Quantum Mechanics

(non-relativistic)

- basics
- operators
- momentum eigenstates
- energy eigenstates

Some basic notions

- *object*: what we study (particle, atom, ...)
- *observable*: a measurable property of the object (E, p, ...)
- *parameter t*: this 'time' is not a property of the object
- *state* of an object: the values of its observables
- *wavefunction*: the mathematical expression of a state, e.g.

$$\psi(x) = \sin(x)$$

We use the wavefunctions to find the state of an object , i.e. to find the value of its observables. How is this done?

Operators ('type 1')

an operator is something that can be applied to
a mathematical function and give a mathematical function

e.g. $\boldsymbol{A} = \dfrac{d^2}{dx^2}$ is an operator: $\boldsymbol{A}\psi = (-1)(k \cdot \sin x)$

Postulate of Quantum Mechanics:
observables are described by operators

<u>Operators ('type 2')</u>

an operator can perform a transformation
(in space or in time or both)

<u>example</u>: the transformation

$$(x, y, z) \rightarrow (-x, -y, -z)$$

is performed by the operator ***P*** (definition):

$$\boldsymbol{P}\psi(x, y, z) = \psi(-x, -y, -z)$$

the operator ***P*** is called the 'parity operator'

Eigenstates and eigenvalues

Assume an operator $\boldsymbol{Q}$ of 'type 1' or 'type 2'.
If $\boldsymbol{Q}\psi = q\psi$, where q=constant, we say that
ψ is eigenstate of the operator $\boldsymbol{Q}$ with eigenvalue q

- each eigenstate has *only one* eigenvalue
- *degenerate* eigenvalues (common eigenvalues)
- *spectrum* of an operator: the set of its eigenvalues
- *Hermitian* operator: has only real eigenvalues

We use Hermitian operators to express observables

Example: parity eigenvalues

We apply the parity operator (transformation) twice

$$(x, y, z) \rightarrow (-x, -y, -z) \rightarrow (x, y, z)$$

i.e. $\boldsymbol{P}(\boldsymbol{P}\psi) = \psi$

If $\boldsymbol{P}\psi = \lambda\psi$, then

$$\boldsymbol{P}(\boldsymbol{P}\psi) = \boldsymbol{P}(\lambda\psi) = \lambda(\boldsymbol{P}\psi) = \lambda(\lambda\psi) = \lambda^2\psi = \psi$$

i.e. $\lambda = \pm 1$ = the parity of the state ψ

($\lambda = 1$: even parity, $\lambda = -1$: odd parity)

Measurements are eigenvalues

Assume an observable A, represented by the operator $\boldsymbol{A}$.
We measure A in a state ψ. What will we get?

case 1: ψ is eigenstate of $\boldsymbol{A}$ with eigenvalue a.
result: all measurements are equal to a.

case 2: ψ is not an eigenstate of $\boldsymbol{A}$.
result: a measurement will be any eigenvalue of $\boldsymbol{A}$ but we *cannot* predict which one.

(We never use the same state to measure twice)

Probability interpretation

wavefunctions are normalized to unity:

$$\int |\psi(x)|^2 dx = 1$$

this allows us to say that

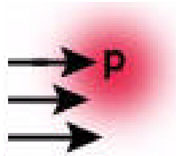
$$P = |\psi(x)|^2 dx$$

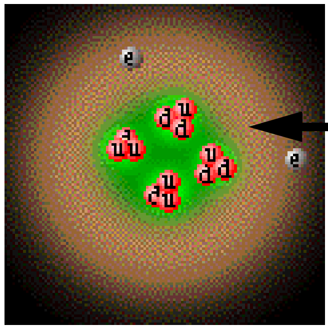
is the probability to find the object at (x,x+dx)

Indeterminacy in the position of an object

it actually means that objects can be found
where we don't expect them to be

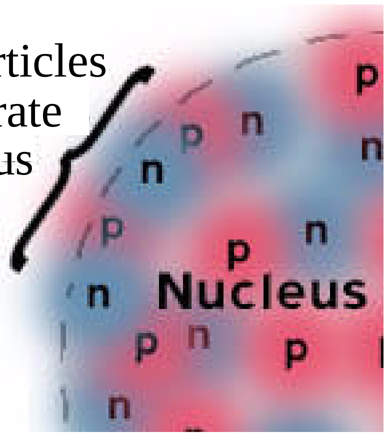
the proton is most likely here
or it could be here
or even here





the electrons are
here someplace

these four particles
might migrate
outside the nucleus



Example: a particle in a box