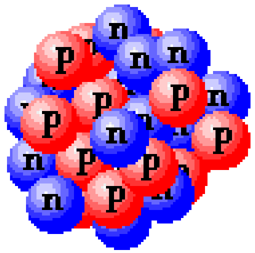
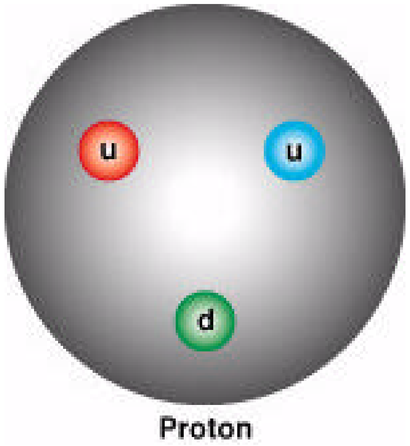
The nucleus



- Notation
- Forces within the nucleus
- Binding energy of nuclei
- The quantum mechanical system
- Angular momentum
- Parity
- Multipoles
- Radioactivity
- alpha, beta decays
- isomeric transitions (gamma decays)

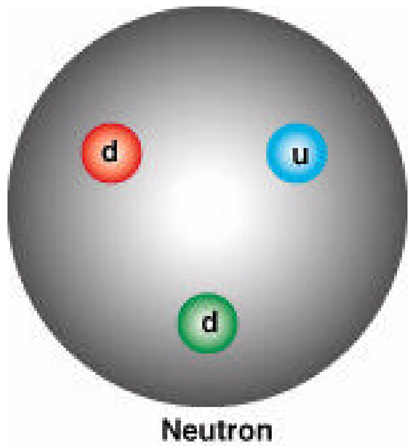
Notation and terminology



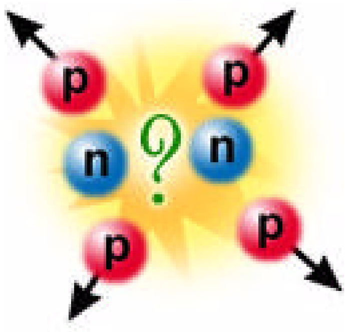
- nucleus = Z protons + N neutrons = A nucleons
- nuclide = nuclear species = same Z and N
- isotopes = nuclides with same Z, different N

- isotope notation: ${}^{A}_{Z}X_{N}$ or ${}^{A}X$

- isobars = nuclides with the same A
- isotones = nuclides with the same N



Forces in the nucleus



- strong interaction: attractive nuclear force between nucleons
- similarity between p and n
- size of nuclei ~ 1 fm

- electromagnetic interaction: repulsion between protons

- no analytical expression for nuclear forces: models

Binding energy

B = (mass of nucleons) - (mass of nucleus)

e.g. $m_n + m_p - m_D = B > 0$

the potential energy of the nucleus is negative

$$m_n + m_p + (-|E_{int}|) = m_D \quad \text{so} \quad B = |E_{int}|$$

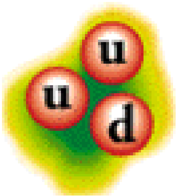
the binding energy per nucleon B/A varies with A and is in the interval 5 MeV - 10 MeV (Krane)

- A>62: B/A decreases with A : fission
- A<62: B/A increases with A : fusion

The nucleus as a quantum mechanical system

- nuclear states: energy, L, S, P eigenstates
- nuclear energies: constant , quantized
- nuclei have a ground state (possibly excited states)
- transitions between states:
    - alpha particles (alpha decay)
    - electrons or positrons (beta decay)
    - photons (gamma- or X-rays)

Angular momentum and parity (J^+ or J^-)



- protons and neutrons have spin s=1/2
- protons and neutrons move inside the nucleus and have orbital angular momentum *l*

How do we add the *l*'s and *s*'s of the nucleons?

$J = \Sigma\, j_{\mathrm{nucleon}}$ 'angular momentum' or 'spin'

parity of the nucleus: $\lambda = (-1)^l$

where *l* is the total orbital angular momentum of the nucleus

Nuclear e/m moments (Krane §3.5)

Electric multipole moments:

- 0^{th} or monopole moment: the electric field varies as r^{-2} (L=0)
- 1^{st} or dipole moment: the electric field varies as r^{-3} (L=1)
- 2^{nd} or quadrupole moment: the electric field varies as r^{-4} (L=2), etc.

where L is the order of the moment

We have the same for magnetic multipole moments except that there is no monopole moment

Parity of electric moments = $(-1)^L$

Parity of magnetic moments = $(-1)^{L+1}$

Radioactivity

'spontaneous transition from one nuclear state to another'
During the transition: quantum numbers may change
In alpha and beta decays: both Z and N change

- exponential decay law: $N = N_0 e^{-\lambda t}$
- activity of the sample: number of decays per unit time
- unit for activity: 1 becquerel, Bq = 1 decay per second
- decay constant, λ: probability for decay per time unit
- half-time: $t_{1/2} = \ln 2/\lambda$
- lifetime: $\tau = 1/\lambda$

Conservation rules in nuclear decays

- Baryon and lepton number always conserved
- Parity of the nucleus may be violated in a weak decay
- Total energy, total angular momentum (orbital+spin), total linear momentum are always conserved

Alpha decay

$$^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + \alpha$$

Q value of the decay:

$$Q = m(^{A}X) - m(^{A-4}Y) - m(^{4}He)$$ (atomic masses)

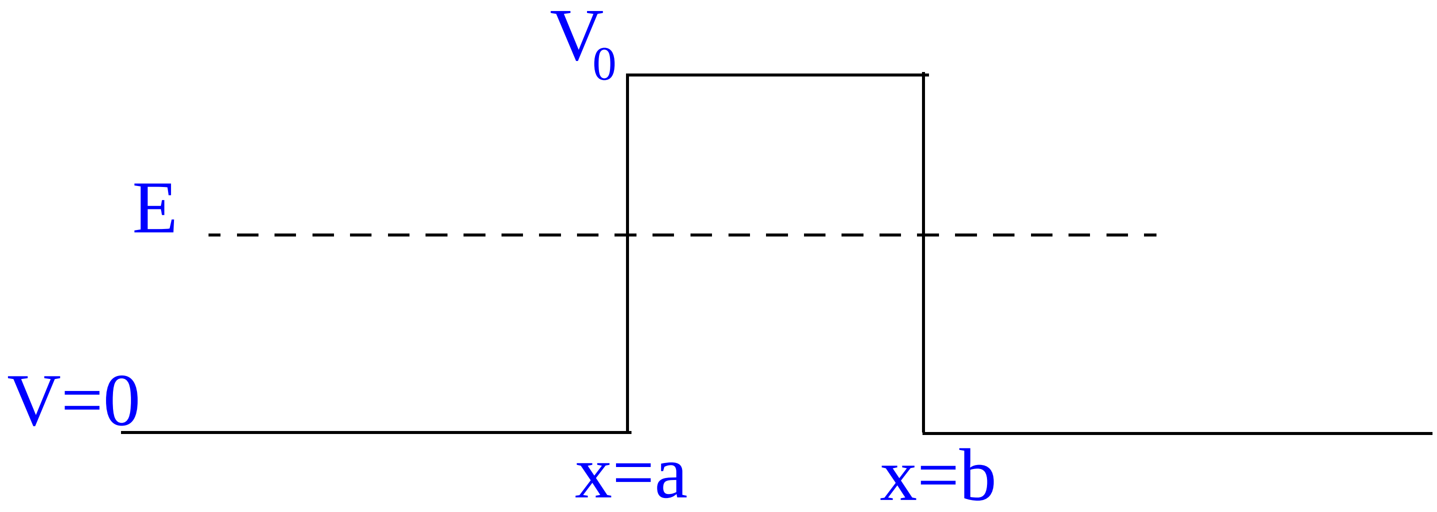
In the frame of the decayed nucleus:

$$Q = K_{Y} + K_{\alpha}$$

The kinetic energy of the alpha (a few MeV):

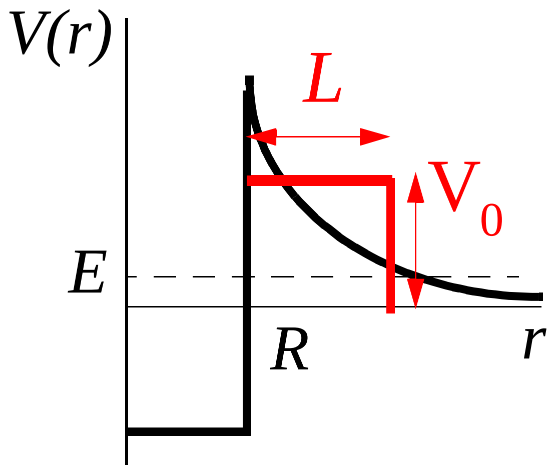
$$K_{\alpha} \cong \frac{A-4}{A} Q$$

The tunnel effect



A particle with E<V is coming from the left.
What happens when the particle reaches x=a?

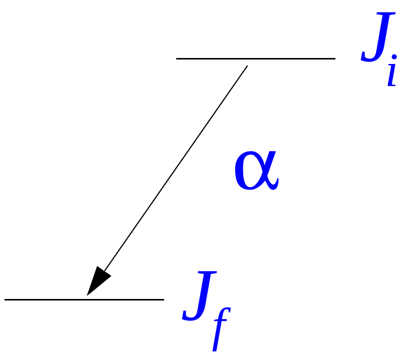
The tunnel effect in a-decay



probability for α-decay:

$$\lambda = \frac{v}{2R} e^{-2kL}, \; k = \sqrt{2m(2\pi/h)^2(V_0 - E)}$$

Alpha decay: selection rules (Krane §8.5)



α: l between $| J_i - J_f |$ and $J_i + J_f$

the wavefunction of the α is Y_{lm}
so the parity of the α is $(-1)^l$

Parity is conserved in alpha decay

The following transitions are allowed:

- the initial and final state have same parity and l is even
- the initial and final state have different parity and l is odd

Beta decay: electron emission

$${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}Y_{N-1} + e^{-} + \bar{\nu}_{e}$$

by neutron decay $n \rightarrow p + e^{-} + \bar{\nu}_{e}$

Q value:

$Q = m({}^{A}X) - m({}^{A}Y)$ (atomic masses)

neglecting the recoil energy of the proton, we have

$Q = K_{\bar{\nu}} + K_{e-}$ (e and ν relativistic)

The kinetic energy of the electron has a continuous spectrum with maximum value $(K_{e-})_{max} \approx Q$

Beta decay: positron emission

$$^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}Y_{N+1} + e^{+} + \nu_{e}$$

by the transformation $p \rightarrow n + e^{+} + \nu_{e}$
(this is not a proton decay!)

Q value:

$$Q = m(^{A}X) - m(^{A}Y) - 2m_{e}$$ (atomic masses)

As in electron emission, we have $Q = K_{\nu} + K_{e+}$ and the kinetic energy of the positron is a continuous distribution with maximum value $(K_{e+})_{max} \approx Q$

Beta decay: selection rules (Krane §9.4)

We consider the e and ν produced at r=0 so that l=0

- Fermi decay (e and ν spins are antiparallel, S=0):

$$\Delta J_{nucleus}=0$$

- Gamow-Teller decay (e and ν spins are parallel, S=1):

$$\Delta J_{nucleus}=0,\ 1\ (\text{except for } J_{initial}=J_{final}=0)$$

Since l=0 for the electron-neutrino system, we must have

$$\Delta P_{nucleus}=0$$

'First-forbidden decays': $\Delta J_{nucleus}=0, 1, 2$ and $P_{nucleus}$ changes

Electron capture

$${}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}Y_{N+1} + \nu_e \text{ with } Q = m({}^{A}X) - m({}^{A}Y)$$

by the process

$$p + e^{-} \rightarrow n + \nu_e$$

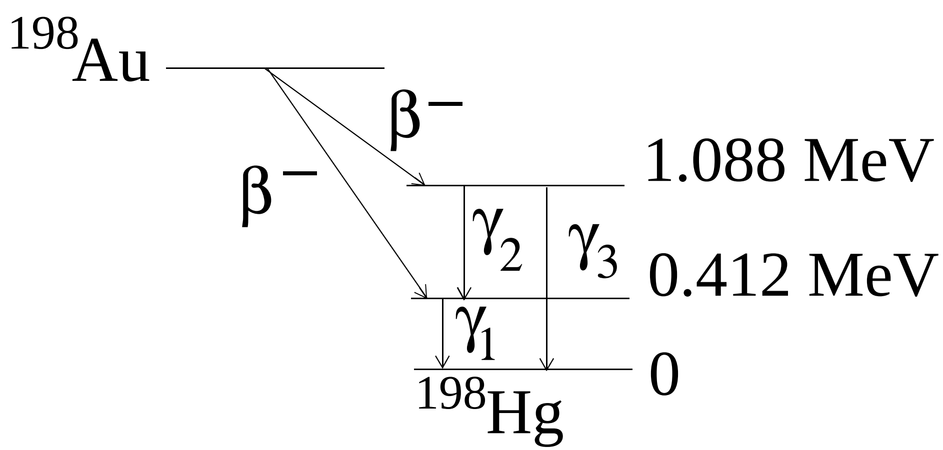
where the electron is an *atomic* electron from a shell with low principal quantum number *n*

For *n*=1, the electron comes from the K shell and the above process is called ‘K-shell capture’, etc.

Isomeric transitions

Transitions from an excited nuclear state to a state of lower energy (by photon emission)

Such transition can occur after an α- or β-decay, e.g.



Isomeric transitions: selection rules

A photon in a multipole carries L(h/2π)
so, in a transition L is between $| J_i - J_f |$ and $J_i + J_f$

The total parity is conserved

- If the parity of the nucleus changes, the radiation field must have odd parity, e.g. E1, E3,..., or M2, M4,...
- If the parity of the nucleus does not change, the radiation field must have even parity, e.g. E2, E4,..., or M1, M3,...