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Parity violation in muon decays and the muon lifetime

Version 4

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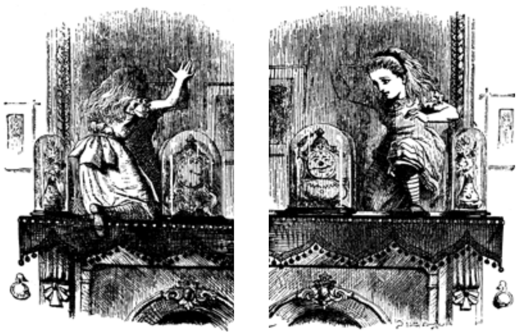

Abstract

An experiment detecting the decay of cosmic muons has been installed for the third-year students at the University of Lund. The setup of the experiment makes it possible to detect the parity violation in the decay process and to measure the lifetime of the muon.

PART I : Lab manual

1 Introduction

In this lab, we detect cosmic muons and their charged decay products, positrons. The aim of the lab is :

1. to measure the muon lifetime, i.e. the average duration of the life of a muon in its rest-frame,
2. confirm that there is forward-backward asymmetry in the emission of the positrons with respect to the muon spin direction (and therefore parity violation in the decay).

2 Right-handed and left-handed particles

In our experiment, we are interested in antimuons, positrons, neutrinos and antineutrinos. All these (anti)particles have spin[1] 1/2. If we consider massless (anti)particles in weak interactions, only left-handed particles and right-handed antiparticles interact with the W. Neutrinos and antineutrinos have a very tiny mass but in the Standard Model they are considered to be massless. Therefore we can say that the neutrinos in our experiment are always left-handed and the antineutrinos are always right-handed. Electrons and positrons are heavier than the neutrinos, but still their mass is small. Therefore, we can say that electrons are almost always left-handed and positrons are almost always right-handed. Negative muons (particles) and positive muons (antiparticles) are rather heavy, so they both have rather large probabilities to be either left-handed or right-handed (see also pp. 16 and 18 of the book [1]).

The forward-backward asymmetry in the muon decay is due to these properties.

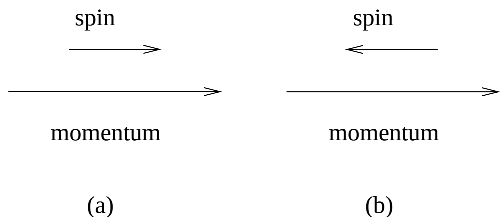

Figure 1: Spin and momentum directions for (a) a right-handed particle and (b) a left-handed particle.

3 Cosmic muons and their decay

Cosmic rays (mainly protons) are particles produced in the stars of our galaxy. When cosmic rays enter the Earth's atmosphere they produce many new particles. Most of the particles that reach the ground are muons, μ^-, and antimuons, μ^+. These are called 'cosmic muons' because of their origin. In this exercise, we are interested in positive muons (antimuons). Antimuons decay as follows:

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

Most of the antimuons decay in the atmosphere but some have high enough energies to reach our detector. Depending on their energy, they will either go through it or stop in the aluminium plate, which is located in the middle of the detector (fig. 10), where they will eventually decay.

[1] The spin is the intrinsic angular momentum of the particle.

In this exercise, we study the decay of the antimuons that stop in our detector. Most of them have a spin which points upwards, as shown in fig. 2. We divide the space around the antimuon in two regions: the forward region is the region where the antimuon spin is pointing to; the backward region is in the opposite direction.

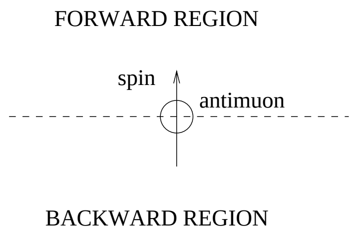

Figure 2: Antimuon at rest inside the detector. The forward and backward regions are defined with respect to the antimuon spin direction. The dashed line shows a plane vertical to the spin vector.

The antimuon will decay to a positron, a neutrino and an antineutrino. The momentum directions of these particles are only constrained by momentum conservation and the angles between their momenta can have various values. To make our study simpler, we have setup the detector in such a way that it 'sees' only the most energetic positrons. In these cases, the neutrino and antineutrino are emitted almost parallel to each other and the positron is emitted in the opposite direction. If we consider the spin of the antimuon as well, this leads to two possibilities (fig. 3): first, the positron is emitted in the forward region and second, the positron is emitted in the backward region.

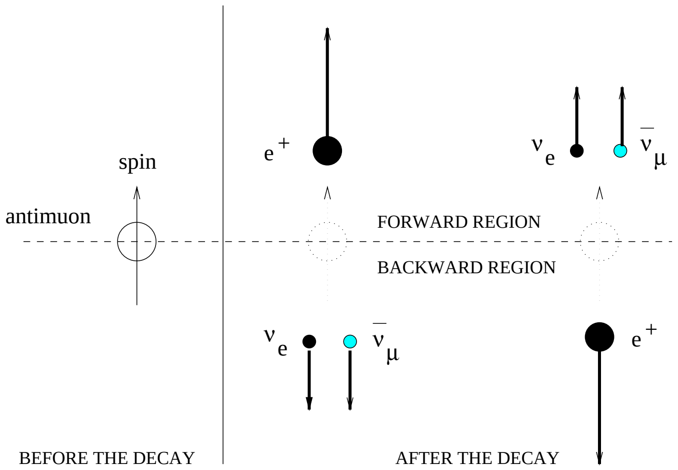

Figure 3: The two possibilities for the antimuon decay. Thick arrows denote momenta.

In fig. 3, the momenta have been drawn vertically but any other angle is possible; what is of importance is in <u>which region</u> (forward or backward) the positron is emitted and not its angle of emission.

The question now arises of whether the positron prefers to be emitted in the forward or in the backward region. We can answer this question by considering the properties discussed in section 2 and the conservation of angular momentum before and after the decay. The angular momentum of the antimuon is the sum of its spin and its orbital angular momentum. The orbital angular momentum of the antimuon is zero because the antimuon is at rest. Therefore the total angular momentum before the decay is equal to the spin on the antimuon. It can be shown that the orbital angular momentum of the final state (positron, neutrino and antineutrino) is also zero. Hence, the sum of the spins of the positron, neutrino and antineutrino must be equal to the spin of the antimuon. All these particles have spins with equal magnitudes, $(h/(2\pi))\sqrt{S(S+1)}$, where h is the Planck constant and S is the spin quantum number (1/2).

As the neutrino and the antineutrino are the lightest particles in the final state, we will start by assigning their spins first. This must be done so that the neutrino is always left-handed and the antineutrino is always right-handed (section 2). This gives us total spin zero for the neutrino and the antineutrino. By angular momentum conservation, it follows that the spin of the positron must be equal to the spin of the antimuon, i.e. the spin of the positron must be pointing upwards (fig. 4).

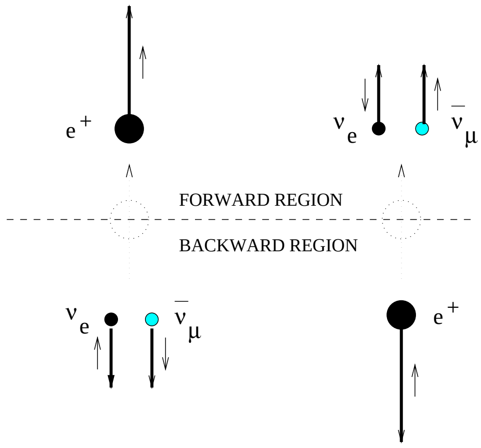

Figure 4: Spin directions for the final state particles. Thick arrows show momenta and thin arrows show spin vectors.

We see that angular momentum conservation and (anti)neutrino spin orientation leaves two choices: (a) either a right-handed positron in the forward region or (b) a left-handed positron in the backward region. In section 2, we mentioned that positrons participating in weak decays are almost always right-handed. This means that choice (a) is favoured and therefore the positrons will be emitted almost always in the forward region. This means **forward-backward asymmetry in the positron emission**. (Forward-backward symmetry would mean equal number of positrons emitted in the forward and in the backward region.)

Questions:

1. Which particles do we study in this experiment? Are they all fundamental? In which family do they belong (see p. 1 of the book [1])? What are their masses (see p. 59 of the book)? Which of them can interact stongly, electromagnetically, weakly?
2. What are the spin quantum numbers of the particles we are studying? What is spin? What is the magnitude of the spin vector for a particle with spin (quantum number) 1/2? Can you draw a particle and its spin? Show how the particle rotates if the spin points upwards or downwards. Can the spin help you out when you want to put two positrons in the (otherwise) same state (see p. 1 of the book)?
3. Is the decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ a (a) strong, (b) electromagnetic, (c) weak decay? Why? Knowing this, what range do you expect the muon lifetime to be in (see p. 3 and ex. 3 on p. 21 of the book)?
4. Check that the decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ is allowed. Examine the points 1, 3, 6 on p. 4 of the book. Comment on point 8.
5. Make a diagram for the decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ as on p. 6 of the book.
6. Which particles are right-handed and which are left-handed? What does that mean? Refer also to the book, pp. 16, 18 to explain why a muon is more likely to be right-handed than an electron (hint: coupling strength to the Higgs medium).
7. Explain how fig. 4 is obtained.
8. What does it mean to have 'forward-backward asymmetry in the positron emission'? Why does that happen?

Discussion subjects for the first laboration (refer to Part II for help):

1. What are cosmic rays? How are the antimuons we study produced (see section D1)?
2. Discuss the positive pion decay (see section D4). What kind of decay is it? Is it allowed? Draw a diagram (see chapter 2 and p. 13 of the book).
3. Study the questions in section D4.
4. The antimuons in fig. 19 (b) are left-handed. Is that allowed for an antiparticle?
5. What is the lifetime of a particle (see section D2)?

4 The positron angular distribution

To measure the phenomenon of forward-backward asymmetry in our experiment, we need to phrase it in terms of mathematical formulae. Let us define the positron *emission angle* θ as shown in fig. 5: θ is the angle between the antimuon spin and the positron momentum vectors. In the forward region, we have $0 \leq \theta < 90^{o}$. In the backward region, we have $90^{o} < \theta \leq 180^{o}$.

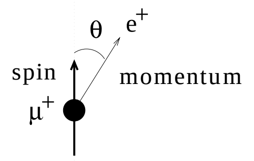

Figure 5: The emission angle of the positron θ is defined as the angle between the antimuon spin vector and the positron momentum vector.

Our detector can only see charged particles, so we will not discuss the neutrino and antineutrino of the final state. We will concentrate on the angular distribution of the positrons, i.e. on the number of positrons, $N(\theta)$, as a function of their emission angle, θ.Detailed calculations give the angular distribution

$$N(\theta) \propto 1 + \alpha' \cos\theta \qquad (1)$$

where α' is a constant. Eq. (1) is plotted in fig. 6. It clearly shows a forward-backward asymmetric positron emission.

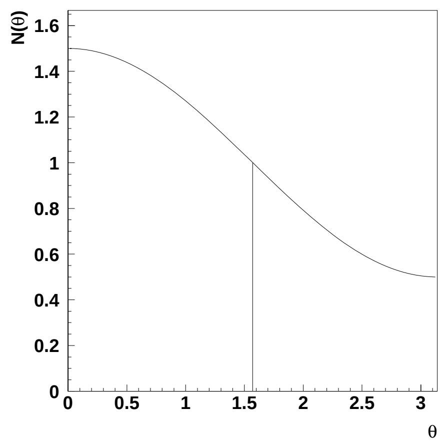

Figure 6: Forward-backward asymmetric angular distribution for the positrons (eq. (1)). The vertical axis measures the number of positrons with a specific value of emission angle. The area between the curve and the horizontal axis shows the number of positrons emitted in (a) the forward region (on the left of the vertical line) and (b) the backward region (on the right of the vertical line). The vertical line is plotted at emission angle equal to 90°. (The emission angle in the plot is in radians.)

A forward-backward symmetric positron emission would have a distribution of the form

$$n(\theta) \propto 1 + \eta \cos^2\theta \qquad (2)$$

where η is a constant. Eq. (2) is plotted in fig. 7.

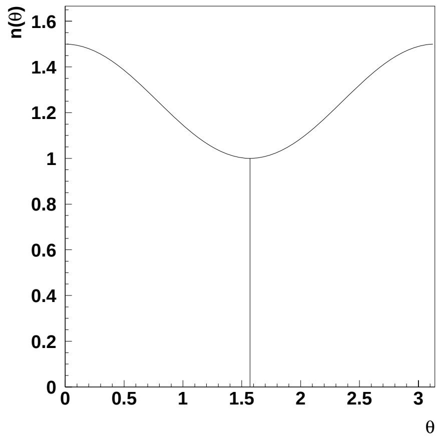

Figure 7: Forward-backward symmetric angular distribution for the positrons (eq. (2)). The areas between the curve and the horizontal axis (a) on the left of the vertical line and (b) on the right of the vertical line are equal, i.e. the same number of positrons is emitted in (a) the forward and (b) the backward region.

What we would need for our experiment is a detector that can measure the number of positrons as a function of the emission angle and examine whether the measured distribution agrees with eq. (1) (asymmetry) or with eq. (2) (symmetry). As such a detector would be very expensive to built, we resort to the following simpler solution: we take a detector that can tell us whether the positron was emitted upwards (in the forward region) or downwards (in the backward region). Then we force the antimuon spin direction to rotate around the horizontal axis[2], i.e. to move from the forward to the backward region and vice versa. Then we measure the positron distributions up and down as a function of time (instead of as a function of their emission angle). If the two distributions are the same (as a function of time), we have forward-backward symmetry (up-down symmetry) (fig. 8). If the two distributions are different, we have forward-backward asymmetry (up-down asymmetry) (fig. 9). The simplest way to see whether the distributions are the same is to divide them and check whether their ratio can be described by a constant function of time (the ratio will not be equal to unity because the detector is not perfect).

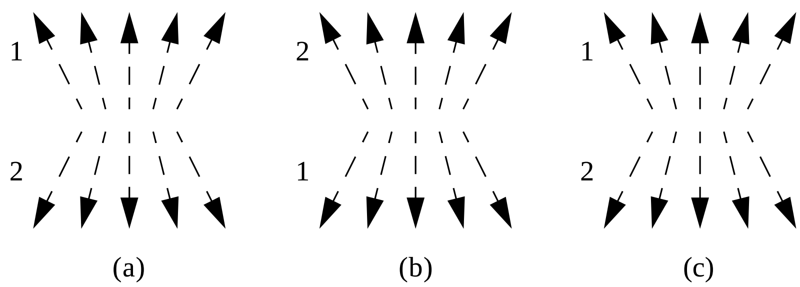

Figure 8: Example snapshots of a forward-backward (up-down) symmetric positron emission. (a) at t=0 (when the antimuon stops in the aluminium plate), (b) after half a turn of the antimuon spin, (c) after one turn of the antimuon spin. Side 1 and side 2 change places after half a turn.

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[2] This is done by introducing a horizontal magnetic field. The reason why we choose to rotate around the horizontal axis is because the forward-backward (up-down) asymmetry is an asymmetry around the horizontal plane.

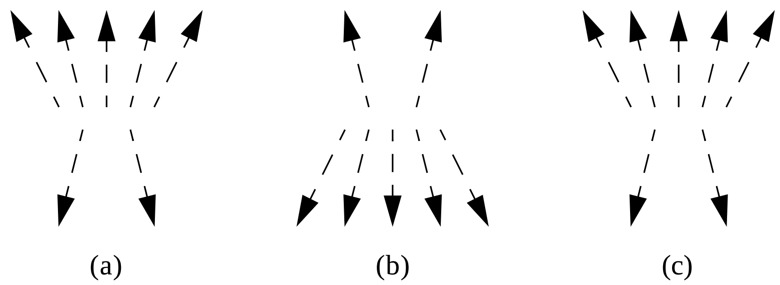

Figure 9: Example snapshots of a forward-backward (up-down) asymmetric positron emission. (a) at t=0, (b) after half a turn of the antimuon spin, (b) after one turn of the antimuon spin.

Starting from eq. (1), it can be shown that the number of positrons emitted upwards (in the forward region), N^{up}, and the number of positrons emitted downwards (in the backward region), N^{down}, as a function of time, t, that the antimuon has spent in the detector are given by the following relations:

$$N^{up}(t) = N_0 e^{-\frac{t}{\tau}}(1 + \alpha\cos(\omega t + \delta)) \qquad (3)$$

$$N^{down}(t) = N_1 e^{-\frac{t}{\tau}}(1 + \alpha\cos(\omega t + \delta + \pi)) \qquad (4)$$

with N_0, N_1, α and δ are constants, ω is the angular velocity of the antimuon spin rotation and τ is the muon lifetime. The constants N_0 and N_1 would be equal for a perfect detector. If we divide eqs. (3) and (4), we will obtain a ratio which is not constant with time (it has a cosine dependence).

If we start from eq. (2), the cosine terms of eqs. (3) and (4) will be replaced by cosine-squared terms, and therefore the two equations will be equal (perfect detector). In a non-perfect detector, they would give a ratio constant with time.

Questions:

1. How do we define the emission angle of the positron? Which values of the emission angle correspond to the forward / backward region?
2. What does the angular distribution of the positrons express (as a mathematical function)?
3. What are the possible formulae for the angular distribution of the positrons? What do they mean in terms of symmetry?
4. What kind of (a)symmetry do we try to measure with our detector? Can we measure it if we detect only one or very few positrons? Can the symmetry be expressed in terms of a rotation (of many positrons) around some plane? Which plane is that? What happens if you rotate around the other two planes in the case of (a) symmetry, (b) asymmetry?
5. Assume you have millions of antimuons (with spin up) stopping in the aluminium plate at the same time (t=0). Some of them will decay quickly and some will decay later. Let us say that they will all have decayed within time Δt. Before this time Δt has passed, all muon spins will have made a few turns in the magnetic field. Imagine that during this time Δt you have a continuous beam of positrons going up and another continuous beam of positrons going down because of the antimuons' decays. How do

you expect the two beams to look like? Imagine you rotate them around the horizontal plane. How many positrons will your detector be measuring up and down while you rotate the two beams? For how long can you perform this rotation?

5 Experimental evaluation of the muon lifetime

The muon lifetime τ appears in eqs. (3) and (4). Our detectror measures both distributions, so we can extract two values for the (anti)muon lifetime[3]: one from the distribution of the positrons emitted upwards and one from the distribution of the positrons emitted downwards. We can make the approximations

$$N^{up}(t) \approx N_0 e^{-t/\tau_1} \tag{5}$$

$$N^{down}(t) \approx N_1 e^{-t/\tau_2} \tag{6}$$

where τ_1 and τ_2 are the measured[4] lifetime values that we will find for the samples up and down. We will calculate the lifetime of the muon as the average of the two results

$$\tau = \frac{\tau_1 + \tau_2}{2} \tag{7}$$

Questions:

1. Why do you expect N_0 and N_1 to be different in the measured distributions (data)?
2. Why do you expect τ_1 and τ_2 to be different in the data?

6 Experimental observation of forward-backward asymmetry in the positron emission

By plotting the ratio of the positron distributions (numbers) up and down, we can see whether the positron emission is forward-backward (up-down) symmetric or not, as explained above. The ratio of eqs. (3) and (4) is

$$\frac{N^{up}}{N^{down}} = \frac{N_0}{N_1}\left(\frac{e^{-t/\tau_1}}{e^{-t/\tau_2}}\right)\frac{1 + \alpha\cos(\omega t + \delta)}{1 - \alpha\cos(\omega t + \delta)} \tag{8}$$

where τ_1 and τ_2 are the measured lifetime values that we will find for the samples up and down (section 5).We define

$$\frac{1}{k} = \frac{1}{\tau_1} - \frac{1}{\tau_2} \tag{9}$$

where k is a constant that we can calculate from τ_1 and τ_2. From eqs. (8) and (9), we have

$$\frac{N^{up}}{N^{down}} = \frac{N_0}{N_1}(e^{-t/k})\frac{1 + \alpha\cos(\omega t + \delta)}{1 - \alpha\cos(\omega t + \delta)} \tag{10}$$

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[3] Particles and antiparticles have the same lifetime.

[4] τ_1 and τ_2 can be somewhat different due to statistical fluctuations.

Eq. (10) can be approximated if we take into account that α is a constant between 0 and 1 (this can be established from the up or down distributions). The term cos(ω*t*+δ) is also a number between 0 and 1, therefore we assume that the product of their squares is much smaller than 1 and we write:

$$\frac{N^{up}}{N^{down}} = N_2 e^{-t/k} \frac{(1+\alpha\cos(\omega t+\delta))^2}{1-\alpha^2\cos^2(\omega t+\delta)} \approx N_2 e^{-t/k}(1+\alpha\cos(\omega t+\delta))^2 \approx$$

$$\approx N_2 e^{-t/k}(1+(\alpha\cos(\omega t+\delta))^2+2\alpha\cos(\omega t+\delta)) \approx N_2 e^{-t/k}(1+2\alpha\cos(\omega t+\delta)) \qquad (11)$$

where N_2 is the ratio of the constants N_0 and N_1.

Questions:

1. How do you expect the ratio in eq. (10) to behave with time *t*? What does the time *t* show?
2. How can the ratio in eq. (10) tell you whether you have a symmetric or an asymmetric angular distribution for the positron emission?
3. How is the parity transformation defined (see section D3)? How does it affect (a) the position vector, (b) the velocity, (c) the momentum, (d) the orbital angular momentum, (e) the spin of a particle?
4. What does it mean ‘a physical system does not conserve parity’?
5. How can we examine how an interaction behaves under the parity transformation?
6. How does the parity transformation affect the initial state in our experiment (imagine many antimuons at the same time)?
7. How does the parity transformation affect the final state in our experiment (imagine many positrons at the same time)?
8. From questions 6 and 7, what do you conclude about the way the antimuon decay behaves under parity transformation?
9. Is parity conserved or violated in weak interactions?
10. How does the asymmetry in the positron angular distribution relate to the parity violation in the antimuon decay?

7 The detector

Our detector is shown in fig. 10. Most of the muons pass right through the detector, however a fraction is stopped in the aluminium absorber, where they will decay. The muons come from above and will pass two scintillators (SC1 and SC2) before they stop. The positron from the decay can either be detected leaving the set-up upwards, as in the figure, or downwards. The absorber is placed inside a magnetic field produced by a coil.

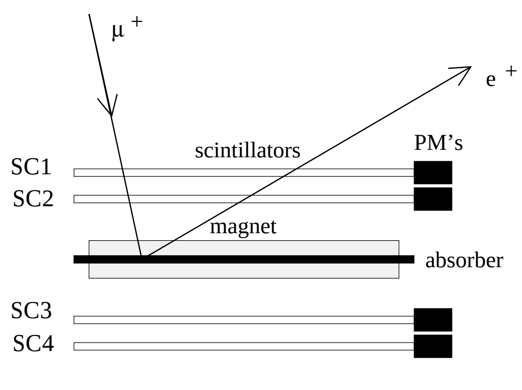


Figure 10: The setup of the experiment (side view). The electronics is connected to the output of the photomultipliers (PM’s).

7.1 Scintillators

Scintillators are devices that detect the passage of charged particles. Our scintillators are made of plastic. When a charged particles passes through the scintillator it excites (gives energy to) the molecules of the scintillator. Soon afterwards the molecules emit light and return to their ground state. A single charged particle will typically cause around 20000 photons to be emitted per cm of traversed scintillator. The scintillators are so called because the light which is emitted from their molecules is ‘a scintillation’, i.e. a small flash of light. The emitted light is directed to the photomultiplier.

7.2 Photomultipliers

A photomultiplier is mounted after each scintillator and collects the light that was emitted by the molecules of the scintillator. The function of the photomultiplier is to convert the light to an electric pulse (analog signal). This is done as follows: the incoming light (photons) hit a photocathode (a piece of material at a negative electric potential). 10 to 30% of the photons cause the emission of an electron (photoelectric effect). The electrons are accelerated between a series of ‘dynodes’ (pieces of metal at increasingly positive electric potentials). When an electron hits a dynode, 5 new electrons are emitted from the dynode. As the electrons traverse the dynodes, an avalanche of electrons is produced. Finally, all electrons are collected at the anode, where their initial number has been multiplied by a factor 10^6 or more. This is the pulse (analog signal) that we obtain from the photomultiplier. The photomultiplier is so called because it takes originally light (photo-) that converts to electrons which it ‘multiplies’ (in the dynodes).

The design of a photomultiplier can be seen in fig. 11. The outputs from the photomultipliers are connected to preamplifiers which give a signal strong enough to be detected by the logic.

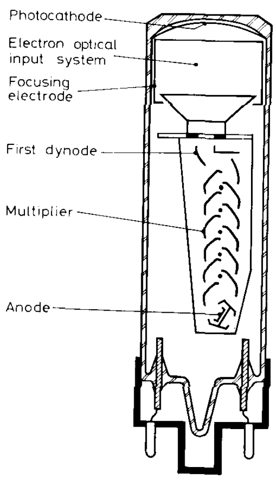


Figure 11: A schematic view of a photomultiplier. Light comes in from above and emitted electrons are multiplied as they move downwards through the dynodes.

7.3 Amplifiers

After the photomultipliers, there are amplifiers. The amplifiers take the signals from the photomultipliers and ‘enlarge’ them (‘amplify’ them) so that they can be handled by the electronics. Both imput and output signals are analog (i.e. proportional to the energy of the particle that crossed the corresponding scintillator).

7.4 Discriminators

In this experiment, we are interested in selecting particles of very high energy, therefore we use the output signal of the amplifier as an input to a discriminator. A discriminator is a device which converts analog to digital (0 or 1) signals as shown in fig. 12. When the signal A is given as input to the discriminator, the discriminator will produce a digital pulse C because the signal A exceeds the threshold of the discriminator. A low signal B which is smaller than the threshold, will be ignored and the discriminator will ‘give’ a 0 pulse as output. The output of the discriminator is always a digital signal (square pulse).

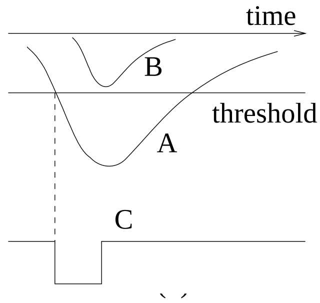


Figure 12: Operation of a discriminator.

7.5 Triggers and event registration

We will now combine the (digital) signals from the four scintillators discriminators in order to define the triggers for our measurements. Particles and antiparticles are treated in the same way by our detector. For convenience, we mention below only antimuons and positrons but the same considerations apply to (negative) muons and electrons.

We are interested in detecting antimuons that stop in the aluminium and the positrons that come from their decays. In order to register interesting events[5] we construct three triggers (a ‘trigger’ is a ‘decision’ made by the electronics): (a) one for stopping antimuons, T1, (b) one for associated up-going positrons, T2 and (c) one for associated down-going positrons, T3. In order to associate a positron with a previous muon, we accept a positron only if it appeared at most 20 μs after the muon (20 μs are enough for most muons to decay). An associated positron is a positron that most probably came from the decay of the specific antimuon.

A trigger T1 means that an antimuon crossed the scintillatiors SC1 and SC2 (fig. 10) but it did not cross the other two scintillators, SC3 and SC4. This situation is shown by the first four signals in fig. 13(a) and fig. 13(b). When the signals from SC1 and SC2 overlap in time (coincidence), we have a T1. At the same time, we start a timer of 20 μs. (This timer is called GATE in fig. 13.) The function of this timer is to enable the electronics to accept coincidences from SC1 and SC2 or from SC3 and SC4 during these 20 μs: such coincidences correspond to T2 and T3. A coincidence from e.g. SC1 and SC2 that comes after the timer is finished is not accepted as a T2. This is done to redude the number of accidental associations between an antimuon and a positron. In fig. 13(a) (fig. 13(b)), the sixth, seventh, eighth and nineth signals are due to an up-going (down-going) positron. As at the time of overlap the timer is active, we accept this coincidence as a T2 (T3). When we have a T2 or a T3, we start the TDC. This is a time-to-digital-converter device, which we use as a clock. It is stopped when the timer is stopped. For each event (=an antimuon and an associated positron), we register the number of pulses emitted by the TDC. This number is called ‘the number of TDC counts’. We know what is the width of one TDC count, called ‘the TDC constant’ (=39 ns). From the number of TDC counts and the TDC constant, we can calculate (for each event) the time measured by the TDC, t_{TDC}. The time difference tmuon=20 μs - t_{TDC} gives approximately the time that the antimuon spent in the aluminium plate before decaying. This time is measured in the rest frame of the antimuon, therefore, we can use it to extract the lifetime of the particle.

The way the three triggers are realized in the electronics is shown in fig. 14. For each event, we save the following information in a data file (text file): the value of tmuon and the region (up or

[5] By ‘event’ we mean a stopping muon and an associated positron.

down) the positron was detected (emitted) in. The data registration is done by a MacIntosh computer.

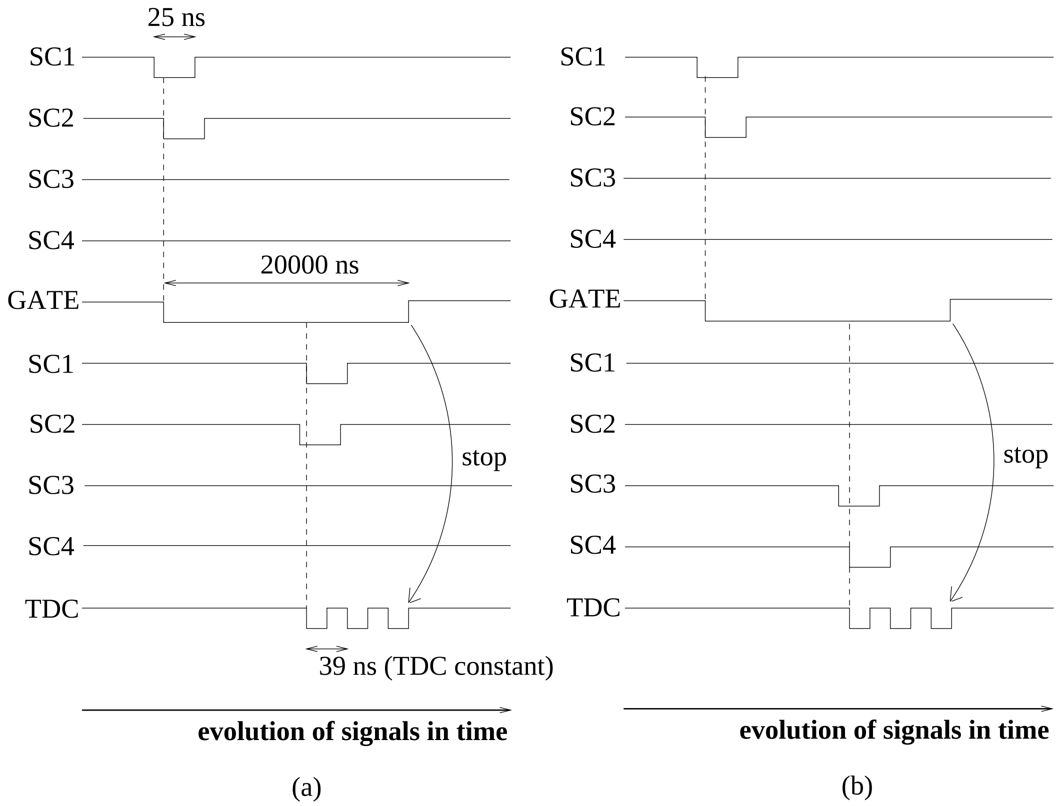


Figure 13: The signals for (a) a decay up and (b) a decay down (after the discriminators). The straight lines indicate absence of particles in the scintillators (digital 0 pulse). If a very high energy charged particle crosses a scintillator, the digital signal from the discriminator is a square pulse (digital 1 pulse).

The operation of the gates AND, NAND and OR is explained in appendix A.

Questions:

1. What is the detector made of? Make a drawing.
2. How does a scintillator work?
3. How does a photomultiplier work?
4. How does an amplifier work?
5. How does a discriminator work?
6. What is 'one event'?
7. What do we call T1, T2 and T3?
8. How do the following gates work: (a) AND, (b), NAND, (c) OR?
9. What does the timer do?
10. What does the TDC do?
11. What does the detector measure for each event?

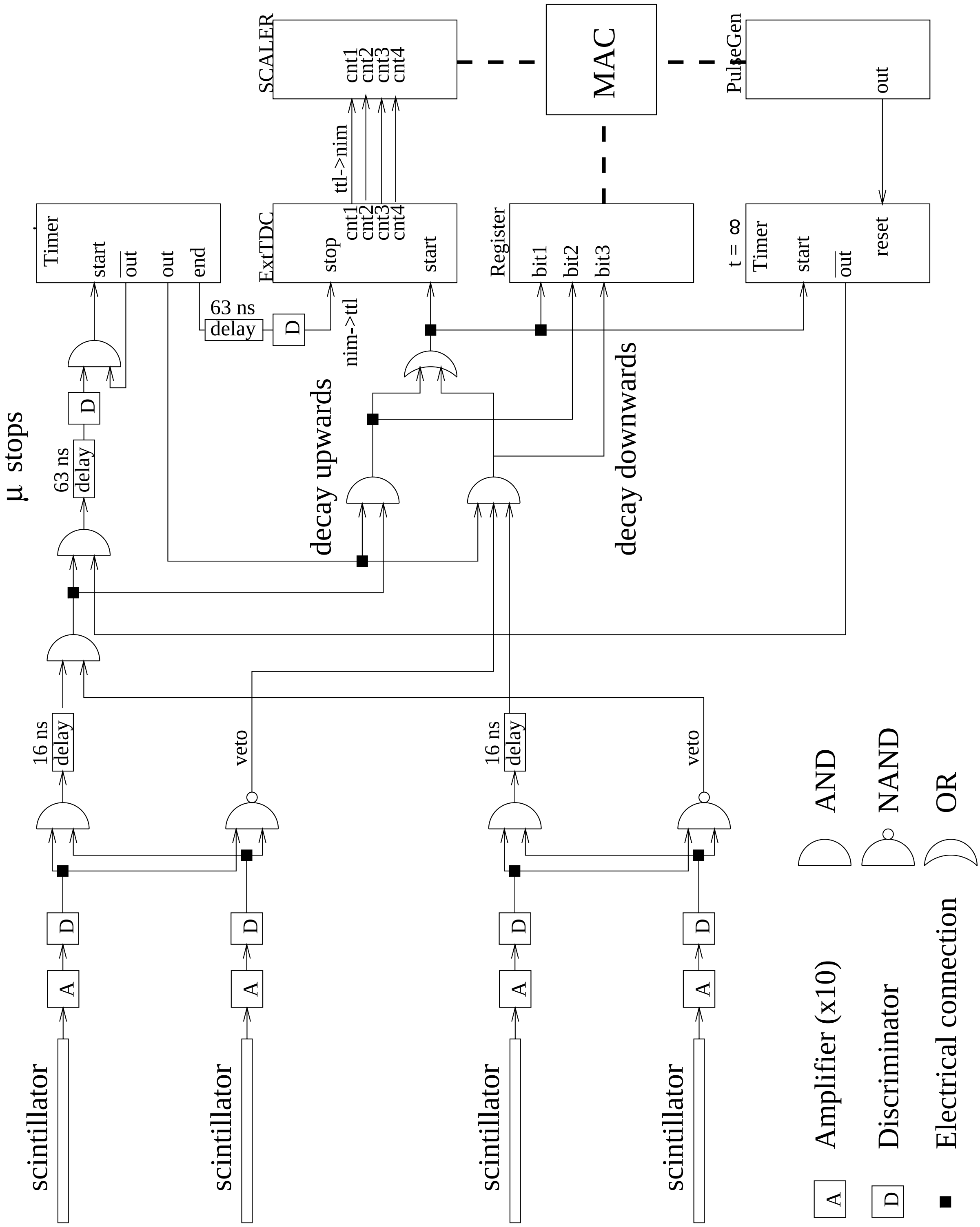


Figure 14: The electronics of the experiment. The first input is the signals from the discriminators. After having gone through the electronics, the data are stored in the hard disc of the computer (MacIntosh).

Exercise:

Use fig. 14 to write down signals (digital 0's and 1's) for the following cases (aasume that the particles have enough energy to pass the discriminators). What does the detector measure in each case?

- (a) an antimuon crosses all four scintillators
- (b) an antimuon crosses SC1 and SC2 and stops in the aluminium plate
- (c) as in (b) and the positron is emitted upwards
- (d) as in (b) and the positron is emitted downwards
- (e) as in (b) and the positron leaves the detector without crossing a scintillator

While doing this exercise, remember that digital pulses last for about 25 ns (except the pulses from the NAND gates, which last for about 100-150 ns, and the timer, which stays open for 20 μs).

8 Using Kurslab computers

The data analysis will be done in the Kurslab computer room (H321). Here are given some instructions on how to connect to the elementary particle division (EPF) machines in order to use the analysis program paw++. The example assumes that the Kurslab machine is called planck and the EPF machine is called toker:

12. login on the Kurslab machine. The userid is `muon`.
13. open a window by clicking on the terminal icon on the screen. The prompt in the window gives the name of the machine, e.g. `planck%` means that the machine is called planck.fysik.lu.se.
14. before connecting to the EPF machine called toker, type `xhost + toker.quark.lu.se`
15. login on toker by typing `ssh1 toker.quark.lu.se -l lab`
16. type `setenv DISPLAY planck.fysik.lu.se:0`
17. to see the subdirectories type `ll`
18. go to the subdirectory that has your name, e.g. type `cd christina`
19. open paw++ by typing `paw++`
20. do the paw++ exercises (in the first laboration) or do your analysis (by yourself)
21. close paw++
22. logout from toker by typing `exit`
23. go to your own directory on planck, e.g. type `cd christina`
24. copy your files (figures) from toker to planck by typing
    `scp1 lab@toker.quark.lu.se:christina/filename filename`
25. print the files by typing `lpr filename`
26. logout from planck (click on footprint icon and select 'logout').

Other EPF machines that can be used are: glader, kloker, prosit, prinsen and snovit.

9 First laboration: exercises for data analysis

1. Creating a histogram. Suppose you have a data file as shown here:

event number	up	tmuon, μs
1	0	3.9
2	1	0.1
3	0	2.1
4	0	7.4
5	1	3.1
6	1	1.4
7	0	9.3
8	1	0.3
9	1	2.6
10	1	0.9
11	1	1.1
12	1	4.3
13	1	8.2
14	1	10.6
15	1	4.4
16	1	0.2
17	1	1.8
18	1	12.3
19	0	12.7
20	1	3.7
21	1	6.1
22	1	5.8
23	0	16.4
24	1	2.3
25	0	2.5
26	1	1.9
27	1	15.0
28	1	7.6
29	1	2.2
30	1	5.2

The first column gives the event number. By ‘event’, we mean the detection of an antimuon and the detection of a positron within an interval of 20 μs. The second column tells us if the positron was seen by the upper two scintillators (1) or not (0). The third column contains the time difference (in μs) between the detection of the antimuon and the detection of the positron. Make the distribution of the variable tmuon for the <u>events with a decay upwards</u>. The distribution should be a histogram with the x-axis starting at 0 μs and ending at 16 μs . There should be 8 bins in the histogram so that the binning of the histogram is 2 μs. Give your histogram the identification number (id) 1. How many entries do you have in each bin? How many are the total entries? How many are the events with decay up?

2. Connect to an EPF machine and go to your directory, as explained in section 8. Open paw++. You will get three windows: the executive window (will not be used), the main browser and the graphics window.
3. Go to the main browser window. There is a menu on the top of the window. Go to **Commands** and select **Function** and then **Plot**. You will get a new window where you can type the function you want to plot, e.g. 10*exp(-x/2)+5 from x=0 to x=10. Try different numbers for the constants to see how the curve changes. Can you find the values of the constants just by looking at the curves? Try to plot exp(-x/2) and 1-x/2 superimposed (option S in the FUNCTION/PLOT window).
4. Go back to the histogram of step 1. What does it mean to do a fit to a histogram? Do an exponential fit to histogram 1, i.e. a fit of the form f(tmuon)=P1+P2*exp(-tmuon/P3), where P1, P2 and P3 are constants. What is the value of f(tmuon) in the limit of tmuon going to infinity? What is the value of f(tmuon) in the limit of tmuon going to zero? Can you extract the values of the constants P1, P2 and P3 from your fit? Think of the function g(tmuon)=1-tmuon/P3 (see last question of point 3 above). Is your evaluation accurate? How can you improve the accuracy of your calculation?
5. Go to the main browser window. There is a menu on the left side of the window. Click on **Hbook** to look for the histogram file. The file **muon.hbook** will appear.
6. Double-click on the icon (**H**) of the histogram file to open it. The link **LUN1** appears.
7. Click on **LUN1** to see what histograms you have in the histogram file. You get three icons symbolizing histograms with identification numbers (id’s) **1** and **2**.
8. Plot histogram 1 by double-clicking on its icon. It is the same as the histogram 1 you made in step 1 but it has many more entries (events).
9. Plot histogram 2. It is the same as histogram 1 but for the events where the positron was emitted <u>downwards</u>.
10. Click on **Macro** to see the fits you can do with paw++. There are two fits: the fit **expfit** fits a histogram to the function P1+P2*exp(-tmuon/P3), where P1, P2 and P3 are constants. The fit **expcosfit** fits to the function P1*exp(-tmuon/P2)*(1+2*P3*cos(P4*tmuon+P5))+P6.
11. Fit histogram 1 exponentially: click on the icon of **expfit** with the left button to select it. Click with the right button and go to **Exec...** to give the id of the histogram you want to fit. What are the values (and errors) of the fitting constants?
12. Plot the functions cos(x), exp(-x), exp(-x)*cos(x), 1-x superimposed. What do you see?
13. Divide histograms 1 and 2 to get histogram 3. To do this go to the main browser window and select the menu **Commands**->**Histograms**->**Operations**->**Divide**. If the first histogram is 1 and the second histogram is 2, then the operation is 1/2. The

option should be E to calculate the errors of the new histogram (3). How are histograms divided?

14. Click on **PAWC** (paw++ memory). Plot histogram 3 and do the **expcosfit**. What are the values of the six parameter constants?

10 Data analysis

The data will be analysed by the students after the first laboration. The results will be presented and discussed in the second laboration.

The aim of the data analysis is to measure the muon lifetime and to confirm the forward-backward asymmetry in the positron emission (which means parity violation in the antimuon decay, see section D3). To do this, follow these steps:

1. Connect to an EPF machine and go to your directory as explained in section 8. Open paw++. For the analysis, you will use the histograms 1 and 2 your histogram file muon.hbook (see section 9).
2. Decide which formulae describe histograms 1 and 2. You will use them to evaluate the muon lifetime. For this purpose, can you think of a suitable approximation? (think in which part of the formula the muon lifetime appears.) Explain why your approximation is good (use point 12 of section 9). Extract the muon lifetime by fitting histogram 1. Do the same for histogram 2. Are the two values different? Why? Do they agree within errors? Calculate their mean value. What is the error of the mean value? What is the relative error of the mean value? How do we judge the quality of a measurement? Is your measurement good? Does it agree with the expected value (2.19 μs)?
3. Divide the histograms 1 and 2. Use the id number 3 for the result of the division. Is it meaningful to divide histograms 1 and 2? What does histogram 3 represent, i.e. what do we measure with the horizontal and vertical axes? What is the mathematical function that describes histogram 3? In the function there is a term that tells us whether there is forward-backward asymmetry in the positron emission. Which is that term? What value do you expect it to have if there were forward-backward symmetry in the positron emission? Why? What value do you expect it to have if there is asymmetry (as expected)? Why? Fit histogram 3 to the function that describes it. Check that you use the correct fit by comparing the function of the fit and the function that describes your histogram. Identify the fitting parameters. What are there values? Did you confirm the forward-backward asymmetry? Why is it necessary to do this fit in order to see the asymmetry? Is it not enough to see that we have more events with a decay upwards than with a decays downwards? Why?
4. Is parity violated in the muon decay? Why?

Error calculation

In this section, we discuss statistical errors. Assume that we have measured a quantity y with an error δy and a quantity x with an error δx. Then the quantities (a) $z=(y+x)/2$, (b) $w=cx$, where c=constant, will have errors:

(a) $\delta z = \pm\sqrt{\left(\frac{\delta y}{2}\right)^2 + \left(\frac{\delta x}{2}\right)^2}$ and (b) $\delta w = \pm c \cdot \delta x$

When we calculate errors we don't keep all the decimal digits in the value of the error. We keep only the first digit which is not equal to zero, unless this digit is equal to 1 or 2, in which case we also keep the next digit. For example, an error ±3.287653 is written as ±3 only, whereas an error ±1.87462 is written as ±1.9. We must always write ± in front of an error. In the measured quantity (mean value), we keep the same number of decimal digits as we have kept in the error. For example, if the error of *x* is ±0.6 m and *x*=483.756 m, we write *x*=(483.8 ± 0.6) m. When we make a number round, we increase the last digit we keep if the next digit is 5 or larger. E.g. ±8.32 becomes ±8 but ±8.69 becomes ±9.

11 Report

The report should show a general understanding of the theory and how the set-up can be used to measure the muon lifetime and the parity violation in the muon decay.

The report should include the following:

THEORY:

- Where do the (anti)muons we study come from? How do they decay?
- What is the lifetime of a particle? In which reference frame is it defined? What does it express in terms of probability for the particle to decay? Do all particles live as long as their lifetime is?
- Explain why we have forward-backward asymmetry in the positron emission.
- Explain why parity is violated in the muon decay.

EXPERIMENT:

- Describe the detector. Explain how the following devices work: (a) a scintillator, (b) a discriminator, (c) a photomultiplier.
- Explain how the gates AND, NAND and OR work.
- In our experiment, what do we call 'an event'?
- When does the timer start? When does it stop? What is its width? Is it the same for all events? What is the purpose of the timer?
- When does the TDC start? When does it stop? What does the TDC signal look like? Does the TDC signal have the same duration in all events?
- When do we store an event? What does the detector measure for each event?

ANALYSIS:

- Describe how the analysis is done (see section 10) and answer the questions of that section.

NOTE:

The report should be delivered no later than two weeks after the last lecture.

PART II : Discussion

D1 Origin of the muons

In this experiment, we observe ‘cosmic muons’. Cosmic rays are high energy particles produced in the Sun and in supernovae and neutron stars of our galaxy. About 85% of the cosmic rays are protons and 12% are alpha particles (helium nuclei). The remainder are electrons and nuclei of heavier atoms. The cosmic rays travel in the space between the stars. In their path they interact with atomic nuclei and they produce new cosmic rays, like antiprotons, positrons, photons and neutrinos. The cosmic rays that reach our atmosphere are called ‘*primary cosmic rays*’.

The energy spectrum of primary cosmic rays, i.e. the number of particles as a function of energy, has been measured over an enormous range. For the nuclear component, it is shown in fig. 15. A good fit to the data, except at the lowest energies, is

$$I(E) \propto E^{-2.6}$$

where $I(E)$ is the intensity (flux) of the nuclear component at energy E.

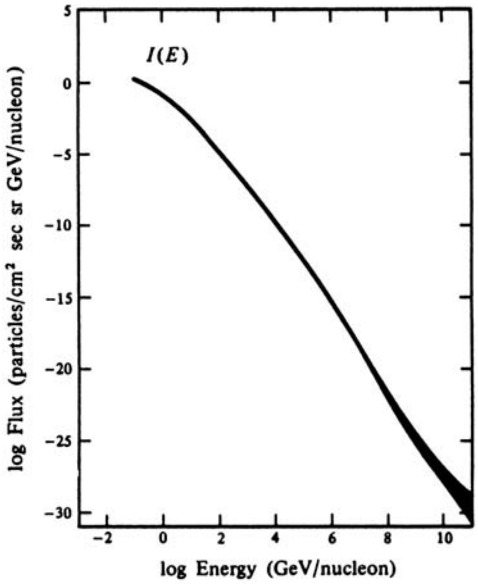
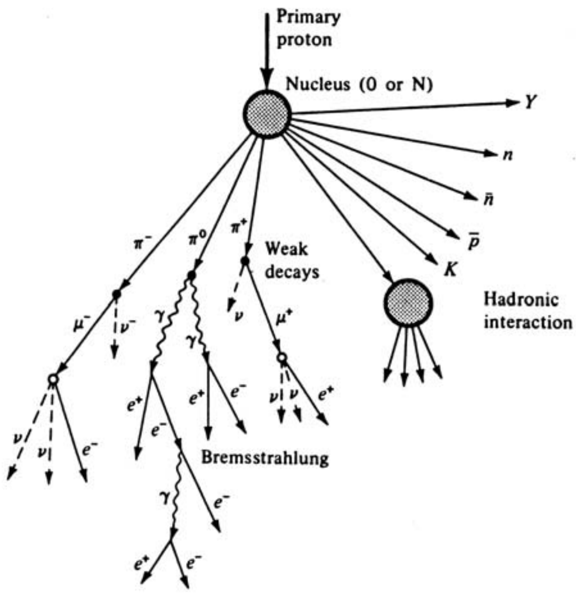


Figure 15: Energy spectrum of the nuclear component of the primary cosmic rays.

When a primary cosmic ray, e.g. a proton, enters the Earth’s atmosphere it interacts with the nuclei of the atmosphere’s atoms (mainly oxygen and nitrogen). From the interactions, new particles are produced (‘*secondary cosmic rays*’), inducing in their turn new reactions with atoms of the atmosphere. This creates a ‘hadronic shower’. The word ‘hadronic’ signifies that the produced particles are hadrons, i.e. strongly interacting particles, like protons, pions, kaons, etc. The word ‘shower’ refers to the way the particle production develops in space (fig. 16). Antiparticles, like antiprotons, are also produced in this process.



Unstable hadrons then decay weakly to electrons, muons and neutrinos. Photons are also produced, e.g. by π^0 decays or electron bremsstrahlung. ‘Electromagnetic showers’ are created by electrons or by photons converting into electron-positron pairs, which emit new photons etc. Overall, a very high energy proton can produce a very extensive shower, covering many km^2 of the Earth’s surface. By the time the showers reach the ground, they mainly consist of electron- and muon-neutrinos and muons. These are the ‘cosmic muons’ that we observe in our experiment.

Figure 16: A high-energy proton strikes an oxygen (O) or nitrogen (N) nucleus in the top of the atmosphere and produces a shower of particles.

D2 What is the lifetime of a particle?

The lifetime of a particle is something that we define in the rest frame of the particle. We will take the muon as an example of a decaying particle but the following discussion is a general one and applies to all unstable particles.

If we take a muon and observe it in its rest frame, we can never tell in advance at exactly which moment the muon will decay, even if we know exactly when the muon was created. This means that the muon does not have a fixed time of life. The only thing that we can say is that at a specific moment in time, t, there is a probability, P(t), for the muon to decay. We therefore say that the muon decay has a 'statistical nature'.

Let us assume that, at time t_0=0, we have N_0 muons. Each one of them has the same probability to decay in a specific time interval. For example, in the interval (t_0, t_0+10 *ns*), each muon will have a probability P_1 to decay. It is convenient to define the decay probability per unit time, λ, which is a constant. We can now find the decay probability for any time interval *dt*: this will be equal to λdt, as we can check from the units of the variables λ and *dt* ([probability / time]x[time]). What does it mean 'the decay probability in time *dt* is equal to λdt'? It means that, if at time *t* we have *N(t)* muons, at time *t+dt*, $N(t)\lambda dt$ muons will have decayed. This means that the number of muons has decreased by a quantity $dN=-N(t)\lambda dt$. The minus sign is there because $dN=N_{final}-N_{initial}<0$. If we integrate this relation, we find the *exponential decay law*[6]

$$N(t) = N_0 e^{-\lambda t} \qquad \text{(D1)}$$

From quantum-mechanics, we know how to use the above relation in order to calculate the mean life of the muon:

$$\langle t\rangle = \frac{\int_{N_0}^{0} t\,dN}{\int_{N_0}^{0} dN} = \frac{\int_0^\infty t\left(\frac{dN}{dt}dt\right)}{\int_0^\infty \frac{dN}{dt}dt} = \frac{1}{N_0}\int_0^\infty t\lambda N dt = \frac{1}{N_0}\int_0^\infty N_o e^{-\lambda t}\lambda t dt = \frac{1}{\lambda} \qquad \text{(D2)}$$

The variable <*t*> is also denoted by τ and is called the 'lifetime' of the muon. It is a constant because λ is a constant. We see that the 'lifetime' of the muon is not a quantity that we can measure by detecting the decay of one muon only. We need to observe how an initial number of (many) muons decreases with time.

Let us now take eq. (D1) and insert the lifetime of the muon, τ=2.2 μs:

$$R(t) = \frac{N(t)}{N_0} = e^{-t/2.2} \qquad \text{(D3)}$$

where *R(t)* is the ratio of the remaining muons at time *t* over the initial number of muons and *t* is measured in μs. We see this distribution in fig. 17.

In principle, the number of muons will never be zero, as the exponential never crosses the horizontal axis. In real life, what happens is that there should be (almost) no muons left after 20 μs

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[6] This relation is also called the 'radioactive decay law' for historical reasons because it was first found to describe the decay of radioactive nuclei.

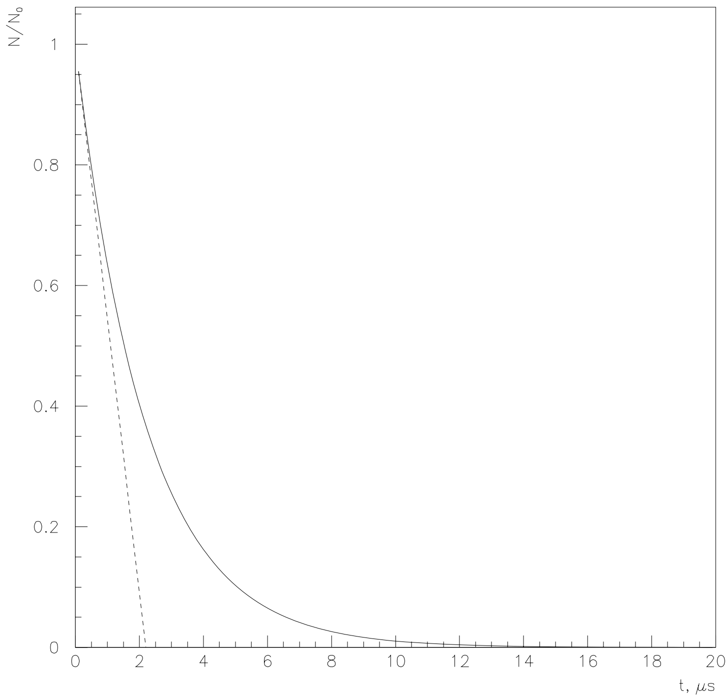


Figure 17: Exponential decay law for muons (solid curve) (eq. (D3)). The dashed line shows the hypothetical linear decay law of eq. (D5).

or so. (Remember the gate of 20 μs .) If we take the derivative of eq. (D3) we find the rate at which the ratio R(t) decreases with time:

$$\frac{dR}{dt} = e^{-t/2.2}(-1/(2.2\ \mu s)) \qquad \text{(D4)}$$

Then we find that, at *t=0*, the rate is $1/(2.2\ \mu s)$. Suppose that this rate stayed constant with time. This would give us a linear decay law

$$\tilde{R}(t) = \frac{\tilde{N}(t)}{N_0} = 1 - t/(2.2\ \mu s) \qquad \text{(D5)}$$

which is shown by the dashed line in fig. 17. Eq. (D5) tells us that if the number of muons that decayed per unit time was always equal to the value it had at t=0, then at time t=τ there would be no muons left.

D3 Transformations and parity violation in (anti)muon decay

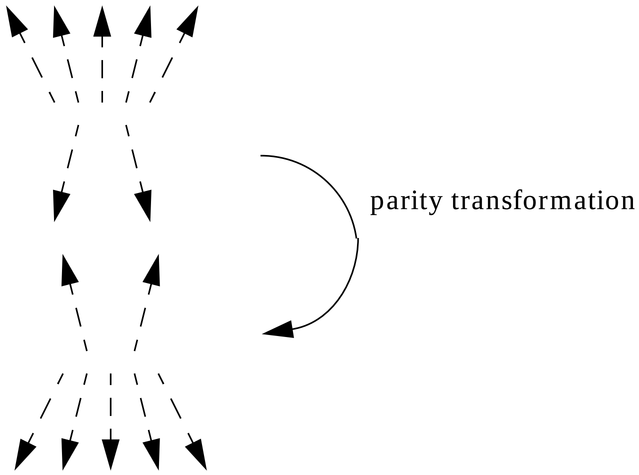
Transformations are very important in elementary particle physics because they reveal symmetries of physical systems and interactions and relate to conservation laws. For example, a translational transformation is defined as $\vec{r} \to \vec{r} + \vec{a}$, where $\vec{r}$ is the position vector of a particle and $\vec{a}$ is a constant vector. This transformation does not alter the properties of a closed system (i.e. a system on which no external forces are acting). We say that the system has translational symmetry. This symmetry leads to conservation of the total linear momentum of the particles in the system.

The parity transformation is another example of a transformation that can be performed on a system of particles. It is defined as $\vec{r} \to -\vec{r}$ so it means that the position vector $\vec{r}$ of every particle of the system is reflected in the origin. For example, if a physical system, e.g. a positron, is described by the function $\Psi(x) = 1 + \sin x$ the parity transformation of this system will give us a system that is described by the function $F(x) = 1 - \sin x$. The function is now different, so we say that this system is not symmetric under parity transformation.

Apart from examining how the parity transformation affects a physical system, we can also investigate how interactions behave under the parity transformation. In our experiment, we study the antimuon decay, which is a weak interaction. This interaction connects two states: the initial state (antimuon at rest) and the final state (positron, neutrino and antineutrino with total orbital momentum zero). We need to examine both states in order to see how the interaction itself behaves under parity transformation.

We start with the initial state, i.e. antimuons at rest (in the aluminium). Each antimuon has a spin but no velocity, therefore, a parity transformation of the initial state corresponds to a parity transformation of the muon's spin. In general, the definition of a particle's angular momentum is $\vec{L} = \vec{r} \times \vec{p}$, where $\vec{r}$ is the position vector of the particle and $\vec{p}$ is the momentum of the particle. A parity transformation (x,y,z) $\rightarrow$ (-x,-y-z) will change $\vec{r}$ to $-\vec{r}$ and $\vec{p}$ to $-\vec{p}$, therefore, $\vec{L}$ will not be changed. As spin is also angular momentum, we conclude that the parity transformation does not affect our initial state. We say that the initial state in our experiment is 'parity symmetric'.

Let us now examine the final state in our experiment, i.e. the positrons. They have both spin and momentum. A parity transformation of the final state will not affect their spins but it will reflect their momenta in the origin (in the following drawing, one arrow shows one positron momentum vector):



As a consequence, the final state is not parity symmetric. Since we have an initial state which is parity symmetric and a final state which is not parity symmetric, it follows that the process that brought us from the initial state to the final state does not conserve parity, i.e. the muon decay is a parity violating process. Since the muon decay is a weak decay, we generalize and say that there is parity violation in weak processes.

D4 Questions and answers

In this section, we explain certain points that we mentioned in section 3.

Question 1: In the cosmic ray showers, the muons come mainly from decays of pions. Consider a positive pion decaying at rest. Assume that the orbital angular momentum of the final state is zero and find the direction of the spin of the antimuon. Assume that the antimuon is emitted upwards.

The decay of the charged pion is the weak process

$$\pi^+ \to \mu^+ + \nu_\mu$$

The pion has spin zero. From angular momentum (spin) conservation, it follows that e.g. in the rest frame of a π^+, the neutrino and the muon must have spins in opposite directions, fig. 18. Since the ν_μ is always left-handed, the spin of the μ^+ must point downwards (fig. 18).

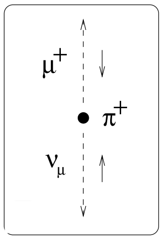


Figure 18: Pion decay in the rest frame of the pion. Dashed arrows denote momenta. The spin of a particle is shown by a solid arrow.

Question 2: As in question 1, find the spin direction of the antimuon when this is emitted downwards. Transform form the pion's rest frame to the lab frame (where the detector is 'at rest'). Remember that the pion is moving downwards with respect to the detector.

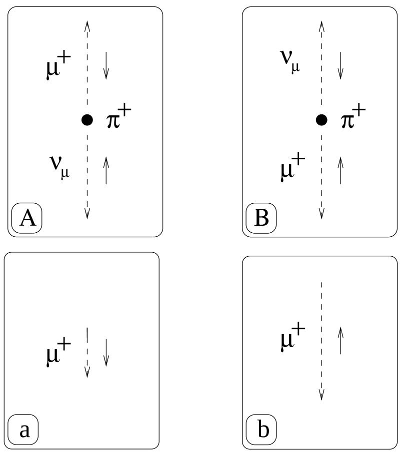


Figure 19: Pion decay in which the muon is emitted upwards (A, a) or downwards (B, b) (for the two extreme cases) in the pion's rest-frame. The drawings A and B refer to the rest frame of the pion. The drawings a and b are the corresponding situations in the lab frame. Dashed arrows denote momenta. The length of such an arrow symbolises the magnitude of the particle's momentum. The spin of a particle is shown by the solid arrow.

We see that in the lab frame, the momentum of the antimuon will be different depending on whether it was emitted along or opposite the momentum direction of the pion. The left-handed antimuons (with spin up) will have higher momenta than the right-handed antimuons (with spin down).

Question 3: Explain why most of the antimuons that we see in our detector have spin up.

The antimuons that we detect in our experiment travel a long distance in the atmosphere before reaching our detector. Their lifetime is not enough to travel this distance. The reason why some of them still manage to come to our detector is the relativistic time dilatation. Therefore we mainly see antimuons with very high momenta, i.e. antimuons with spin up (fig. 19(b)).

Question 4: Explain why we are intrested only in the decay of antimuons.

The fate of μ^+ and μ^- is very different when they come to the aluminium plate of our detector. The μ^- can decay as a free particle, $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, or be captured by a nucleus: most of the negative muons will orbit around aluminium nuclei, just like electrons do. The muon orbits lie very close to the nuclei and the μ^- will rapidly loose its initial spin direction. The μ^+ cannot be bound in an atom and it behaves as a free muon, decaying as follows: $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$. The different behaviour makes it possible to study the forward-backward asymmetry in the decay of the antimuons because the negative muons give a continuous background.

PART III : Appendices

A AND, NAND and OR gates

In fig. 14, we see the electronics of our experiment. There are three kinds of gates that we use: AND gates, NAND gates and an OR gate. A gate is a device that takes as input two (or more) digital signals (0's and 1's) and combines them into one digital signal. The value of this signal depends on the kind of the gate. In fig. 20, we denote by A and B the input signals and by C, D and E the output signals of the gates. In Table 1, the values of C, D and E are given for all possible combinations of A and B.

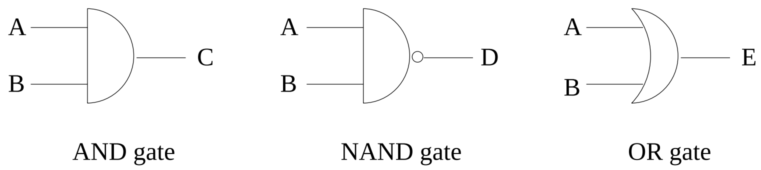


Figure 20: The three kinds of gates that we use in our electronics: AND, NAND and OR.

A (first input)	**B (second input)**	**C (A AND B)**	**D (Not (A AND B))**	**E (A OR B)**
0	0	0	1	0
1	0	0	1	1
0	1	0	1	1
1	1	1	0	1

Table 1: Input (A, B) and output (C, D, E) signals for the gates of fig. 27.

From Table 1, we see that the output of the AND gate is 1 when both A AND B are 1. The output of the NAND gate is the opposite of the output of the AND gate ('N' in NAND stands for 'Not'). The output of the OR gate is 1 when A OR B are 1. Therefore, the name of the gate denotes the operation of the gate.

It is possible to have gates that receive more than two inputs. In fig. 14, there is such an AND gate, which takes three input signals. The output of this gate is 1 when all three inputs are 1 and 0 otherwise.

B Units in high energy physics

The fundamental units in physics are of length, mass and time. In the familiar SI system, these are expressed as meter (m), kilogram (kg) and second (s). In high energy physics, these units are not very useful because the distances are much shorter than 1 m, the particles have masses much smaller than 1 kg and the duration of processes is much smaller than 1 s. Therefore, we need to introduce new units or modify the old ones.

Length is usually expressed in 'femtometers' (or 'fermis'). We write '1 fermi' as '1 fm' and it corresponds to 10^{-15} m. The radius of a proton is about 1 fm. The radius of a nucleus is a few fm.

Time is usually measured in 'nanoseconds' or 'microseconds'. One nanosecond is 10^{-9} s and we write it as '1 ns'. One microsecond is 10^{-6} s or 1000 ns and it is written as '1 μs'.

The unit for energy is based on the so-called 'electron-volt', 'eV', which is defined as follows: an electron which is accelerated by a potential difference (voltage) of 1 Volt gains an amount of energy equal to 1 eV. As this is a very small amount, we usually multiply it by 10^6 to form 1 mega-electron-volt, ‘MeV’. If we multiply by 10^9 instead we obtain a giga-electron-volt, ‘GeV’. The mass of a particle is measured, for example, in MeV/c^2, where c is the velocity of light. This comes from Einstein’s relation $E=mc^2$. As it is not convenient to divide by c^2 in the calculations, we use a unit-system where c=1 by definition. This is called ‘the system of natural units’. For example, the mass of an electron is about 0.5 MeV and the mass of the proton is about 938 MeV. The particles’ momenta are also expressed in units of eV.

Bibliography

1 L. Gislén, H.-U. Bengtsson, G. Gustafson, *Particle Physics and Cosmology,* spring 2000.

2 T. K. Gaisser, *Cosmic rays and particle physics*, Cambridge University Press, 1990.

3 B. R. Martin, G. Shaw, ‘*Particle physics*’, John Wiley, 1997.

4 W. R. Leo, Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag, 1987.