Introduction to Phenomenology and Experiment of Particle Physics (PEPP)

Cycle 1

Matrix element description of hard processes

Automatic matrix element generation

One would think that simulating a given process at a particle collider should be quite straight forward. After all we know the Standard Model Lagrangian, so for any set of incoming and outgoing particles we should be able to write down all Feynman diagrams, construct the corresponding amplitudes, sum them and square them to get a cross section. Then we can simply sample the phase space for the particles and weight the events with the cross section. There is really no reason why this cannot be done completely automatically in a computer program.

OK, we can't simulate full events since the Standard Model Lagrangian describes quarks and gluons, while at experiments we only see hadrons. And, of course, it is difficult to include loop diagram, and the tree-level diagrams diverges for situations where we eg. have soft or collinear gluons. But still. Wouldn't it be nice to have such a program.

Literature

Most of this course is covered by

> R.K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press (1996).

and for this cycle the most relevant chapters are 1 and 3. Additional input can be found in lecture notes from various summer schools, eg.

> T. Sjöstrand, *Monte Carlo Generators*, `hep-ph/0611247`.

Also the manual for the Madgraph and CompHEP program contains useful input, especially relevant for completing the exercises:

> The MadGraph program: http://madgraph.hep.uiuc.edu/
> The CompHep program: http://comphep.sinp.msu.ru/

Goals

- Order-of-magnitude estimates of cross sections
- Simple $2 \to 1$ and $2 \to 2$ processes with Mandelstam variables
- n-body phase space and convenient transformations thereof
- the basic QCD and electroweak processes

Comments to exercises

This cycle was planned to extend over two weeks, which is why the amount of exercises is larger than for the following cycles.

Exercises (first week)

1. Assume a resonance of mass M at rest. It decays isotropically to two massless particles. Calculate the $p_\perp$ spectrum of these particles and plot it schematically. (The answer is the famous Jacobian peak, once used to discover the W through the channel $W \to e\nu$.)

2. Study the three-body phase space in a process such as $Z^0 \to q(1)\bar{q}(2)g(3)$, with vanishing quark (and gluon) masses.
   a) Introduce the energy fractions $x_i = 2E_i/E_{\mathrm{cm}}$ in the Z^0 rest frame and show that they are related to the Lorentz invariants $y_{jk} = m_{jk}^2/s$, where $s = E_{\mathrm{cm}}^2$, by $x_i = 1 - y_{jk}$, i, j, k cyclically permuted.
   b) Show that the three-body phase space can be written as
   $$\mathrm{d}(LIPS) \propto s\,\mathrm{d}(cos\theta_1)\,\mathrm{d}\varphi_1\,\mathrm{d}\varphi_{12}\,\mathrm{d}x_1\,\mathrm{d}x_2 \tag{1}$$
   where θ_1, φ_1 give the direction of the quark and φ_{12} the azimuthal angle of the anti-quark around the quark direction.

3. a) Show that the one-body phase-space expression
   $$\frac{\mathrm{d}^3p}{E} = \mathrm{d}^2p_\perp\,\mathrm{d}y \tag{2}$$
   where $p_\perp$ is counted transverse to the rapidity axis.
   b) The nice properties of rapidity under longitudinal boosts leads to the multiplicity distribution $\mathrm{d}n/\mathrm{d}y$ being roughly constant in the central region around $y = 0$, and also to the transverse-momentum spectrum $\mathrm{d}n/\mathrm{d}p_\perp$ being almost the same for different central rapidity slices. What can one then say qualitatively about the $\mathrm{d}n/\mathrm{d}\eta$ spectrum around $\eta = 0$?
   Hint: use that $\mathrm{d}y/\mathrm{d}\eta = (\mathrm{d}y/\mathrm{d}p_z)/(\mathrm{d}\eta/\mathrm{d}p_z)$.

4. Show that, in a $2 \to 2$ process with massive incoming and outgoing particles,
   $s + t + u = \sum m_i^2$.

5. Study a $2 \to 2$ process with massive outgoing particles, but massless incoming ones. Calculate the kinematically allowed t range, and show that
   $$p_\perp^2 = \frac{tu - m_3^2 m_4^2}{s} \tag{3}$$

6. Assume two massless particles characterized by their $E_\perp$, η and φ values.
a) Show that a four-vector representation is

$$p = (E_\perp \cosh\eta; E_\perp \cos\varphi, E_\perp \sin\varphi, E_\perp \sinh\eta) \tag{4}$$

b) For fixed $E_{\perp 1}$ and $E_{\perp 2}$ values and small angular separation, show that their invariant mass only depends on $R = \sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2}$ rather than on $\Delta\eta$ and $\Delta\varphi$ separately.

7. The process $e^+e^- \to \gamma\gamma$ has the cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \alpha_{\mathrm{em}}^2 \frac{\pi}{s^2} \frac{t^2 + u^2}{tu} \; . \tag{5}$$

(Since the final-state particles are identical, one may include a further factor of 2 in the $\mathrm{d}\sigma/\mathrm{d}t$ expression but, in order not to double-count, one would then only integrate over half the normal phase space.)
a) Enumerate which Feynman graphs contribute and show that a rule-of-thumb estimate is consistent with the above expression.
b) Estimate, by suitable analytic approximations, the cross section at LEP2, $\sqrt{s} \approx 200$ GeV, for transverse momenta $p_\perp > 1$ GeV.

8. The three most important QCD $2 \to 2$ processes $qq' \to qq'$ $(= q\overline{q}' \to q\overline{q}')$, $qg \to qg$ and $gg \to gg$ are all dominated by gluon exchange. The cross sections

$$\begin{aligned}
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}(qq' \to qq') &= \frac{\pi\alpha_{\mathrm{s}}^2}{\hat{s}^2} \frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \\
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}(qg \to qg) &= \frac{\pi\alpha_{\mathrm{s}}^2}{\hat{s}^2} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - \frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} \right) \\
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}(gg \to gg) &= \frac{\pi\alpha_{\mathrm{s}}^2}{\hat{s}^2} \frac{9}{4} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right)
\end{aligned}$$

are thus approximately proportional. (A factor 1/2 for identical final particles has already been included in the last expression, so don't worry about such aspects.)
Find a 'structure function', i.e. a linear combination of parton distributions that it is therefore possible to access in $p\overline{p}/pp$ collisions.

Exercises (second week)

1. One of the search channels for a Higgs will be $gg \to h^0 \to \gamma\gamma$. This channel has a background coming e.g. from $q\overline{q} \to \gamma\gamma$.
   a) Use MadGraph to compare these two cross sections, e.g. for a 120 GeV h^0 and assuming a 2 GeV experimental mass window for the background. Each of the photons is expected to have a $p_\perp > 30$ GeV.
   b) Higher-order corrections implies that additional jets may be produced. Assume you allow one extra gluon jet in the final state, with $p_\perp > 40$ GeV. Can such a requirement improve your signal-to-background ratio? At what price in terms of number of events, e.g. for one year of running as 10^{34} cm^{-2}s^{-1} luminosity? Note that "our" photons will have to be isolated from the jet(s) not to be confused with photons e.g. from π^0 decays inside the jets.

2. Often quark masses can be neglected in jet cross sections. However, if one is studying $b\overline{b}$ or $t\overline{t}$ production at small $p_\perp$ this is not the case. Therefore find the analytical expression $\mathrm{d}\hat{\sigma}/\mathrm{d}\hat{t}$ for $q\overline{q} \to Q\overline{Q}$ and/or $gg \to Q\overline{Q}$, $Q = b$ or t. Combine with phase space to say something about the shape of $\mathrm{d}\hat{\sigma}/\mathrm{d}p_\perp^2$ at small $p_\perp$.

3. Pick a Supersymmetry scenario and study the cross section for a process such as $qg \to \tilde{q}\tilde{g}$, e.g. plotted as a function of $p_\perp$. Compare that with the $p_\perp$ spectrum of the standard-model one $qg \to qg$. How much different are they? How much of this is phase space and how much the processes themselves. (Let the sparticle masses become small!) Is there some variable that better than $p_\perp$ shows the similarities also for non-negligible sparticle masses?

4. MadGraph can be used to produce a Les Houches Event File of parton-level processes that can be generated in full with Pythia. Use this to find the expected charged-multiplicity distributions for LHC events of the kind $q\overline{q} \to Z^0 b\overline{b}$, given some reasonable cuts on the process. (This process is a background to $q\overline{q} \to Z^0 h^0$ with $h^0 \to b\overline{b}$.)