II. Leptons, quarks and hadrons

- **Leptons** are spin-1/2 fermions, not subject to strong interaction

\[
\begin{pmatrix}
\nu_e \\
e^-
\end{pmatrix},
\begin{pmatrix}
\nu_\mu \\
\mu^-
\end{pmatrix},
\begin{pmatrix}
\nu_\tau \\
\tau^-
\end{pmatrix}
\]

\[M_e < M_\mu < M_\tau\]

- Electron \(e^-\), muon \(\mu^-\) and tau-lepton \(\tau^-\) have corresponding neutrinos \(\nu_e, \nu_\mu\) and \(\nu_\tau\).

- Electron, muon and tau have electric charge of \(-e\). Neutrinos are neutral.

- Neutrinos have **very small** masses (were thought to be massless).

- For neutrinos, only weak interactions have been observed so far.
Antileptons are: positron $e^+$, positive muon $\mu^+$, positive tau-lepton $\tau^+$, and antineutrinos:

$$\left(\begin{array}{c}
e^+ \\
-\nu_e
\end{array}\right), \left(\begin{array}{c}
\mu^+ \\
-\nu_\mu
\end{array}\right), \left(\begin{array}{c}
\tau^+ \\
-\nu_\tau
\end{array}\right)$$

Neutrinos and antineutrinos differ by the *lepton number*. Leptons possess lepton numbers $L_\alpha=1$ ($\alpha$ stands for $e$, $\mu$ or $\tau$), and antileptons have $L_\alpha=-1$.

*Lepton numbers are conserved in all interactions!*

Neutrinos can not be directly registered by any detector, there are only indirect measurements of their properties.

First indication of neutrino existence came from $\beta$-decays of nuclei, $N$:

$$N(Z,A) \rightarrow N(Z+1,A) + e^- + \bar{\nu}_e$$
\( \beta \)-decay is simply one of the neutrons decaying:

\[
n \rightarrow p + e^- + \bar{\nu}_e
\]

Experimentally, only proton and electron can be observed, and a fraction of energy and angular momentum is “missing”. W. Pauli in 1930 suggested that these are carried out by an undetectable neutral particle, a neutrino.

- Note that for the sake of the lepton number conservation, electron must be accompanied by an electron-type antineutrino!

The \( \bar{\nu}_e \) mass can be estimated from the electron energy in the \( \beta \)-decay:

\[
m_e \leq E_e \leq \Delta M_N - m_{\bar{\nu}_e}
\]

Current results from the tritium decay indicate a very small upper limit:

\[
^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e \quad m_{\bar{\nu}_e} \leq 3 \text{ eV/c}^2
\]

- Recently observed neutrino mixing suggests \textit{non-zero} mass
An inverse $\beta$-decay (neutrino “capture”) also takes place:

$$\nu_e + n \rightarrow e^- + p$$

or

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

However, the probabilities of these processes is very low, therefore to register any, one needs a very intense flux of neutrinos.

Reines and Cowan experiment (1956)

Reactions of type (22) or (23) provide direct evidence of neutrinos.

By “capturing” antineutrinos produced in a nuclear reactor, it is possible to obtain around 2 events of type (23) per hour.

To separate the signal from background noise, the “delayed coincidence” scheme was used: signal from neutron comes later than one from positron.
(a) Antineutrino interacts with a proton in water (hydrogen nucleus), producing neutron and positron
(b) Positron annihilation with an atomic electron produces fast photon $\gamma$; Coulomb scattering gives rise to softer photons
(c) Neutron is captured by a $Cd$ nucleus, releasing more photons.

Figure 16: Schematic representation of the Reines and Cowan experiment. Aqueous solution of $CdCl_2$ used as the target ($Cd$ captures neutrons).
Figure 17: The Savannah River neutrino detector which registered neutrinos in 1956
Muons were first observed in 1936, in *cosmic rays*

Cosmic rays have two components:

1) *primaries*, which are high-energy particles coming from the outer space, mostly hydrogen nuclei

2) *secondaries*, the particles which are produced in collisions of primaries with nuclei in the Earth atmosphere; muons belong to this component

Figure 18: Schematic representation of cosmic rays
Muons are 200 times heavier than electrons and are very penetrating particles.

Electromagnetic properties of muon are identical to those of electron (except the mass difference)

- Tau is the heaviest lepton, discovered in $e^+e^-$ annihilation experiments in 1975

![Figure 19: $\tau$ pair production in $e^+e^-$ annihilation](image)

- Electron is a stable particle, while $\mu$ and $\tau$ have finite lifetimes:

$$\tau_\mu = 2.2 \times 10^{-6} \, \text{s} \quad \text{and} \quad \tau_\tau = 2.9 \times 10^{-13} \, \text{s}$$

Muon decays in a purely leptonic mode:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$ (24)
Tau has a mass sufficient to decay into hadrons, but it has leptonic decay modes as well:

\[
\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau
\]  \hspace{1cm} (25)

\[
\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau
\]  \hspace{1cm} (26)

$v$ Note: lepton numbers are conserved in all reactions ever observed

$\triangleright$ Fraction of a given decay mode with respect to all possible decays is called branching ratio, denoted by $B$.

$\triangleright$ Decay rate: $\Gamma = B/\tau$, where $\tau$ is decaying particle’s lifetime.

Branching ratio of the process (25) is 17.84%, and of (26) – 17.37%.

Important assumptions:

$\triangleright$ Weak interactions of leptons are identical, just like electromagnetic ones ("universality of weak interactions")

$\triangleright$ One can neglect final state lepton masses for many basic calculations
Decay rate $\Gamma$ of a muon is given by the expression:

$$\Gamma(\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu) = \frac{G_F^2 m_\mu^5}{195\pi^3}$$

(27)

where $G_F$ is the Fermi constant and $m_\mu$ is muon mass.

Substituting $m_\mu$ with $m_\tau$ in (27), one obtains decay rates of leptonic tau decays.

- Since only decaying particle mass enters (27), decay rates are equal for processes (25) and (26).
- It explains why branching ratios of these processes have such close values.

Lifetime of a lepton can be calculated using measured decay rate:

$$\tau_l = \frac{B(l^- \rightarrow e^- \bar{\nu}_e \nu_l)}{\Gamma(l^- \rightarrow e^- \bar{\nu}_e \nu_l)}$$

(28)

Here $l$ stands for $\mu$ or $\tau$.  

---
Since muons have basically only one decay mode, \( B=1 \) in their case. Using experimental values of \( B \) and formula (27), one obtains the ratio of muon and tau lifetimes:

\[
\frac{\tau_\tau}{\tau_\mu} \approx 0.178 \cdot \left( \frac{m_\mu}{m_\tau} \right)^5 \approx 1.3 \times 10^{-7}
\]

This again is in a very good agreement with independent experimental measurements.

- Universality of lepton interactions is proved to a great extent. That means that there is basically no difference between lepton generations, apart of the mass and the lepton numbers.
Quarks are spin-1/2 fermions, subject to all interactions. Quarks have fractional electric charges.

Quarks and their bound states are the only particles which interact strongly (via strong force).

Some historical background:

- Proton and neutron ("nucleons") were known to interact strongly.
- In 1947, in cosmic rays, new heavy particles were detected ("hadrons").
- By 1960s, in accelerator experiments, many dozens of hadrons were discovered
- An urge to find a kind of “periodic system” lead to the “Eightfold Way” classification, invented by Gell-Mann and Ne‘eman in 1961, based on the SU(3) symmetry group and describing hadrons in terms of “building blocks”.
- In 1964, Gell-Mann invented quarks as the building blocks (and Zweig invented “aces”).
The quark model: *baryons* and *antibaryons* are bound states of three quarks, and *mesons* are bound states of a quark and antiquark.

*Hadrons* is a common name for baryons and mesons.

Like leptons, quarks and antiquarks occur in three generations:

\[
\begin{pmatrix}
  u \\
  d
\end{pmatrix}, \begin{pmatrix}
  c \\
  s
\end{pmatrix}, \begin{pmatrix}
  t \\
  b
\end{pmatrix}, \begin{pmatrix}
  \bar{d} \\
  \bar{u}
\end{pmatrix}, \begin{pmatrix}
  \bar{s} \\
  \bar{c}
\end{pmatrix}, \begin{pmatrix}
  \bar{b} \\
  \bar{t}
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>Name (&quot;Flavour&quot;)</th>
<th>Symbol</th>
<th>Charge (units of e)</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>d</td>
<td>-1/3</td>
<td>4-8 MeV/c²</td>
</tr>
<tr>
<td>Up</td>
<td>u</td>
<td>+2/3</td>
<td>1.5-4.0 MeV/c²</td>
</tr>
<tr>
<td>Strange</td>
<td>s</td>
<td>-1/3</td>
<td>80-130 MeV/c²</td>
</tr>
<tr>
<td>Charmed</td>
<td>c</td>
<td>+2/3</td>
<td>1.15-1.35 GeV/c²</td>
</tr>
<tr>
<td>Bottom</td>
<td>b</td>
<td>-1/3</td>
<td>4.1-4.9 GeV/c²</td>
</tr>
<tr>
<td>Top</td>
<td>t</td>
<td>+2/3</td>
<td>≈178 GeV/c²</td>
</tr>
</tbody>
</table>
Despite numerous attempts, free quarks could never be observed

There is an elegant explanation for this:

- Every quark possesses a new quantum number: the *colour*. There are three different colours, thus each quark can have three distinct colour states. Colours are called red (R), green (G) and blue (B).

- Coloured objects can not exist as free observable particles.
  - Therefore quarks must confine into colourless hadrons.

- Colourless (“blank”) combinations are: 3-colour states RGB (RGB), or colour-anticolour states RR, GG, BB, and their linear combinations.
  - **Baryons** are bound states of three quarks *of different colours*.
  - **Mesons** consist of *colour-anticolour* quark pairs.

- Each quark *flavour* is associated with a different quantum number, which is conserved in strong and electromagnetic interactions, but *not* in weak ones.
Quark *flavour quantum numbers* are defined as: *strangeness* $S = -1$ for s-quark; *charm* $C = 1$ for c-quark; *beauty* $\tilde{B} = -1$ for b-quark.

- top-quark has lifetime too short to form hadrons before decaying, thus $truth T = 0$ for all hadrons
- Up and down quarks have nameless flavour quantum numbers

### Some examples of baryons:

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (GeV/c²)</th>
<th>Quark composition</th>
<th>Q (units of e)</th>
<th>S</th>
<th>C</th>
<th>$\tilde{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>0.938</td>
<td>uud</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>n</td>
<td>0.940</td>
<td>udd</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Λ</td>
<td>1.116</td>
<td>uds</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Λ_c</td>
<td>2.285</td>
<td>udc</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Baryons are assigned own *baryon quantum number* $B = (N(q) - N(\bar{q}))/3$, which gives $B = 1$ for baryons, $B = -1$ for antibaryons (for mesons, $B = 0$).

- $B$ is conserved in all interactions, thus the lightest baryon, proton, is stable.
Some examples of mesons:

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (Gev/c^2)</th>
<th>Quark composition</th>
<th>Q (units of e)</th>
<th>S</th>
<th>C</th>
<th>( \tilde{B} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ )</td>
<td>0.140</td>
<td>ud</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K^-</td>
<td>0.494</td>
<td>su</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D^-</td>
<td>1.869</td>
<td>dc</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( D_s^+ )</td>
<td>1.969</td>
<td>cs</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B^-</td>
<td>5.279</td>
<td>bu</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Y</td>
<td>9.460</td>
<td>bb</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Majority of hadrons are unstable and tend to decay by the strong interaction to the state with the lowest possible mass (lifetime about \( 10^{-23} \) s).

- Hadrons with the lowest possible mass for each quark number (S, C, etc.) may live significantly longer before decaying weakly (lifetimes \( 10^{-7} - 10^{-13} \) s) or electromagnetically (mesons, lifetimes \( 10^{-16} - 10^{-21} \) s). Such hadrons are called *long-lived particles* (sometimes even “stable”).

- The only truly stable hadron is proton – *that is, if baryon number conservation is not violated.*
Brief history of hadron discoveries

- First known hadrons were proton and neutron.
- The lightest are pions $\pi$ ("pi-mesons"). There are charged pions $\pi^+$, $\pi^-$ with mass of 0.140 GeV/c$^2$, and neutral ones $\pi^0$, mass 0.135 GeV/c$^2$.

- Pions and nucleons are the lightest particles containing u- and d-quarks only.

Pions were discovered in 1947 in cosmic rays, using photoemulsions to detect particles.

Some reactions induced by cosmic rays primaries:

\[
p + p \rightarrow p + n + \pi^+ \\
\rightarrow p + p + \pi^0 \\
\rightarrow p + p + \pi^+ + \pi^-
\]

Same reactions can be reproduced in accelerators, with higher rates, although cosmic rays may provide higher energies.
Figure 20: First observed pions: a $\pi^+$ stops in the emulsion and decays to a $\mu^+$ and $\nu_\mu$, followed by the decay of $\mu^+$. In emulsions, pions were identified by much more dense ionization along the track, as compared to electron tracks.
Figure 20: examples of the reaction

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

where the pion comes to rest, producing muons which in turn decay by the reaction $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$.

- Charged pions decay mainly to the muon-neutrino pair (branching ratio about 99.99%), having lifetimes of $2.6 \times 10^{-8}$ s. In quark terms:

  $$(u\bar{d}) \rightarrow \mu^+ + \nu_\mu$$

- The decay occurs through weak interaction, hence quark quantum numbers are not conserved. $B$ and $L$ are conserved.

- Neutral pions decay mostly by the electromagnetic interaction, having shorter lifetime of $0.8 \times 10^{-16}$ s:

  $$\pi^0 \rightarrow \gamma + \gamma$$
Discovered pions were fitting very well into Yukawa’s theory – they were thought to be responsible for the nuclear forces:

The resulting potential for this kind of exchange is of Yukawa type (19), and at the longest range reproduces observed nuclear forces very well, including even spin effects.

However, at the ranges comparable with the size of nucleons, this description fails, and the internal structure of hadrons must be taken into account.

Figure 21: Yukawa model of direct (a) and exchange (b,c) nuclear forces
Strange mesons and baryons

were called so, because they were produced in strong interactions, and yet had quite long lifetimes, and decayed weakly.

The lightest particles containing s-quarks are:

- mesons $K^+$, $K^-$ and $K^0$, $\bar{K}^0$: "kaons", lifetime of $K^+$ is $1.2 \times 10^{-8}$ s
- baryon $\Lambda$, lifetime of $2.6 \times 10^{-10}$ s

Principal decay modes of strange hadrons:

\[
K^+ \rightarrow \mu^+ + \nu_\mu \quad (B=0.64) \\
K^+ \rightarrow \pi^+ + \pi^0 \quad (B=0.21) \\
\Lambda \rightarrow \pi^- + p \quad (B=0.64) \\
\Lambda \rightarrow \pi^0 + n \quad (B=0.36)
\]

The first decay is clearly a weak one. Decays of $\Lambda$ have too long lifetime to be strong: if $\Lambda$ were (udd), the decay $(udd) \rightarrow (du) + (uud)$ should have had a lifetime of order $10^{-23}$ s. $\Lambda$ cannot be (udd) like the neutron.
Figure 22: “Strange” particle discovery (neutral kaon) by Rochester and Butler, 1947
Solution: to invent a new “strange” quark, bearing a new quark number, “strangeness”, which does not have to be conserved in weak interactions.

\[
\begin{array}{c|c}
S = 1 & S = -1 \\
\hline
\Lambda (1116) = uds & \Lambda (1116) = uds \\
K^+ (494) = us & K^- (494) = s\bar{u} \\
K^0 (498) = d\bar{s} & \bar{K}^0 (498) = s\bar{d}
\end{array}
\]

- In strong interactions, strange particles have to be produced in pairs in order to conserve total strangeness ("associated production"): 
  \[ \pi^- + p \rightarrow K^0 + \Lambda \]  
  \hspace{1cm} (30)

In 1952, bubble chambers were invented as particle detectors, and also worked as targets, providing, in particular, the proton target for reaction (30).
A bubble chamber is filled with a liquid (hydrogen, propane, freons) under pressure, heated above its boiling point.

Particles ionize the liquid along their passage.

Volume expands $\Rightarrow$ pressure drops $\Rightarrow$ liquid starts boiling along the ionization trails.

Visible bubbles are stereo-photographed.

Figure 23: A bubble chamber picture of the reaction (30)
Bubble chambers were great tools in particle discoveries, providing physicists with numerous hadrons, all of them fitting u-d-s quark scheme until 1974.

In 1974, a new particle was discovered, which demanded a new flavour to be introduced. Since it was detected simultaneously by two groups in Brookhaven (BNL) and Stanford (SLAC), it received a double name: J/ψ (3097), a cc meson. The new quark was called “charmed”, and the corresponding quark number is charm, C. Since J/ψ itself has C=0, it is said to contain “hidden charm”.

Shortly after that particles with “open charm” were discovered as well:

$$D^+(1869) = c\bar{d}, \quad D^0(1865) = c\bar{u}$$

$$D^-(1869) = d\bar{c}, \quad \bar{D}^0(1865) = u\bar{c}$$

$$\Lambda_c^+(2285) = udc$$
Even heavier charmed mesons were found – those which contained strange quark as well:

\[ D_s^+ (1969) = c\bar{s}, \ D_s^- (1969) = s\bar{c} \]

Lifetimes of the lightest charmed particles are of order \(10^{-13}\) s, well in the expected range of weak decays.

- Discovery of “charmed” particles was a triumph for the electroweak theory, which demanded number of quarks and leptons to be equal.

In 1977, “beautiful” mesons were discovered:

\[ Y(9460) = b\bar{b} \]
\[ B^+(5279) = u\bar{b}, \ B^0(5279) = d\bar{b} \]
\[ B^-(5279) = b\bar{u}, \ \bar{B}^0(5279) = b\bar{d} \]

and the lightest b-baryon: \( \Lambda_b^0 (5461) = udb \)

And this is the limit: top-quark is too unstable to form observable hadrons.