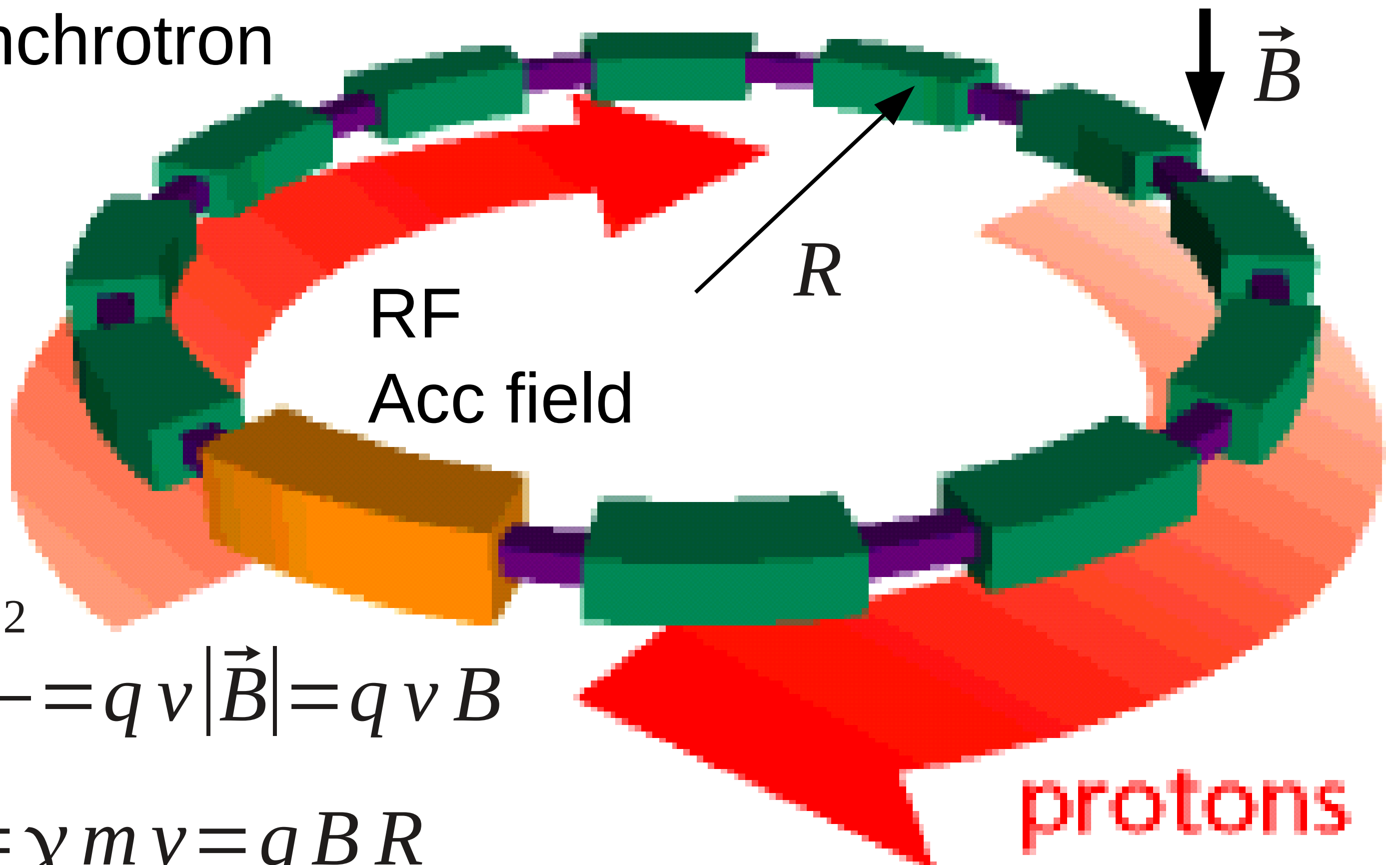
Lectures on accelerator physics

- Lecture 5 and 6: Advanced topics
  - Transverse motion, strong focusing, and LHC
- Material borrowed from
  - Lecture by Anders Oskarsson
  - Lecture by Eric Torrence (University of Oregon)
  - LHC lectures by Danillo Vranic (GSI)
- Weak focusing follows “Principles of Charged Particle Acceleration” by Stanley Humpries Jr. Chapter 7.

Towards the Synchrotron

Synchrotron



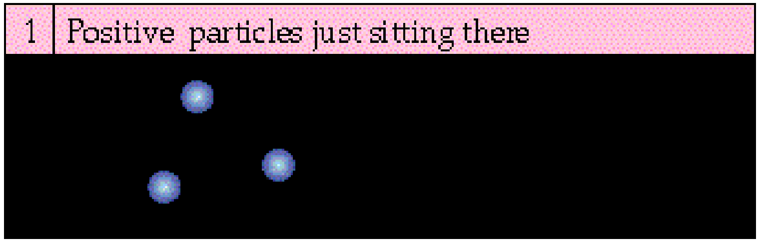
$$\frac{\gamma m v^2}{R} = q v |\vec{B}| = q v B$$

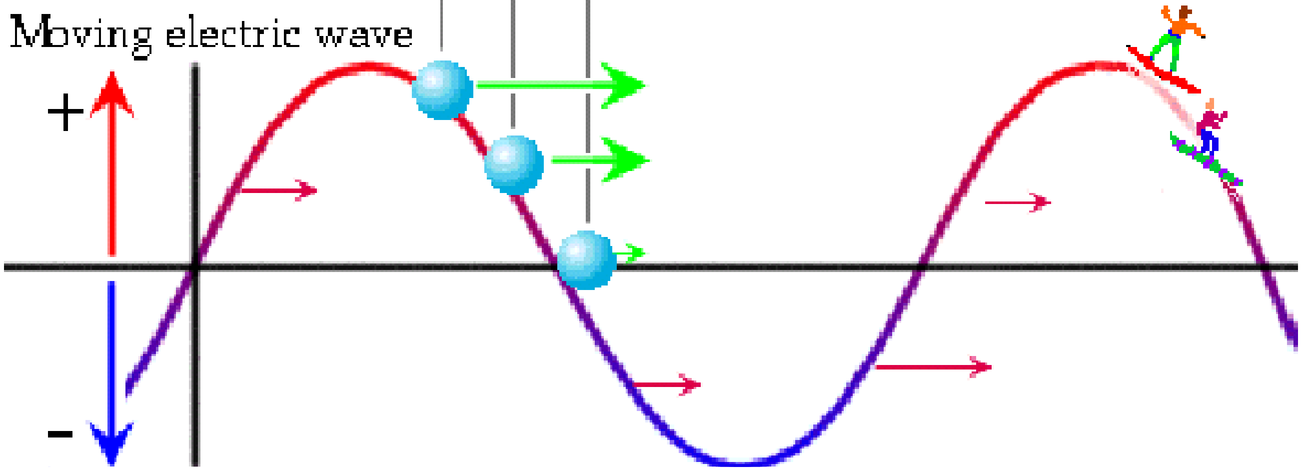
$$p = \gamma m v = q B R$$

 Dipole magnet give circular motion

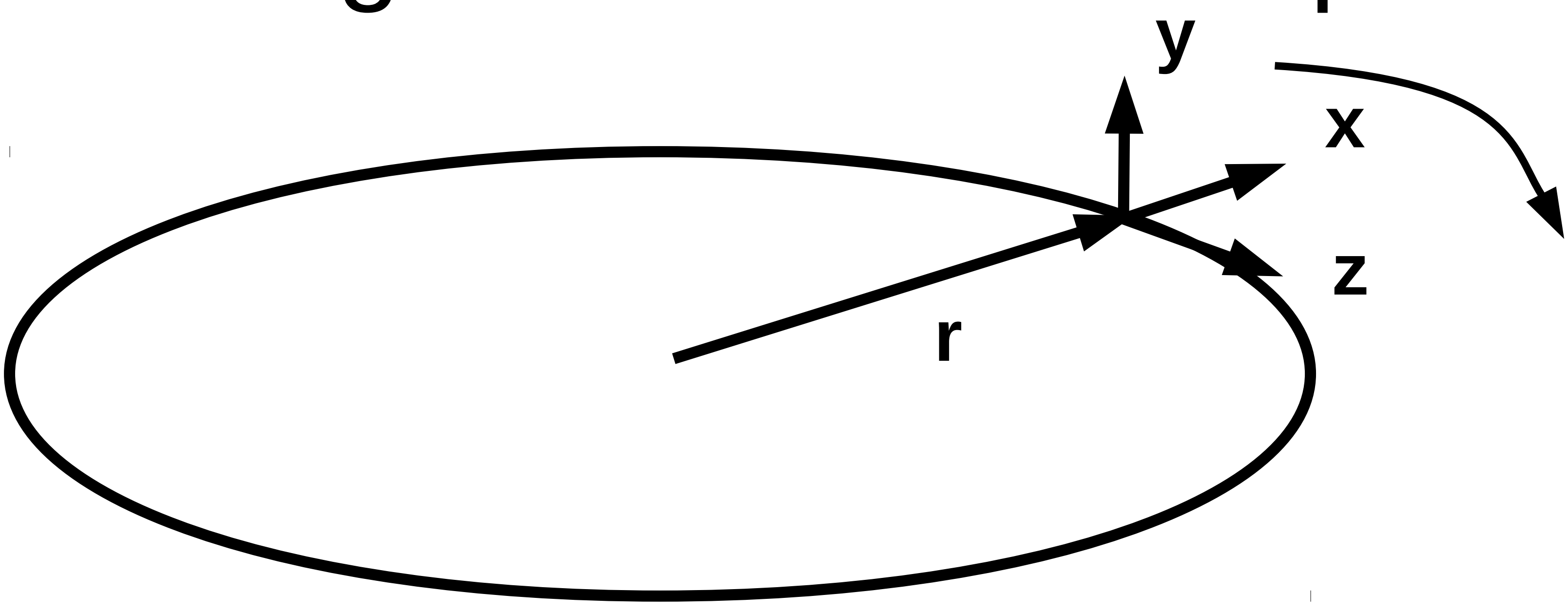
 Acceleration in E-field

The particles are “surfing” the acceleration wave



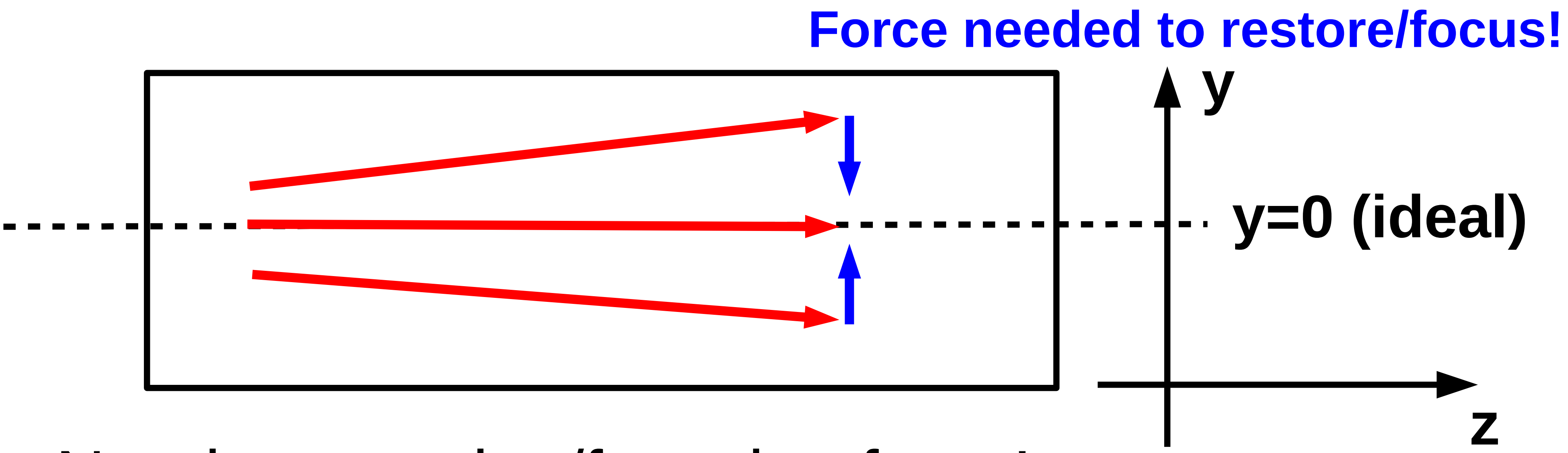


Focusing in the transverse plane



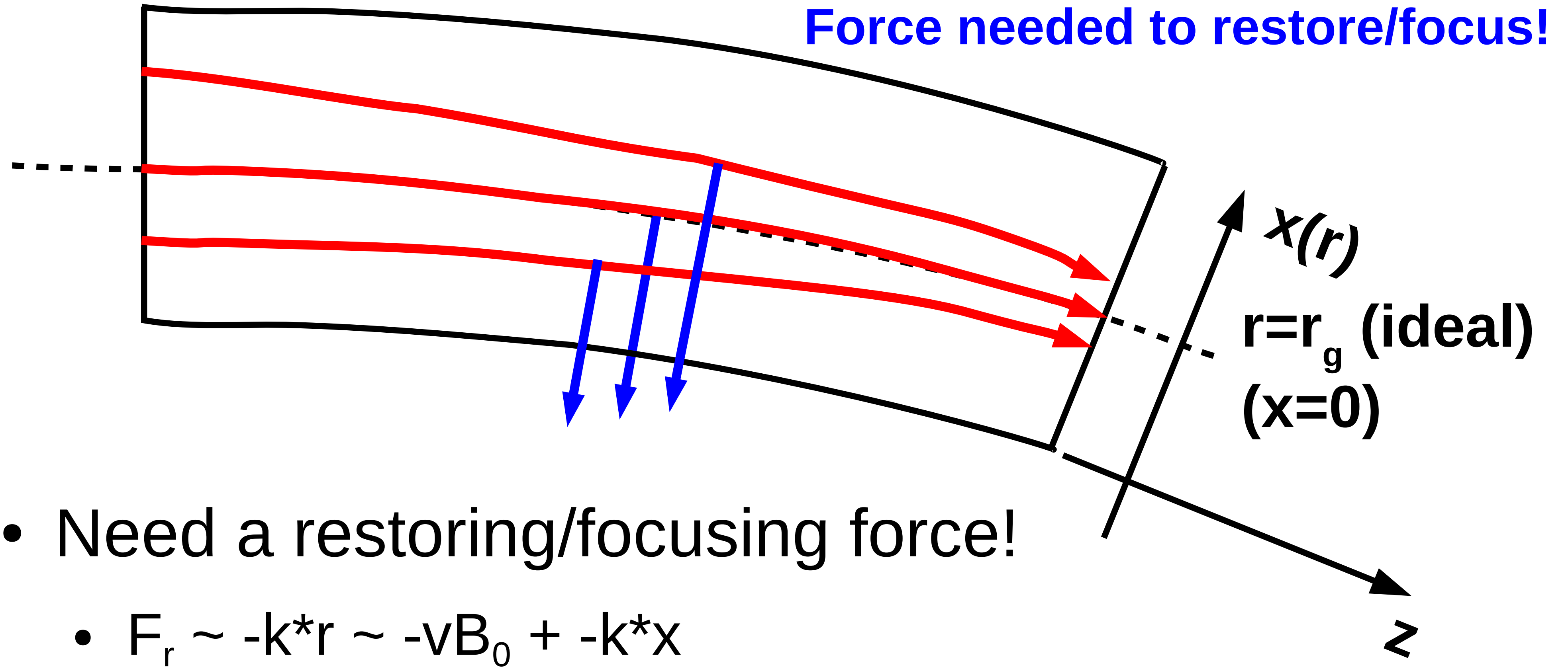
- Assume (v_x,v_y,v_z)~(0,0,v) and v~c = constant!
  - Very good assumption!
- z=vt → t=z/v (~z/c)
  - d/dt ~ v d/dz (~c d/dz)

Transverse focusing in y direction



- Need a restoring/focusing force!
  - $F_y \sim -k*y$
- Harmonic oscillator (like string)

Transverse focusing in x direction



- Need a restoring/focusing force!
  - $F_r \sim -k*r \sim -vB_0 + -k*x$
    = central force (r_g) + harmonic oscillator in x

Let us first solve harmonic equation (ignoring magnet realities!)

$$\gamma m \frac{d^2 y}{dt^2} = \gamma m v^2 \frac{d^2 y}{dz^2} = -ky$$

$$y(z) = y_0 \cos\left(\frac{2\pi}{\lambda} z + \varphi\right),$$

where

$$\lambda = 2\pi \sqrt{\frac{\gamma m v^2}{k}}.$$

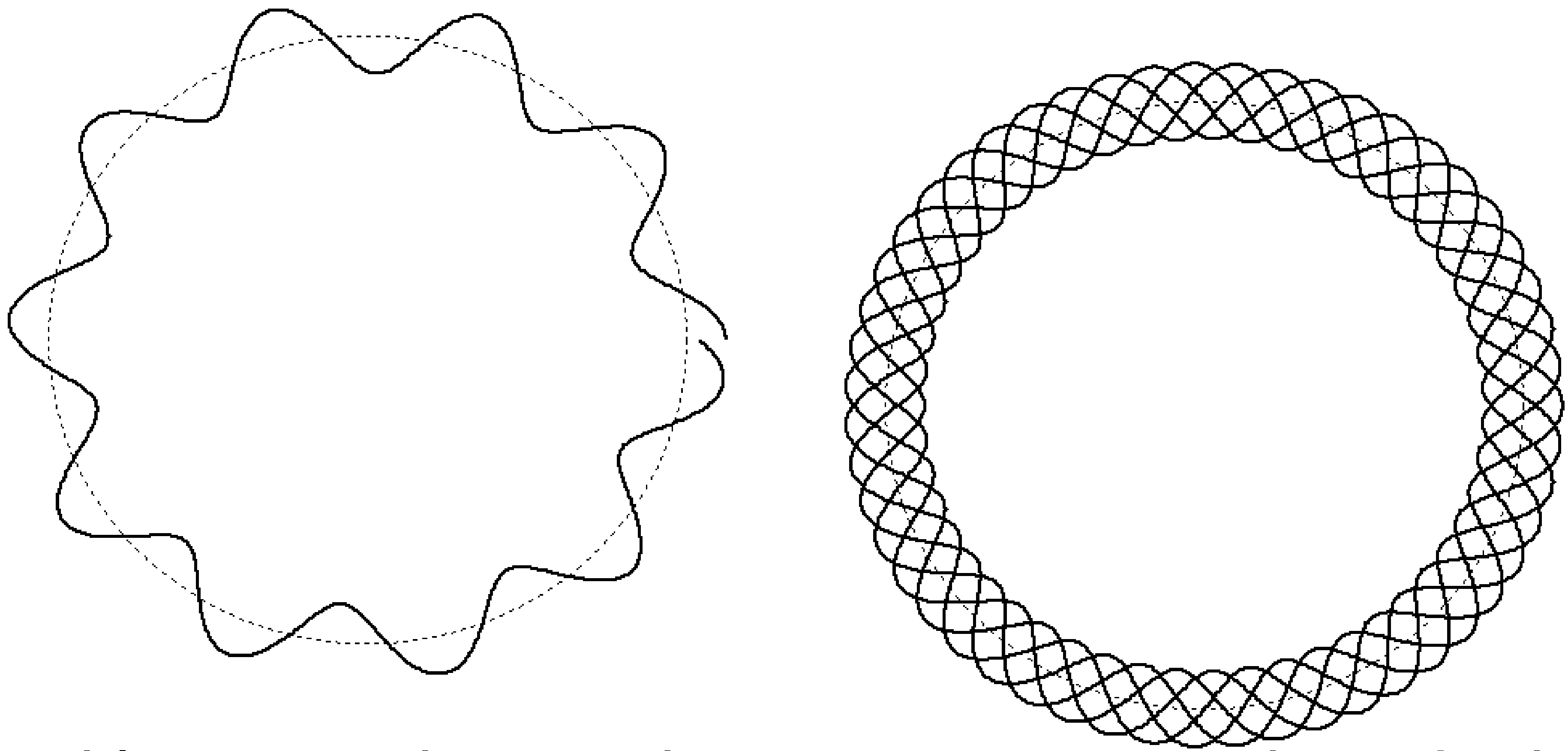
- Note that the wavelength does not depend on the amplitude y_0. There is only one wavelength for all amplitudes!

Tune interlude

- One defines
  - Q (ν[nu]) = C/λ, where $C=2\pi r_g$ is the circumference of the synchrotron ring
- Q is the number of transverse (betatron) oscillations per turn
- It is different for x and y directions
- Very important for beam stability!

Bad harmonic tune (Q=10.2)

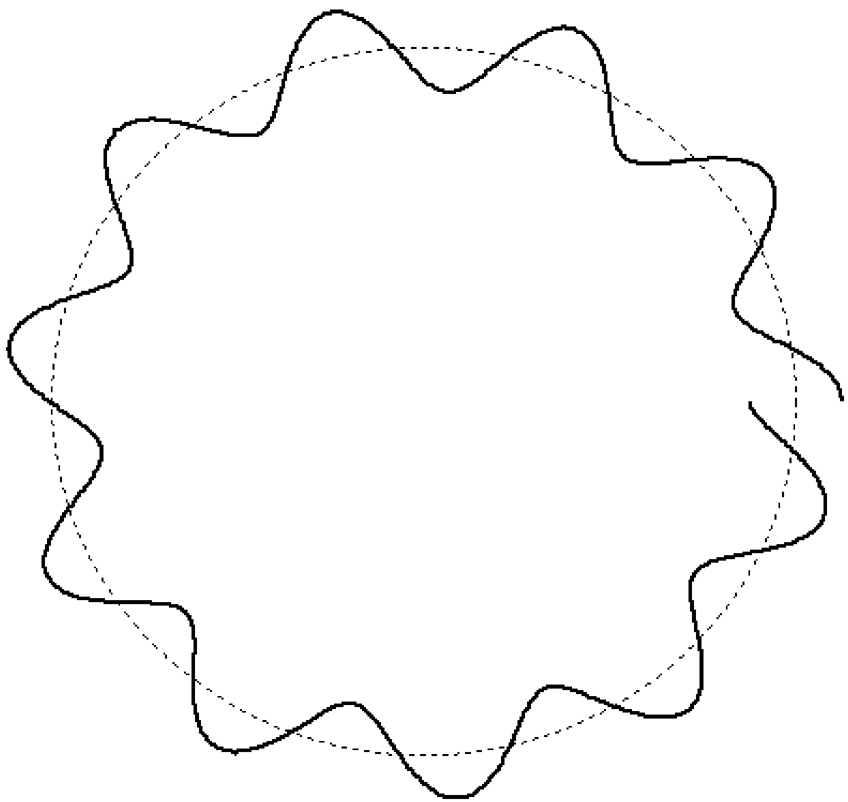
1 turn 100 turns



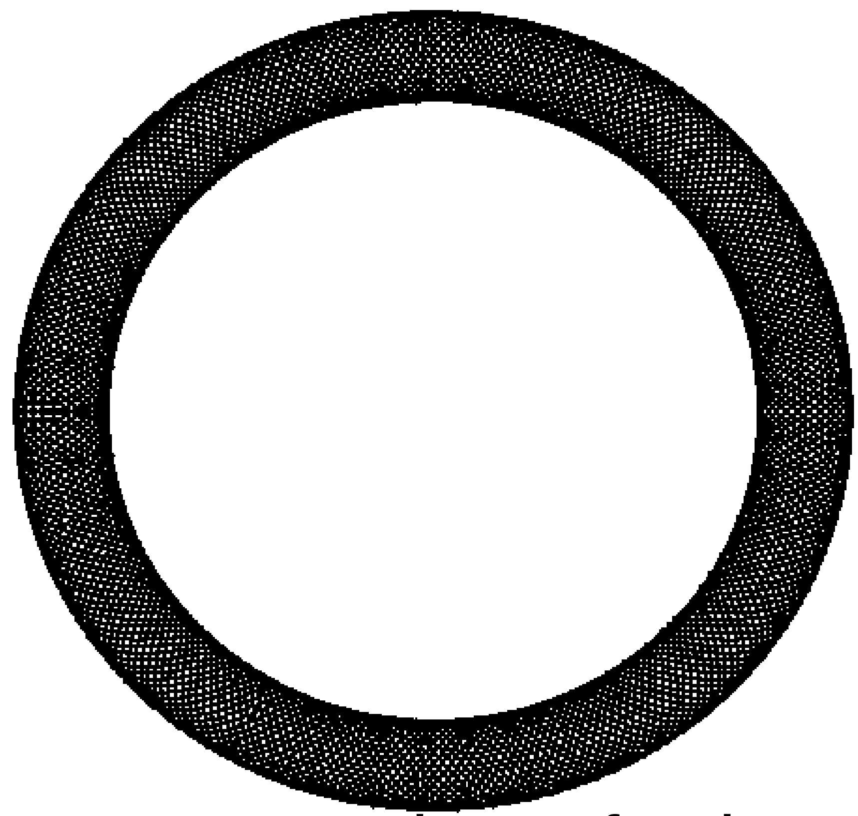
Problem: tune does not integrate out magnet imperfections

Better (less harmonic) tune (Q=10.48)

1 turn

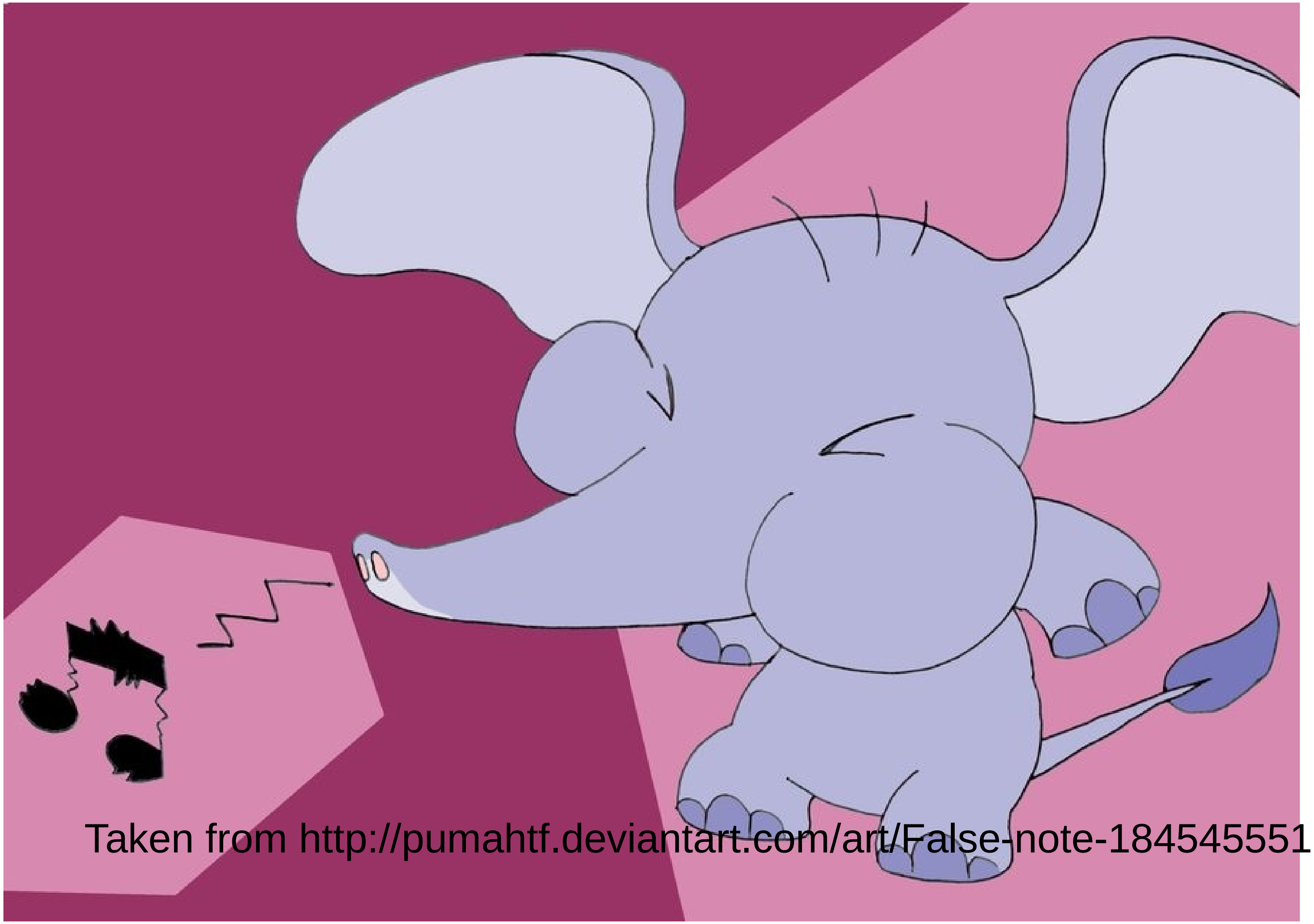


100 turns



Tune is better at integrating out magnet imperfections

Lesson: bad musicians makes great accelerator physicists



Taken from http://pumahtf.deviantart.com/art/False-note-184545551

LHC TUNES

AT 7TeV

HORIZONTAL TUNE: $Q_x = 64.31$

VERTICAL TUNE: $Q_y = 59.32$

$$\Delta Q \leq 3 \cdot 10^{-3}$$

AT 450GeV

HORIZONTAL TUNE: $Q_x = 64.28$

VERTICAL TUNE: $Q_y = 59.31$

Betatron tunes should avoid linear coupling resonances at

$$nQ_x + mQ_y = p$$

Back to transverse motion and magnet realities!

- Taylor expanding the dipole magnetic field AND fulfilling Maxwell equations gives
  - $(B_x,B_y,B_z) \sim (-(n_0B_0/r_g)y, B_0 - (n_0B_0/r_g)*x, 0)$
    - NB! note that – sign is not good!
- Ideally we want n_0 as large as possible to confine the beam!
  - (And make the magnet as small as possible)
- Let us look at solution for x (r)!

The equation of motion for x

$$\gamma m \frac{d^2 r}{dt^2} = \gamma m v^2 \frac{d^2 r}{dz^2} = \gamma m \frac{v^2}{r} - qvB_y$$

$$\frac{d^2 r}{dz^2} = \frac{1}{r} - \frac{q}{\gamma m v} B_y$$

Substituting $x = r - r_g$ + expanding $\frac{1}{r}$:

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{1}{r_g^2} x - \frac{q}{\gamma m v} B_y$$

Inserting the Taylor expansion of B_y:

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{qB_0}{\gamma m v} - \frac{1}{r_g^2} x + \frac{q n_0 B_0}{\gamma m v r_g} x$$

The first two terms gives the solution for the ideal trajectory $\rightarrow$: $\frac{1}{r_g} = \frac{qB_0}{\gamma m v}$ so that:

$$\frac{d^2 x}{dz^2} = -\frac{1}{r_g^2}(1 - n_0)x.$$

Weak focusing: $0 < n_0 < 1$

$$\frac{d^2x}{dz^2} = -\frac{1}{r_g^2}(1 - n_0)x.$$

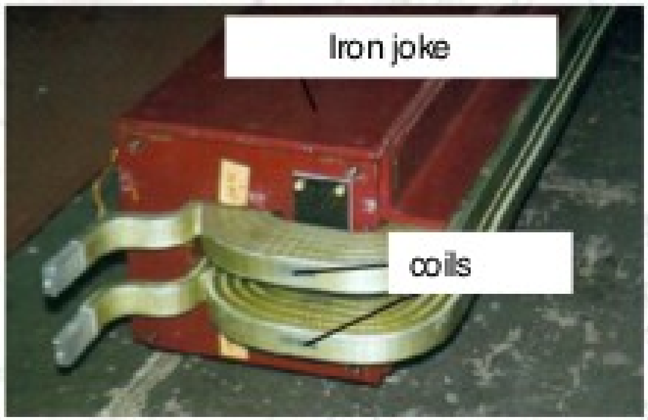
- Only harmonic oscillation solution when $(1-n_0)>0$ (and y equation requires $n_0>0$)
  - Otherwise exponential growth!
- This means that the focusing is limited!
  - That is why this solution is called weak focusing

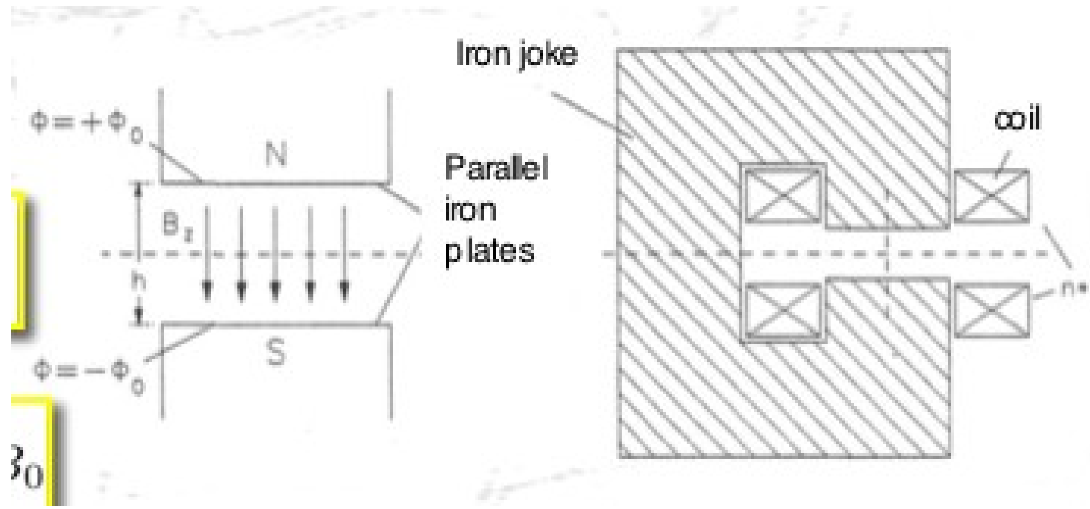
LHC example

- LHC focuses the beam using magnetic fields of 223 T/m
- If one would use weak focusing the magnetic field should be smaller than 8.33T/2800m = 0.003 T/m!!!!
- And so the ring would have to be enormous!
  - And the luminosity would be very small!

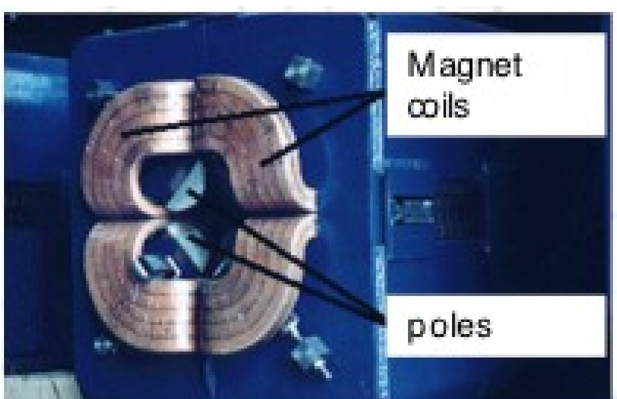
Can we find better focusing? Beamline Elements

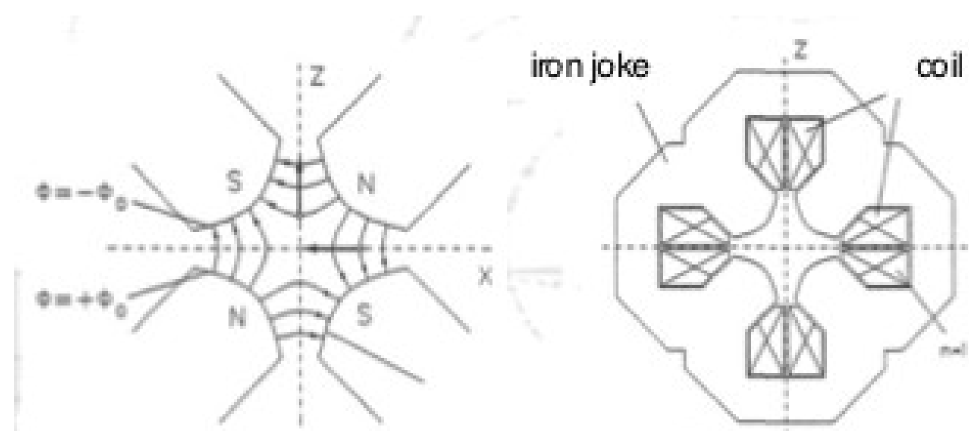
Dipole (bend) magnets





Quadrupole (focusing) magnets



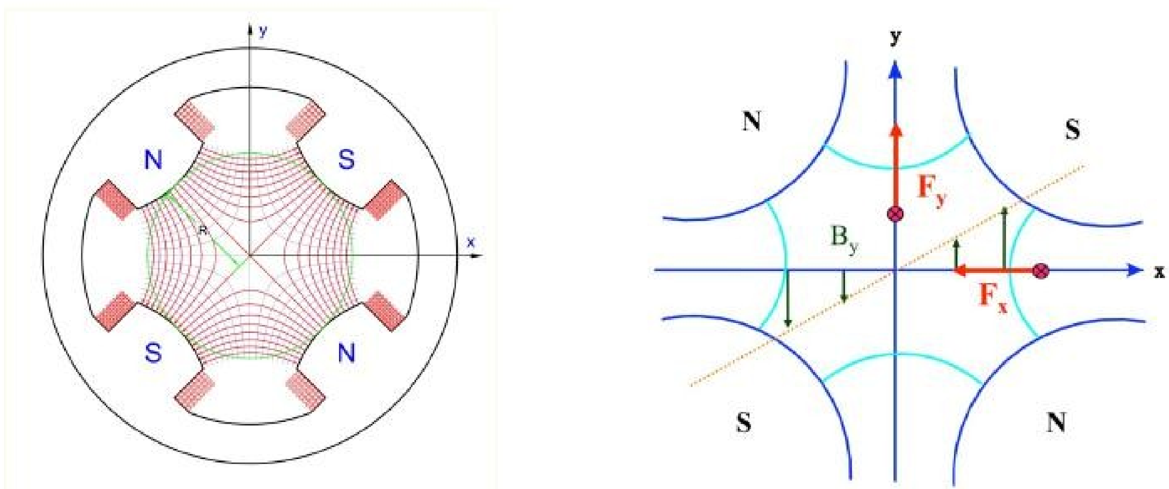


Also Sextupoles and beyond

Quadropoles has similar problem!

FOCUSING OF THE PROTON BEAM

Quadrupole looks good – field increases linarly with distance from the center.



F_y has wrong direction! It doesn't work!

No solution: Maxwell tells us $\nabla\times\mathbf{B}=\mu_0\mathbf{J}$ $\oint_{\partial S}\mathbf{B}\cdot d\boldsymbol{l}=\mu_0 I$

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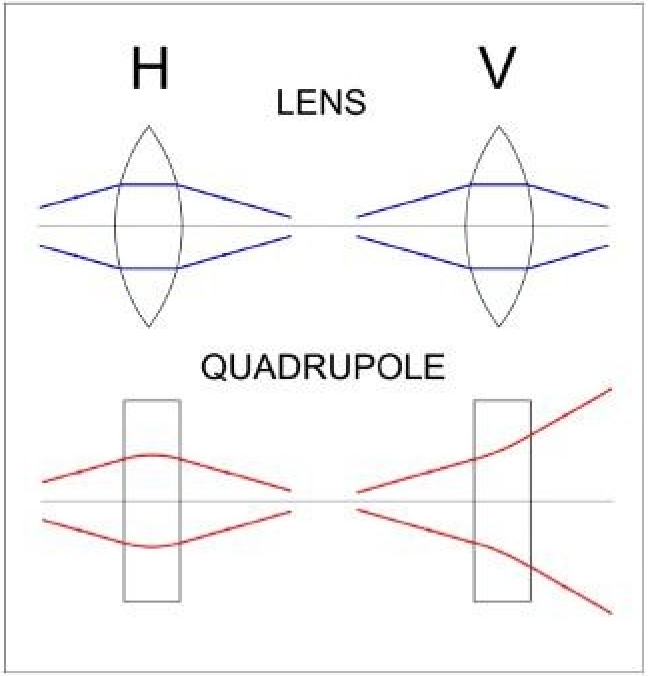
PROBLEM

Quarupole is convergent lens in horizontal, but divergent in vertical direction!



There was no solution until 1952, and it is beautiful and simple:

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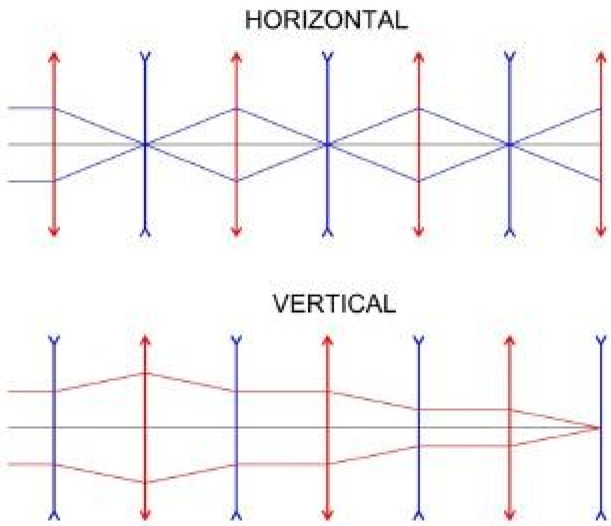
SOLUTION: AG OR STRONG FOCUSING



FODO LATTICE

F - focusing
D - defocusing
O - drift space or dipoles

If we have alternating convergent and divergent lenses with right spacing overall effect is focusing!

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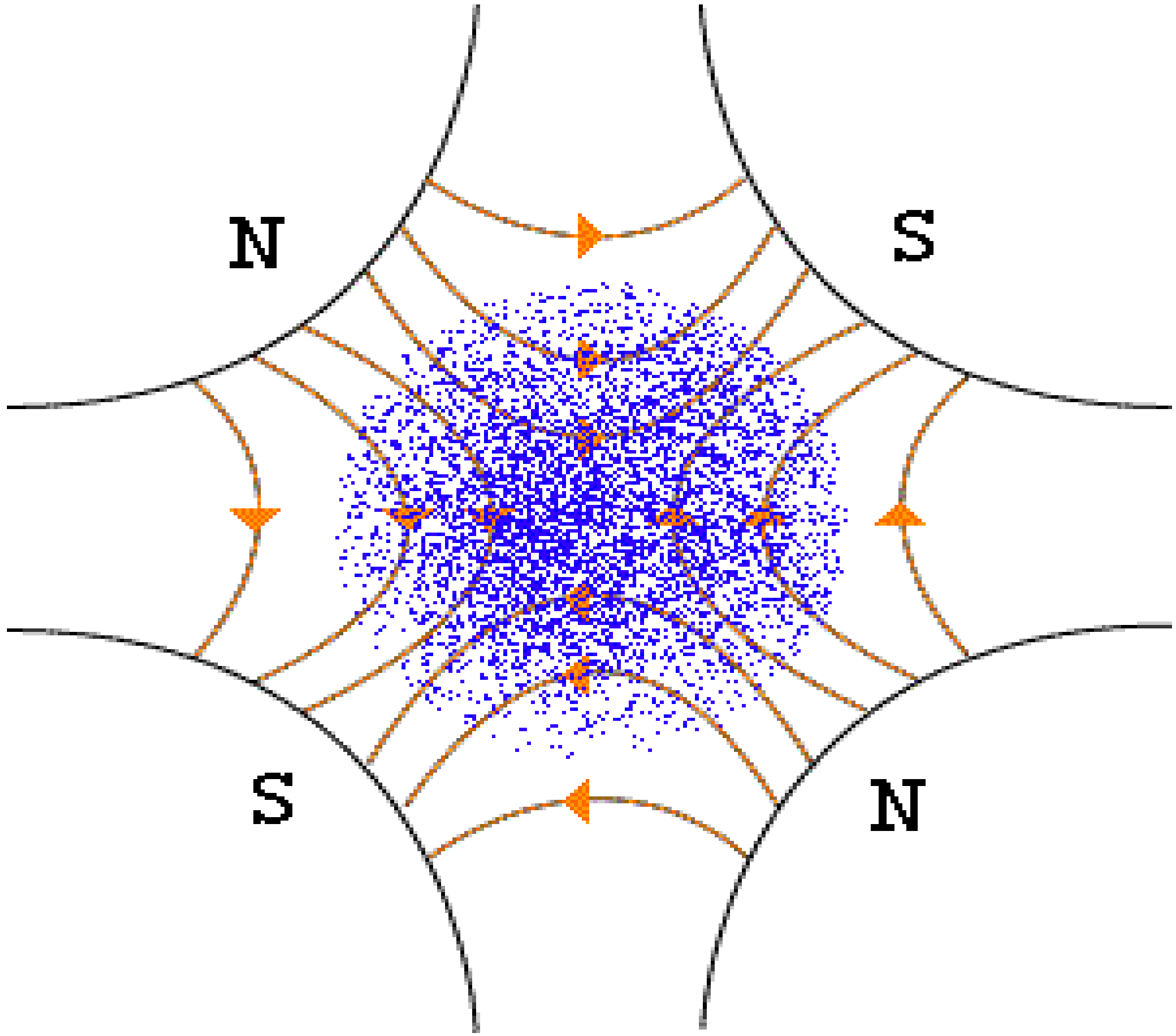
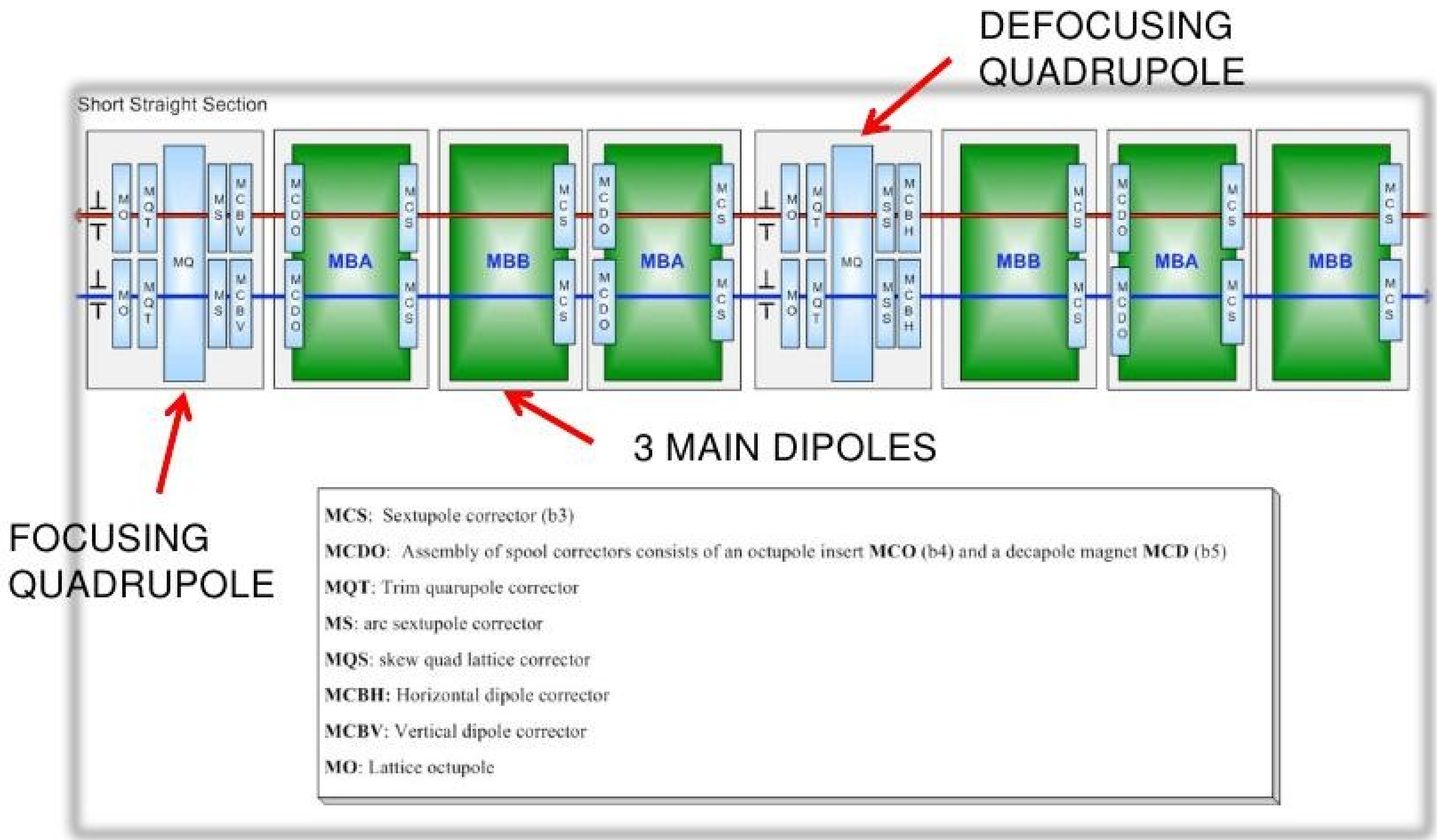


Animation taken from The Physics of Accelerators
Slides by C.R. Prior Rutherford Appleton Laboratory and Trinity College, Oxford

LHC FODO LATTICE CELL (106.9 m)

The pattern of bending and focusing magnets is called lattice.



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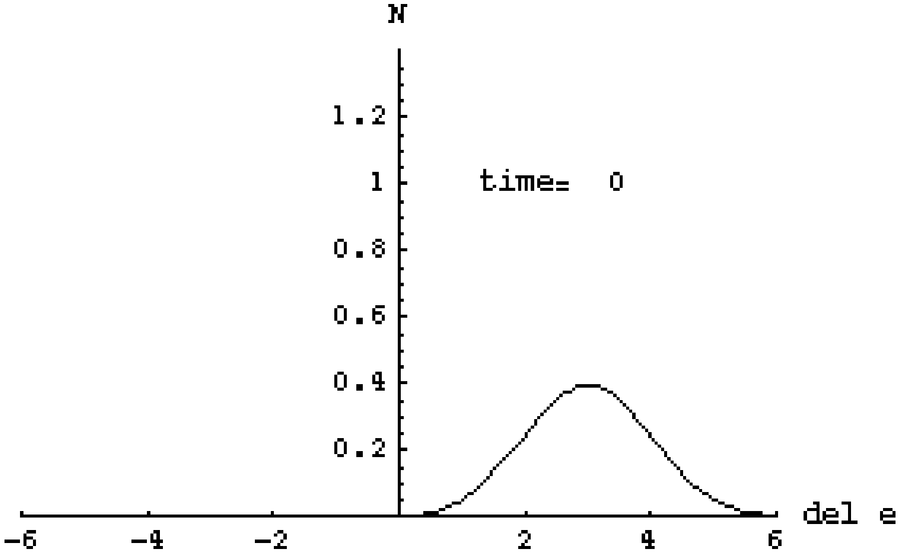
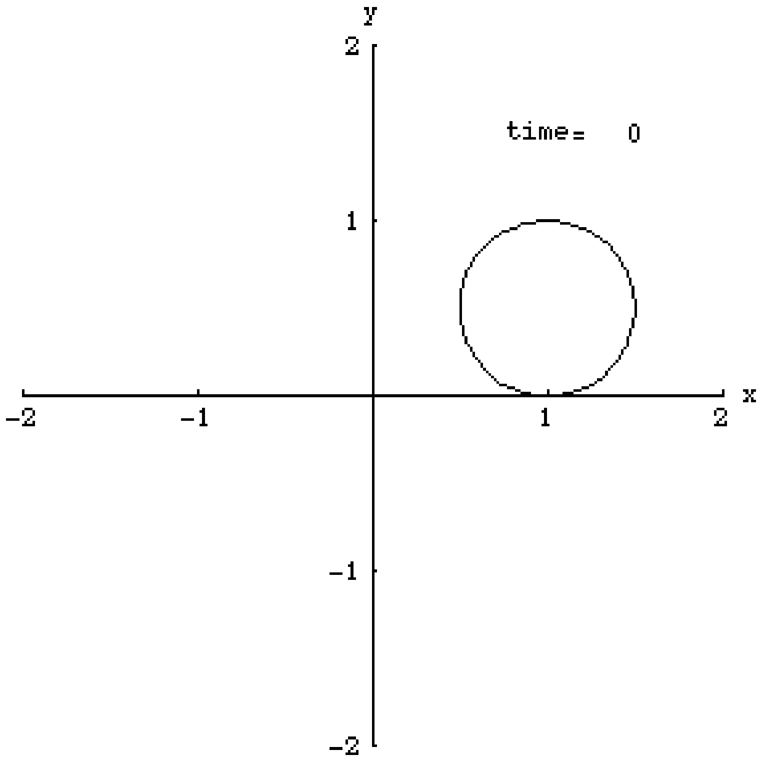
Energy adjusting by AC (longitudinal) & transverse strong focusing

Longitudinal



Transverse

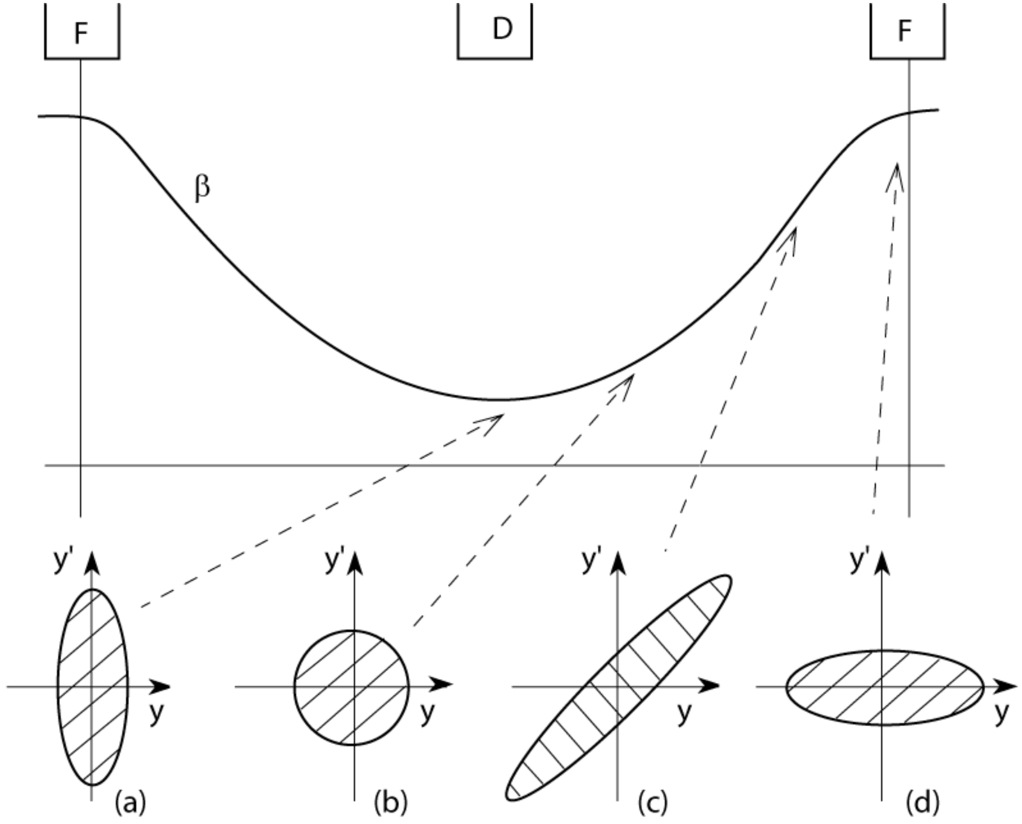


- “Catching the beam” animations taken from
  - http://www.lns.cornell.edu/~dugan/USPAS/

Beam dynamics

- Phase space limits
- The beta function
- Focusing the beam at the interaction point

Phase space limit (1/2)



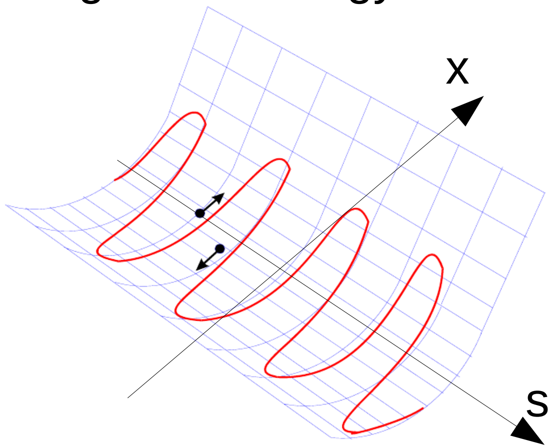
- Liouville's theorem states that for most beams the phase space area cannot change

Phase space limit (2/2)

- In reality the limit is on p_y*y:
  - Constant = p_y * y ~ γmβ_y * y where β_y= dy/dt
    ~ γmβ * y' * y where y' = dy/dz
  - So the phasespace limit implies that the area of the phase space ellipse for y'*y (the beam emittance) decreases as 1/p
  - This is called adiabatic damping. The physical size of the beam decreases as it is accelerated. The width decreases as 1/√p [the other 1/√p is the decreasing divergence].

The beta function (1/2)

The “gutter” analogy



Based on:

ACCELERATORS FOR PEDESTRIANS

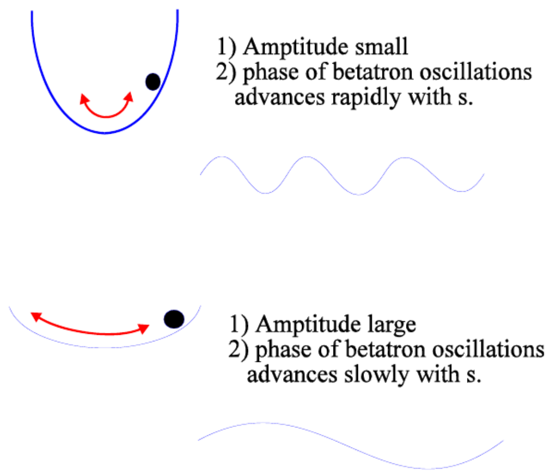
Simon Baird

$$x = \sqrt{\varepsilon \beta(s)} \cos(\Psi(s) + \varphi)$$

- The idea is to separate the transverse motion into two parts:
  - The initial conditions: ε (emittance) and φ
  - A part depending the focusing and de-focusing by magnets: β(s) and Ψ(s)
    - The β(s) function depends on where in the accelerator we are and <u>not related to the velocity β</u>)

The beta function (2/2)

The “gutter” analogy



Based on:
ACCELERATORS FOR PEDESTRIANS

Simon Baird

$$x = \sqrt{\varepsilon \beta(s)} \cos(\Psi(s) + \phi)$$

$$x' = \sqrt{\frac{\varepsilon}{\beta(s)}} \sin(\Psi(s) + \phi)$$

- So the beta function is related to how strong focusing we do and is the one we want to optimize in particular at the interaction points
- Note that x*x'~ε so that the phase space area is still the same

Recall lecture 1 and 2

- Intensity or brightness of an accelerator

$$N = \mathcal{L} \cdot \sigma$$

- Events Seen = Luminosity * cross-section

Rare processes (fb) need lots of luminosity (fb^{-1})

- In a storage ring

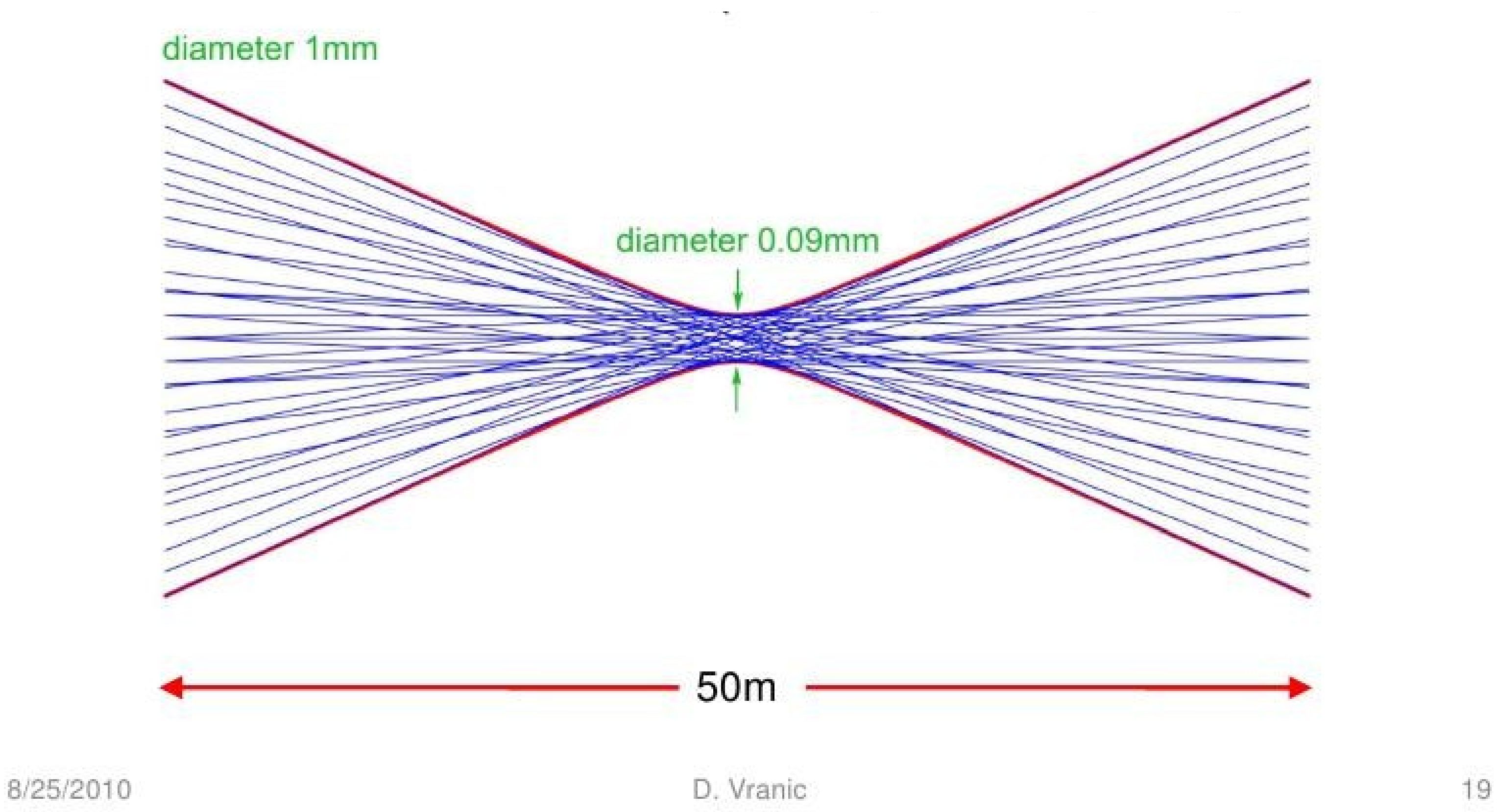
$$\mathcal{L} = \frac{1}{4\pi} \frac{f \cdot N_1 \cdot N_2}{\sigma_x \cdot \sigma_y}$$

"Current"

"Spot size"

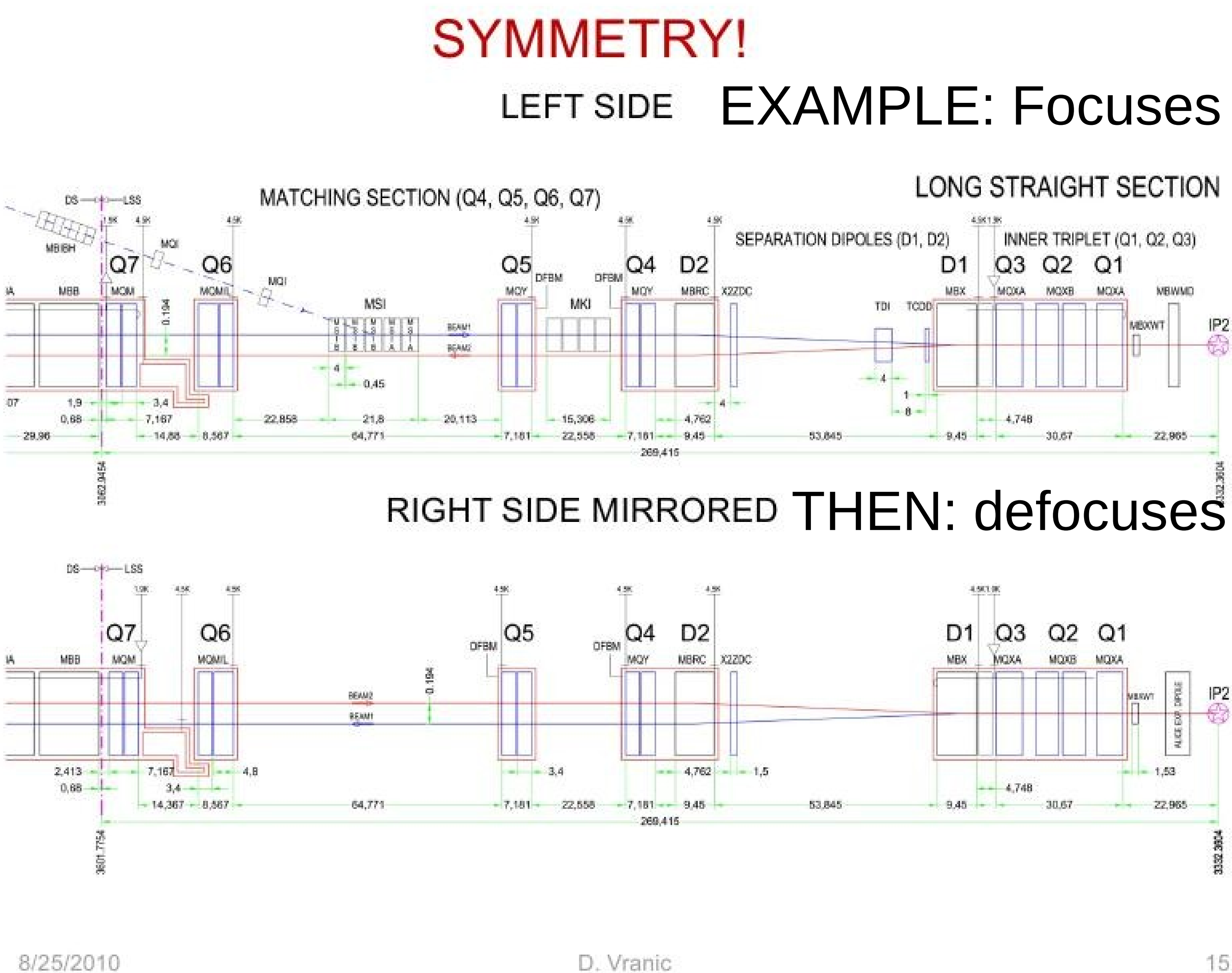
Where *f* is the revelation frequency multiplied by # of colliding bunches
More particles through a smaller area means more collisions

Example of focusing for collisions at P2 (ALICE)



EXAMPLE: Focuses beam!

THEN: defocuses beam!



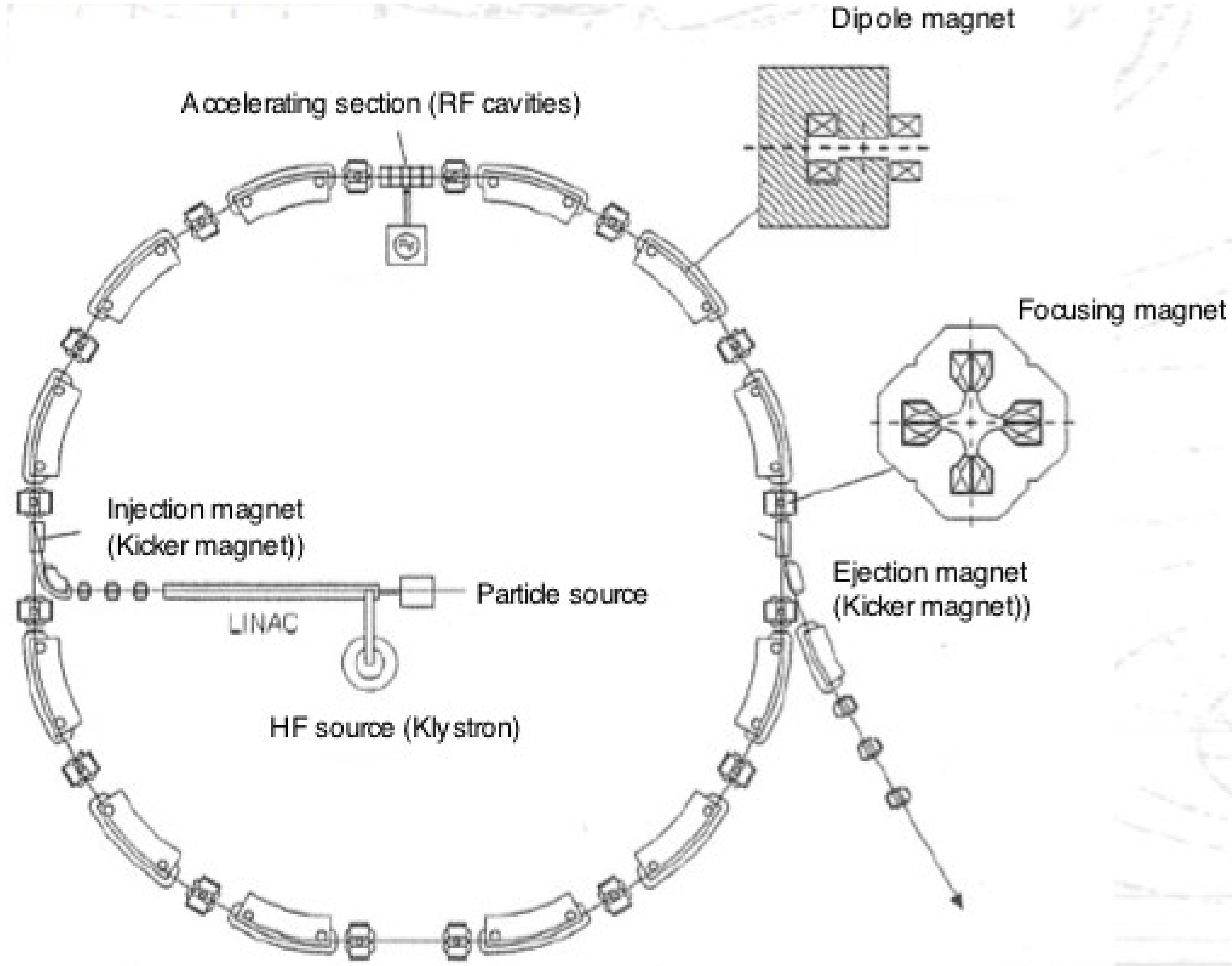
Examples of synchrotrons

Synchrotrons

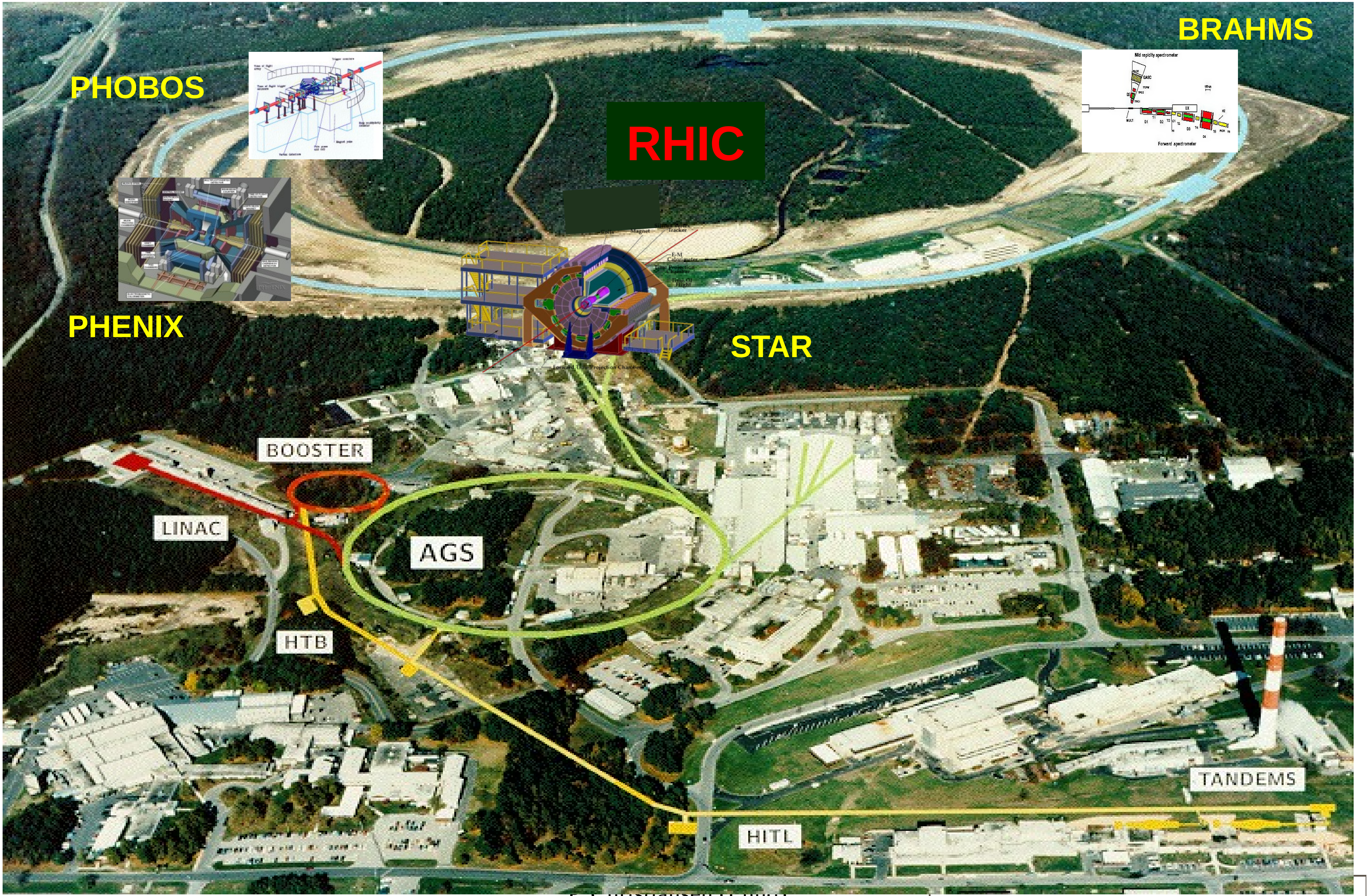
Use smaller magnets in a ring + accelerating station

3 GeV protons
BNL 1950s

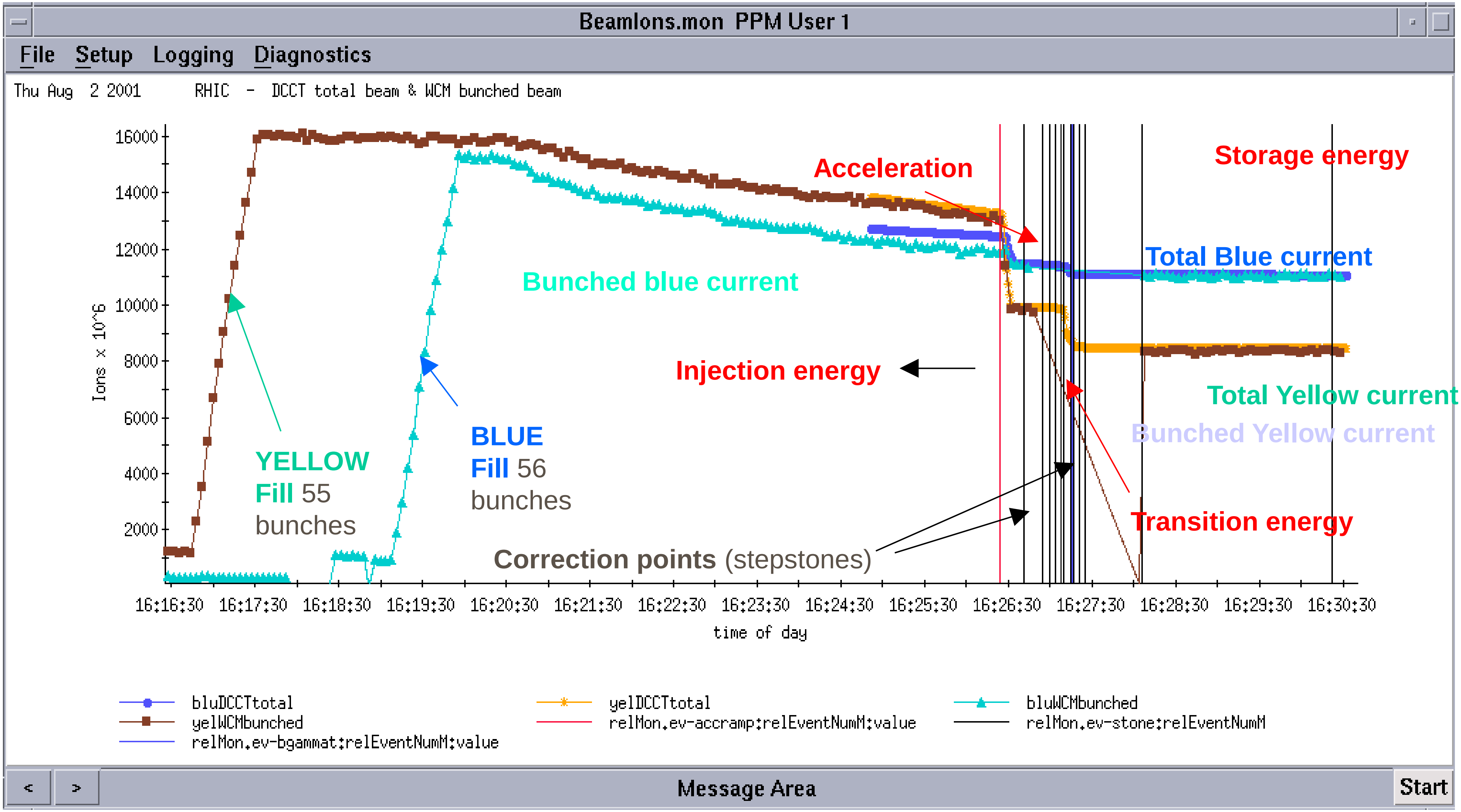
Basis of all circular machines built since



at Brookhaven National

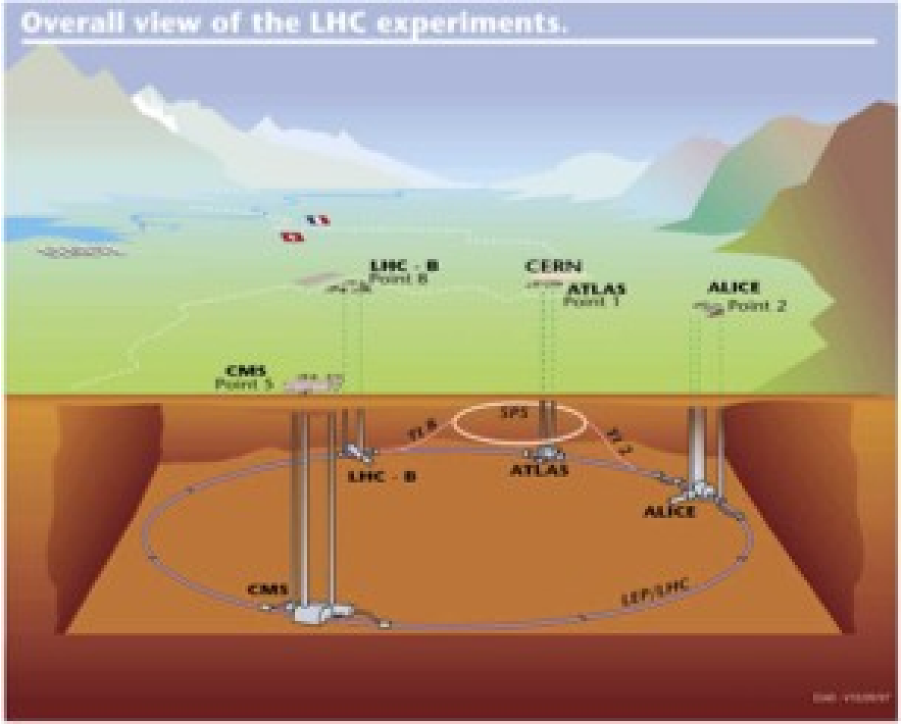


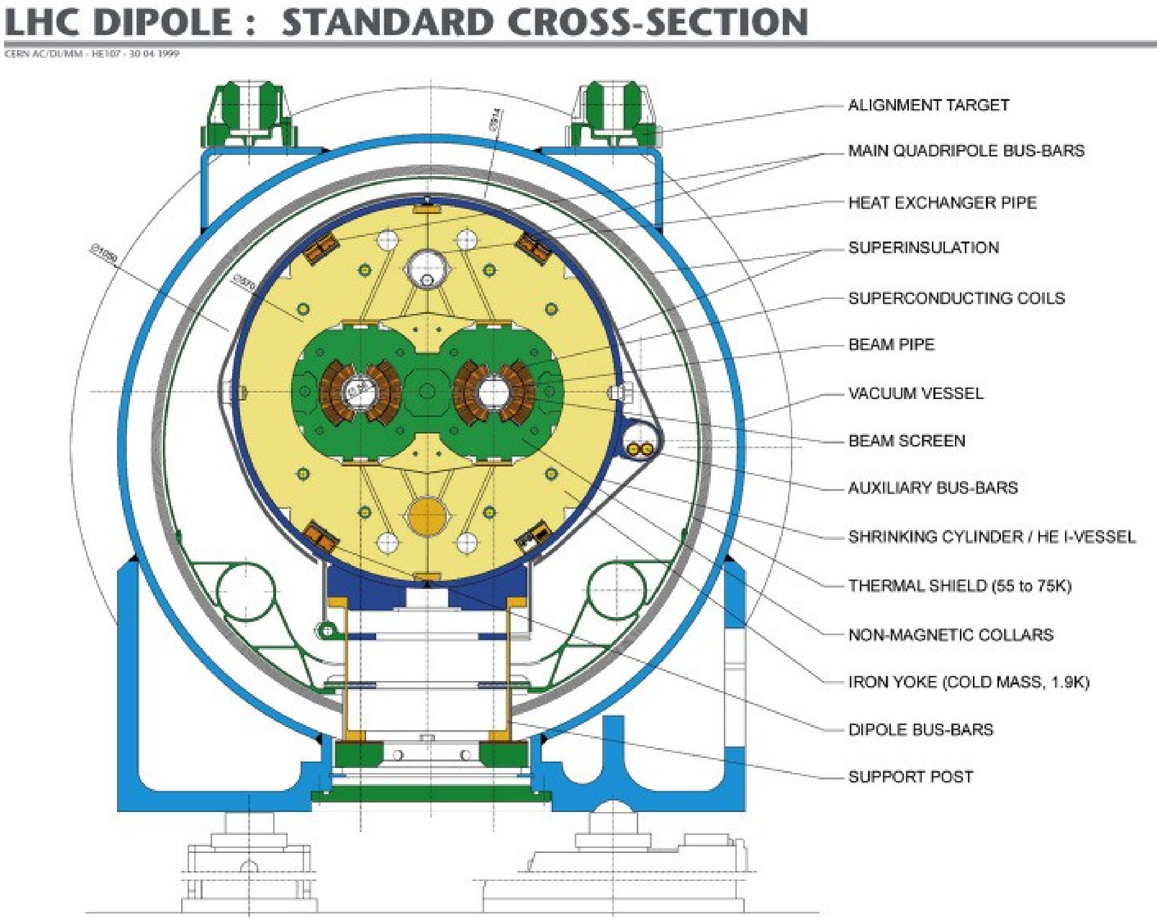
RHIC ramp with 56 bunches

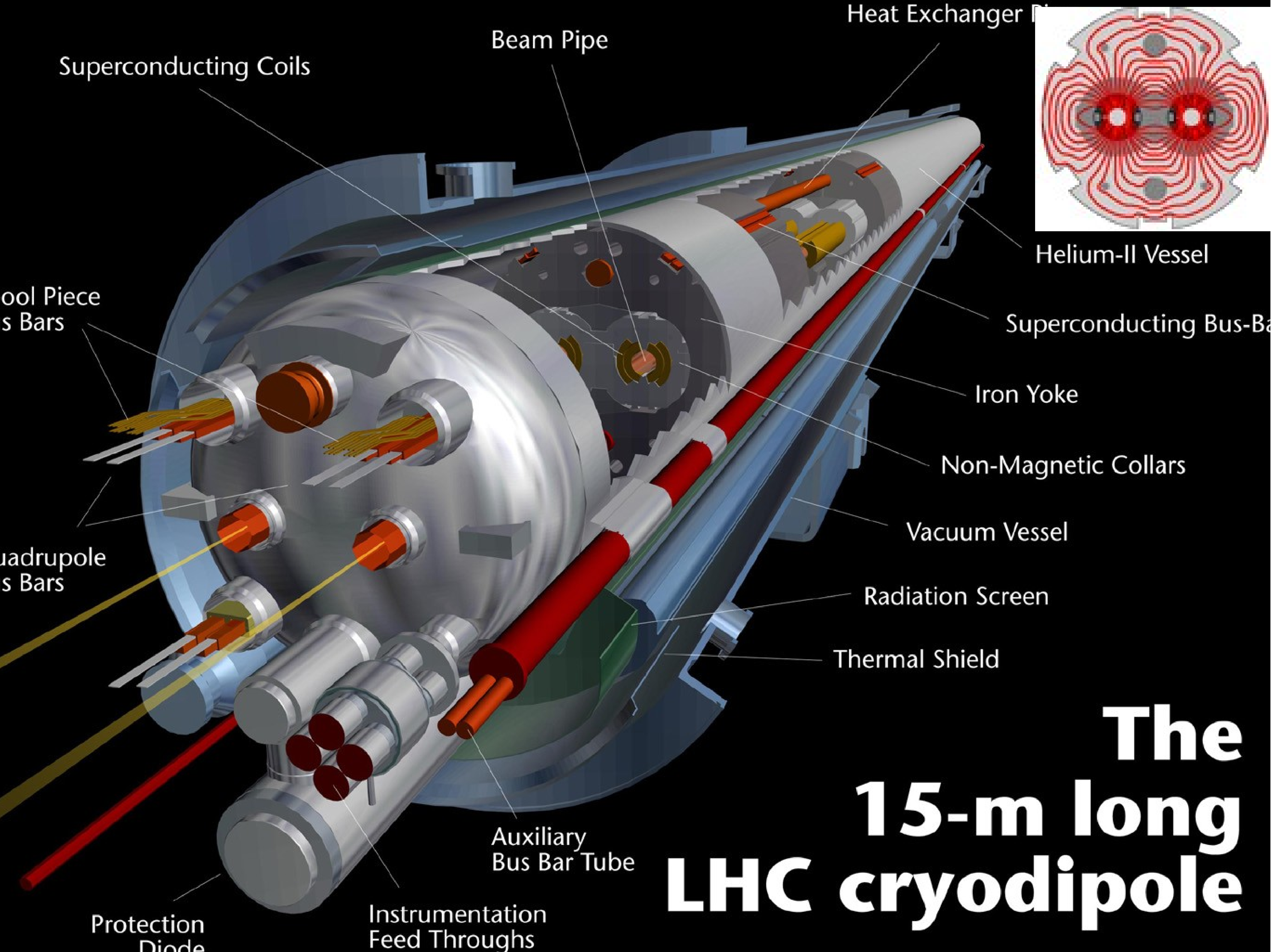


The beam is accelerated from Injection Energy (10 GeV) to Storage Energy (100 GeV). The acceleration process is called “ramp”.

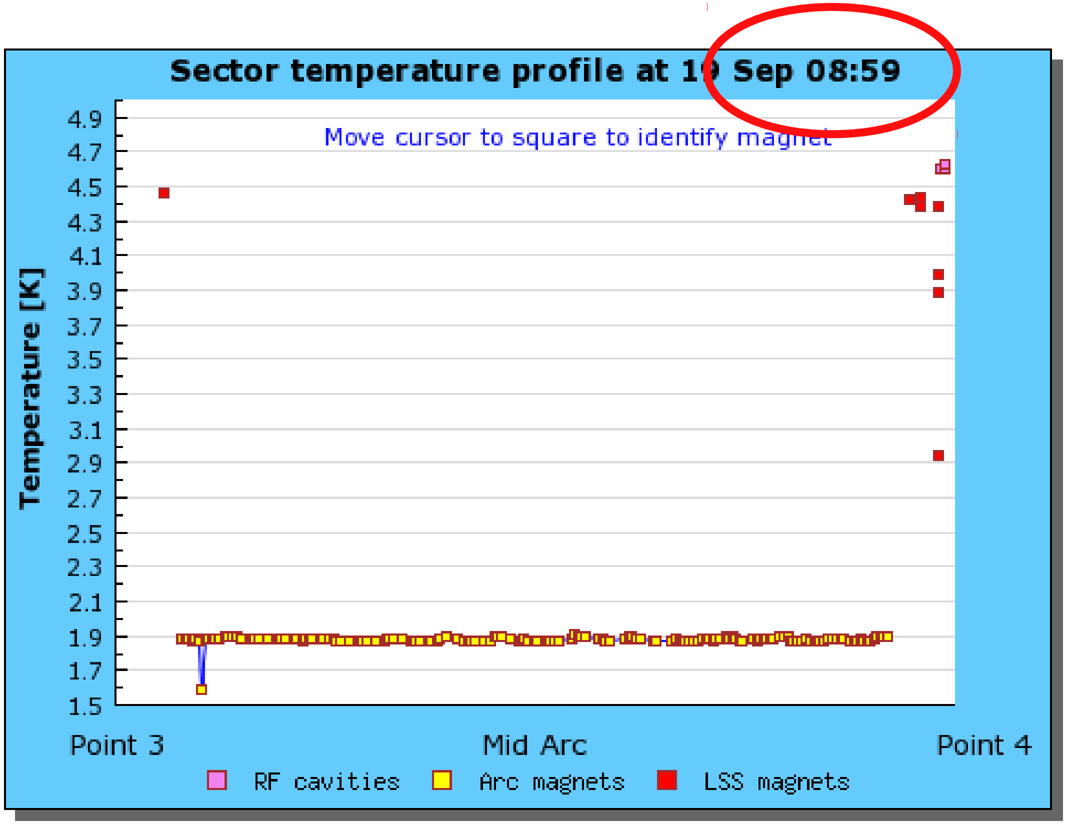
CERN Large Hadron Collider






Heat Exchanger
Beam Pipe
Superconducting Coils
Helium-II Vessel
ool Piece
s Bars
Superconducting Bus-Ba
Iron Yoke
Non-Magnetic Collars
Vacuum Vessel
uadrupole
s Bars
Radiation Screen
Thermal Shield
Auxiliary
Bus Bar Tube
Protection
Instrumentation
Feed Throughs
The
15-m long
LHC cryodipole

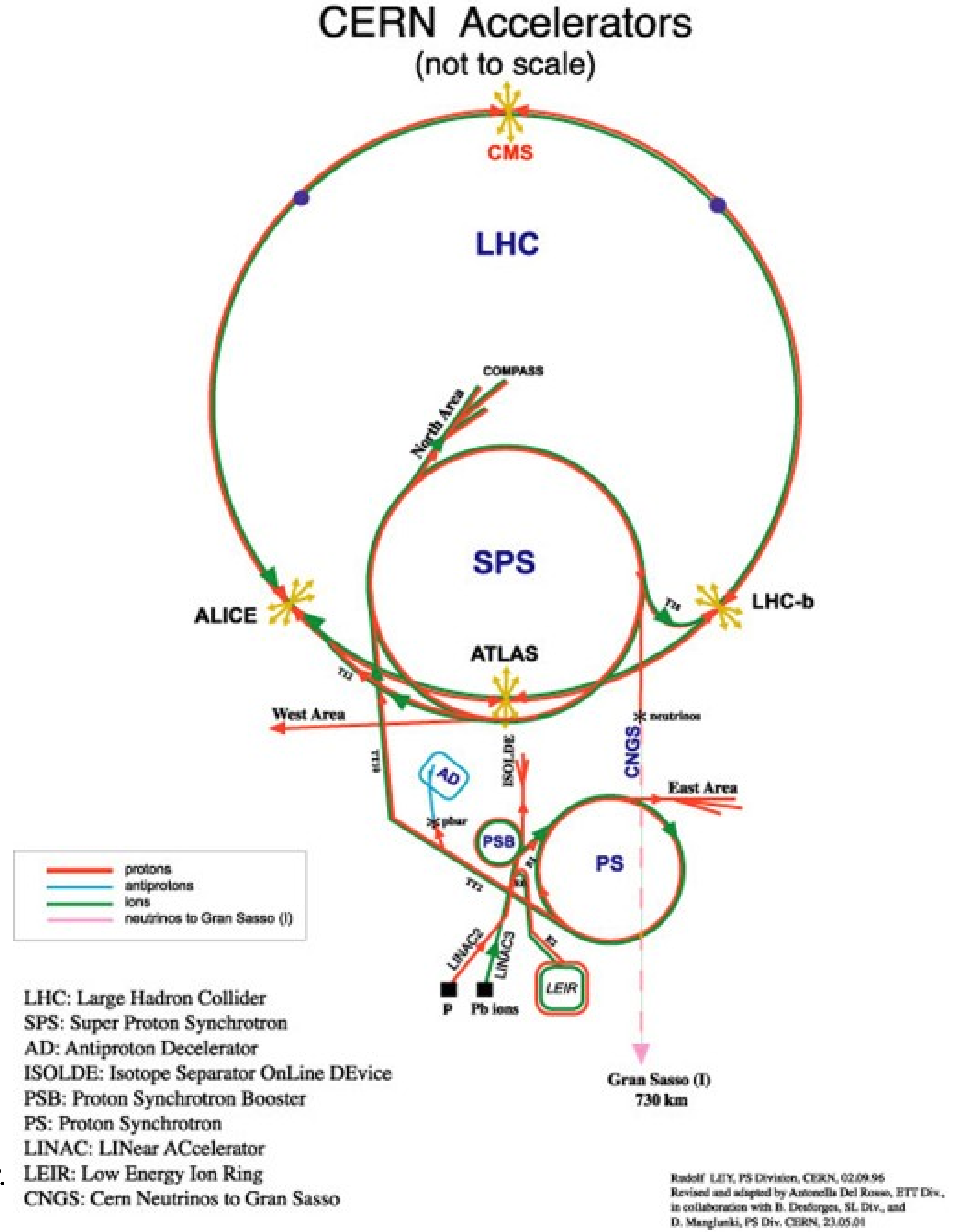
The 19 September 2008 accident



THE 19 SEPTEMBER 2008 INCIDENT

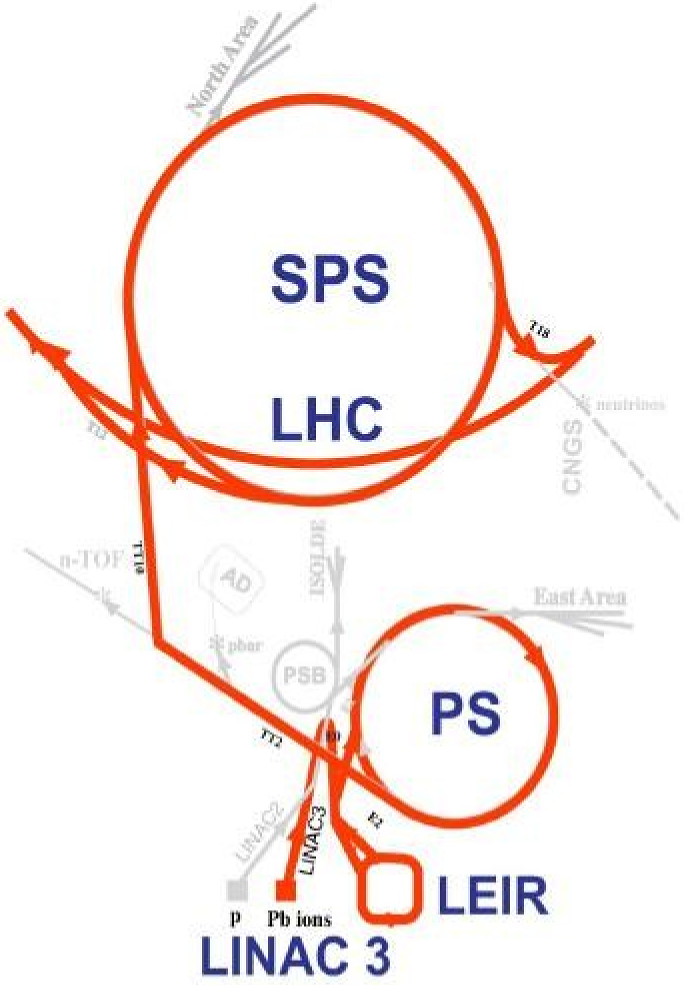
CERN Complex

Old rings still in use
Many different programs

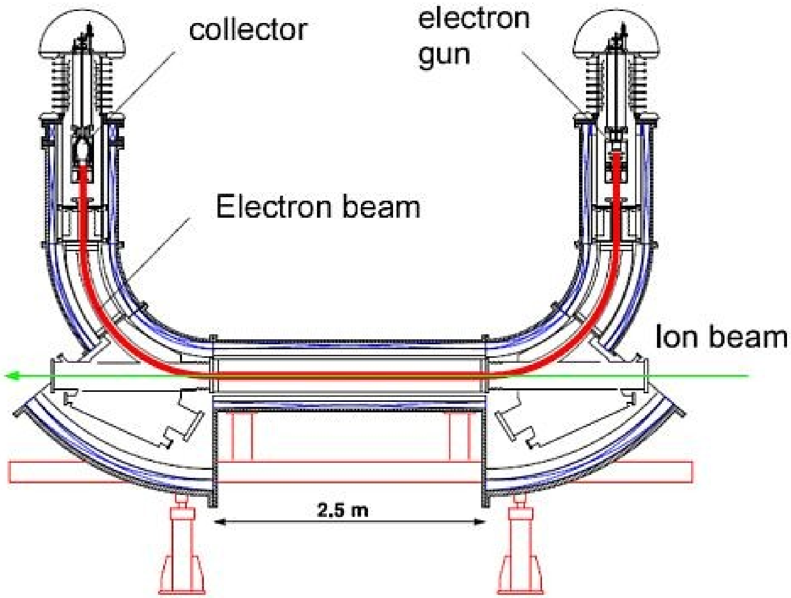


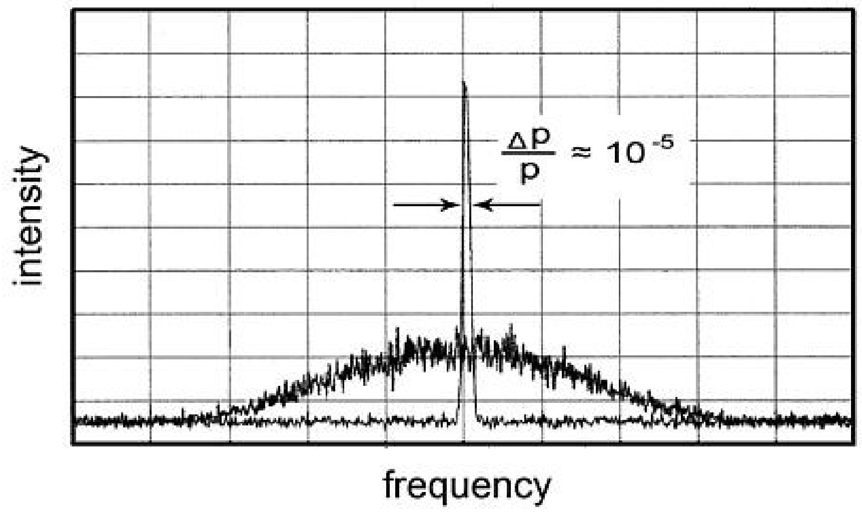
ION BEAM IN THE LHC

- ECR ion source
  - Provide highest possible intensity of Pb29+
- RFQ + Linac 3
  - Adapt to LEIR injection energy
  - strip to Pb54+
- LEIR
  - Accumulate and cool Linac 3 beam
  - Prepare bunch structure for PS
- PS
  - Define LHC bunch structure
  - Strip to Pb82+
- SPS
  - Define filling scheme



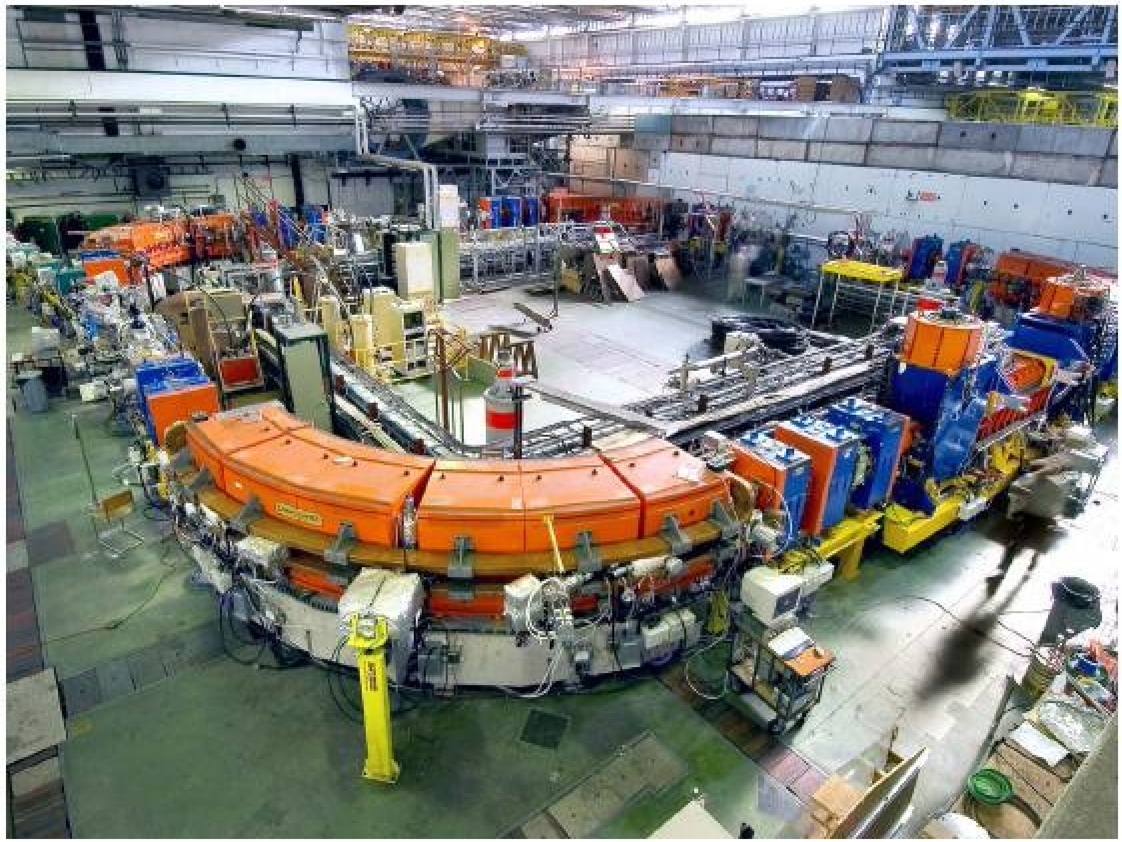
LEIR: Electron cooling example





- From: http://web-docs.gsi.de/
- Elastic collision e+ion will decrease the relative momentum spread in the beam

LEIR



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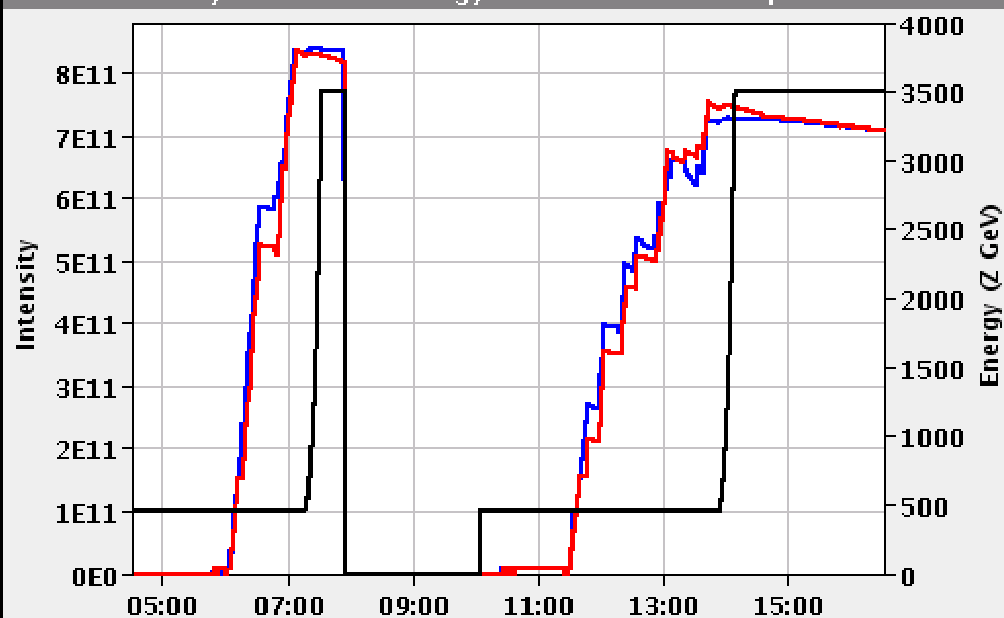
LHC Page1 | Fill: 1530 | E: 3500 Z GeV | 29-11-2010 16:30:51

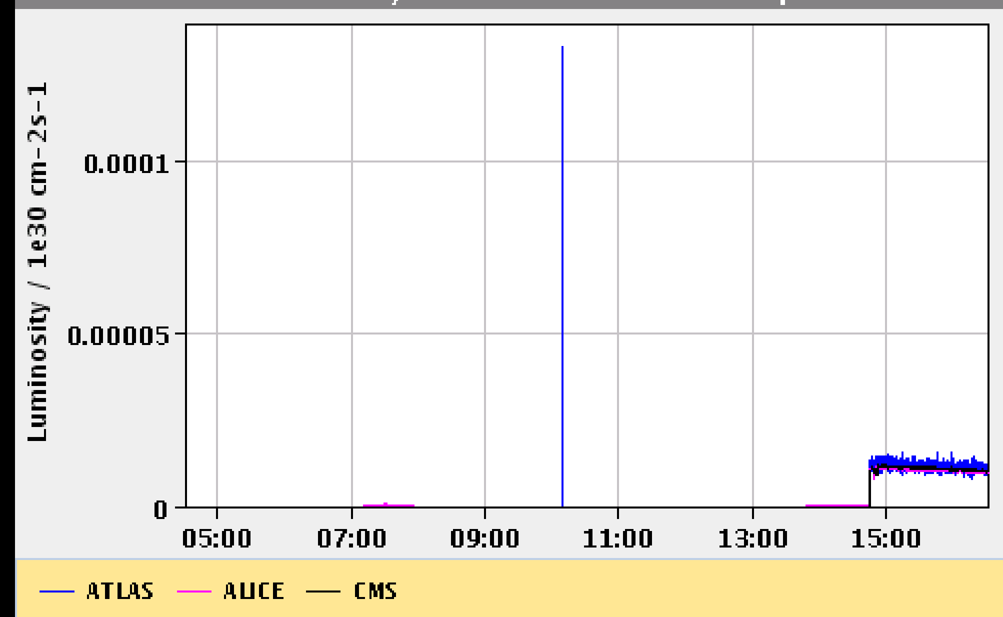
ION PHYSICS: STABLE BEAMS

Energy:	3500 Z GeV	I(B1):	8.32e+11	I(B2):	7.57e+11

FBCT Intensity and Beam Energy — Updated: 16:30:50



Instantaneous Luminosity — Updated: 16:30:48



Comments 29-11-2010 14:54:46 :

*** STABLE BEAMS ***

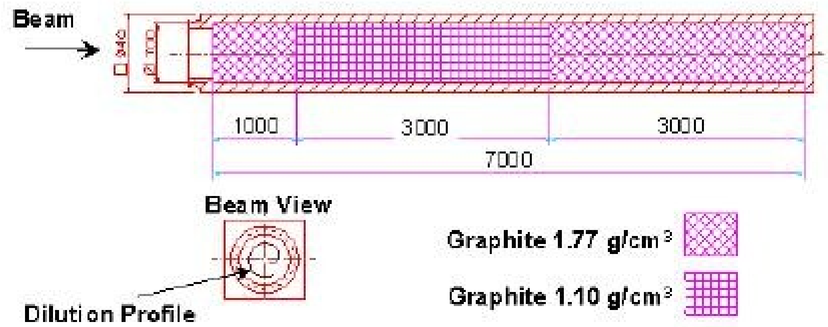
All points optimized

BIS status and SMP flags	B1	B2
Link Status of Beam Permits	true	true
Global Beam Permit	true	true
Setup Beam	false	false
Beam Presence	true	true
Moveable Devices Allowed In	true	true
Stable Beams	true	true

AFS: 500ns_121b_113_114_0_4bpi31inj_IONS

PM Status B1: ENABLED | PM Status B2: ENABLED

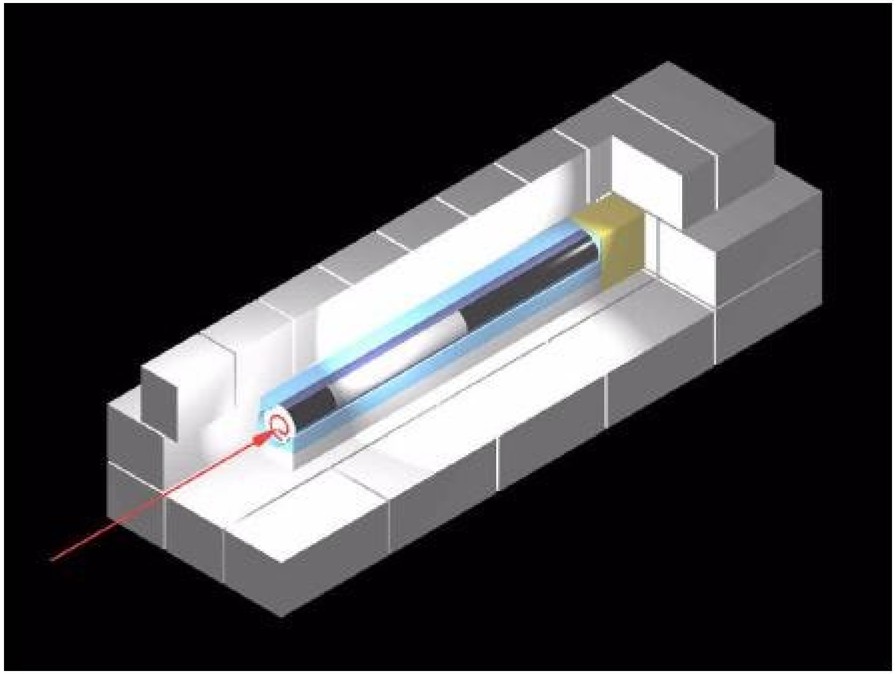
DUMP CORE TDE



7m long C / C-C TDE in steel shrink-cylinder, followed by 1m Al, 2m Fe
~1000 T of concrete shielding

This is the **ONLY element in the LHC that can** withstand the impact of the full 7 TeV beam !
Nevertheless, the dumped beam must be painted to keep the peak energy densities at a tolerable level !

Why graphite? If the material were heavy, all the beam's energy would concentrate in the first half meter of the block.



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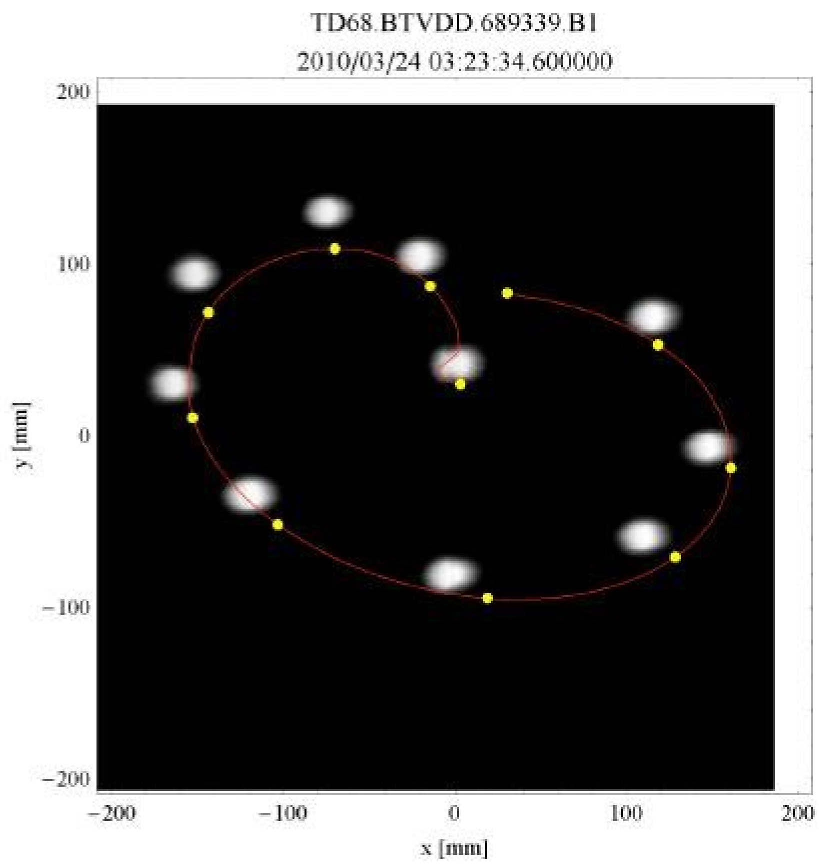
The beam size has increased to an extent where the sigma is 1.6mm in both planes.



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