FYST17 Exercises: QGP and heavy-ion physics

In order to pass one needs to solve at least 3 of the 4 exercises. To obtain the best grade one has to solve all 4 exercises.
Please note on the exercise if you worked together with anyone in solving it. The maximum group size is 4 people.
Each student always has to submit an individually formulated solution to the exercises.

1.1 Kinematics of heavy-ion collisions

a) Show that if the maximum beam energy for pp collisions is $\sqrt{s_{\rm pp}}$, the maximal Pb-Pb beam energy is $\sqrt{s_{\rm Pb-Pb}} = Z\sqrt{s_{\rm pp}}$ (so that the beam energy per nucleon pair is $\sqrt{s_{\rm NN}} = \frac{Z}{A}\sqrt{s_{\rm pp}}$).

b) Consider each nuclei as a disk with radius $r_A = 1.2 \times A^{1/3}$ fm and assume that the two nuclei interact if the disks overlap, i.e., when the impact parameter (b) is less than $2r_A$. What range of impact parameters does the most central 0-5% Pb-Pb collisions correspond to?
(0-5% centrality is the 5% fraction of the total cross section with smallest impact parameters)
Hint: use simple geometry to express the differential cross section $d\sigma/db$ and the total cross section

1.2 Azimuthal anisotropic flow

We assume in the following that the particles in an event is distributed according to:

$$f(\varphi) = \frac{1}{2\pi}\left(1 + \sum_{i=1}^{\infty} 2v_i \cos[i(\varphi - \Psi_i)]\right), \tag{1}$$

where φ is in the range from 0–2π, v_i is the i-th order flow coefficient and Ψ_i is called the i-th order symmetry plane (really an angle in general). We only consider measurements of the elliptic flow, v_2, in the following, so that:

$$f(\varphi) = \frac{1}{2\pi}\left(1 + 2v_2 \cos[2(\varphi - \Psi_2)]\right). \tag{2}$$

a) Make a drawing that illustrates the initial elliptic overlap region, indicating Ψ_2.

b) The event plane method. Show by explicit calculation that Ψ_2 can be estimated as:

$$\Psi_2 = \frac{1}{2}\tan^{-1}\left(\frac{\langle\sin(2\varphi)\rangle}{\langle\cos(2\varphi)\rangle}\right), \tag{3}$$

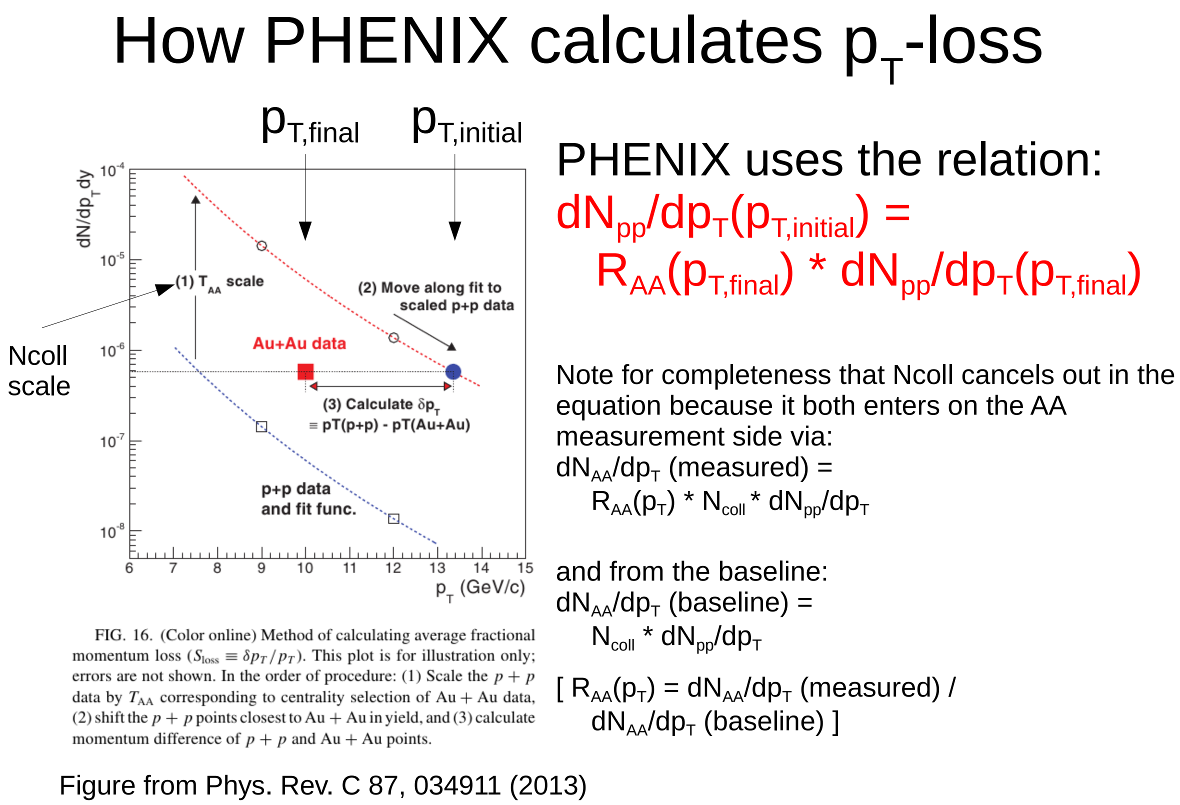
where the brackets indicate averages.
Hint: calculate the means using trigonometric identities.

c) Two particle correlations. The Event Plane method is criticized for the need to first determine the event plane and then measure v_2 since the statistical precision with which we can determine the event plane event-by-event will affect the result (even one typically corrects for this using a resolution function). One can avoid this by studying 2-particle correlations [1]. Show that

$$\langle \cos[2(\varphi_1 - \varphi_2)] \rangle = v_2^2, \tag{4}$$

where the brackets indicate an average over all pairs, and the assumption is that there are no direct correlations between particle 1 and 2, but only indirect correlations through the common event plane Ψ_2.

1.3 Jet quenching



a) The PHENIX experiment at RHIC has developed a simple method for extracting the energy loss from the R_{AA}, see above Figure. If the initial spectrum is given by: $dN/dp_{\mathrm{T}} = a \times p_{\mathrm{T}}^b$ then they assume that the p_{T} is reduced, due to the jet quenching, by a factor $k = 1 - \Delta p_{\mathrm{T}}/p_{\mathrm{T}}$. The R_{AA} is then the ratio between the spectrum where the p_{T} is shifted down due to energy loss and the initial p_{t}-spectrum. Derive a relation between R_{AA}, b, and k.

b) At both RHIC and LHC the R_{AA} is $\approx$0.20 at $p_{\mathrm{T}} = 10$ GeV/c for the most central collisions, but the exponents of the pp spectra are $b = -7.1$ (RHIC) and $b = -5.5$ (LHC). What is the difference in energy loss?

[1] The biggest gain is when we go to higher order correlations, but that is beyond the scope here, see e.g. Phys. Rev. C 83 (2011) 044913.

1.4 The medium temperature

a) The charged particle multiplicity increases by a factor ≈ 2 from RHIC to LHC. Assuming this is true also for the energy density what is the estimated temperature increase of the medium from RHIC to LHC?
Hint: what is the expected relation between energy density and temperature.

b) Try to give an argument for why the increase is so small and reflect on what the gain would be for heavy-ion physics by increasing the beam energy by another factor 10.