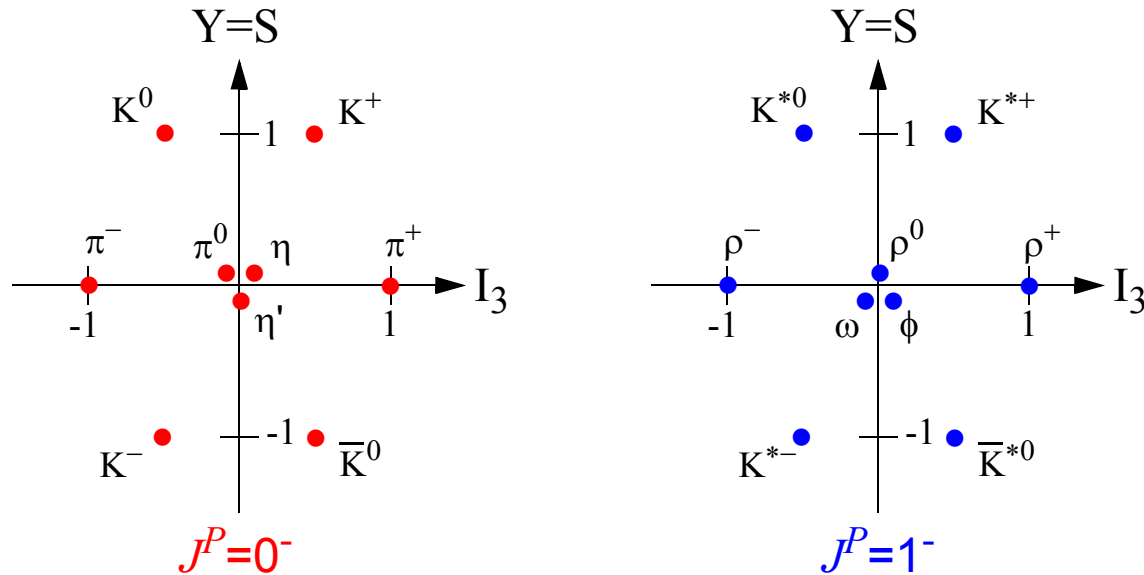


Figure 107: Light meson nonets in (I_3, Y) space (“weight diagrams”)

❖ In each nonet, there are three particles with equal quantum numbers $Y=S=I_3=0$

☉ They correspond to a $q\bar{q}$ pair like $u\bar{u}$, $d\bar{d}$ or a *linear combination* of these states (follows from the isospin operator analysis):

$$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad I = 1, I_3 = 0 \quad (118)$$

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \quad I = 0, I_3 = 0 \quad (119)$$

❖ π^0 and ρ^0 mesons are linear combinations of $u\bar{u}$ and $d\bar{d}$ states (118):

$$(u\bar{u} - d\bar{d})/(\sqrt{2})$$

❖ ω meson is the linear combination (119): $(u\bar{u} + d\bar{d})/(\sqrt{2})$

Inclusion of an $s\bar{s}$ pair leads to further combinations:

$$\eta(547) = \frac{(d\bar{d} + u\bar{u} - 2s\bar{s})}{\sqrt{6}} \quad I = 0, I_3 = 0 \quad (120)$$

$$\eta'(958) = \frac{(d\bar{d} + u\bar{u} + s\bar{s})}{\sqrt{3}} \quad I = 0, I_3 = 0 \quad (121)$$

- ❖ There exists meson $\phi(1019)$, which is a quarkonium $s\bar{s}$, having $I=0$ and $I_3=0$

Light baryons

- ❖ Three-quark states of the lightest quarks (u,d,s) form baryons, which can be arranged in *supermultiplets* (*singlets*, *octets* and *decuplets*).
- ❖ The lightest baryon supermultiplets are *octet* of $J^P = \frac{1}{2}^+$ particles and *decuplet* of $J^P = \frac{3}{2}^+$ particles
- ☉ Weight diagrams of baryons can be deduced from the quark model under assumption that the **combined** space-spin wavefunctions are *symmetric* under interchange of **like** quarks

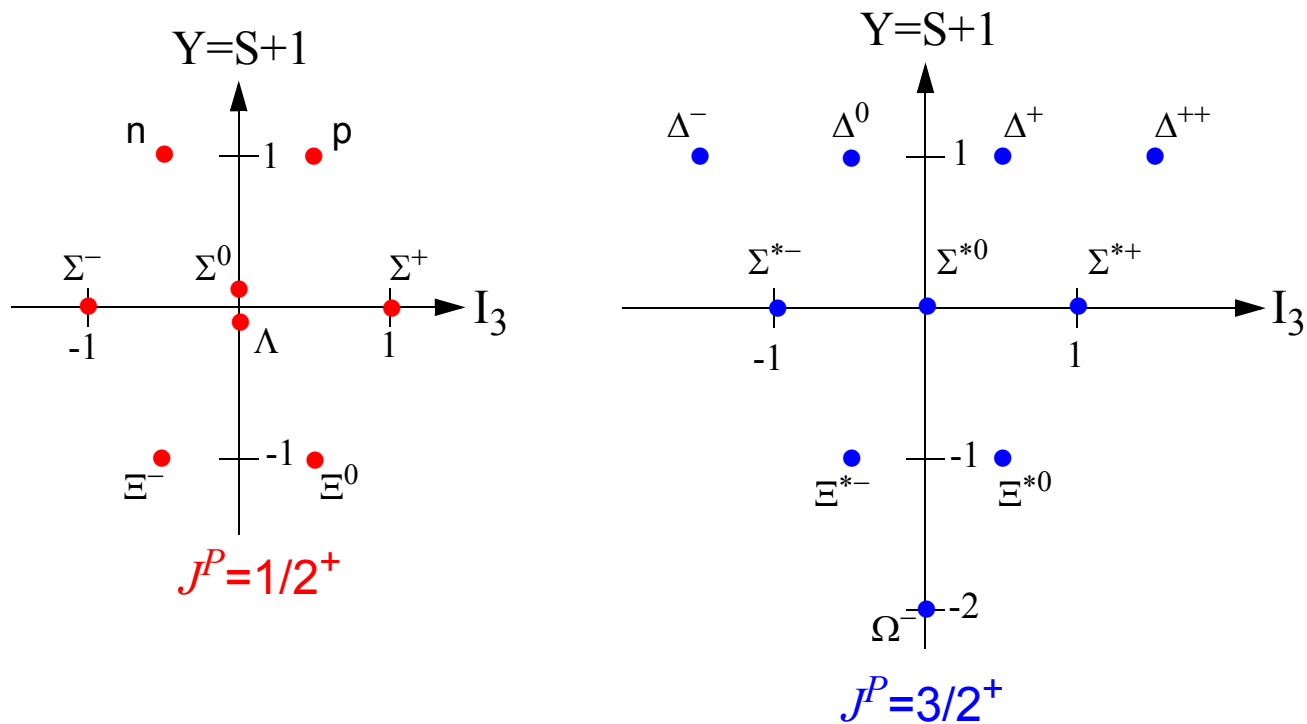


Figure 108: Weight diagrams for light baryons

- Parity of a 3-quark state $q_i q_j q_k$ is $P = P_i P_j P_k = 1$
- Spin of such a state is sum of quark spins
- From presumption of symmetry under exchange of like quarks, any pair of like quarks qq must have total spin-1 (quark spins co-directed)

❖ There are six distinct combinations of the form $q_i q_j q_k$:

$uud, uus, ddu, dds, ssu, ssd$

each of them can have spin $J=1/2$ or $J=3/2$

❖ three combinations of the form $q_i q_i q_i$ are possible:

uuu, ddd, sss

spins of all like-quarks have to be parallel (symmetry presumption), hence $J=3/2$ only

❖ The remaining combination is uds , with two distinct states having spin values $J=1/2$ and one state with $J=3/2$

❖ By adding up numbers, one gets 8 states with $J^P=1/2^+$ and 10 states with $J^P=3/2^+$, exactly what is shown by weight diagrams

❖ Measured masses of baryons show that mass difference between members of same isospin multiplets is much smaller than that between members of different isospin multiplets

☉ In what follows, equal masses of isospin multiplet members are assumed, e.g.,

$$m_p = m_n \equiv m_N$$

Experimentally, more s-quarks contains a particle, heavier it is:

$$\Xi^0(1315) = (uss); \Sigma^+(1189) = (uus); p(938) = (uud)$$

$$\Omega^-(1672) = (sss); \Xi^{*0}(1532) = (uss);$$

$$\Sigma^{*+}(1383) = (uus); \Delta^{++}(1232) = (uuu)$$

❖ There is an evidence that the main contribution to big mass differences comes from the s-quark

☉ Knowing masses of baryons, one can calculate 6 **simplistic** estimates of mass difference between s-quark and light quarks (u,d)

For the $3/2^+$ decuplet:

$$M_{\Omega} - M_{\Xi} = M_{\Xi} - M_{\Sigma} = M_{\Sigma} - M_{\Delta} = m_s - m_{u,d}$$

and for the $1/2^+$ octet:

$$M_{\Xi} - M_{\Sigma} = M_{\Xi} - M_{\Lambda} = M_{\Lambda} - M_N = m_s - m_{u,d}$$

Average value of those differences gives

$$m_s - m_{u,d} \approx 160 \text{ MeV}/c^2 \quad (122)$$

❖ So far, so good – BUT quarks are spin-1/2 particles \Rightarrow fermions \Rightarrow their wavefunctions are *antisymmetric* and all the discussion above **contradicts Pauli principle!**

COLOUR

- ❖ Experimental data confirm predictions based on the assumption of symmetric wave functions
- ❖ That means that apart of space and spin degrees of freedom, quarks carry yet another attribute

In 1964-1965, Greenberg and Nambu with colleagues proposed the new property – the *colour* – with THREE possible states, and associated with the corresponding wavefunction χ^C :

$$\Psi = \psi(\vec{x})\chi\chi^C \quad (123)$$

- ⊙ Conserved quantum numbers associated with χ^C are *colour charges* – in strong interaction they play analogous role to the electric charge in e.m. interaction
- ⊙ Hadrons are presumed to exist only in *colour singlet* states, with total colour charge of zero
- ⊙ Quarks have to be *confined* within the hadrons, since non-zero colour states are forbidden

☉ Three independent colour wavefunctions are represented by “*colour spinors*”:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (124)$$

- ☉ They are acted upon by eight independent “*colour operators*” which are represented by a set of 3-dimensional matrices (analogs of Pauli matrices)
- ☉ Colour charges I_3^C and Y^C are eigenvalues of corresponding operators

In this formalism, colours can be quantified. Values of I_3^C and Y^C for the colour states of quarks and antiquarks are:

	Quarks		Antiquarks		
	I_3^C	Y^C	I_3^C	Y^C	
r (“red”)	1/2	1/3	\bar{r}	-1/2	-1/3
g (“green”)	-1/2	1/3	\bar{g}	1/2	-1/3
b (“blue”)	0	-2/3	\bar{b}	0	2/3

◎ *Colour hypercharge* Y^C and *colour isospin charge* I_3^C are additive quantum numbers, having opposite sign for quark and antiquark

Confinement condition for the total colour charges of a hadron:

$$I_3^C = Y^C = 0 \quad (125)$$

The most general colour wavefunction for a baryon is a linear superposition of six possible combinations:

$$\begin{aligned} \chi_B^C = & \alpha_1 r_1 g_2 b_3 + \alpha_2 g_1 r_2 b_3 + \alpha_3 b_1 r_2 g_3 \\ & + \alpha_4 b_1 g_2 r_3 + \alpha_5 g_1 b_2 r_3 + \alpha_6 r_1 b_2 g_3 \end{aligned} \quad (126)$$

where α_i are constants. It can be shown that the color confinement requires the totally antisymmetric combination:

$$\begin{aligned} \chi_B^C = & \frac{1}{\sqrt{6}} (r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 \\ & - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3) \end{aligned} \quad (127)$$

Colour confinement principle (125) implies certain requirements for states containing both quarks and antiquarks:

- consider an arbitrary combination $q^m \bar{q}^n$ of m quarks and n antiquarks, $m \geq n$
- for a particle with α quarks in r-state, β quarks in g-state, γ quarks in b-state ($\alpha + \beta + \gamma = m$) and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ antiquarks in corresponding antistates ($\bar{\alpha} + \bar{\beta} + \bar{\gamma} = n$), the colour wavefunction is

$$r^\alpha g^\beta b^\gamma \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}} \quad (128)$$

Adding up colour charges (from the table above) and applying the confinement requirement,

$$I_3^C = (\alpha - \bar{\alpha})/2 - (\beta - \bar{\beta})/2 = 0$$

$$Y^C = (\alpha - \bar{\alpha})/3 + (\beta - \bar{\beta})/3 - 2(\gamma - \bar{\gamma})/3 = 0$$

$$\Downarrow$$

$$\alpha - \bar{\alpha} = \beta - \bar{\beta} = \gamma - \bar{\gamma} \equiv p$$

Here p is a non-negative integer \Rightarrow

❖ Numbers of quarks and antiquarks in a colorless state are related as:
 $m - n = 3p$

❖ The only combination $q^m \bar{q}^n$ allowed by the colour confinement principle is

$$(3q)^p (q\bar{q})^n, \quad p, n \geq 0 \quad (129)$$

⊙ Form (129) forbids states with fractional electric charges

⊙ However, it allows exotic combinations like $qqq\bar{q}$, $qqqq\bar{q}$ (like e.g. the pentaquark $\Theta^+ = uud\bar{s}$)

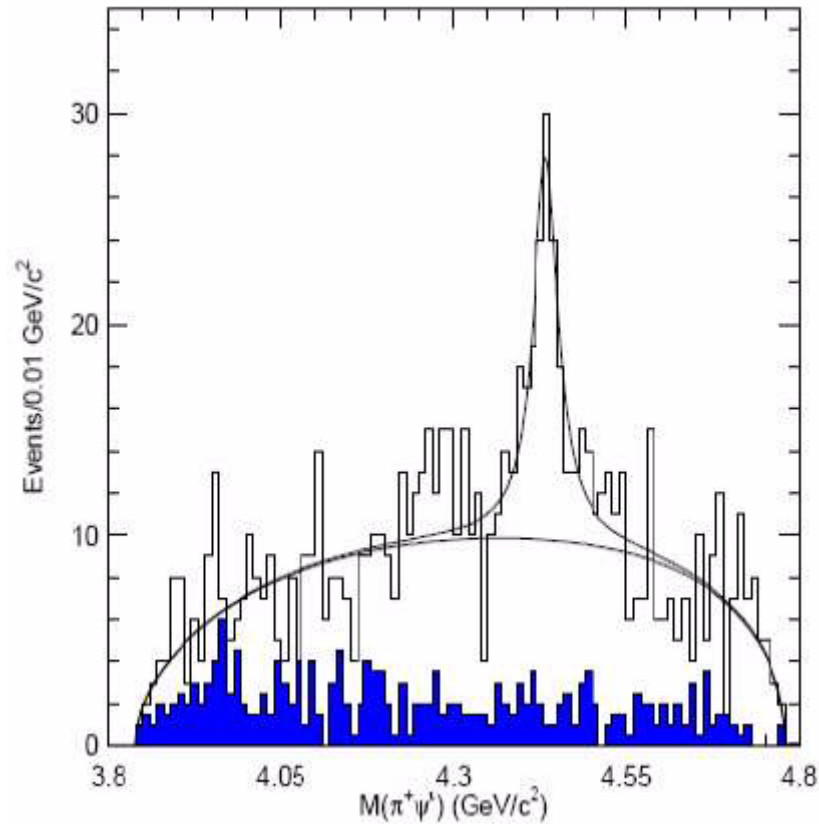


Figure 109: A tetraquark $Z^+(4330)$ candidate ($u\bar{d}c\bar{c}$) as published by the BELLE experiment in 2008