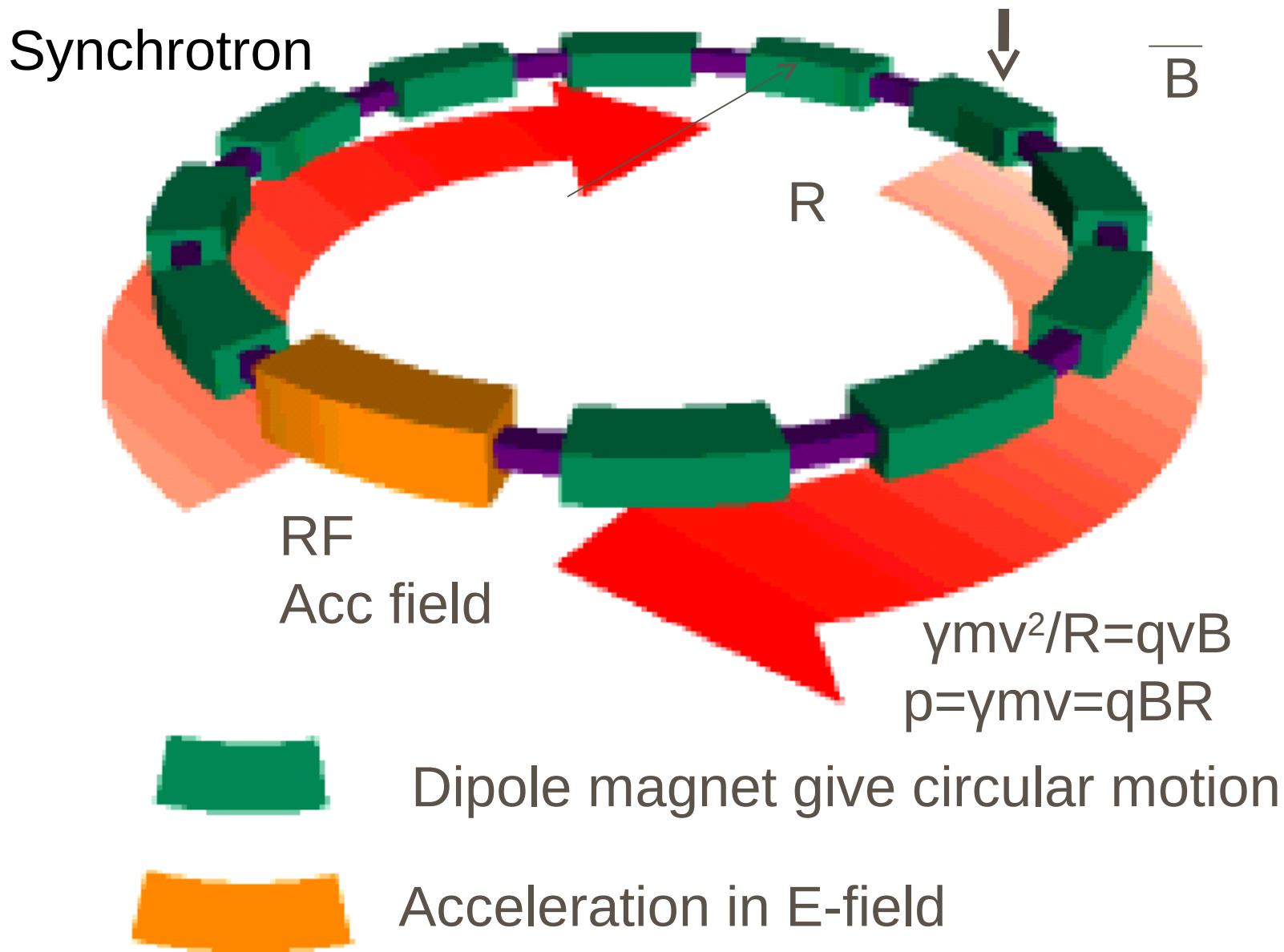


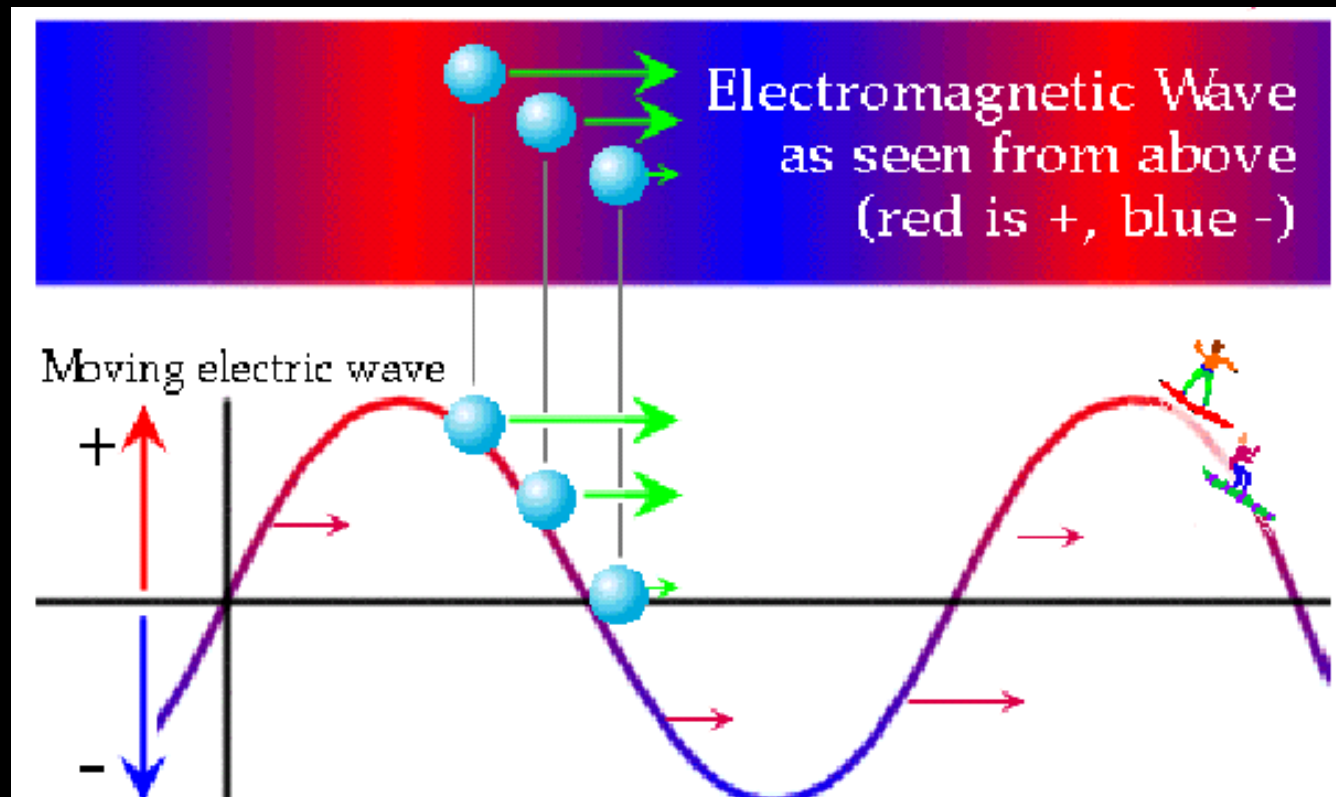
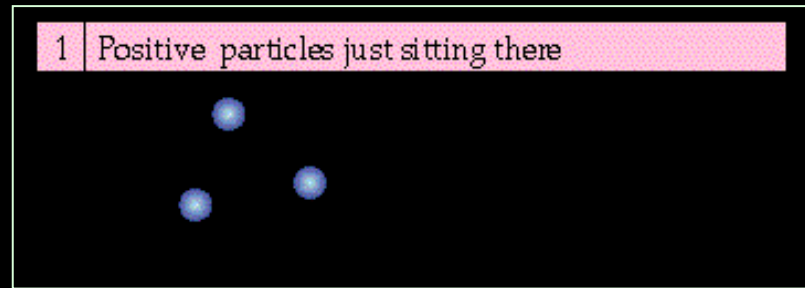
# Lectures on accelerator physics

- Lecture 5 and 6: Advanced topics
  - Transverse motion, strong focusing, and LHC
- Material borrowed from
  - Lecture by Anders Oskarsson
  - Lecture by Eric Torrence (University of Oregon)
  - **LHC lectures by Danillo Vranic (GSI)**
- Weak focusing follows “Principles of Charged Particle Acceleration” by Stanley Humpries Jr. Chapter 7.

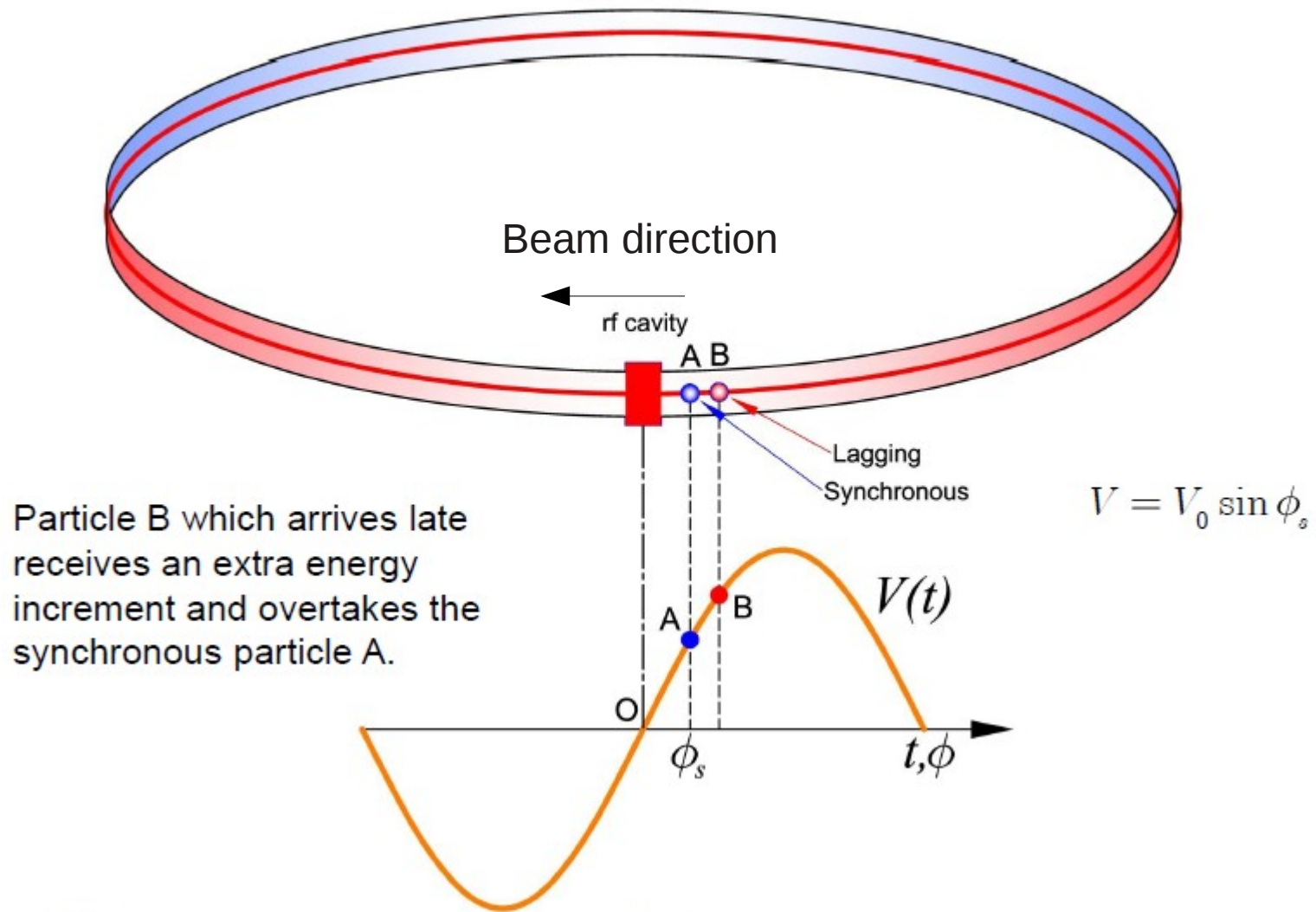
# Towards the synchrotron



# The alternating E-field keeps particles in bunches



# LONGITUDINAL DYNAMICS

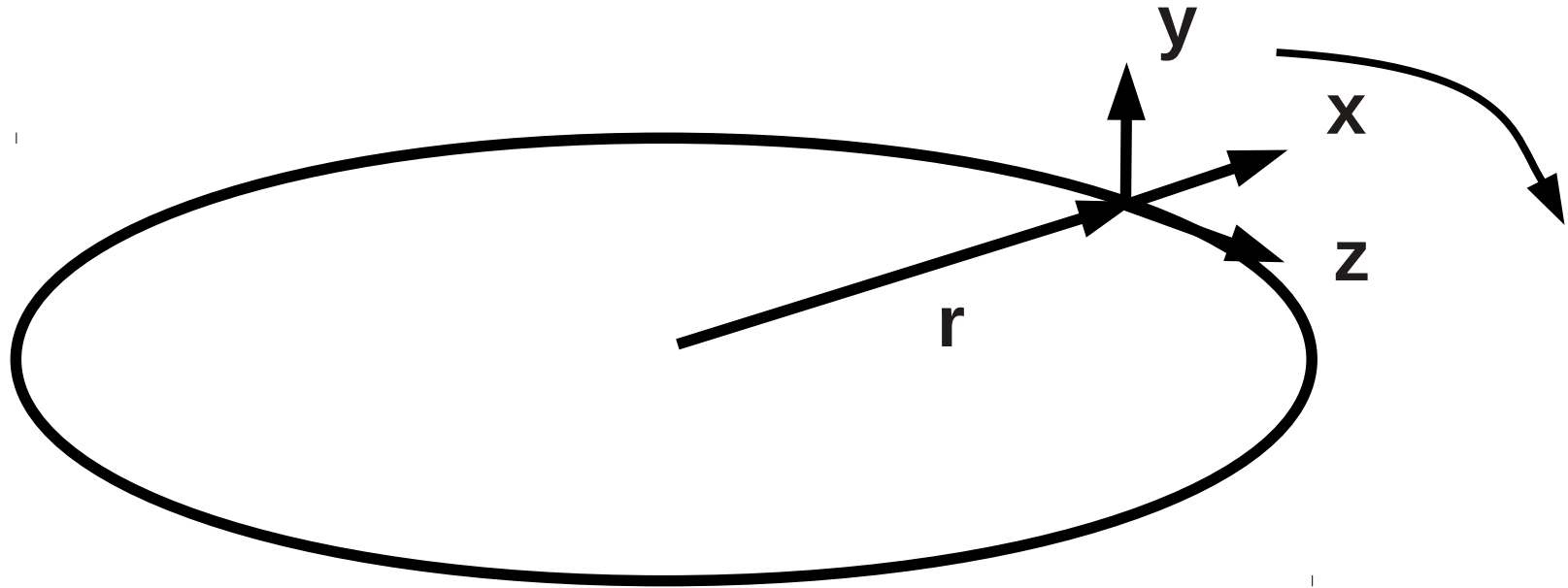


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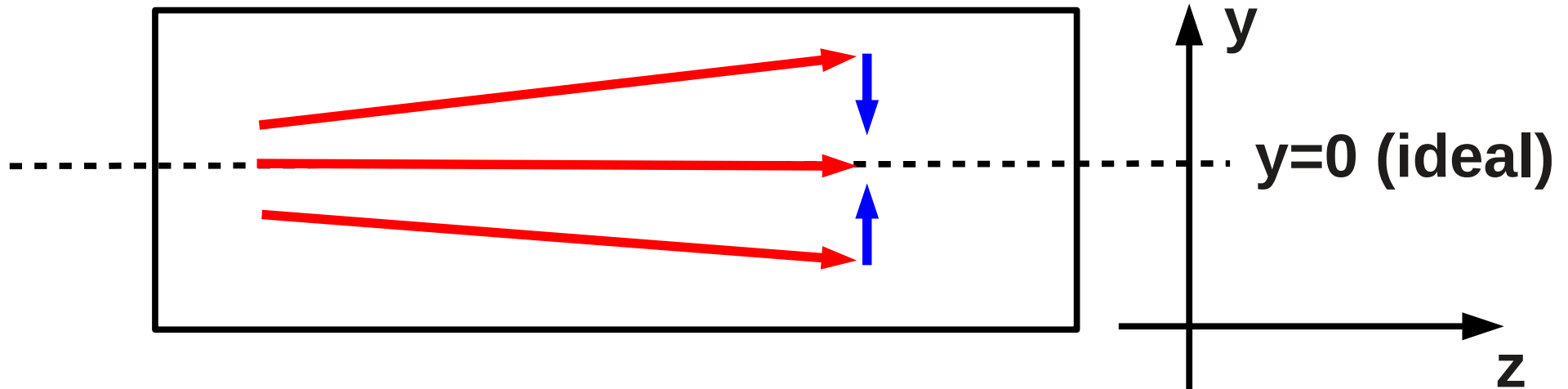
# Focusing in the transverse plane



- Assume  $(v_x, v_y, v_z) \sim (0, 0, v)$  and  $v \sim c = \text{constant!}$ 
  - Very good assumption!
- $z = vt \rightarrow t = z/v \ (\sim z/c)$ 
  - $d/dt \sim v \ d/dz \ (\sim c \ d/dz)$

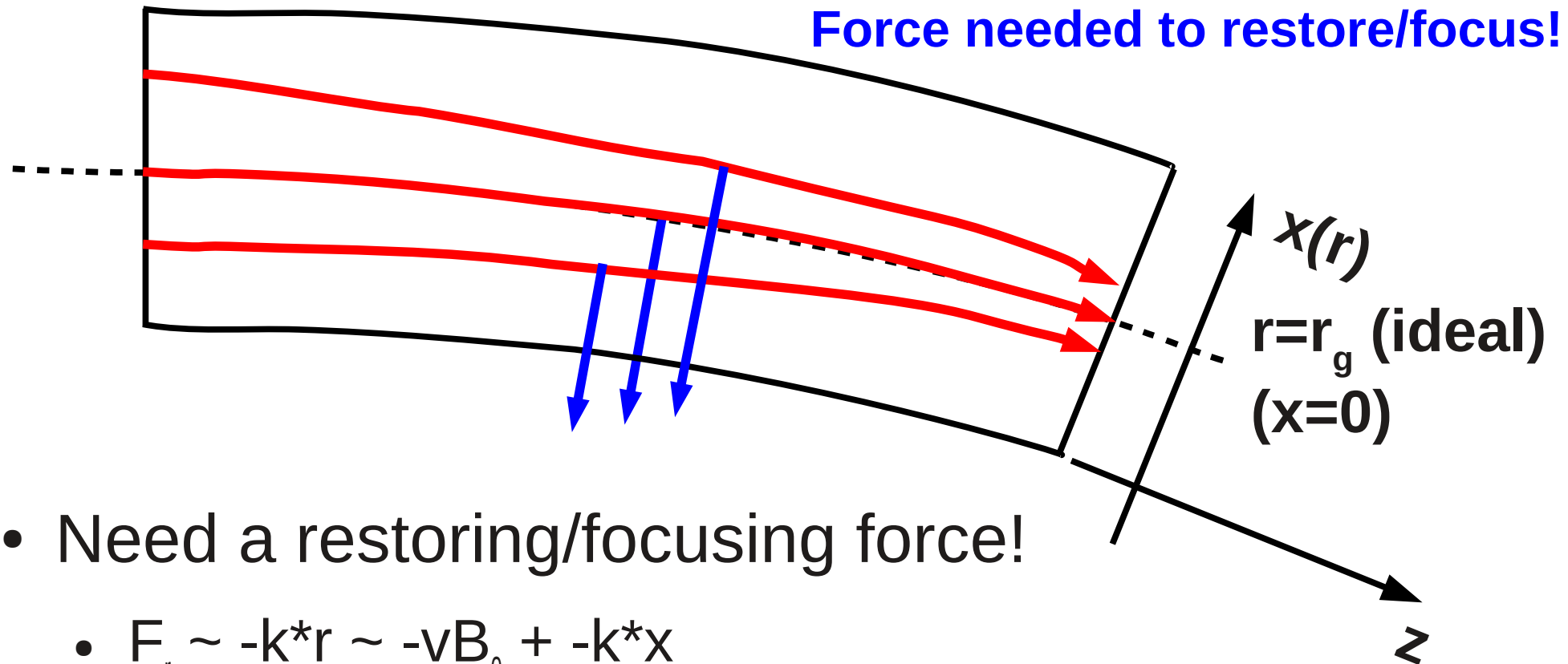
# Transverse focusing in y direction

Force needed to restore/focus!



- Need a restoring/focusing force!
  - $F_y \sim -k*y$
- Harmonic oscillator (like string)

# Transverse focusing in x direction



- Need a restoring/focusing force!
  - $F_r \sim -k*r \sim -vB_0 + -k*x$
  - = central force ( $r_g$ ) + harmonic oscillator in  $x$

# Let us first solve harmonic equation (ignoring magnet realities!)

$$\gamma m \frac{d^2 y}{dt^2} = \gamma m v^2 \frac{d^2 y}{dz^2} = -ky$$

$$y(z) = y_0 \cos\left(\frac{2\pi}{\lambda} z + \varphi\right),$$

where

$$\lambda = 2\pi \sqrt{\frac{\gamma m v^2}{k}}.$$

- Note that the wavelength does not depend on the amplitude  $y_0$ . There is only one wavelength for all amplitudes!



# Tune interlude

- One defines
  - $Q$  ( $\nu$ [nu]) =  $C/\lambda$ , where  $C=2\pi r_g$  is the circumference of the synchrotron ring
- $Q$  is the number of transverse (betatron) oscillations per turn
- It is different for  $x$  and  $y$  directions
- Very important for beam stability!

# TUNE

The tune is the **number of betatron oscillations per turn.**

It is very important that tune is not integer or a simple fraction

$$Q \neq \frac{p}{n} \quad (\text{where } n \text{ and } p \text{ are integers})$$

otherwise, over one or more revolutions, particle will repeat its path in the accelerator and 'see' the same field imperfections. These will then build up a resonant growth and beam will be lost.

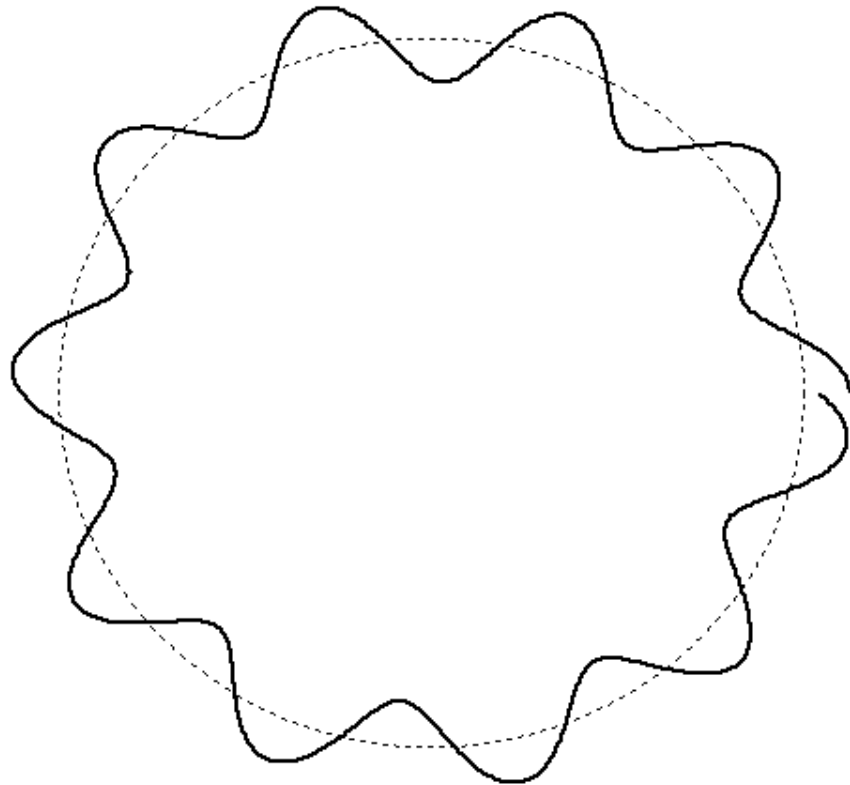
## NO HARMONY!

REMARK:

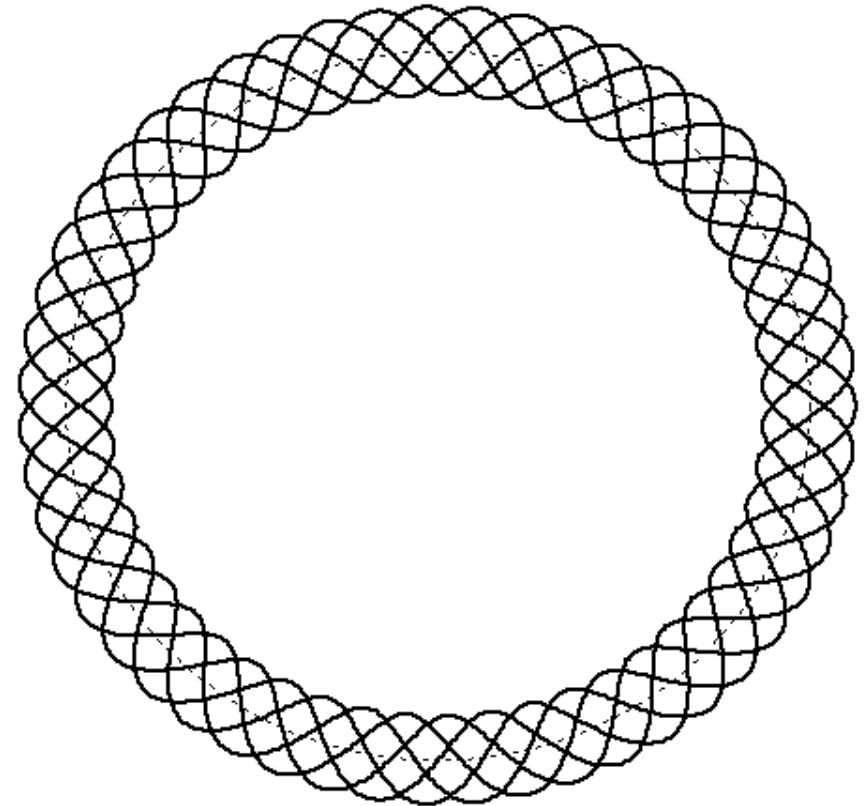
As opposite to old 'theories' about solar system, there is no 'harmony' and that is the reason why it lasts so long. Planet between Mars and Jupiter was 'in the harmony'.

# Bad harmonic tune ( $Q=10.2$ )

1 turn



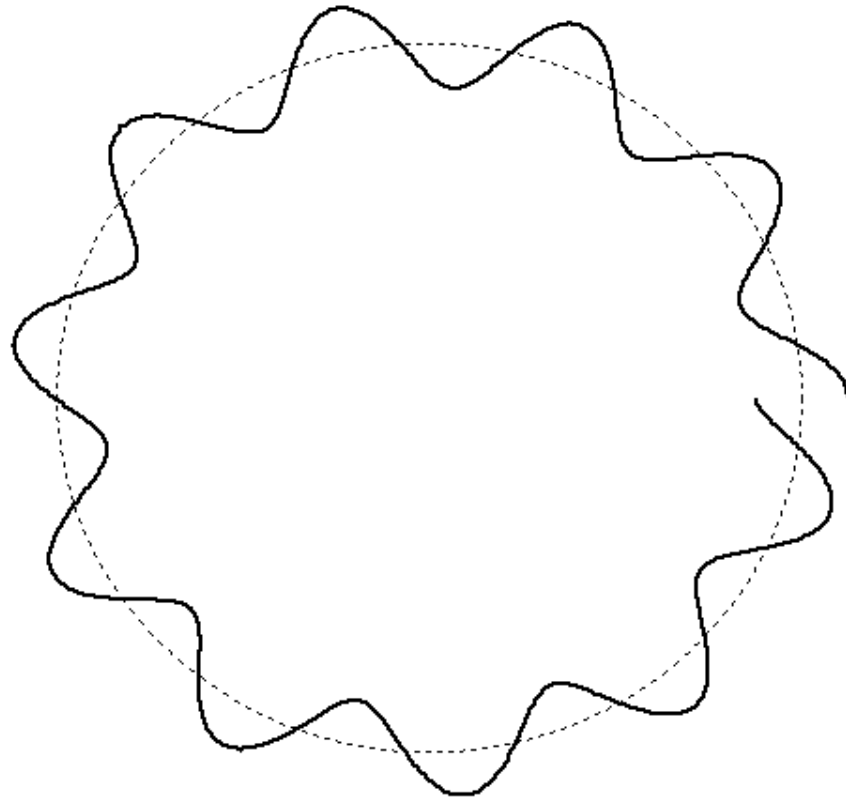
100 turns



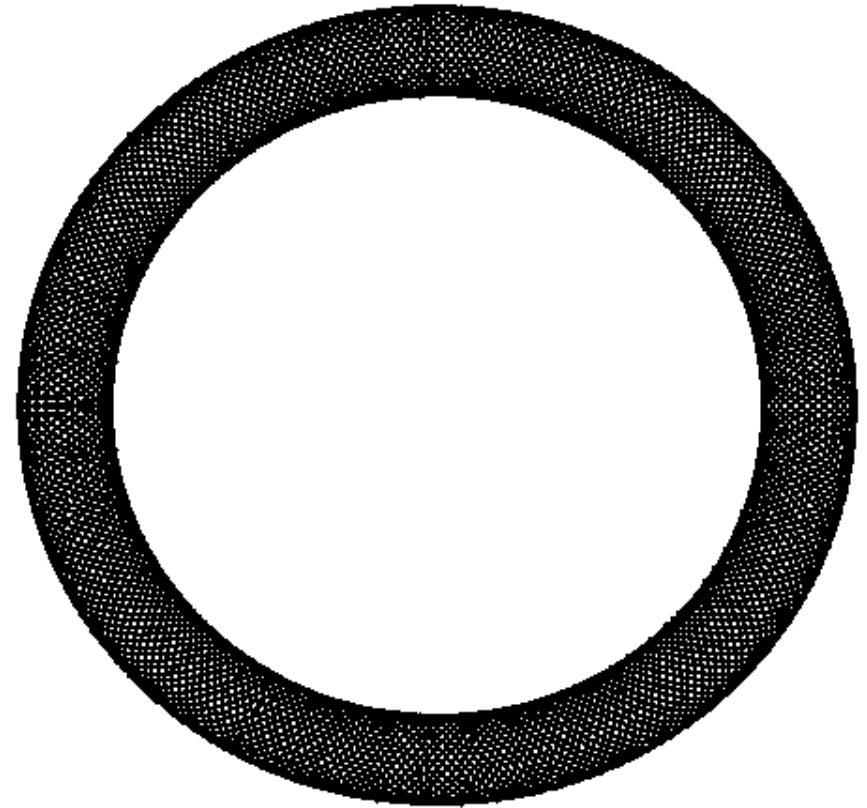
Problem: tune does not integrate out magnet imperfections

# Better (less harmonic) tune ( $Q=10.48$ )

1 turn

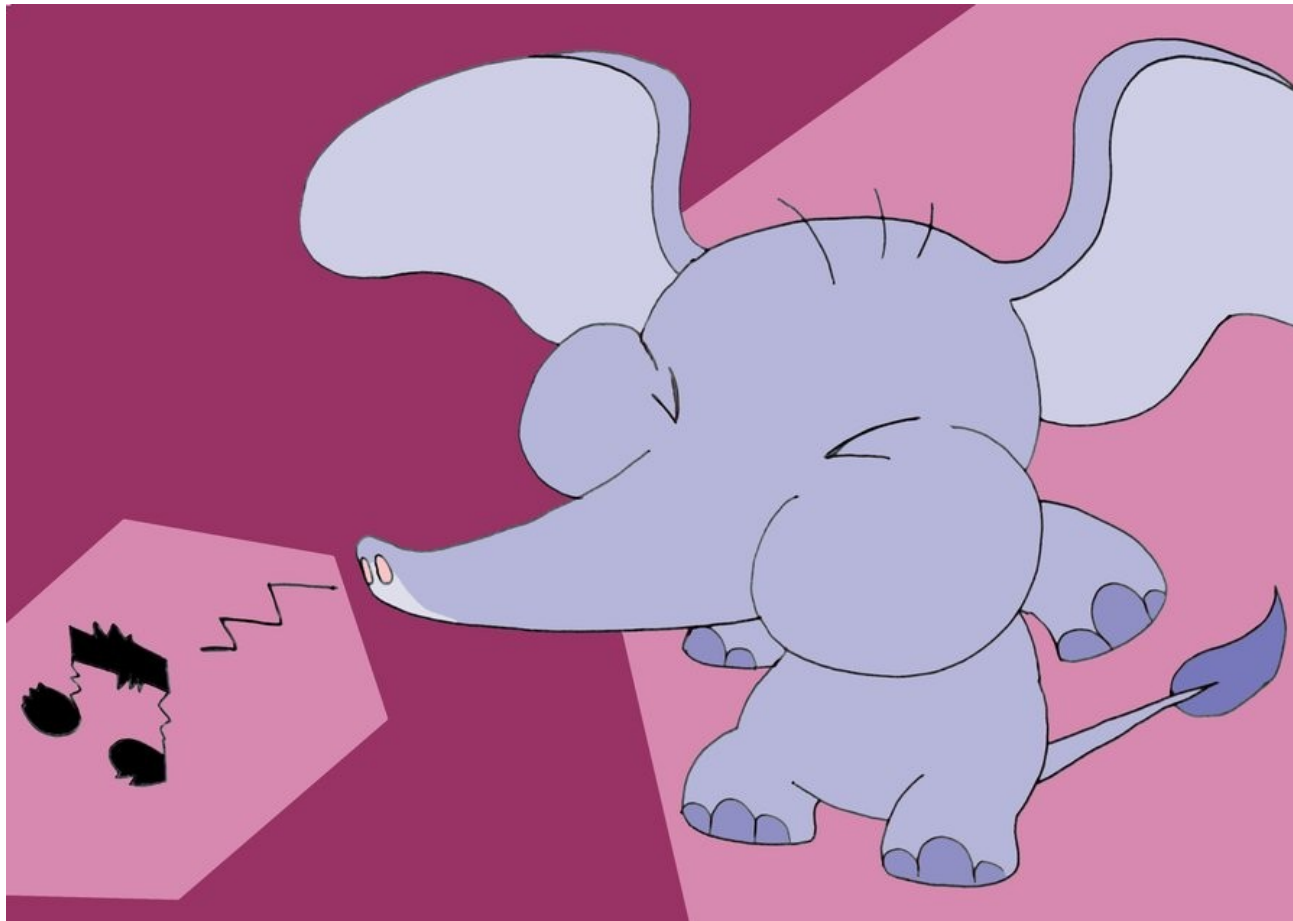


100 turns



Tune is better at integrating out magnet imperfections

# Lesson: bad musicians makes great accelerator physicists



Taken from <http://pumahtf.deviantart.com/art/False-note-184545551>

## LHC TUNES

AT 7TeV

HORIZONTAL TUNE:  $Q_x = 64.31$

VERTICAL TUNE:  $Q_y = 59.32$

$$\Delta Q \leq 3 \cdot 10^{-3}$$

AT 450GeV

HORIZONTAL TUNE:  $Q_x = 64.28$

VERTICAL TUNE:  $Q_y = 59.31$

Betatron tunes should avoid linear coupling resonances at

$$nQ_x + mQ_y = p$$

# Back to transverse motion and magnet realities!

- Taylor expanding the dipole magnetic field AND fulfilling Maxwell equations gives
  - $(B_x, B_y, B_z) \sim (-(n_0 B_0 / r_g) y, B_0 - (n_0 B_0 / r_g) x, 0)$ 
    - NB! note that  $-$  sign is not good!
- Ideally we want  $n_0$  as large as possible to confine the beam!
  - (And make the magnet as small as possible)
- Let us look at solution for  $x(r)$ !

# The equation of motion for x

$$\gamma m \frac{d^2 r}{dt^2} = \gamma m v^2 \frac{d^2 r}{dz^2} = \gamma m \frac{v^2}{r} - qvB_y$$

$$\frac{d^2 r}{dz^2} = \frac{1}{r} - \frac{q}{\gamma m v} B_y$$

Substituting  $x = r - r_g$  + expanding  $\frac{1}{r}$ :

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{1}{r_g^2} x - \frac{q}{\gamma m v} B_y$$

Inserting the Taylor expansion of  $B_y$ :

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{qB_0}{\gamma m v} - \frac{1}{r_g^2} x + \frac{qn_0 B_0}{\gamma m v r_g} x$$

The first two terms gives the solution for the ideal trajectory  $\rightarrow: \frac{1}{r_g} = \frac{qB_0}{\gamma m v}$   
so that:

$$\frac{d^2 x}{dz^2} = -\frac{1}{r_g^2} (1 - n_0) x.$$



# Weak focusing: $0 < n_0 < 1$

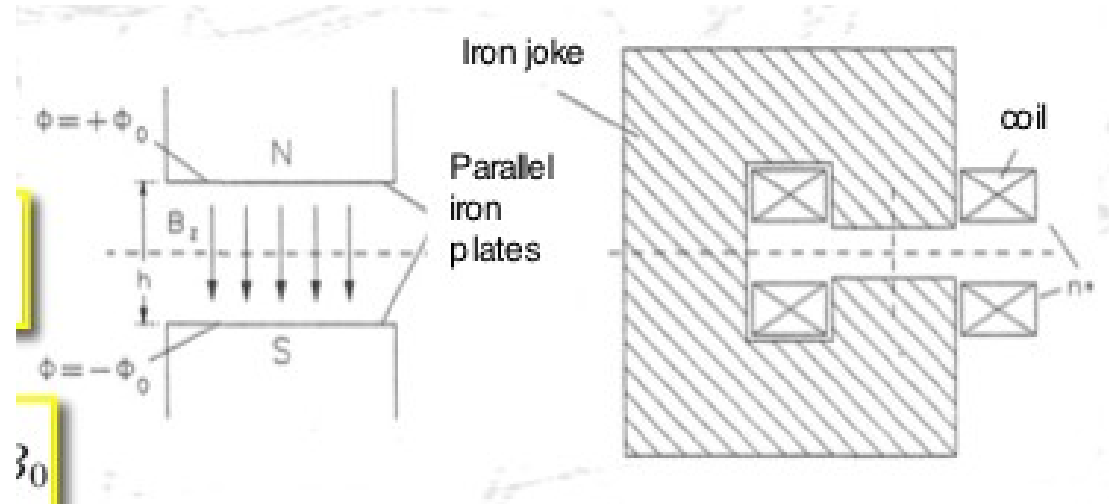
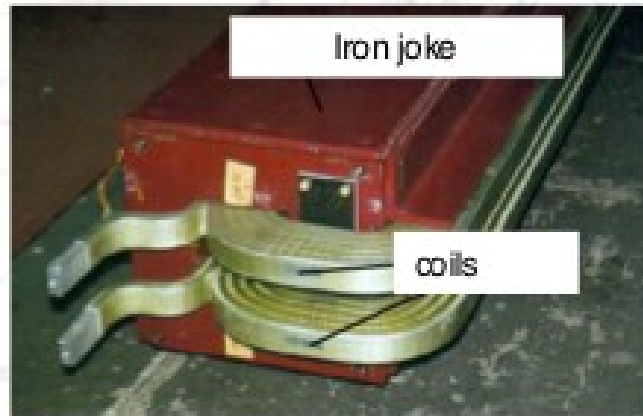
$$\frac{d^2 x}{dz^2} = -\frac{1}{r_g^2} (1 - n_0) x.$$

- Only harmonic oscillation solution when  $(1 - n_0) > 0$  (and y equation requires  $n_0 > 0$ )
  - Otherwise exponential growth!
- This means that the focusing is limited!
  - That is why this solution is called weak focusing

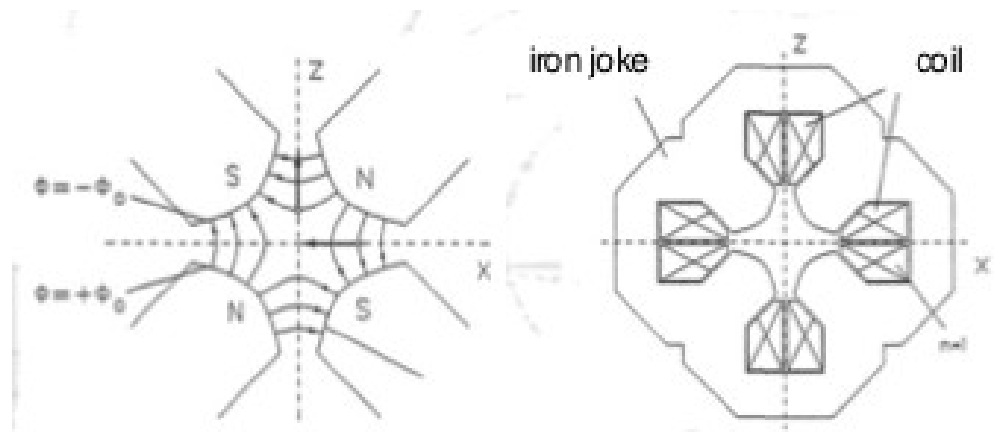
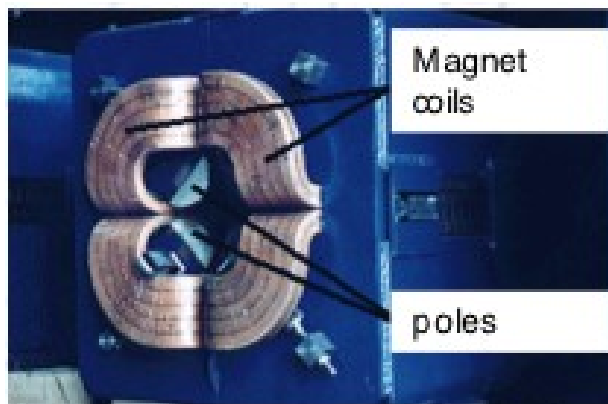
# Can we find better focusing?

## Beamline Elements

Dipole (bend) magnets



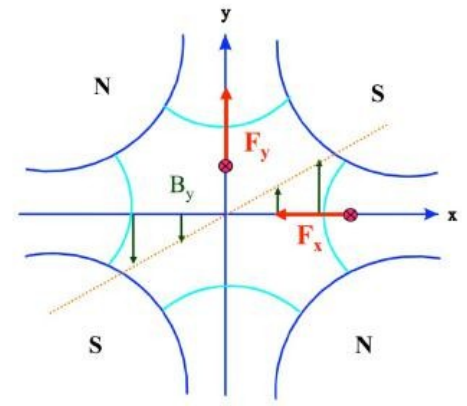
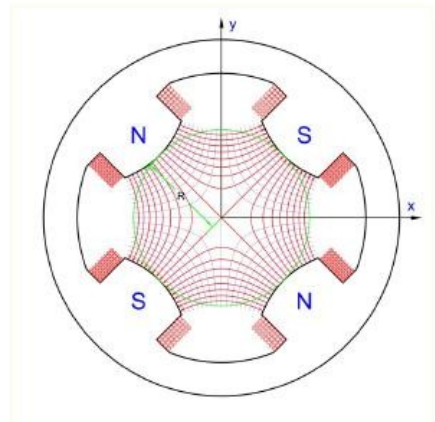
Quadrupole (focusing) magnets



# Quadrupoles has similar problem!

## FOCUSING OF THE PROTON BEAM

Quadrupole looks good – field increases linearly with distance from the center.

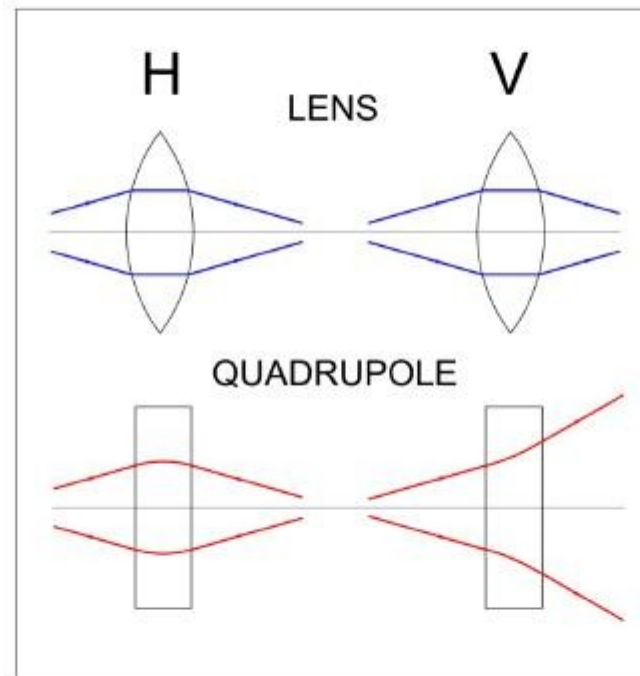


$F_y$  has wrong direction! It doesn't work!

No solution: Maxwell tells us  $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$      $\oint_{\partial S} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

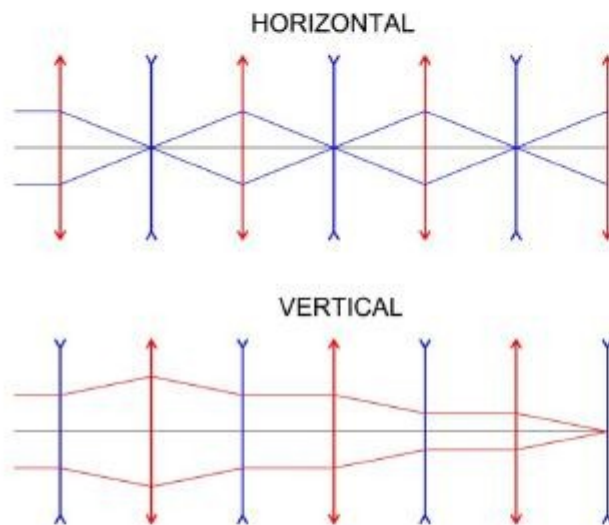
# PROBLEM

Quarupole is convergent lens in horizontal, but divergent in vertical direction!



There was no solution until 1952, and it is beautiful and simple:

## SOLUTION: AG OR STRONG FOCUSING



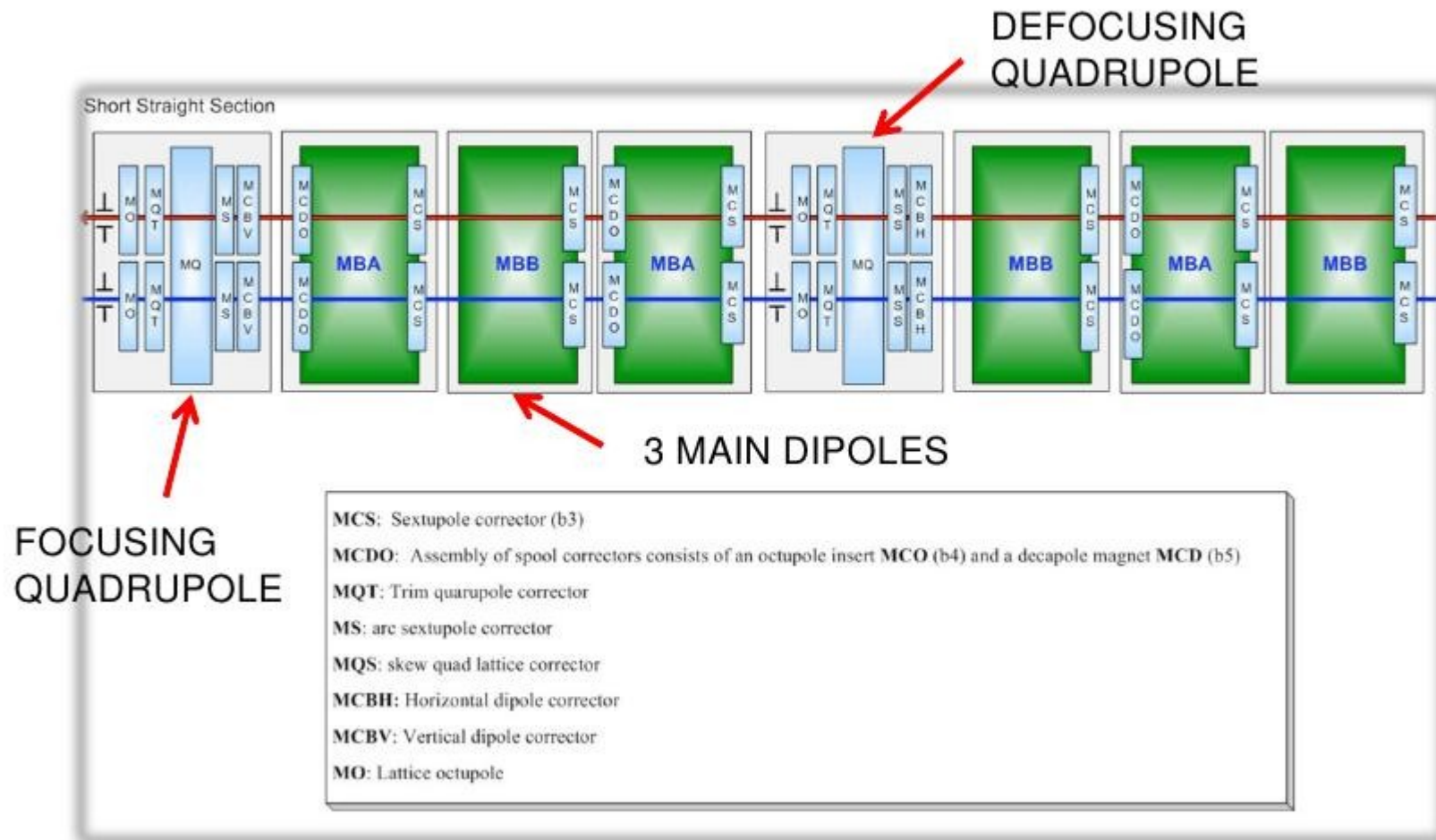
## FODO LATTICE

- F - focusing
- D - defocusing
- O - drift space or dipoles

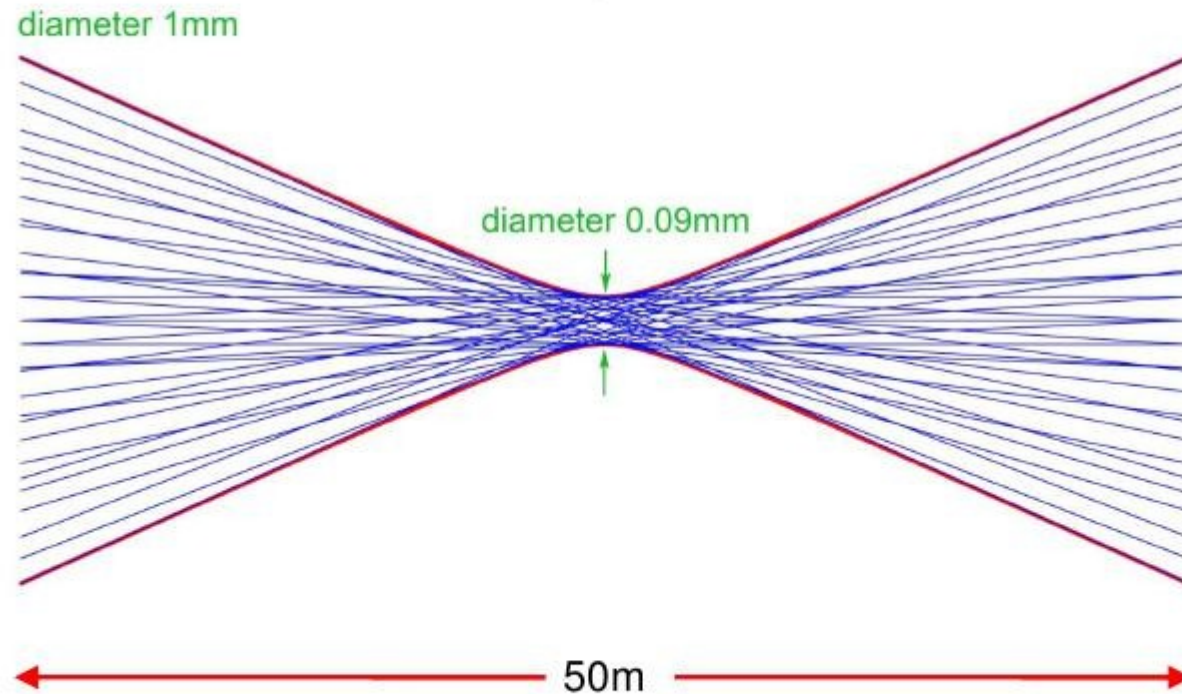
If we have alternating convergent and divergent lenses with right spacing overall effect is focusing!

## LHC FODO LATTICE CELL (106.9 m)

The pattern of bending and focusing magnets is called lattice.



# Example of focusing for collisions at P2 (ALICE)



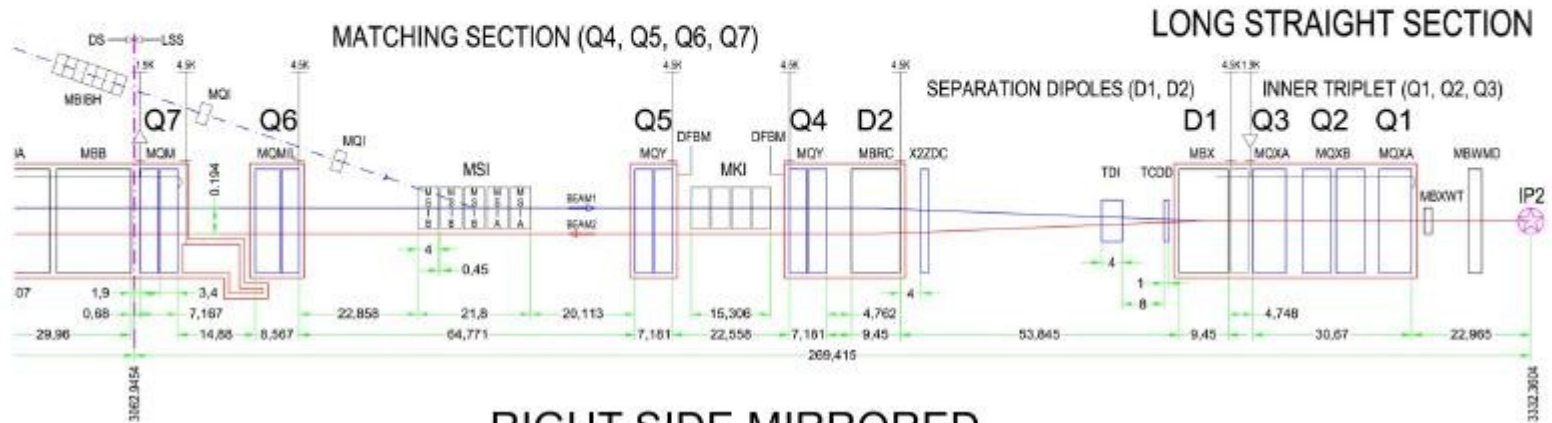
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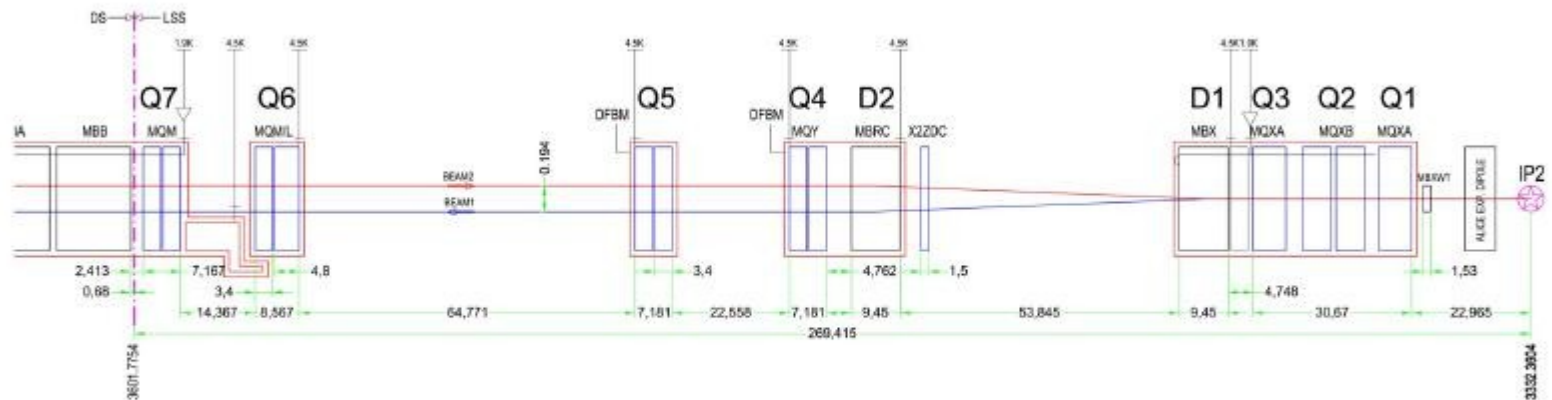
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# SYMMETRY!

## LEFT SIDE



## RIGHT SIDE MIRRORED



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# Recall lecture 1 and 2

**Collision rate** is defined to be the number of 'events' per second, i.e. the number of collisions happening in the center of one of the experiments (depends on the cross section)

The collision rate can be increased if:

- o There is more beam/bunch in the two rings ( $N_B, N_Y$ )
- o There are more bunches colliding ( $k_b$ )
- o **The beam profiles, the size of the beam, at the interaction point, is small ( $\sigma_x, \sigma_y$ )  $\rightarrow \beta^*$**

$$L = \frac{N_B N_Y}{4\pi \sigma_x \sigma_y} k_b f_{rev} \quad (\text{cm}^{-2}\text{s}^{-1})$$

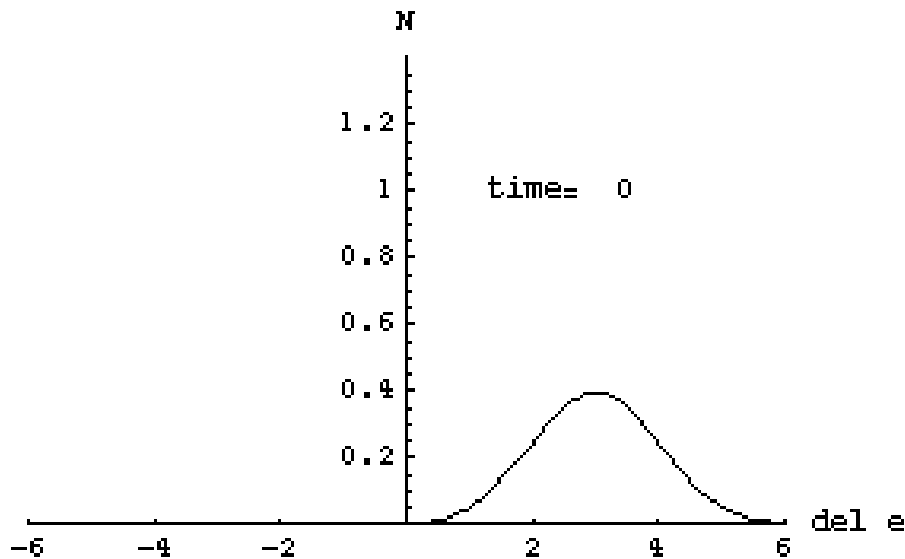
$$R = L \cdot \sigma$$

$\sigma$  is the cross-section

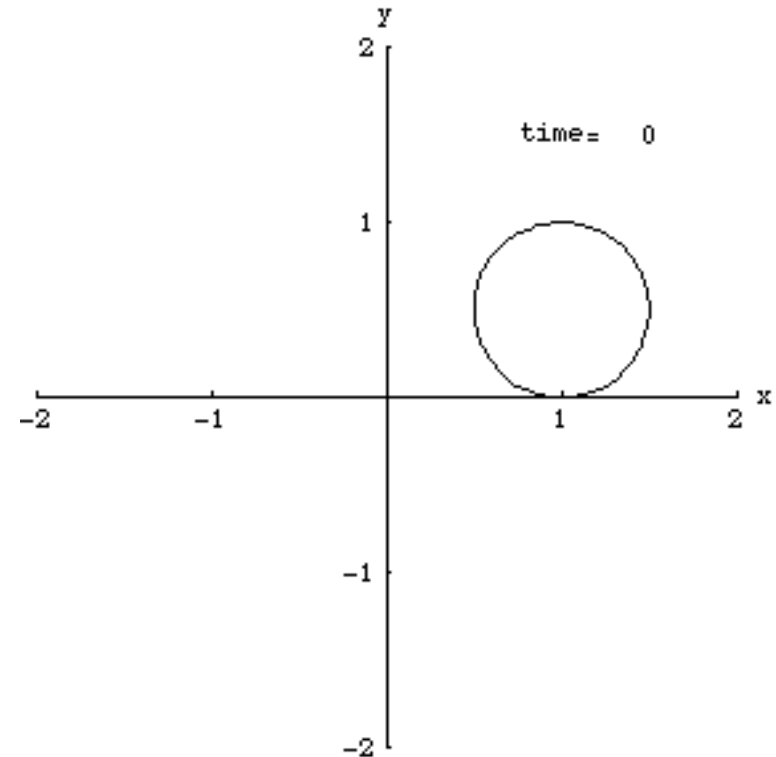
$R$  is the number of events per Second (corresponding to  $\sigma$ )

# Energy adjusting by AC (longitudinal) & transverse strong focusing

## Longitudinal



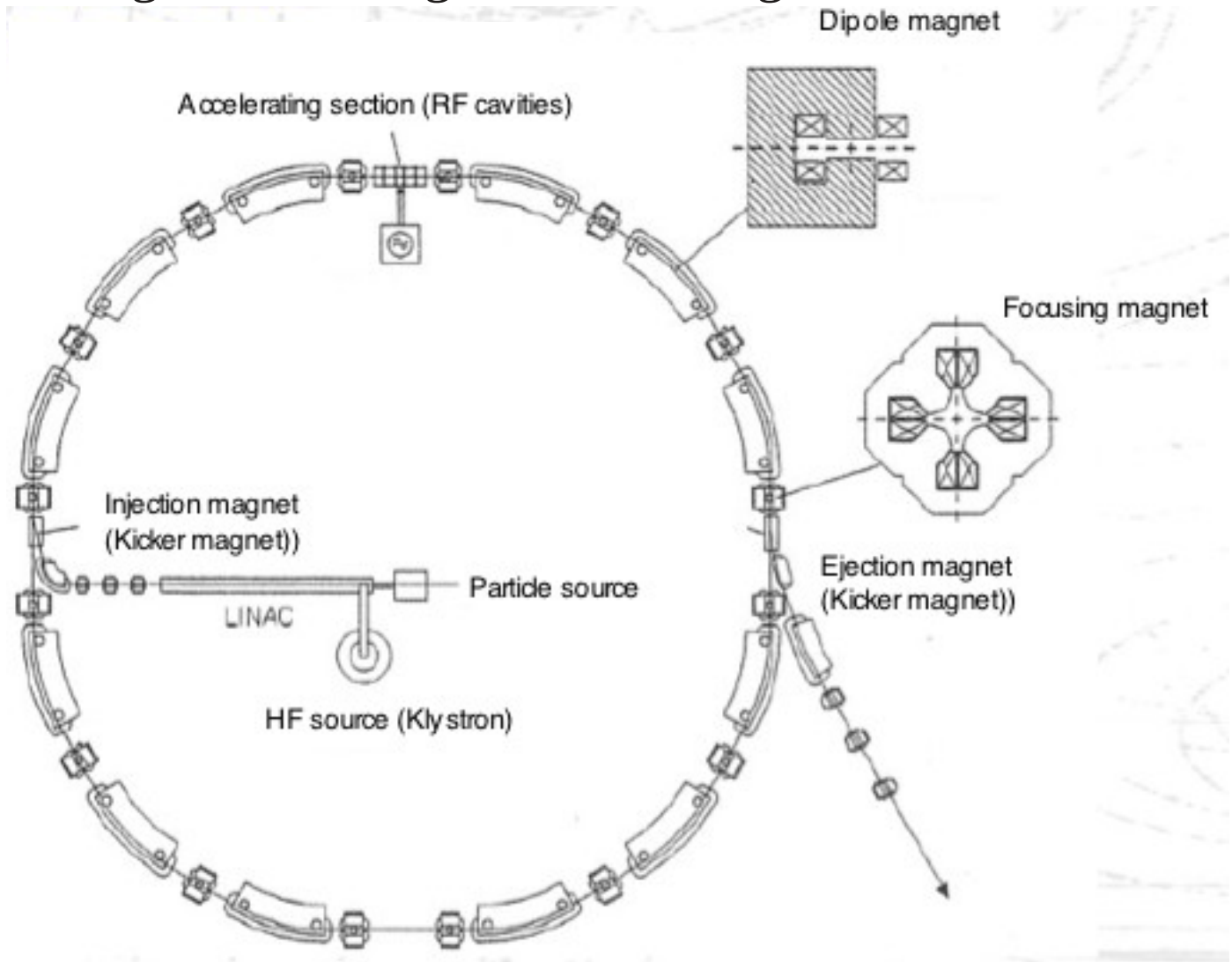
## Transverse



- “Catching the beam” animations taken from
  - <http://www.lns.cornell.edu/~dugan/USPAS/>

# Synchrotrons

Use smaller magnets in a ring + accelerating station

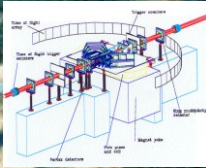


3 GeV protons  
BNL 1950s

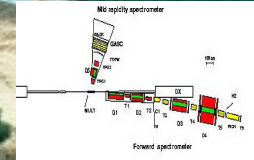
Basis of all circular  
machines built since

# at Brookhaven National

PHOBOS



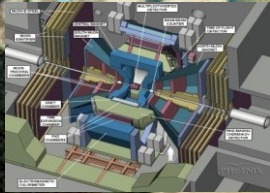
BRAHMS



RHIC



PHENIX



STAR

BOOSTER

LINAC

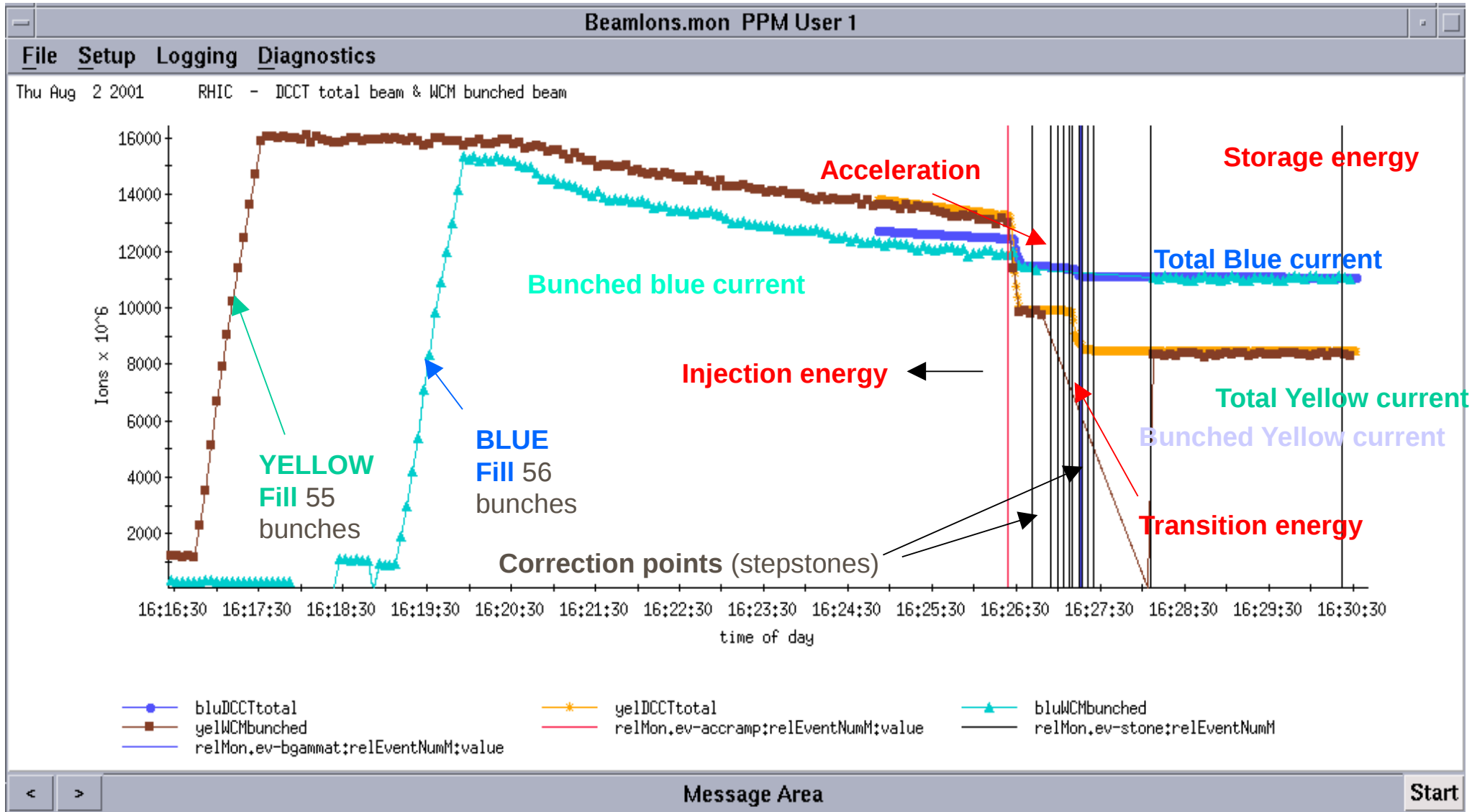
AGS

HTB

HITL

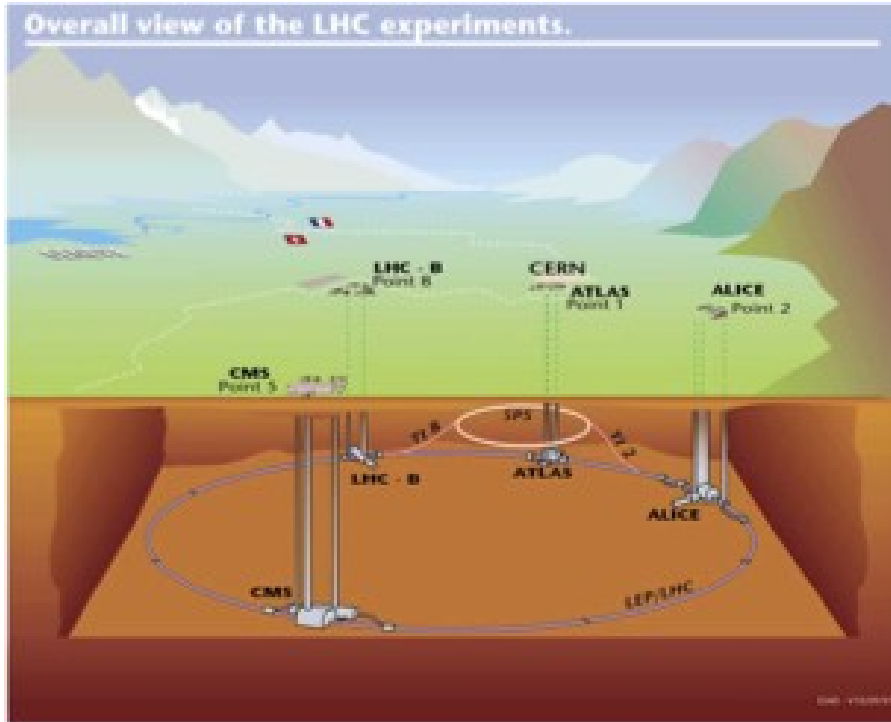
TANDEMS

# RHIC ramp with 56 bunches



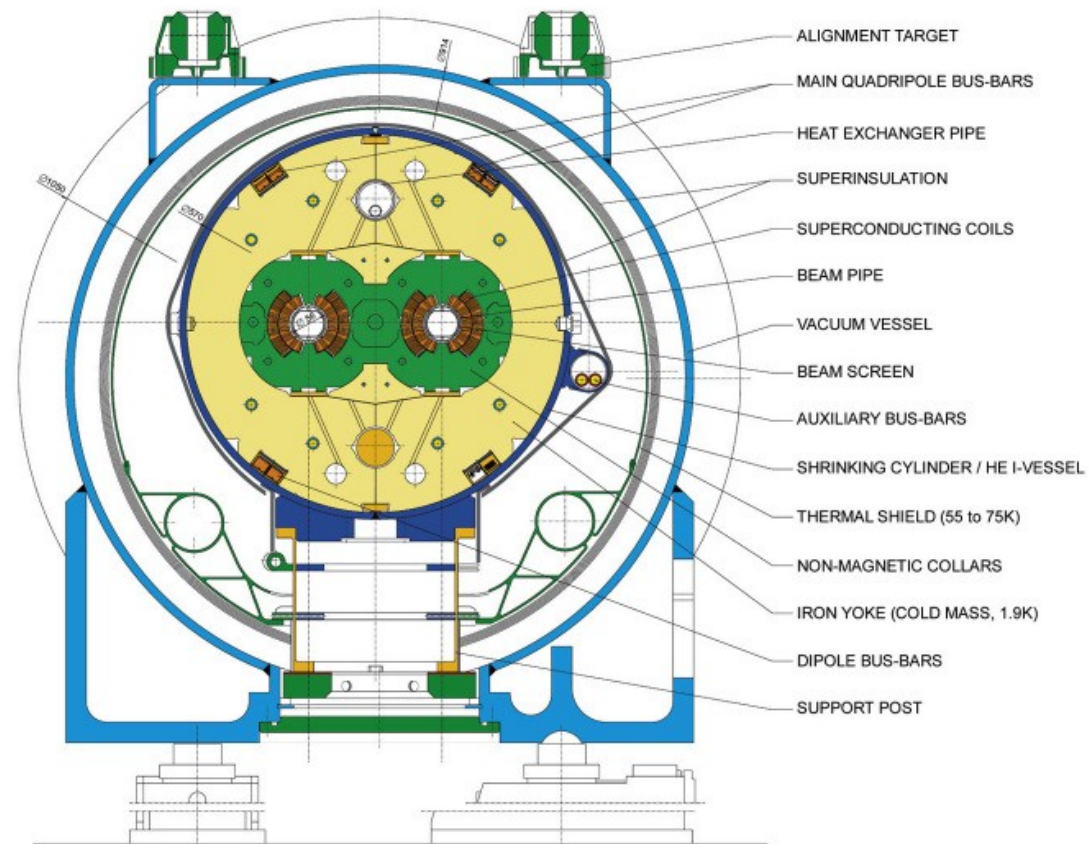
The beam is accelerated from Injection Energy (10 GeV) to Storage Energy (100 GeV). The acceleration process is called “ramp”.

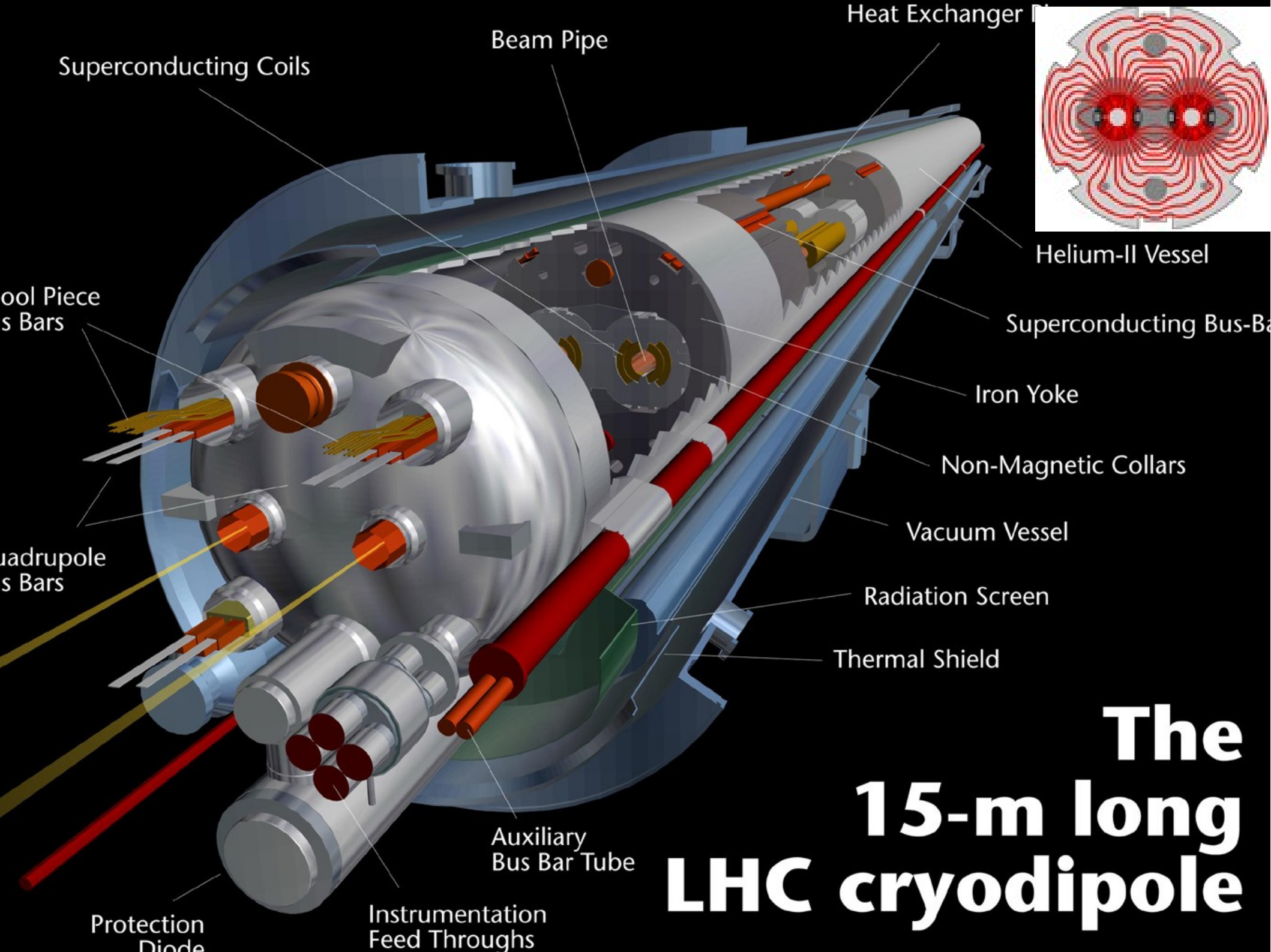
# CERN Large Hadron Collider



## LHC DIPOLE : STANDARD CROSS-SECTION

CERN AC/DUMM - HE107 - 30 04 1999

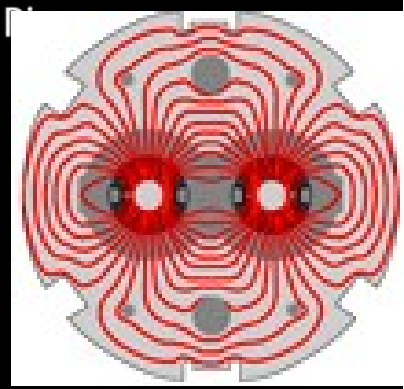




Superconducting Coils

Beam Pipe

Heat Exchanger



Helium-II Vessel

Cool Piece  
Bars

Superconducting Bus-Bar

Iron Yoke

Non-Magnetic Collars

Quadrupole  
Bars

Vacuum Vessel

Radiation Screen

Thermal Shield

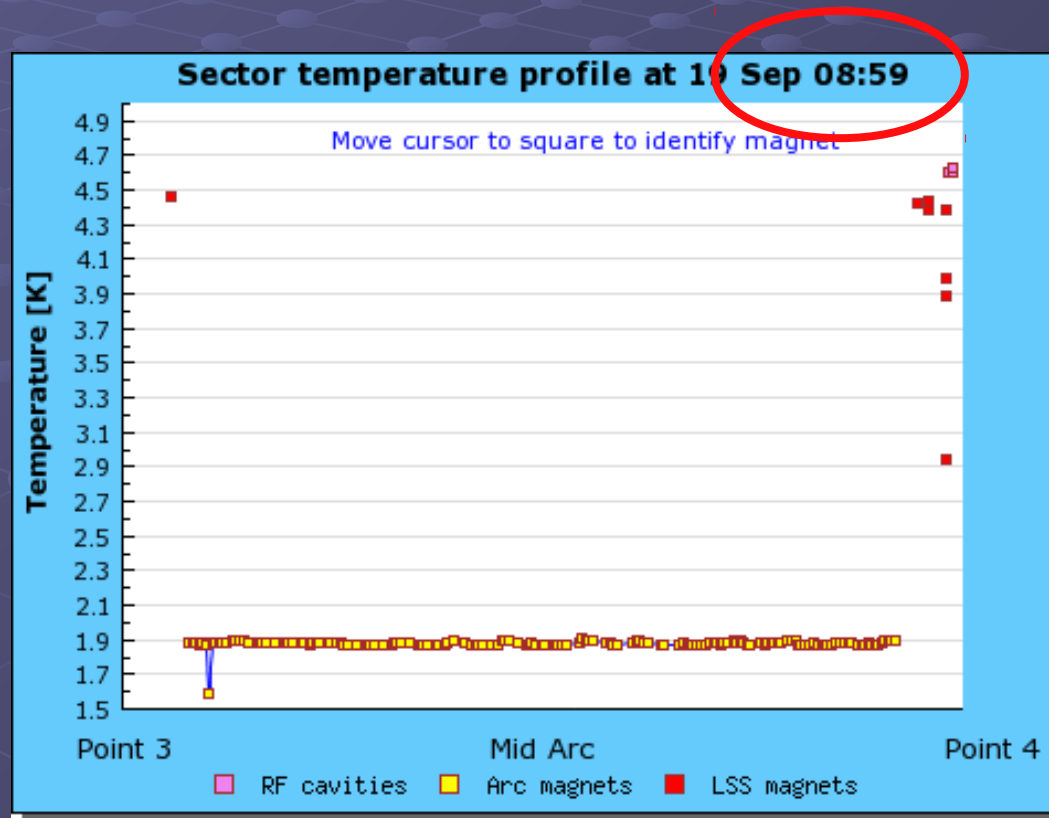
Auxiliary  
Bus Bar Tube

# The 15-m long LHC cryodipole

Protection  
Diode

Instrumentation  
Feed Throughs

# The 19 September 2008 accident



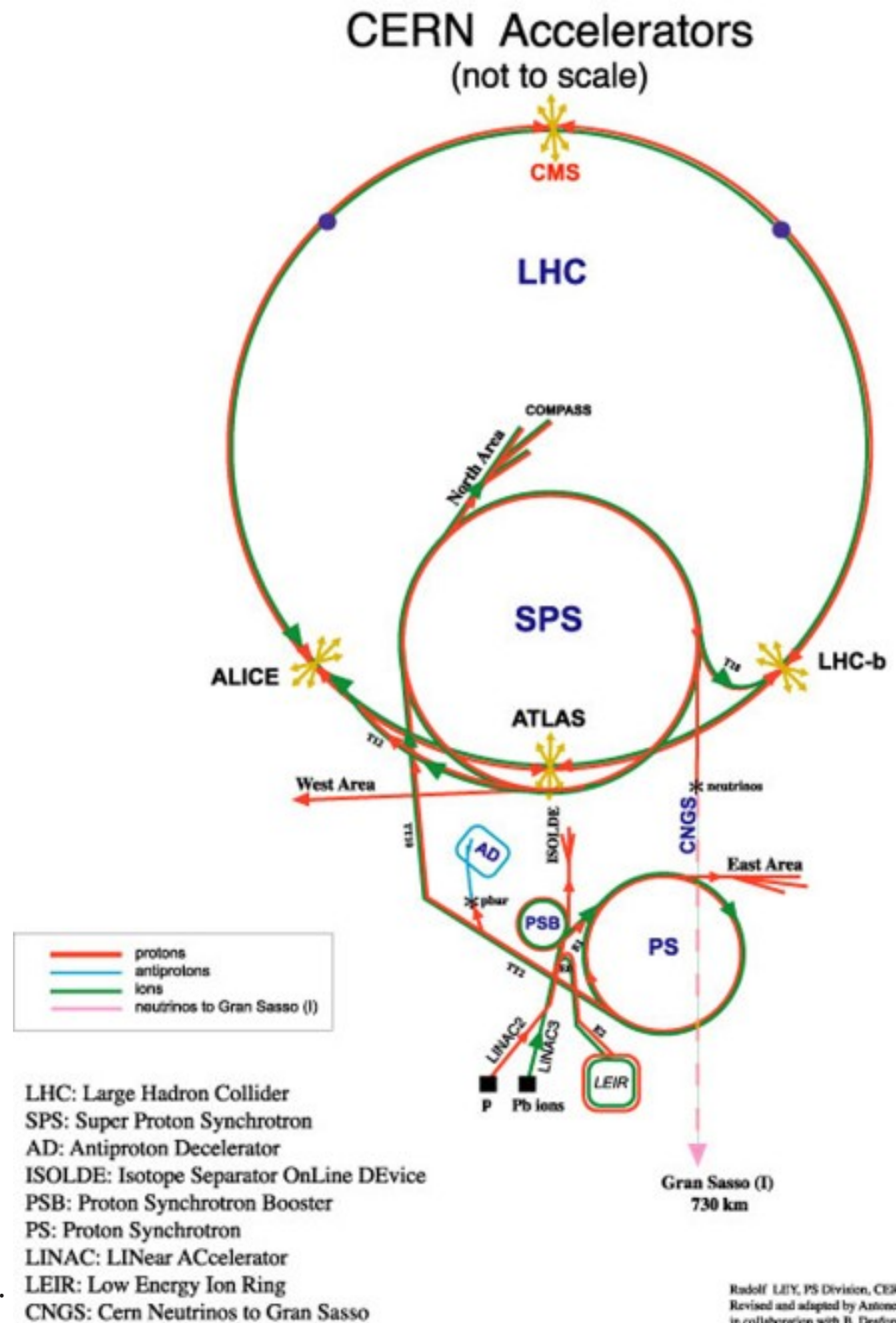


# THE 19 SEPTEMBER 2008 INCIDENT



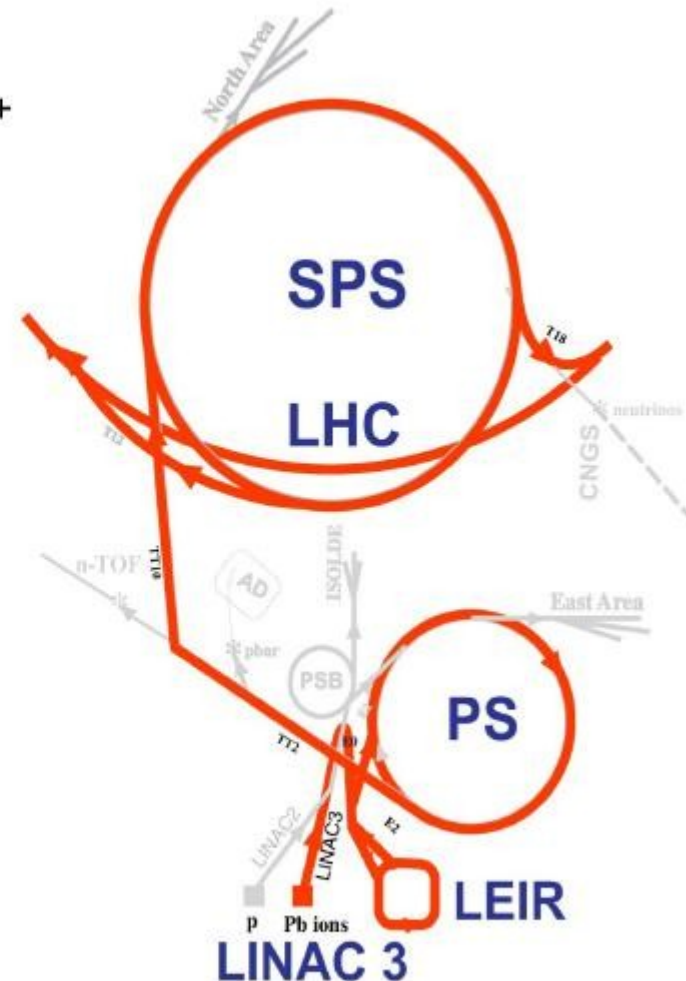
# CERN Complex

Old rings still in use  
Many different programs



# ION BEAM IN THE LHC

- ECR ion source
  - Provide highest possible intensity of Pb29+
- RFQ + Linac 3
  - Adapt to LEIR injection energy
  - strip to Pb54+
- LEIR
  - Accumulate and cool Linac 3 beam
  - Prepare bunch structure for PS
- PS
  - Define LHC bunch structure
  - Strip to Pb82+
- SPS
  - Define filling scheme

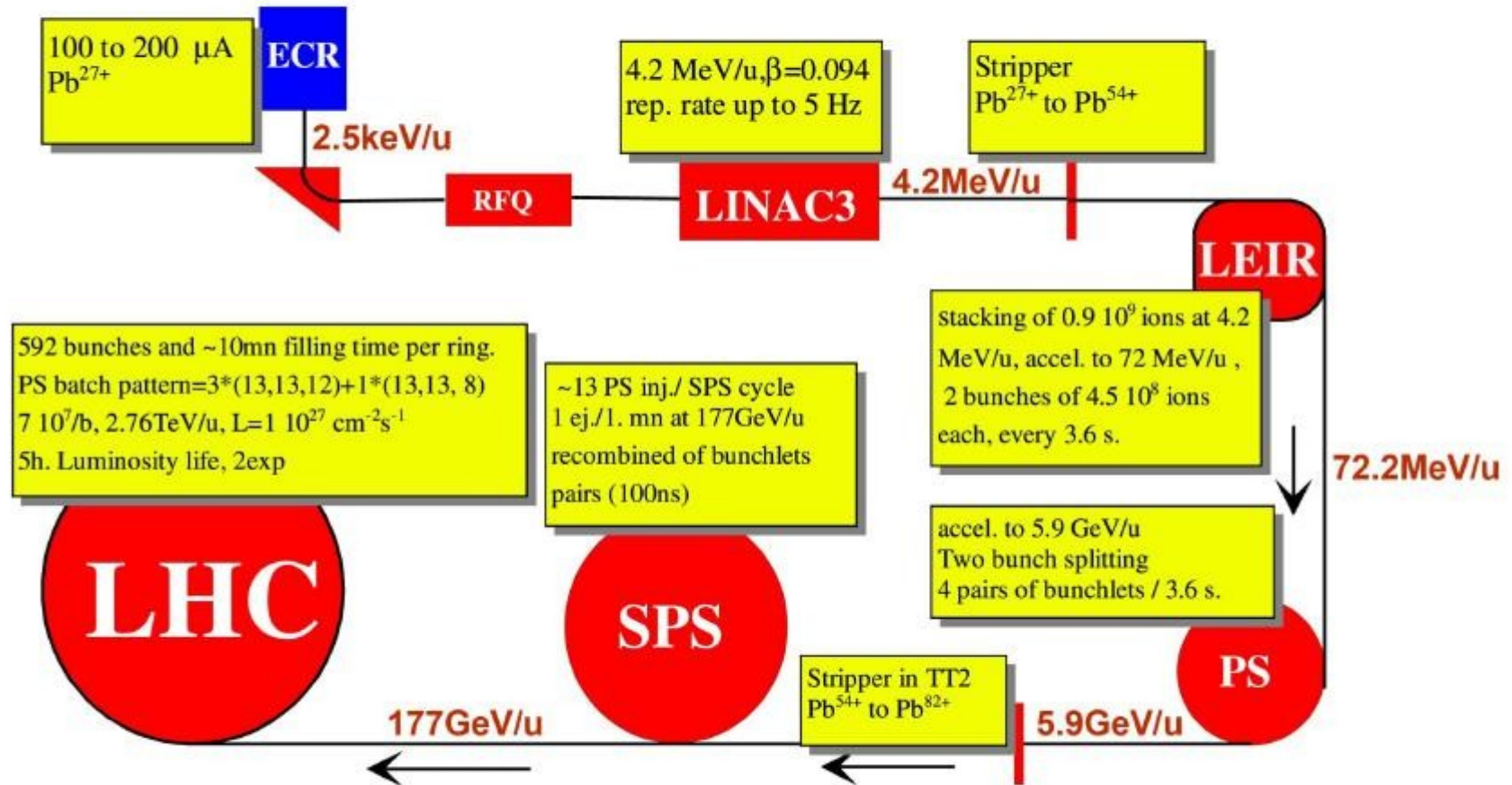


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# ION BEAM IN THE LHC



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# ION BEAM IN THE LHC

## How is a beam of lead ions produced at CERN?

Lead ions are produced when lead atoms are stripped of electrons. A highly purified lead sample is heated to a temperature of about 500 °C.

The lead vapour is then ionized by an electron current. Many different charge states are produced, with a maximum around Pb29+.

These ions are selected and accelerated to 4.2 MeV/u (energy per nucleon) before passing through a carbon foil, which strips most of them to Pb54+.

The Pb54+ beam is accumulated, then accelerated to 72 MeV/u in the Low Energy Ion Ring (LEIR), which transfers them to the PS.

The PS accelerates the beam to 5.9 GeV/u and sends it to the SPS after first passing it through a second foil where it is fully stripped to Pb82+.

The SPS accelerates it to 177 GeV/u then sends it to the LHC, which accelerates it to 2.76 TeV/u.

# LEIR

LEIR is a central part of the injector chain to supply lead ions to the LHC.

## THE ROLES OF LEIR

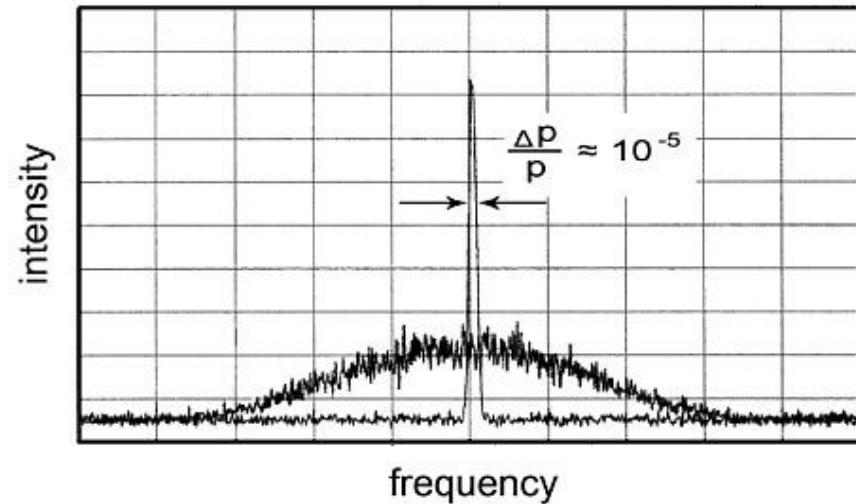
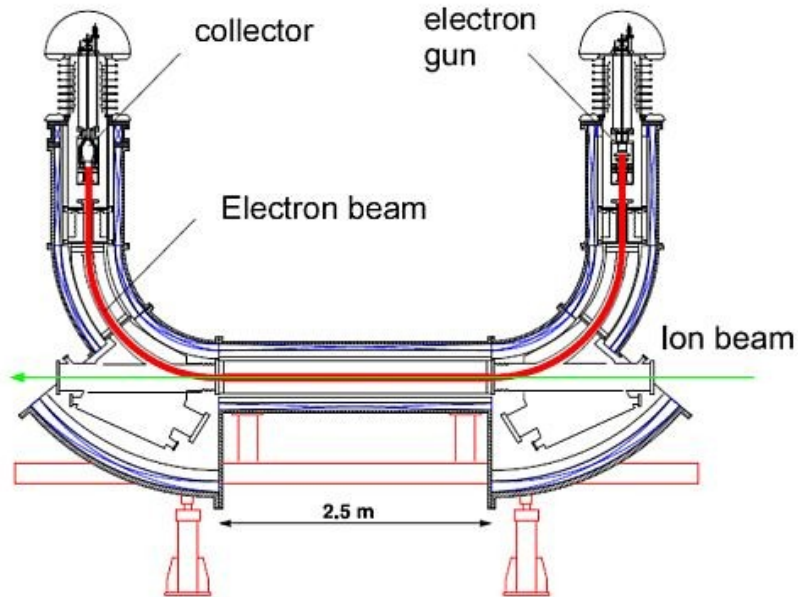
- Accumulate enough ions for LHC bunches
  - Keep their H, V and // emittances small
  - Bring Linac3 ion beam to PS injection energy
  - Prepare bunch structure for PS
- ▶ 3 plane stacking
  - ▶ Cooling
  - ▶ Acceleration

## ELECTRON COOLING

Electron cooling is used to merge the newly injected particles with the stack. The relatively small transverse emittance facilitates rapid cooling of the injected beam.

- Principle: an electron beam with same velocity as the ion beam is merged with it over a fraction of the circumference (~3%)
- In the moving frame, collisions between electrons and ions correspond to the mixture of a hot ion gas with a cool electron gas.
- The heat exchange leads to cooling, i.e. emittance reduction, in all 3 planes (H,V, ||) of the ion beam.

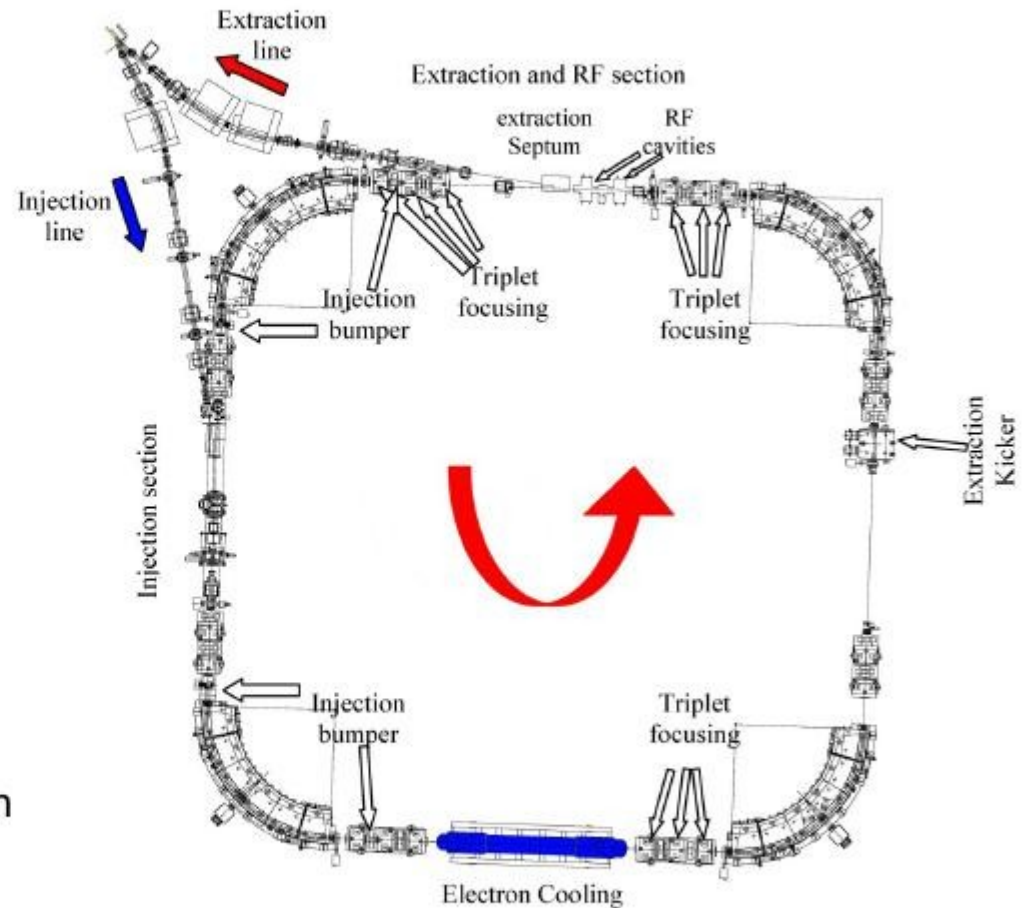
# Electron cooling example



- From: <http://web-docs.gsi.de/>
- Elastic collision e+ion will decrease the relative momentum spread in the beam

# LEIR

- Square shaped “circular machine”
- Circumference = 78.54m  
= PS/8 = SPS/88
- Operated below transition  
 $\gamma t \approx 2.87$
- 4x90° bending magnets
- 2 SS's with Q doublets, 2  
SS's with Q triplets,
- Common injection/ejection  
line
- Electron cooling



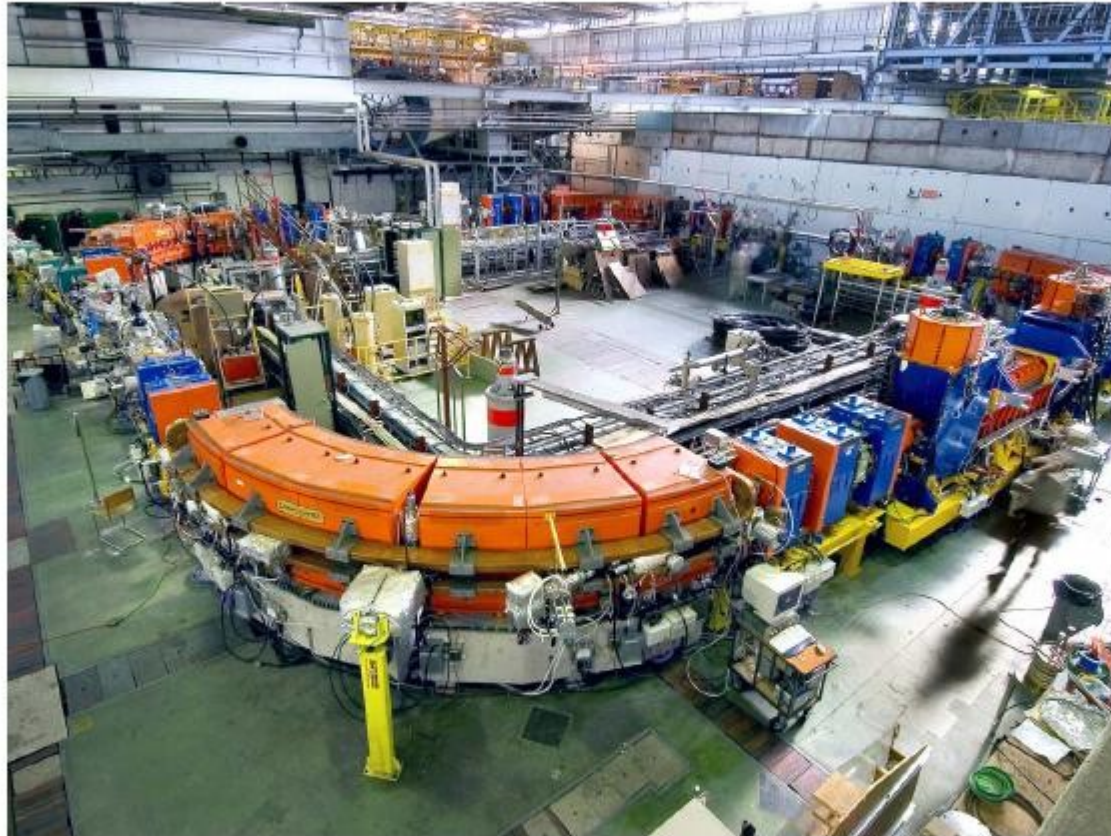
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# LEIR



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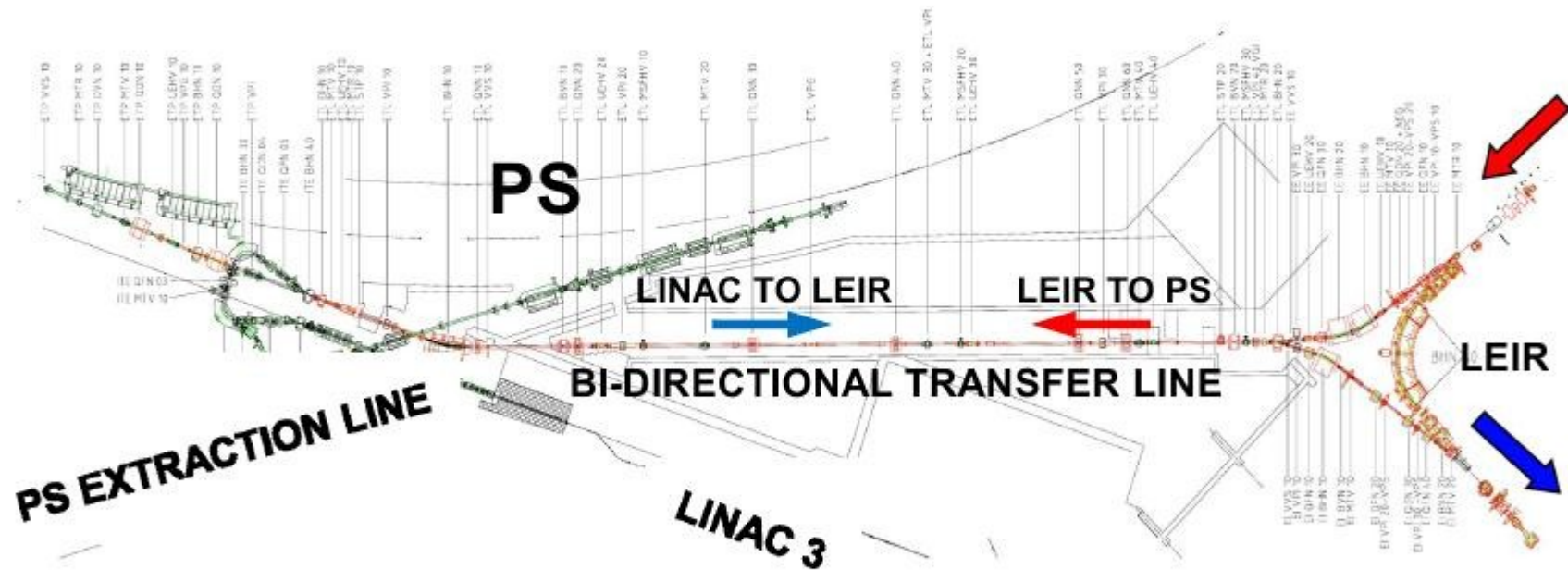
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19/11-12

Lecture 5 and 6  
P. Christiansen (Lund)

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# Geometry of the LINAC3, LEIR and PS accelerators and the transfer lines between them



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# HARMONIC NUMBER

A particle circulates around the machine with period:  $\tau = \frac{L}{\beta c}$

Then the number of turns (circulation frequency) is:  $f_r = \frac{1}{\tau}$

$L$  is the circumference and  $\beta c$  is the velocity.

For LHC  $L = 26658.8832m$  and  $\tau = 88.92\mu s \rightarrow f_r \approx 11245.5Hz$

The **synchronous particle** is defined as that particle which always arrives at the desired synchronous phase lag  $\phi_s$  behind the zero-crossing of the rf wave. For this to occur, the rf frequency  $f_a$  must be an integer multiple of  $f_r$

$$f_a = h \cdot f_r$$

where integer  $h$  is known as the **harmonic number**.

$$h = \frac{\text{RF frequency}}{\text{Circulation frequency}}$$

For LHC  $h$  is chosen to be **35640**. Then we have  $f_a \approx 400.8MHz$

## BUNCHES AND BUCKETS

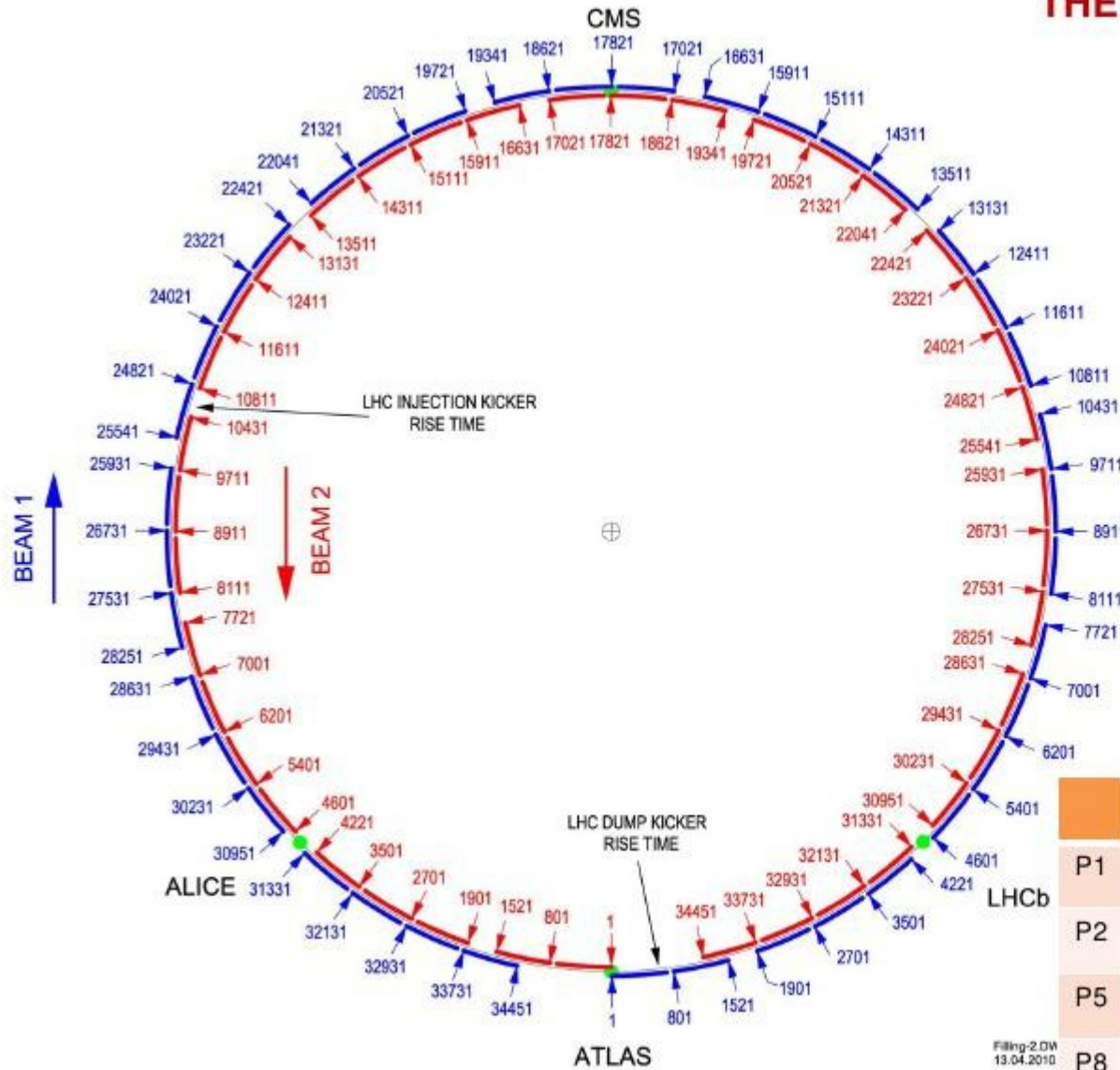
Harmonic number determines number of places on the circumference where a particle could be located and arrive synchronously in the accelerating cavity. The segments of the circumference centred on these points are called **buckets**. The groups of particles in these buckets are called **bunches**. Not all buckets need to be filled with bunches. In LHC only 2808 out of 35640 buckets will be filled with minimal distance of 10 buckets.

Number of buckets = $h$	35640
Bucket spacing (time)	2.5 ns
Bucket spacing (space)	74.8cm
Max umber of bunches	2808
Min bunch spacing (time)	25 ns
Min bunch spacing (space)	7.48 m
RMS bunch length	7.5 cm

**Bucket spacing (space) is constant:**

$$\text{LHC circumference} / \text{Number of buckets} = 26658.8832 / 35640 = 0.748\text{m}$$

## THE INITIAL POSITIONS OF THE BUNCHES



EACH BEAM 39x72  
=2808 BUNCHES

PS=1/11 SPS  
SPS=7/27 LHC

### IP POSITIONS

	BEAM 1	BEAM 2
P1	1	1
P2	31186(EMPTY)	4456(EMPTY)
P5	17821	17821
P8	4471(EMPTY)	31171(EMPTY)

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# ION PHYSICS: STABLE BEAMS

Energy:

3500 Z GeV

I(B1):

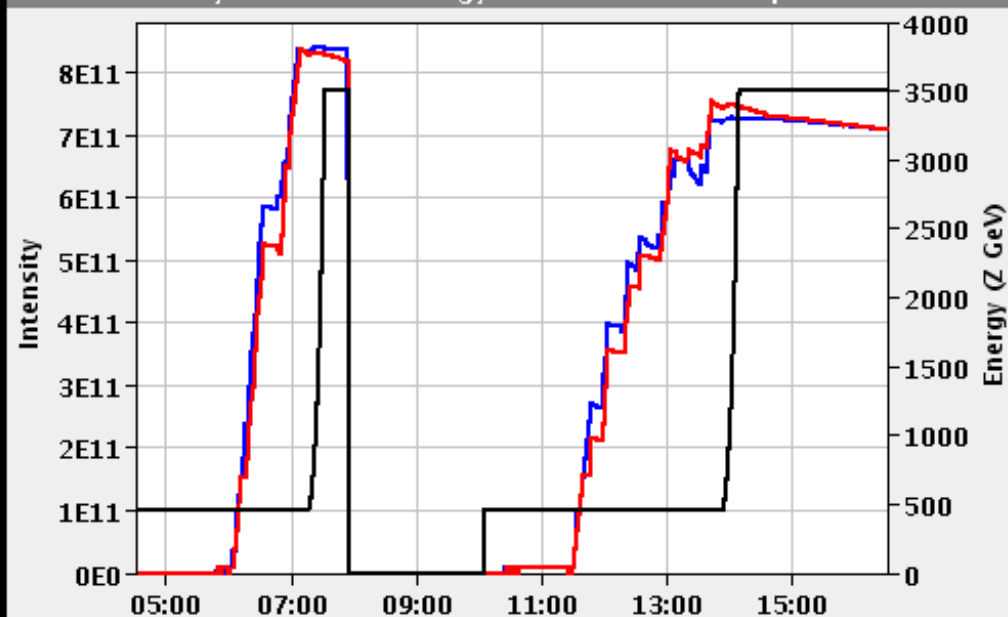
8.32e+11

I(B2):

7.57e+11

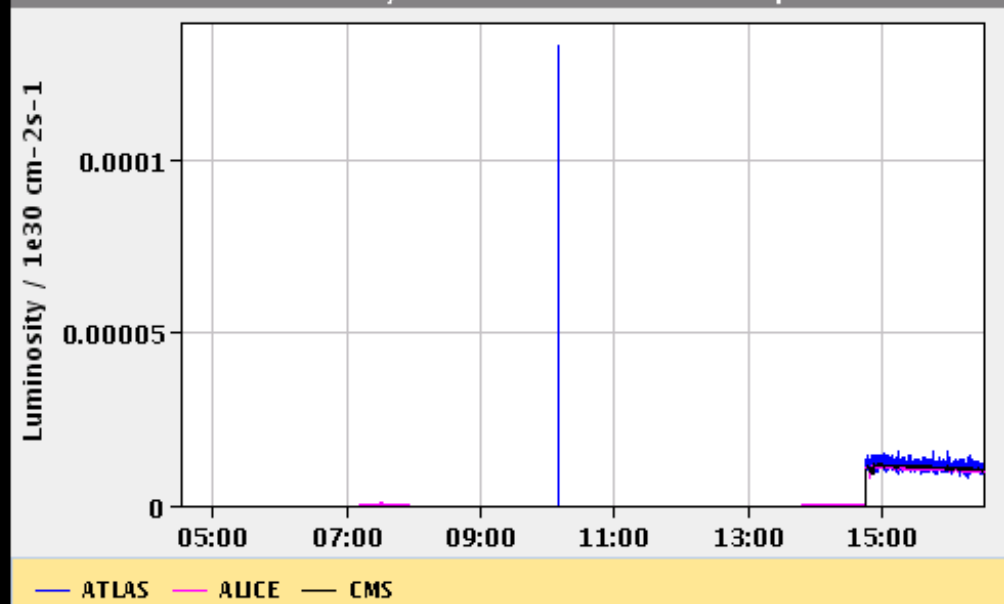
FBCT Intensity and Beam Energy

Updated: 16:30:50



Instantaneous Luminosity

Updated: 16:30:48



Comments 29-11-2010 14:54:46 :

\*\*\* STABLE BEAMS \*\*\*

All points optimized

BIS status and SMP flags

B1

B2

Link Status of Beam Permits

true

true

Global Beam Permit

true

true

Setup Beam

false

false

Beam Presence

true

true

Moveable Devices Allowed In

true

true

Stable Beams

true

true

AFS: 500ns\_121b\_113\_114\_0\_4bpi31inj\_IONS

PM Status B1

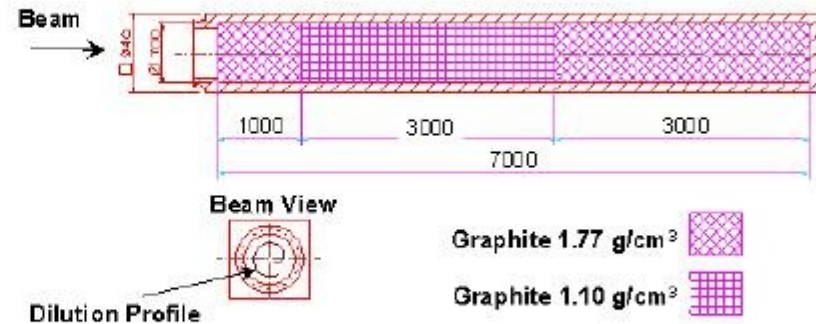
ENABLED

PM Status B2

ENABLED

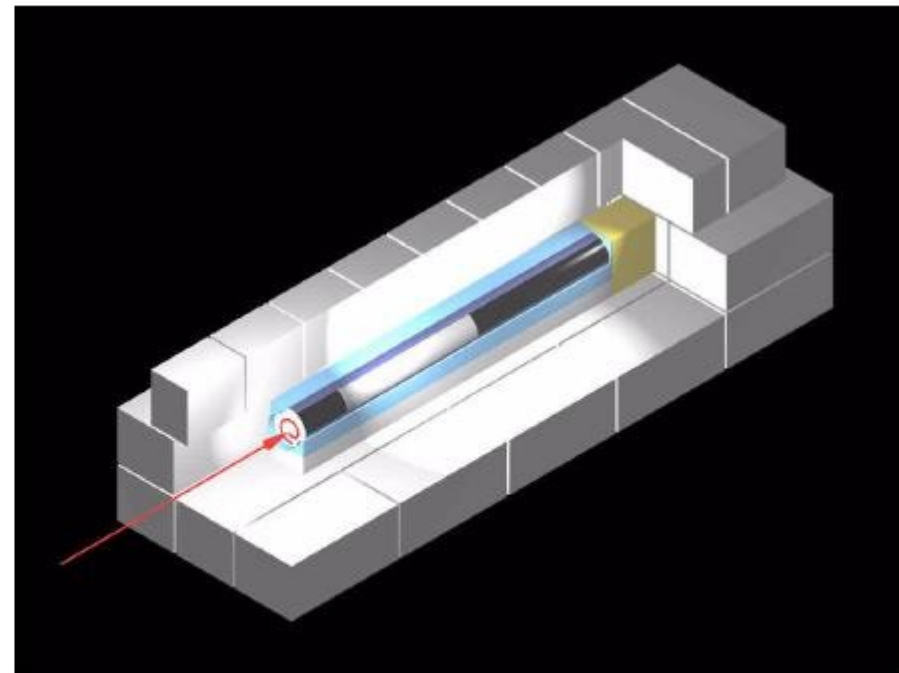
## DUMP CORE TDE

7m long C / C-C TDE in steel shrink-cylinder, followed by 1m Al, 2m Fe  
~1000 T of concrete shielding



This is the **ONLY** element in the **LHC that can** withstand the impact of the full 7 TeV beam !  
Nevertheless, the dumped beam must be painted to keep the peak energy densities at a tolerable level !

Why graphite? If the material were heavy, all the beam's energy would concentrate in the first half meter of the block.

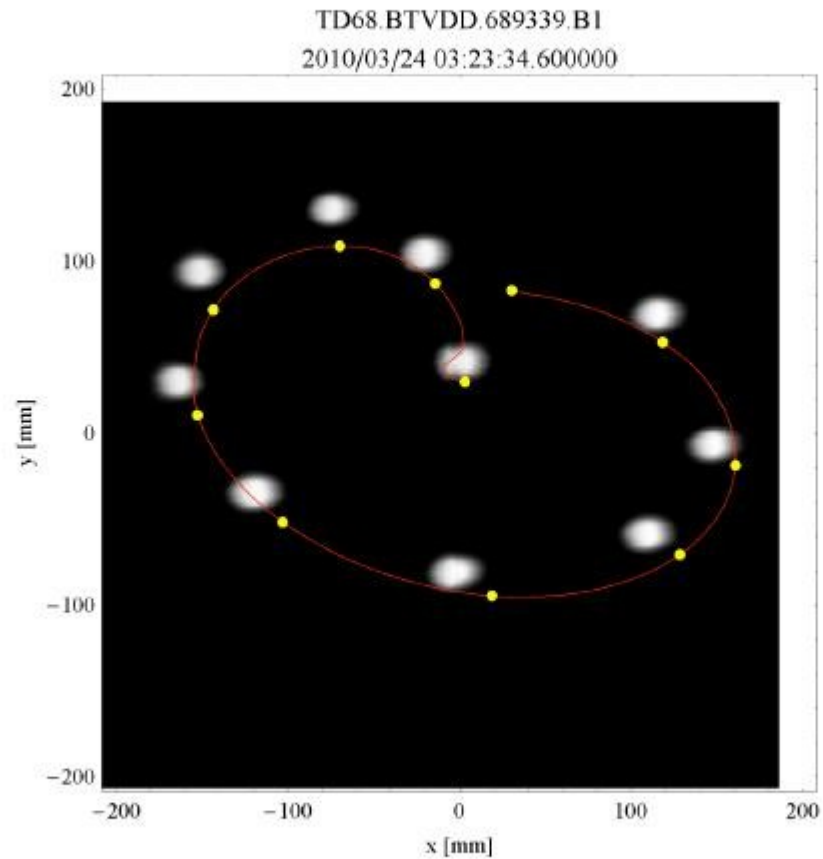


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The beam size has increased to an extent where the sigma is 1.6mm in both planes.



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D. Vranic

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