# VII. Quark states and colours

- Forces acting between quarks in hadrons can be investigated by studying a simple quark-antiquark system
- Systems of heavy quarks, like $c\bar{c}$ (*charmonium*) and $b\bar{b}$ (*bottomonium*), are essentially non-relativistic (masses of quarks are much bigger than their kinetic energies)
  - Charmonium and bottomonium (*quarkonium*) are analogous to a hydrogen atom in a sense that they manifest many energy levels
  - While the hydrogen atom is governed by the electromagnetic force, the quarkonium system is dominated by the <u>strong force</u>

Like in a hydrogen atom, energy states of a quarkonium can be labelled by their principal (radial) quantum number $n$, and $J$, $L$, $S$, where $L \leq n$-1 and $S$ can be either 0 or 1.

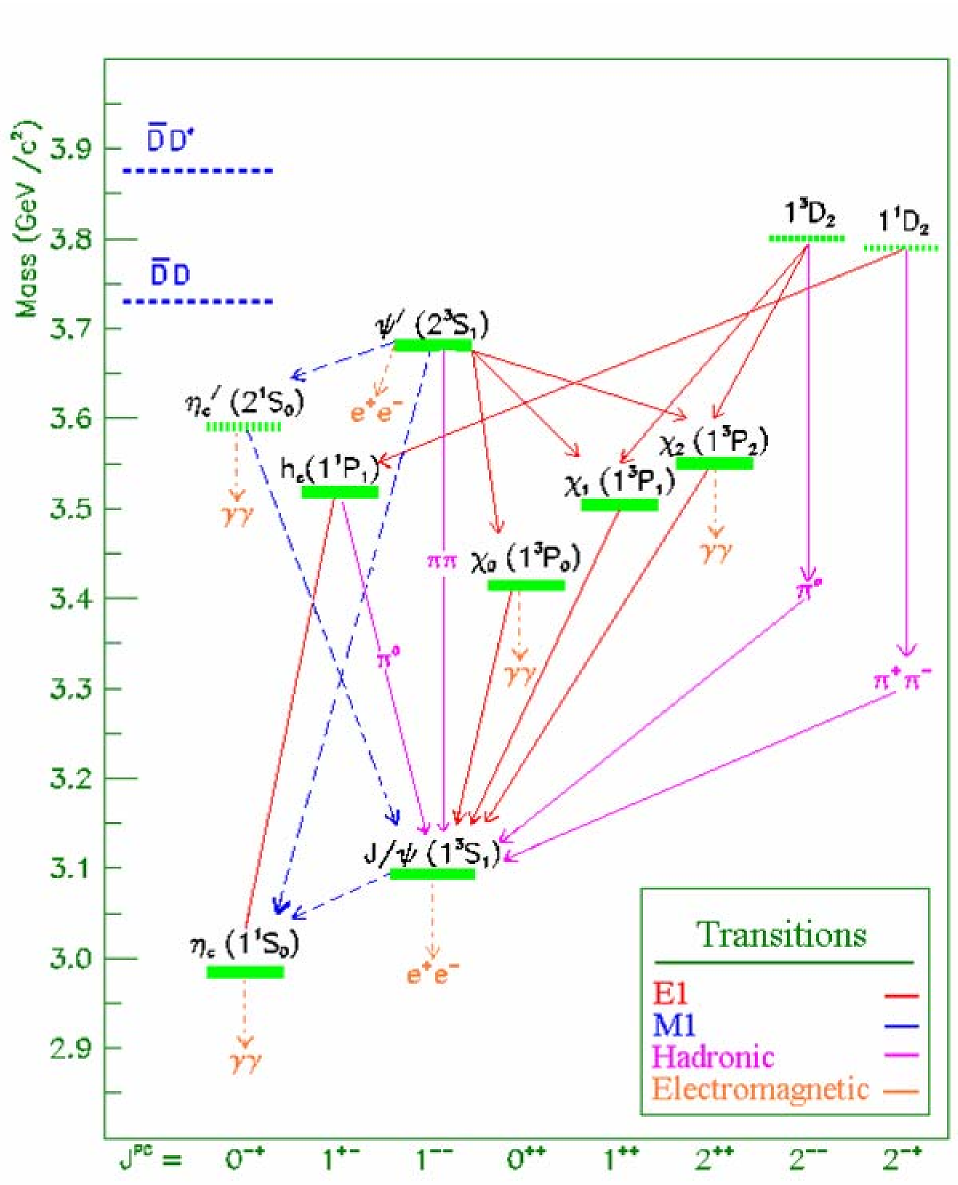


Figure 101: The charmonium spectrum

From Equations (63) and (74) , parity and C-parity of a quarkonium are:

$$P = P_q P_{\bar{q}}(-1)^L = (-1)^{L+1} \;\; ; \;\; C = (-1)^{L+S}$$

Predicted and observed charmonium and bottomonium states for $n$=1 and $n$=2:

| | | $J^{PC}$ | $c\bar{c}$ state | $b\bar{b}$ state |
|---|---|---|---|---|
| n=1 | $^1S_0$ | $0^{-+}$ | $\eta_c(2980)$ | $\eta_b(9389)$ (?) |
| n=1 | $^3S_1$ | $1^{--}$ | $J/\psi(3097)$ | $\Upsilon(9460)$ |
| n=2 | $^1S_0$ | $0^{-+}$ | $\eta_c(3637)$ (?) | – |
| n=2 | $^3S_1$ | $1^{--}$ | $\psi(3686)$ | $\Upsilon(10023)$ |
| n=2 | $^3P_0$ | $0^{++}$ | $\chi_{c0}(3415)$ | $\chi_{b0}(9860)$ |
| n=2 | $^3P_1$ | $1^{++}$ | $\chi_{c1}(3511)$ | $\chi_{b1}(9892)$ |
| n=2 | $^3P_2$ | $2^{++}$ | $\chi_{c2}(3556)$ | $\chi_{b2}(9913)$ |
| n=2 | $^1P_1$ | $1^{+-}$ | $h_c(3526)$ (?) | |

- States $J/\psi$ and $\psi$ have the same $J^{PC}$ quantum numbers as a photon: $1^{--}$, and the most common way to form them is through $e^+e^-$-annihilation, where virtual photon converts to a charmonium state

# Electron-positron collisions, cross-section

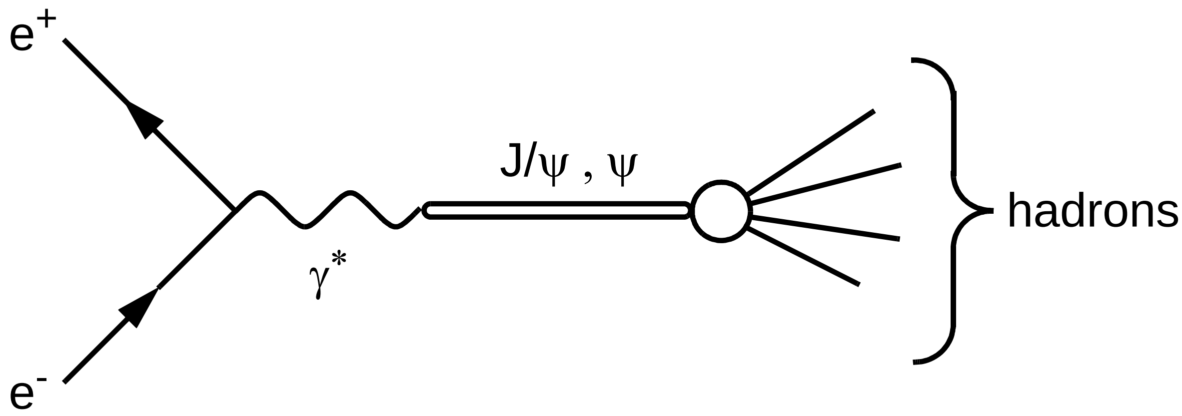


Figure 102: Formation and decay of $J/\psi$ ($\psi$) mesons in $e^+e^-$ annihilation

- If centre-of-mass energy of incident $e^+$ and $e^-$ is equal to the quarkonium mass, formation of the latter is highly probable, which leads to the peak in the cross-section $\sigma(e^+e^-\to hadrons)$.

- Cross-section $\sigma$ in a collision is defined through

$$N = \sigma \times L \tag{104}$$

Here $N$ is the count of reactions (*events*) in a time period, and $L$ is the integrated *luminosity* – density of colliding particles integrated over this time period

- Cross-section is measured in barns:

$$[\sigma] = 1 \text{ barn } (1\text{ b}) \equiv 10^{-24}\text{ cm}^2 \Rightarrow [L] = \text{cm}^{-2} \text{ or } 1\text{ barn}^{-1}\ (1\text{ b}^{-1})$$

An example:

- an LHC collider run will last $10^7$ s, with instantaneous luminosity of $10^{34}$ cm$^{-2}$s$^{-1}$ $\Rightarrow L = 10^{41}$ cm$^{-2}$ = 100 fb$^{-1}$.
- The total production cross-section for $b\bar{b}$-pairs is about 500 μb $\Rightarrow$ in $10^7$ s, the number of produced events will be $N$=500 μb ×100 fb$^{-1}$ = 5 ×$10^{13}$

- $e^+e^-$ collisions provide clean study environment; it is convenient to normalize the cross-sections to that of muon production:

$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{105}$$

- Sharp peaks can be observed in $R$ at $E_{cm}$=3.097 GeV (J/ψ) and 3.686 GeV (ψ)

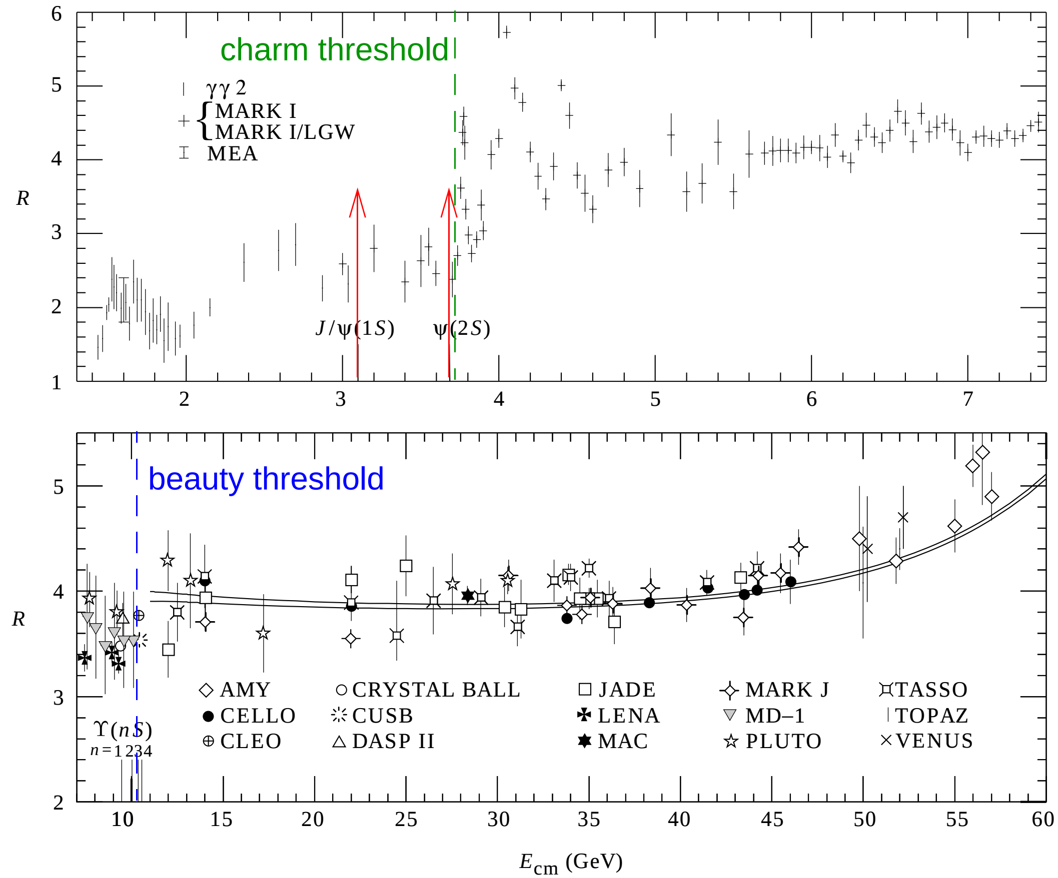


Figure 103: Cross-section ratio R in $e^+e^-$ collision. Arrows indicate the peaks.

- Cross-section for a $\mu^+\mu^-$ final state is known and depends only on $E_{CM}$ and $\alpha$:

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2} \tag{106}$$

- Charm *threshold* (3730 MeV): twice the mass of the lightest charmed meson, D
  - J/$\psi$, $\psi$ are lighter $\Rightarrow$ can not decay into charmed particles $\Rightarrow$ long-living (narrow peaks below charm threshold)
  - Wide peaks above charm threshold: short-living resonances

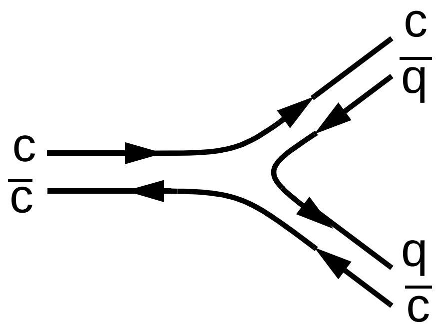


Figure 104: Charmonium resonance decay to charmed mesons

- J/$\psi$ and $\psi$ can only decay via annihilation of $c\bar{c}$ pair
  - Long lifetime since such annihilation is *suppressed* as opposed to light quarks (e.g. in $\pi_0$)
  - J/$\psi$ and $\psi$ can only decay to light hadrons (containing u, d, s), or to $e^+e^-$, or $\mu^+\mu^-$.

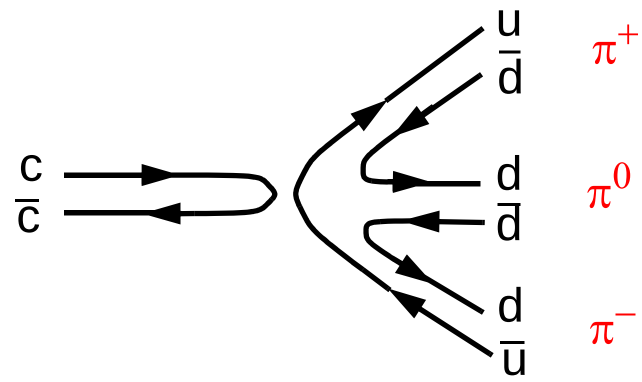


Figure 105: Charmonium decay to light non-charmed mesons

- Charmonium states with quantum numbers different of those of photon can not be produced in $c\bar{c}$ annihilation, but can be found in radiative decays of J/$\psi$ or $\psi$:

$$\psi(3686) \to \chi_{ci} + \gamma \qquad (i=0,1,2) \tag{107}$$

$$\psi(3686) \to \eta_c(2980) + \gamma \tag{108}$$

$$J/\psi(3097) \to \eta_c(2980) + \gamma \tag{109}$$

- Bottomonium spectrum is observed in much the same way as charmonium
- Beauty threshold is at 10560 MeV/$c^2$ (twice the mass of the B meson)

❖ Similarities between spectra of bottomonium and charmonium suggest similarity of forces acting in two systems

# The quark-antiquark potential

- Let's assume the $q\bar{q}$ potential being a central one, V($r$), and the system to be <u>non-relativistic</u>

In the centre-of-mass frame of a $q\bar{q}$ pair, Schrödinger equation is

$$-\frac{1}{2\mu}\nabla^2\psi(\vec{x}) + V(r)\psi(\vec{x}) = E\psi(\vec{x}) \qquad (110)$$

Here $\mu = m_q/2$ is the *reduced mass* of a quark, and $r = |\vec{x}|$ is distance between the quarks.

Mass of a quarkonium state in this framework is

$$M(q\bar{q}) = 2m_q + E \qquad (111)$$

- In the case of a Coulomb-like approximation V($r$)$\propto r^{-1}$, energy levels are quantized, depending only on the *principal quantum number* n:
- In the case of a harmonic oscillator potential V($r$)$\propto r^2$, the degeneracy of energy levels is broken: dependency on *L* arises

$$E_n = -\frac{\mu\alpha^2}{2n^2}$$

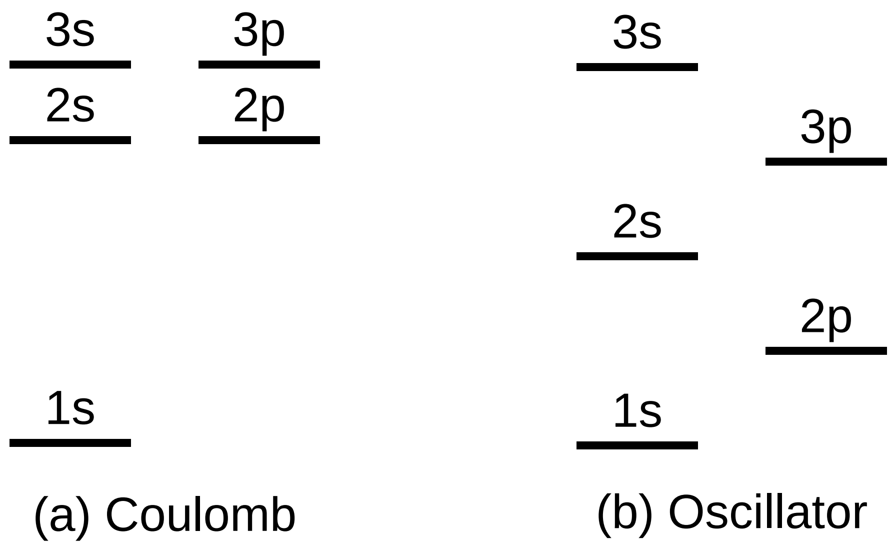


Figure 106: Energy levels arising from Coulomb and harmonic oscillator potentials for $n$=1,2,3

Cf Figure 101: one can see that heavy quarkonia spectra are inbetween the two approximations; the actual potential can be described by:

$$V(r) = -\frac{a}{r} + br \tag{112}$$

Coefficients $a$ and $b$ are determined by solving Equation (110) and fitting results to data:

$$a = 0.48 \qquad\qquad b = 0.18\ \text{GeV}^2$$

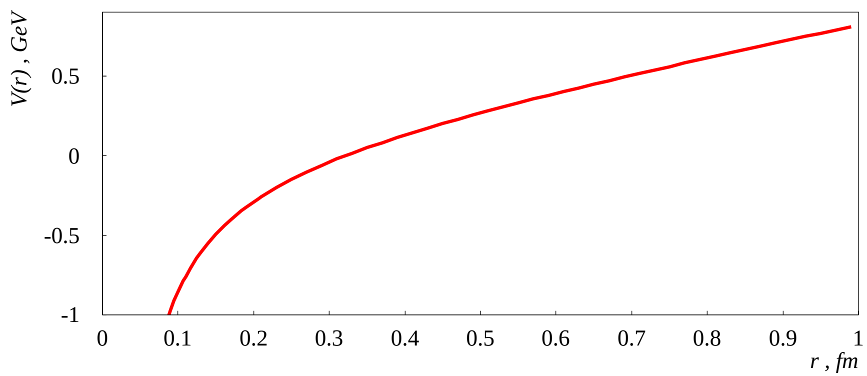


Figure 107: Modified Coulomb potential (112)

Other forms of the potential can give equally good results, for example

$$V(r) = a\ln(br) \tag{113}$$

where parameters appear to be

$$a = 0.7\ \text{GeV} \qquad\qquad b = 0.5\ \text{GeV}$$

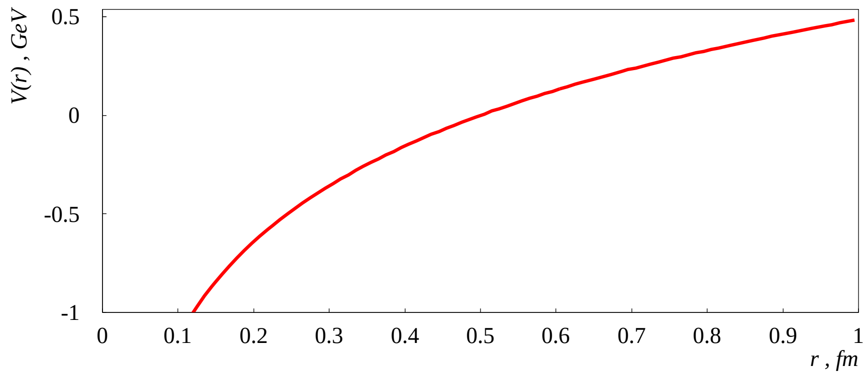


Figure 108: Logarithmic potential (113)

- In the range of $0.2 \le r \le 0.8$ fm potentials (112) and (113) are in good agreement $\Rightarrow$ in this region the quark-antiquark potential can be considered as well-defined

- Simple non-relativistic Schrödinger equation explains quite well existence of several energy states for a given heavy quark-antiquark system

# Light mesons; nonets

- Spins of quarks are counter-directed $\Rightarrow J^P = 0^-$, *pseudoscalar meson nonet* (9 possible $q\bar{q}$ combinations for u,d,s quarks)
- Spins of quarks are co-directed $\Rightarrow J^P = 1^-$, *vector meson* nonet

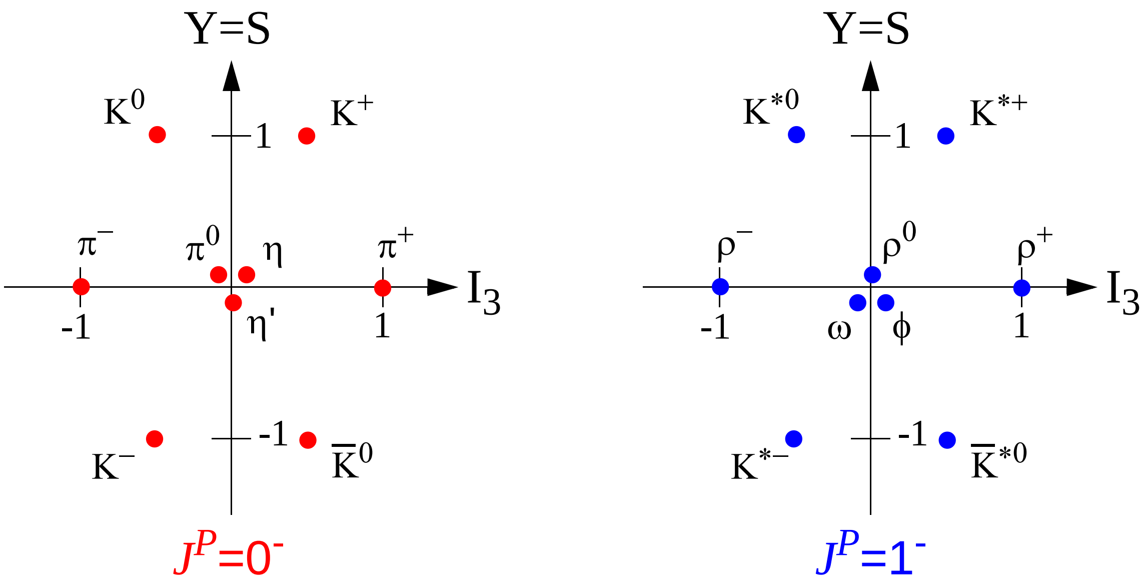


Figure 109: Light meson nonets in $(I_3, Y)$ space ("weight diagrams")

- In each nonet, there are three particles with equal quantum numbers $Y=S=I_3=0$
  - They correspond to a $q\bar{q}$ pair like $u\bar{u}$, $d\bar{d}$ or a *linear combination* of these states (follows from the isospin operator analysis):

$$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I = 1, I_3 = 0 \qquad (114)$$

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad I = 0, I_3 = 0 \qquad (115)$$

- $\pi^0$ and $\rho^0$ mesons are linear combinations of $u\bar{u}$ and $d\bar{d}$ states (114): $(u\bar{u} - d\bar{d})/(\sqrt{2})$

- $\omega$ meson is the linear combination (115): $(u\bar{u} + d\bar{d})/(\sqrt{2})$

Inclusion of an $s\bar{s}$ pair leads to further combinations:

$$\eta(547) = \frac{(d\bar{d} + u\bar{u} - 2s\bar{s})}{\sqrt{6}} \qquad I = 0, I_3 = 0 \qquad (116)$$

$$\eta'(958) = \frac{(d\bar{d} + u\bar{u} + s\bar{s})}{\sqrt{3}} \qquad I = 0, I_3 = 0 \tag{117}$$

- There exists meson $\phi$(1019), which is a quarkonium $s\bar{s}$, having $I$=0 and $I_3$=0

## *Light baryons*

- Three-quark states of the lightest quarks (u,d,s) form baryons, which can be arranged in *supermultiplets* (*singlets, octets* and *decuplets*).

- The lightest baryon supermultiplets are *octet* of $J^P = \frac{1}{2}^+$ particles and *decuplet* of $J^P = \frac{3}{2}^+$ particles

  - Weight diagrams of baryons can be deduced from the quark model under assumption that the **combined** space-spin wavefunctions are *symmetric* under interchange of **like** quarks

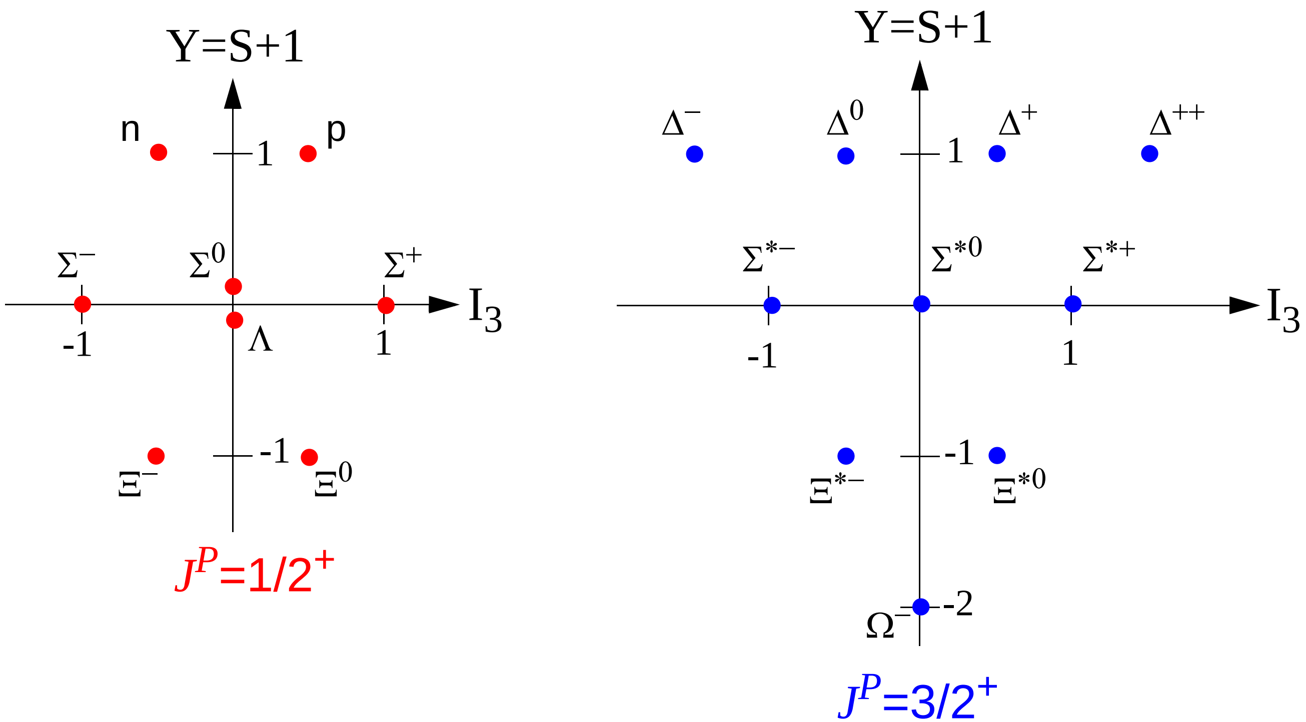


Figure 110: Weight diagrams for light baryons

– Parity of a 3-quark state $q_iq_jq_k$ is $P=P_iP_jP_k=1$

– Spin of such a state is sum of quark spins

– From presumption of symmetry under exchange of like quarks, any pair of like quarks qq must have total spin-1 (quark spins co-directed)

- There are six distinct combination of the form $q_i q_i q_j$:

uud, uus, ddu, dds, ssu, ssd

each of them can have spin $J$=1/2 or $J$=3/2

- three combinations of the form $q_i q_i q_i$ are possible:

uuu, ddd, sss

spins of all like-quarks have to be parallel (symmetry presumption), hence $J$=3/2 only

- The remaining combination is uds, with two distinct states having spin values $J$=1/2 and one state with $J$=3/2

- By adding up numbers, one gets 8 states with $J^P=1/2^+$ and 10 states with $J^P=3/2^+$, exactly what is shown by weight diagrams

- Measured masses of baryons show that mass difference between members of same isospin multiplets is much smaller than that between members of different isospin multiplets
  - In what follows, equal masses of isospin multiplet members are assumed, e.g.,

$$m_p = m_n \equiv m_N$$

Experimentally, more s-quarks contains a particle, heavier it is:

$$\Xi^0(1315)=(uss); \Sigma^+(1189)=(uus); p(938)=(uud)$$

$$\Omega^-(1672)=(sss); \Xi^{*0}(1532)=(uss);$$

$$\Sigma^{*+}(1383)=(uus); \Delta^{++}(1232)=(uuu)$$

- There is an evidence that the main contribution to big mass differences comes from the s-quark
  - Knowing masses of baryons, one can calculate 6 **simplistic** estimates of mass difference between s-quark and light quarks (u,d)

For the $3/2^+$ decuplet:

$$M_\Omega - M_\Xi = M_\Xi - M_\Sigma = M_\Sigma - M_\Delta = m_s - m_{u,d}$$

and for the $1/2^+$ octet:

$$M_\Xi - M_\Sigma = M_\Xi - M_\Lambda = M_\Lambda - M_N = m_s - m_{u,d}$$

Average value of those differences gives

$$m_s - m_{u,d} \approx 160\ MeV/c^2 \tag{118}$$

- So far, so good – BUT quarks are spin-1/2 particles ⇒ fermions ⇒ their wavefunctions are *antisymmetric* and all the discussion above **contradicts Pauli principle!**

# COLOUR

- Experimental data confirm predictions based on the assumption of symmetric wave functions
- That means that apart of space and spin degrees of freedom, quarks carry yet another attribute

In 1964-1965, Greenberg and Nambu with colleagues proposed the new property – the *colour* – with THREE possible states, and associated with the corresponding wavefunction $\chi^C$:

$$\Psi = \psi(\vec{x})\chi\chi^C \qquad (119)$$

- Conserved quantum numbers associated with $\chi^C$ are *colour charges* – in strong interaction they play analogous role to the electric charge in e.m. interaction
- Hadrons are presumed to exist only in *colour singlet* states, with total colour charge of zero
- Quarks have to be *confined* within the hadrons, since non-zero colour states are forbidden

- Three independent colour wavefunctions are represented by *"colour spinors"* :

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{120}$$

- They are acted upon by eight independent *"colour operators"* which are represented by a set of 3-dimensional matrices (analogs of Pauli matrices)
- Colour charges $I_3^C$ and $Y^C$ are eigenvalues of corresponding operators

In this formalism, colours can be quantified. Values of $I_3^C$ and $Y^C$ for the colour states of quarks and antiquarks are:

| **Quarks** | | | **Antiquarks** | | |
|---|---|---|---|---|---|
| | $I_3^C$ | $Y^C$ | | $I_3^C$ | $Y^C$ |
| **r ("red")** | 1/2 | 1/3 | $\bar{r}$ | -1/2 | -1/3 |
| **g ("green")** | -1/2 | 1/3 | $\bar{g}$ | 1/2 | -1/3 |
| **b ("blue")** | 0 | -2/3 | $\bar{b}$ | 0 | 2/3 |

- *Colour hypercharge* $Y^C$ and *colour isospin charge* $I_3^C$ are additive quantum numbers, having opposite sign for quark and antiquark

*Confinement condition* for the total colour charges of a hadron:

$$I_3^C = Y^C = 0 \tag{121}$$

The most general colour wavefunction for a baryon is a linear superposition of six possible combinations:

$$\begin{aligned}\chi_B^C = {} & \alpha_1 r_1 g_2 b_3 + \alpha_2 g_1 r_2 b_3 + \alpha_3 b_1 r_2 g_3 \\ & + \alpha_4 b_1 g_2 r_3 + \alpha_5 g_1 b_2 r_3 + \alpha_6 r_1 b_2 g_3\end{aligned} \tag{122}$$

where $\alpha_i$ are constants. It can be shown that the color confinement requires the totally antisymmetric combination:

$$\begin{aligned}\chi_B^C = {} & \frac{1}{\sqrt{6}}(r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 \\ & - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3)\end{aligned} \tag{123}$$

Colour confinement principle (121) implies certain requirements for states containing both quarks and antiquarks:

– consider an arbitrary combination $q^m\bar{q}^n$ of *m* quarks and *n* antiquarks, $m \geq n$

– for a particle with $\alpha$ quarks in r-state, $\beta$ quarks in g-state, $\gamma$ quarks in b-state ($\alpha+\beta+\gamma=m$) and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ antiquarks in corresponding antistates ($\bar{\alpha}+\bar{\beta}+\bar{\gamma}=n$), the colour wavefunction is

$$r^{\alpha} g^{\beta} b^{\gamma} \bar{r}^{\bar{\alpha}} \bar{g}^{\bar{\beta}} \bar{b}^{\bar{\gamma}} \qquad (124)$$

Adding up colour charges (from the table above) and applying the confinement requirement,

$$I_3^C = (\alpha - \bar{\alpha})/2 - (\beta - \bar{\beta})/2 = 0$$

$$Y^C = (\alpha - \bar{\alpha})/3 + (\beta - \bar{\beta})/3 - 2(\gamma - \bar{\gamma})/3 = 0$$

$$\Downarrow$$

$$\alpha - \bar{\alpha} = \beta - \bar{\beta} = \gamma - \bar{\gamma} \equiv p$$

Here $p$ is a non-negative integer $\Rightarrow$

- Numbers of quarks and antiquarks in a colorless state are related as: $m - n = 3p$

- The only combination $q^m\bar{q}^n$ allowed by the colour confinement principle is

$$(3q)^p(q\bar{q})^n , \qquad p,n \geq 0 \tag{125}$$

  - Form (125) forbids states with fractional electric charges
  - However, it allows exotic combinations like $qq\overline{qq}$, $qqqq\bar{q}$ (like e.g. the pentaquark $\Theta^+ = uudd\bar{s}$)

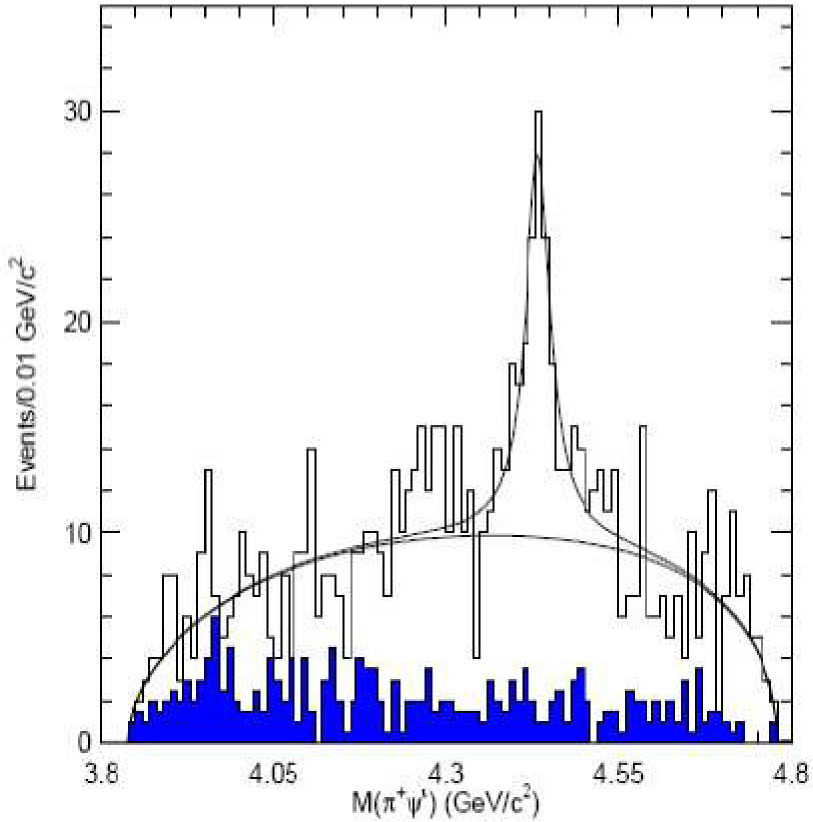


Figure 111: A tetraquark $Z^{+}(4330)$ candidate ($u\bar{d}c\bar{c}$) as published by the BELLE experiment in 2008