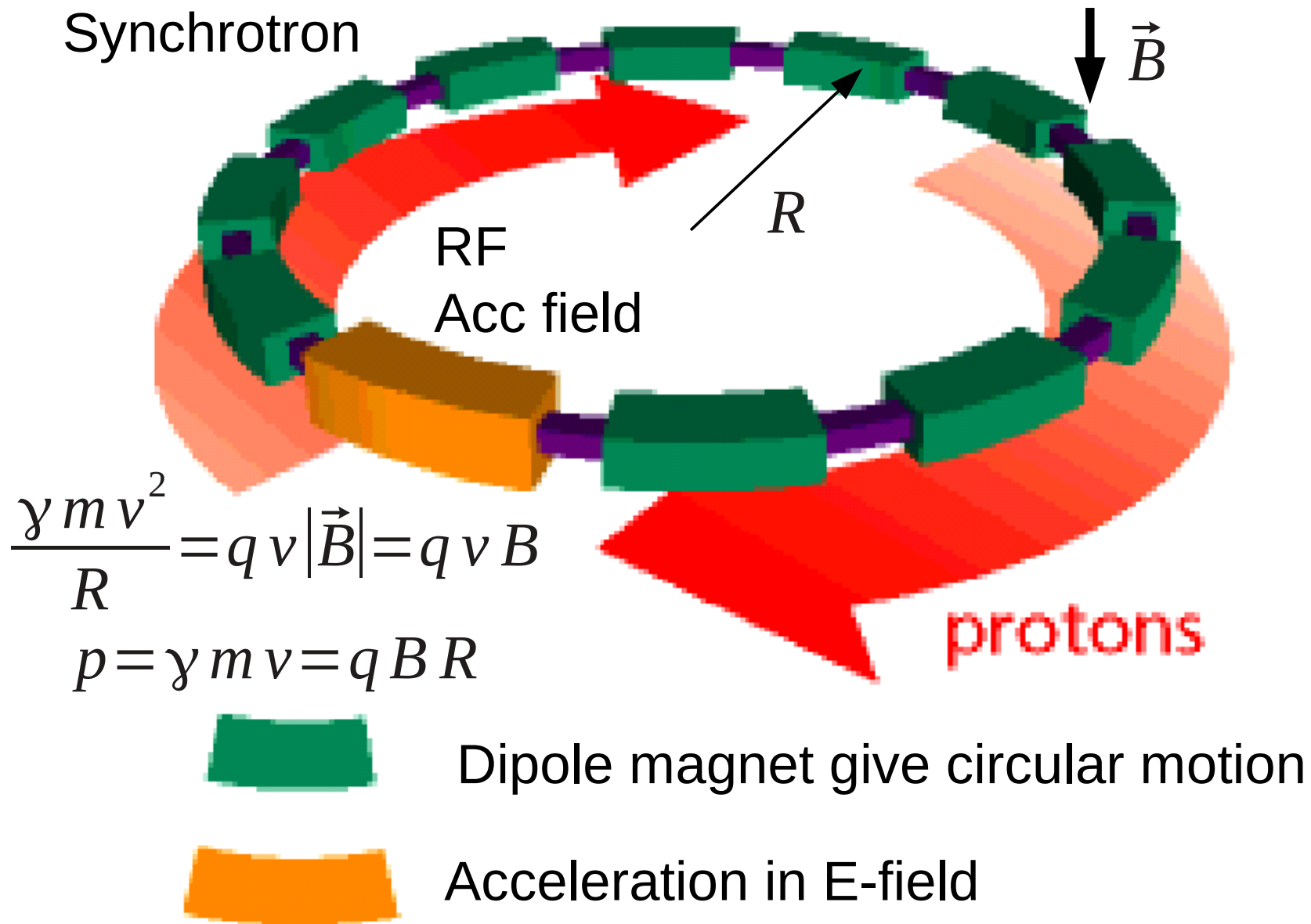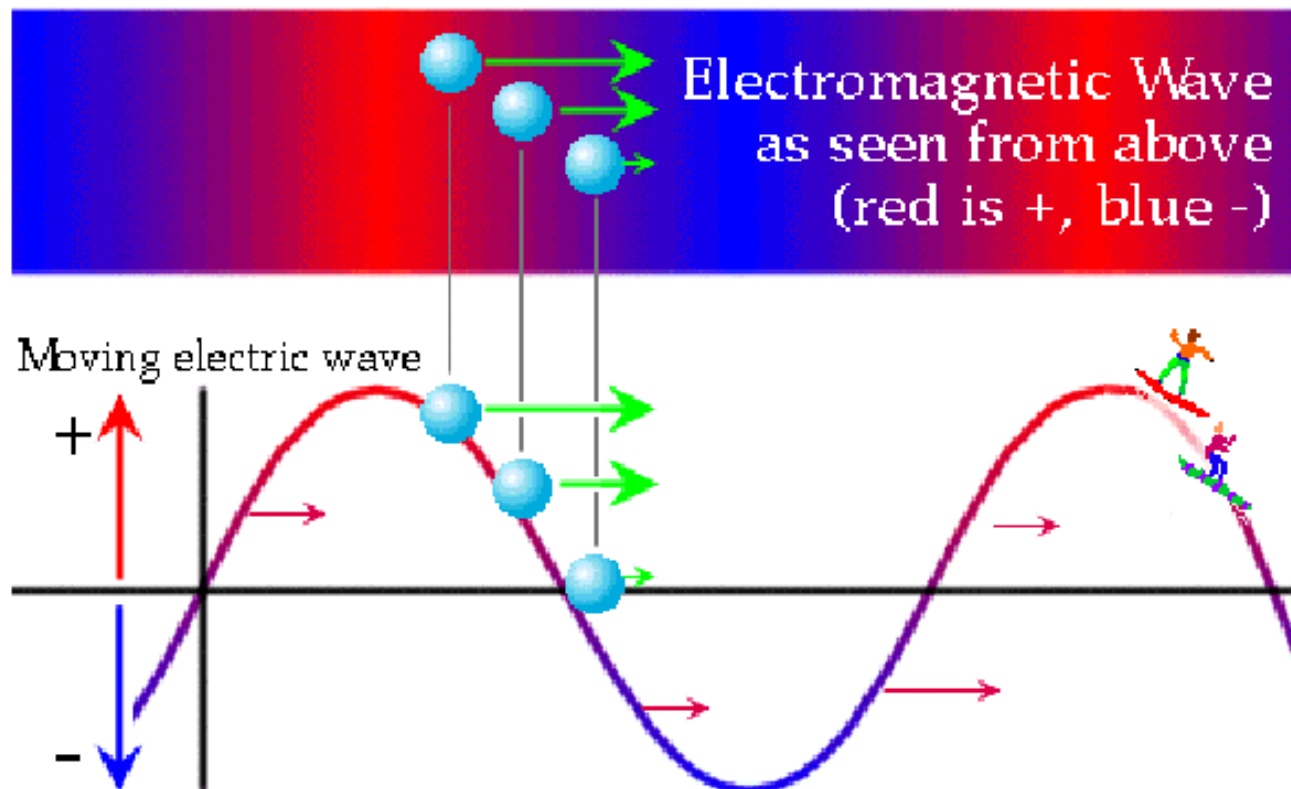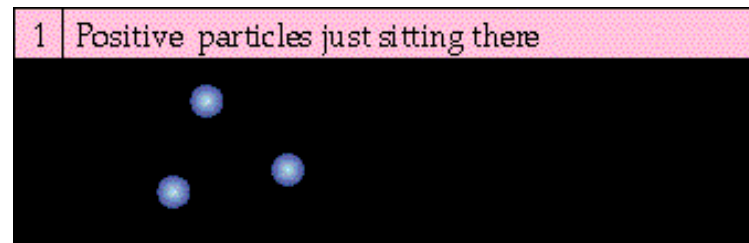# Lectures on accelerator physics

- Lecture 5 and 6: Advanced topics

    - Transverse motion, strong focusing, and LHC

- Material borrowed from

    - Lecture by Anders Oskarsson

    - Lecture by Eric Torrence (University of Oregon)

    - LHC lectures by Danillo Vranic (GSI)

- Weak focusing follows "Principles of Charged Particle Acceleration" by Stanley Humpries Jr. Chapter 7.

Towards the Synchrotron

Synchrotron

$\vec{B}$

RF
Acc field

R

$$\frac{\gamma m v^2}{R} = q v |\vec{B}| = q v B$$

$$p = \gamma m v = q B R$$

protons

Dipole magnet give circular motion

Acceleration in E-field

P. Christiansen (Lund)

The particles are "surfing" the acceleration wave

HARMONIC NUMBER

A particle circulates around the machine with period: $\tau = \dfrac{L}{\beta c}$

Then the number of turns (circulation frequency) is: $f_r = \dfrac{1}{\tau}$

L is the circumference and βc is the velocity.

For LHC $L = 26658.8832m$ and $\tau = 88.92\mu s \rightarrow f_r \approx 11245.5 Hz$

The **synchronous particle** is defined as that particle which always arrives at the

desired synchronous phase lag ϕ_s behind the zero-crossing of the rf wave. For

this to occur, the rf frequency f_a must be an integer multiple of f_r

$$f_a = h \cdot f_r$$

where integer h is known as the **harmonic number.**

$$h = \frac{\text{RF frequency}}{\text{Circulation frequency}}$$

For LHC h is chosen to be **35640**. Then we have $f_a \approx 400.8 MHz$
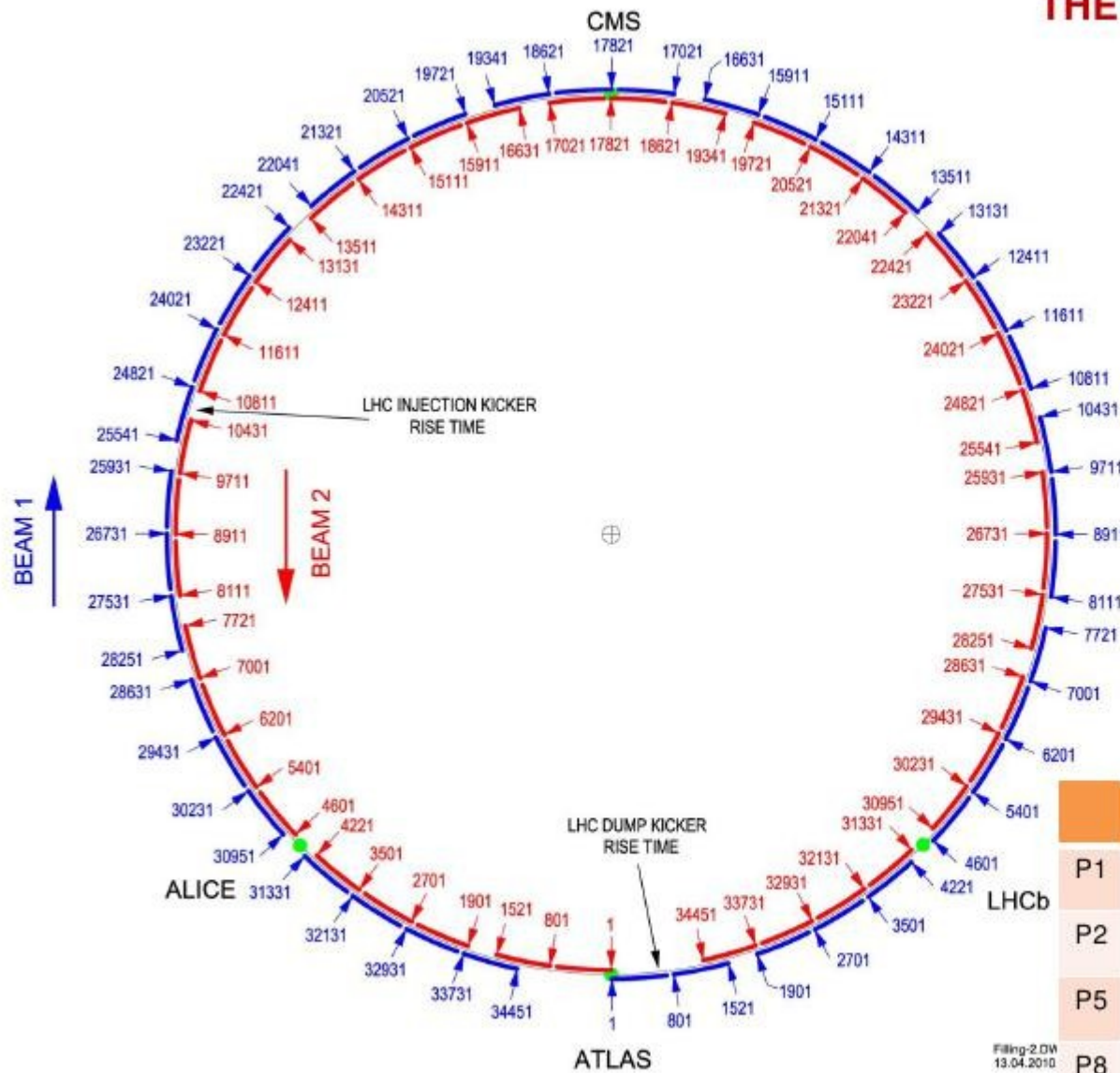
BUNCHES AND BUCKETS

Harmonic number determines number of places on the circumference where a particle could be located and arrive synchronously in the accelerating cavity. The segments of the circumference centred on these points are called **buckets**. The groups of particles in these buckets are called **bunches**. Not all buckets need to be filled with bunches. In LHC only 2808 out of 35640 buckets will be filled with minimal distance of 10 buckets.

Number of buckets = h	35640
Bucket spacing (time)	2.5 ns
Bucket spacing (space)	74.8cm
Max umber of bunches	2808
Min bunch spacing (time)	25 ns
Min bunch spacing (space)	7.48 m
RMS bunch length	7.5 cm

Bucket spacing (space) is constant:

LHC circumference / Number of buckets =26658.8832/35640=0.748m

8/25/2010   D. Vranic   4

THE INITIAL POSITIONS OF THE BUNCHES

CMS

EACH BEAM 39x72 =2808 BUNCHES

PS=1/11 SPS

SPS=7/27 LHC

BEAM 1

BEAM 2

LHC INJECTION KICKER RISE TIME

LHC DUMP KICKER RISE TIME

ALICE

ATLAS

LHCb

IP POSITIONS

	BEAM 1	BEAM 2
P1	1	1
P2	31186(EMPTY)	4456(EMPTY)
P5	17821	17821
P8	4471(EMPTY)	31171(EMPTY)

Filling-2.DW 13.04.2010

Focusing in the transverse plane
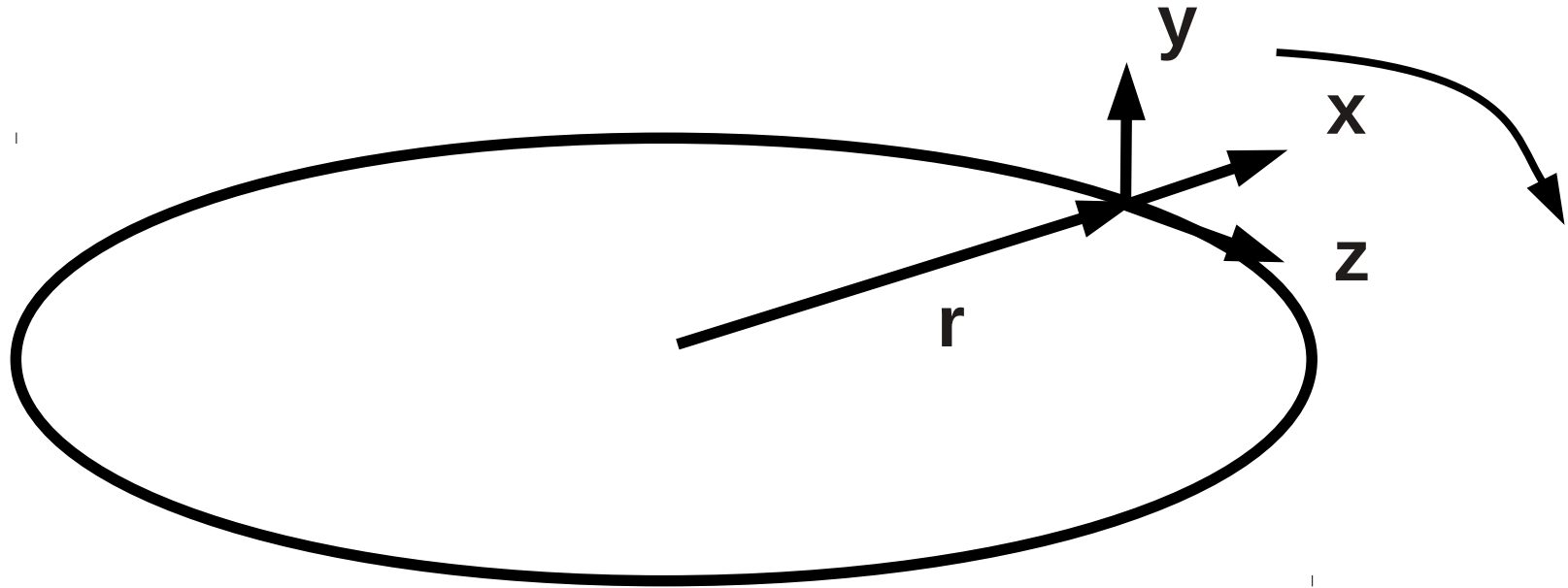
- Assume $(v_x, v_y, v_z) \sim (0,0,v)$ and $v \sim c$ = constant!

  - Very good assumption!

- $z = vt \;\rightarrow\; t = z/v \;(\sim z/c)$

  - $d/dt \sim v\, d/dz \;(\sim c\, d/dz)$
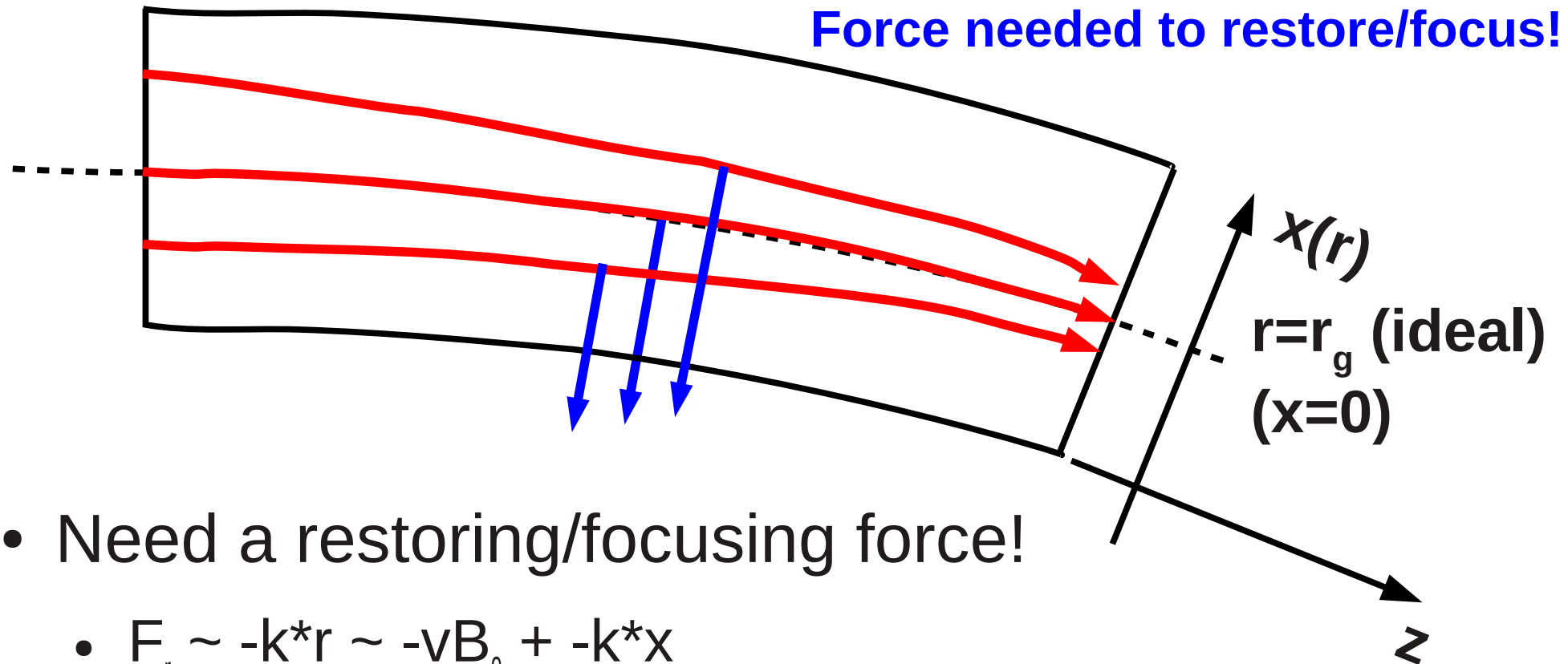
Transverse focusing in y direction

Force needed to restore/focus!

y

y=0 (ideal)

z
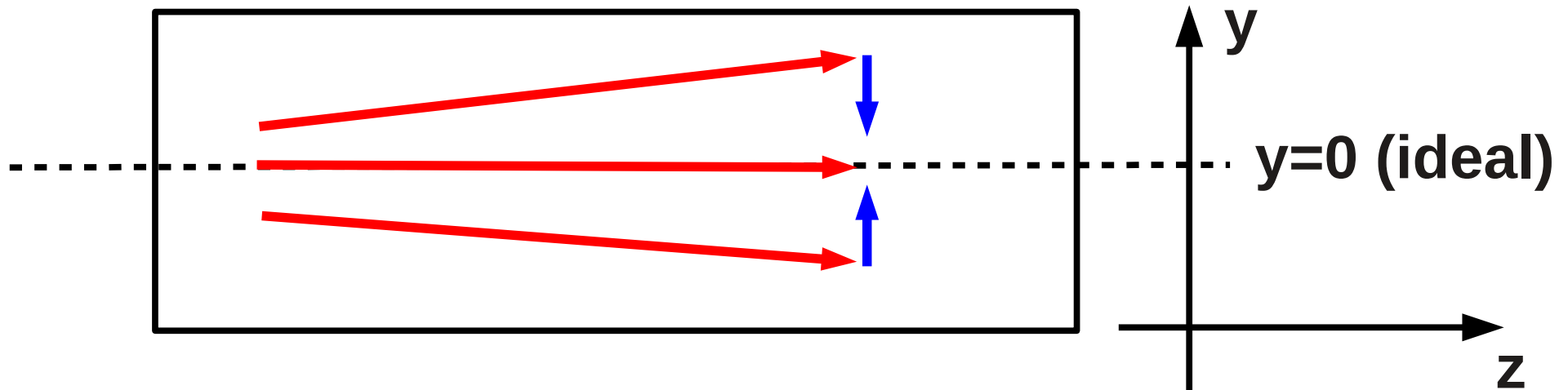
- Need a restoring/focusing force!
  - $F_y \sim -k*y$
- Harmonic oscillator (like string)

Transverse focusing in x direction

Force needed to restore/focus!

$x(r)$

$r = r_g$ (ideal)
(x=0)

z

- Need a restoring/focusing force!
  - $F_r \sim -k*r \sim -vB_0 + -k*x$
  - = central force (r_g) + harmonic oscillator in x

Let us first solve harmonic equation (ignoring magnet realities!)

$$\gamma m \frac{d^2 y}{dt^2} = \gamma m v^2 \frac{d^2 y}{dz^2} = -ky$$

$$y(z) = y_0 \cos\left(\frac{2\pi}{\lambda} z + \varphi\right),$$

where

$$\lambda = 2\pi \sqrt{\frac{\gamma m v^2}{k}}.$$

- Note that the wavelength does not depend on the amplitude y_0. There is only one wavelength for all amplitudes!
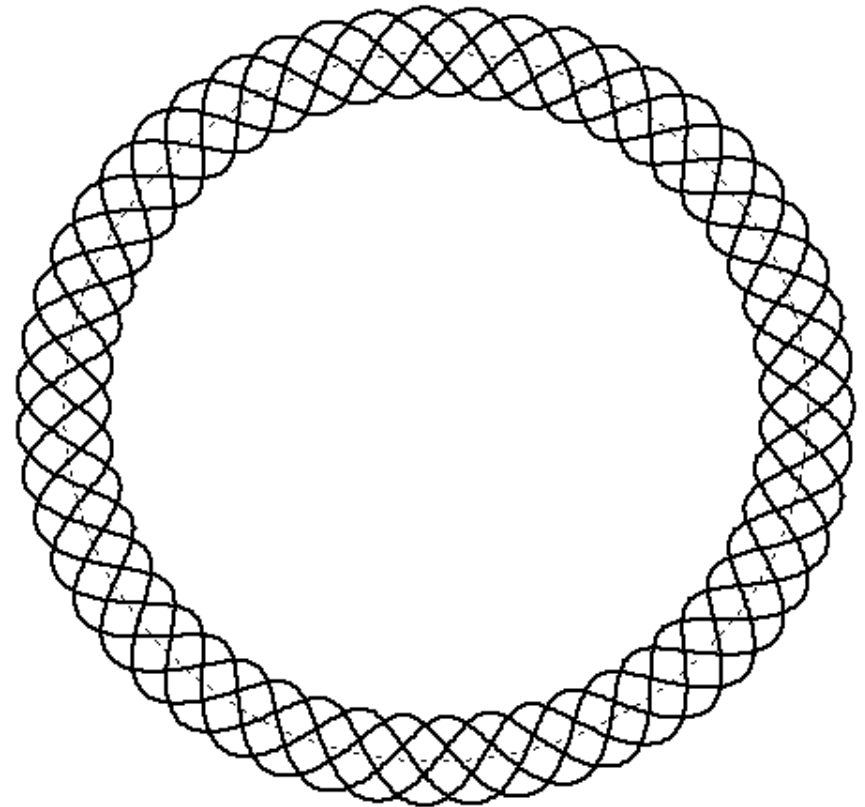
Tune interlude

- One defines
  - Q (ν[nu]) = C/λ, where C=$2\pi r_g$ is the circumference of the synchrotron ring

- Q is the number of transverse (betatron) oscillations per turn

- It is different for x and y directions

- Very important for beam stability!

Bad harmonic tune (Q=10.2)
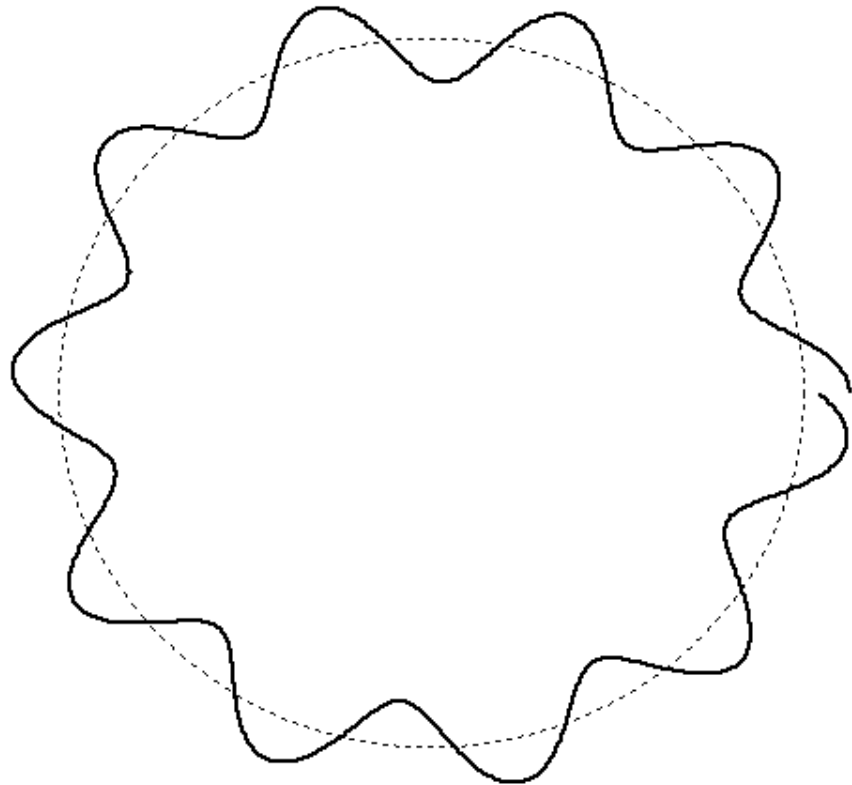
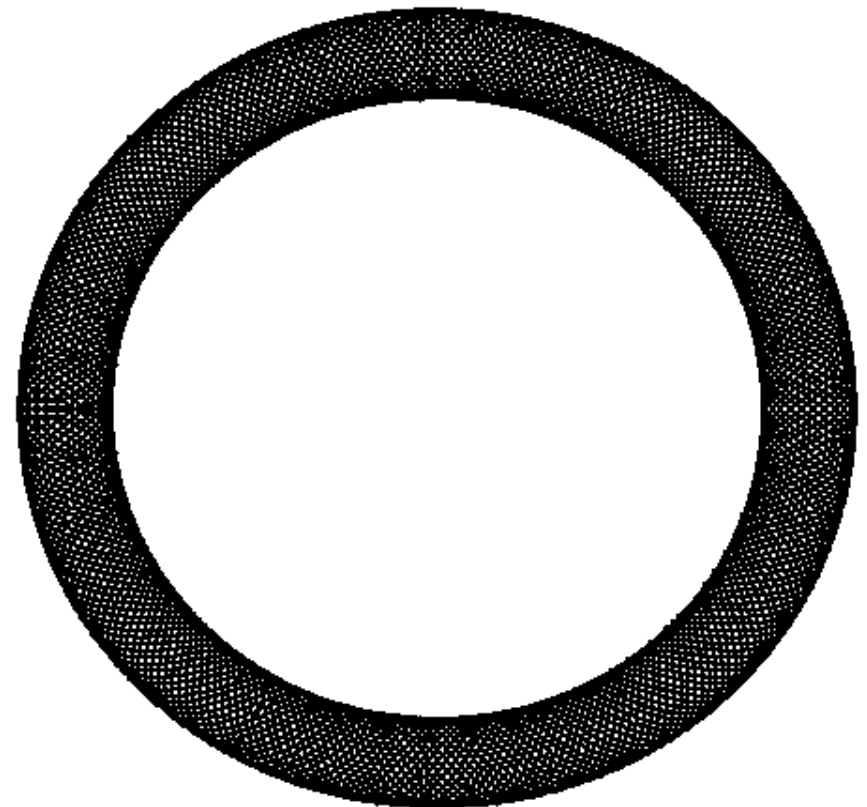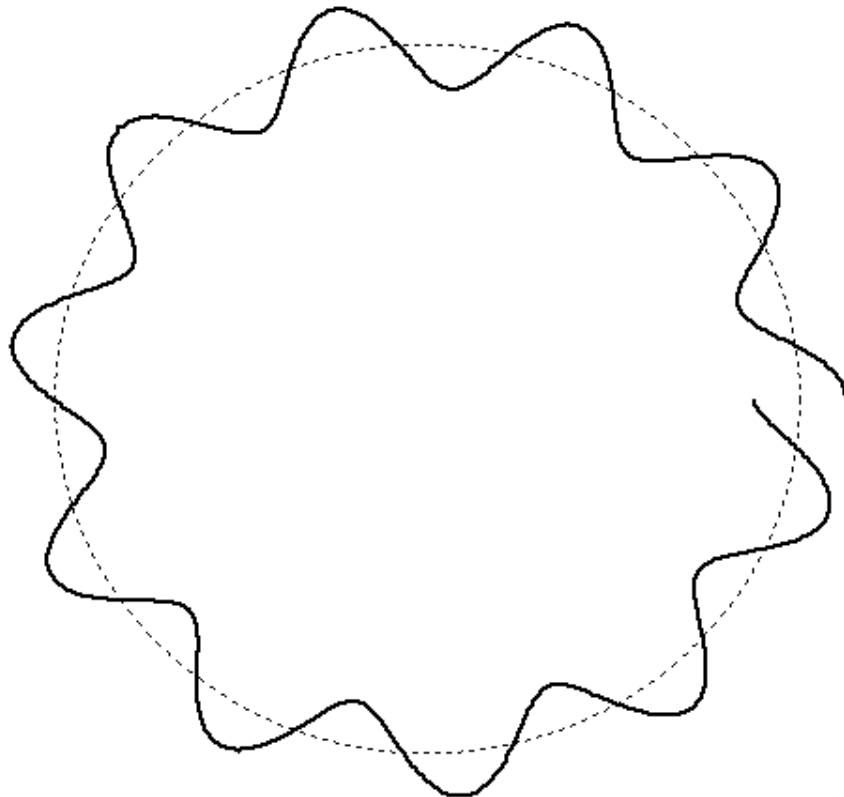1 turn   100 turns

Problem: tune does not integrate out magnet imperfections
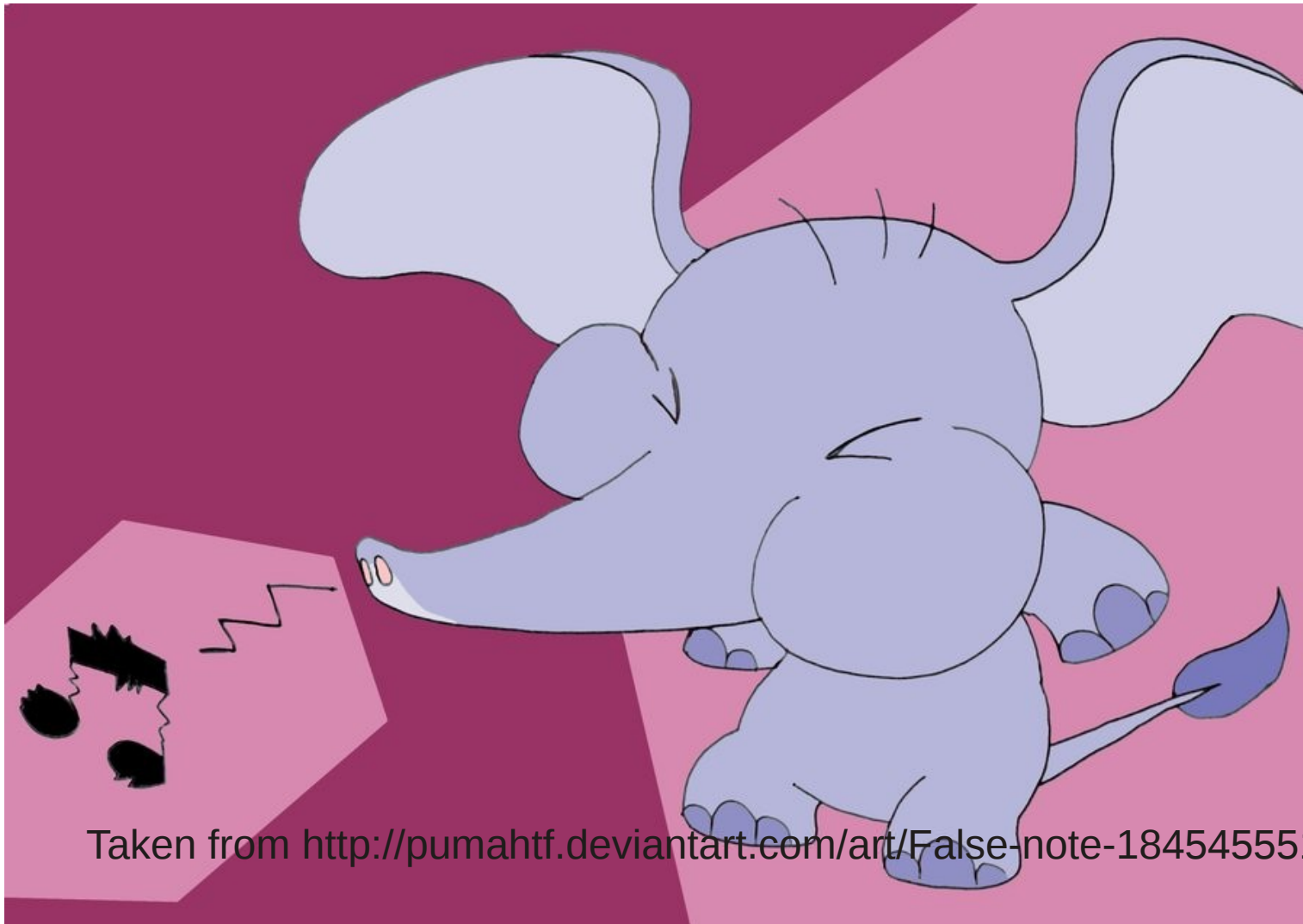
Better (less harmonic) tune (Q=10.48)

1 turn

100 turns

Tune is better at integrating out magnet imperfections

Lesson: bad musicians makes great accelerator physicists

Taken from http://pumahtf.deviantart.com/art/False-note-184545551

Lecture 5 and 6
P. Christiansen (Lund)

LHC TUNES

AT 7TeV

HORIZONTAL TUNE: $Q_x = 64.31$

VERTICAL TUNE: $Q_y = 59.32$

$$\Delta Q \leq 3 \cdot 10^{-3}$$

AT 450GeV

HORIZONTAL TUNE: $Q_x = 64.28$

VERTICAL TUNE: $Q_y = 59.31$

Betatron tunes should avoid linear coupling resonances at

$$n Q_x + m Q_y = p$$

Back to transverse motion and magnet realities!

- Taylor expanding the dipole magnetic field AND fulfilling Maxwell equations gives

  - $(B_x, B_y, B_z) \sim (-(n_0 B_0 / r_g)y, B_0 - (n_0 B_0 / r_g) * x, 0)$

    - NB! note that – sign is not good!

- Ideally we want n_0 as large as possible to confine the beam!

  - (And make the magnet as small as possible)

- Let us look at solution for x (r)!

The equation of motion for x

$$\gamma m \frac{d^2 r}{dt^2} = \gamma m v^2 \frac{d^2 r}{dz^2} = \gamma m \frac{v^2}{r} - q v B_y$$

$$\frac{d^2 r}{dz^2} = \frac{1}{r} - \frac{q}{\gamma m v} B_y$$

Substituing $x = r - r_g$ + expanding $\frac{1}{r}$:

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{1}{r_g^2} x - \frac{q}{\gamma m v} B_y$$

Inserting the Taylor expansion of B_y:

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{q B_0}{\gamma m v} - \frac{1}{r_g^2} x + \frac{q n_0 B_0}{\gamma m v r_g} x$$

The first two terms gives the solution for the ideal trajectory $\rightarrow$: $\frac{1}{r_g} = \frac{q B_0}{\gamma m v}$
so that:

$$\frac{d^2 x}{dz^2} = -\frac{1}{r_g^2} (1 - n_0) x.$$

Weak focusing: $0 < n_0 < 1$

$$\frac{d^2 x}{dz^2} = -\frac{1}{r_g^2}(1 - n_0)x.$$

- Only harmonic oscillation solution when ($1-n_0$)>0 (and y equation requires n_0>0)

  - Otherwise exponential growth!

- This means that the focusing is limited!

  - That is why this solution is called weak focusing

Lecture 5 and 6
P. Christiansen (Lund)

LHC example

- LHC focuses the beam using magnetic fields of 223 T/m

- If one would use weak focusing the magnetic field should be smaller than 8.33T/2800m = 0.003 T/m!!!!

- And so the ring would have to be enormous!

  - And the luminosity would be very small!
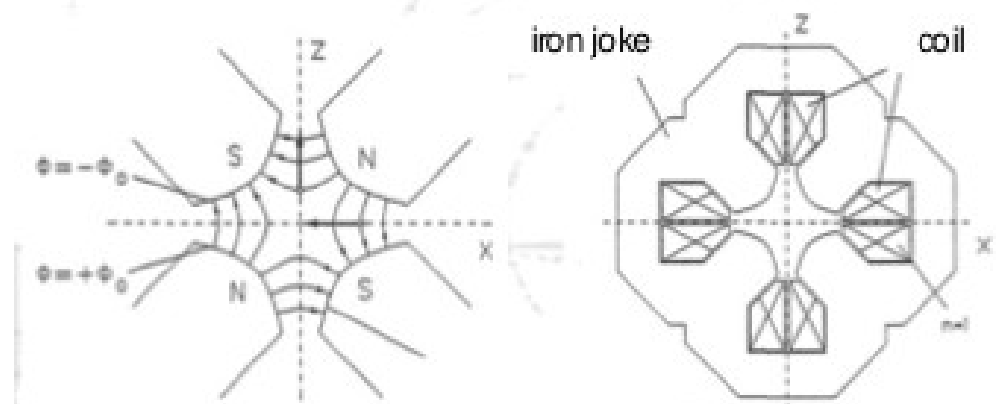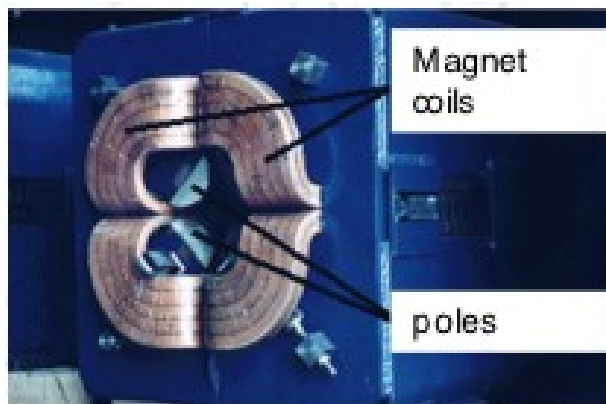
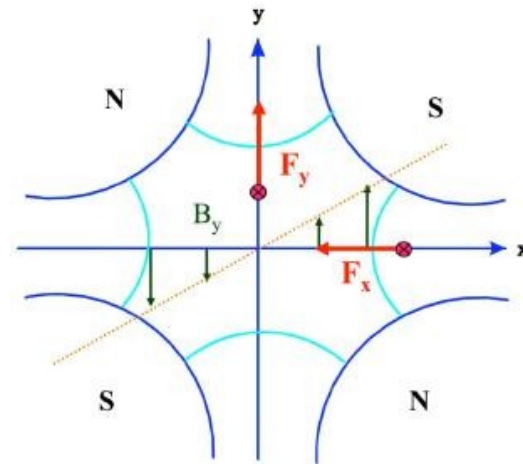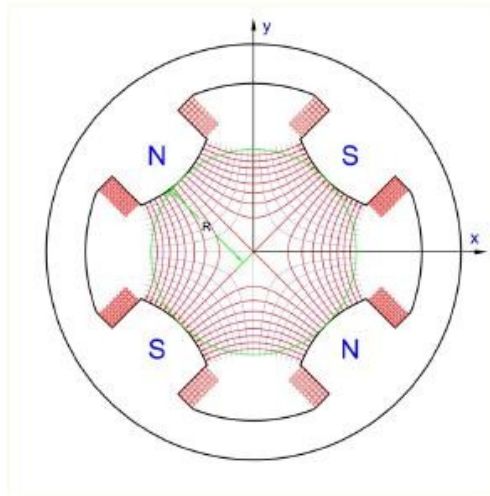# Can we find better focusing?
Beamline Elements

Dipole (bend) magnets
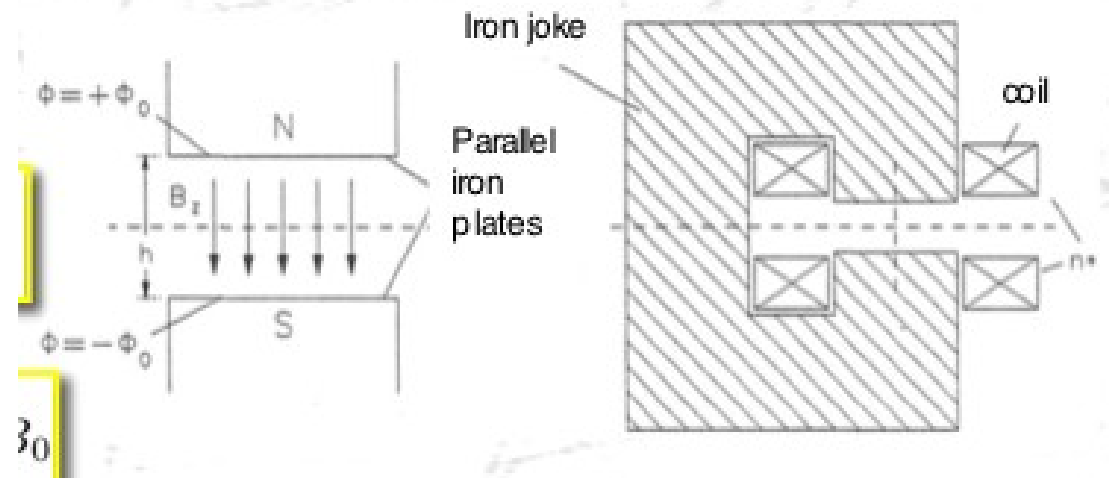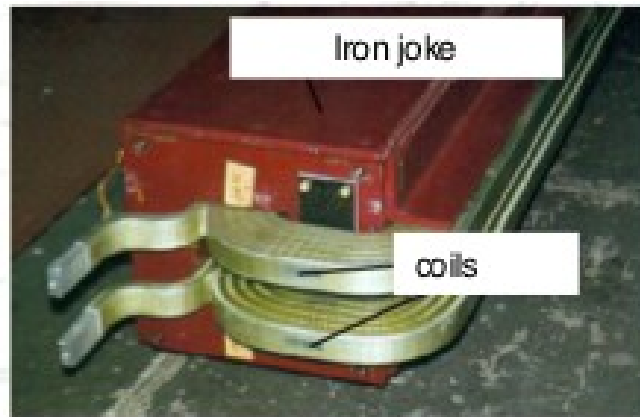
Quadrupole (focusing) magnets

Also Sextupoles and beyond

Lecture 5 and 6
P. Christiansen (Lund)

Quadropoles has similar problem!

FOCUSING OF THE PROTON BEAM

Quadrupole looks good – field increases linarly with distance from the center.
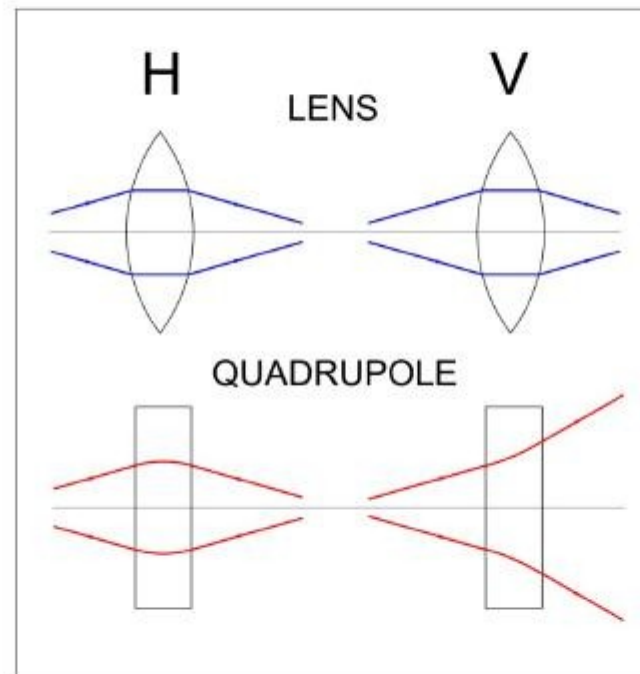
F_y has wrong direction! It doesn't work!

No solution: Maxwell tells us $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ $\oint_{\partial S} \mathbf{B} \cdot d\boldsymbol{l} = \mu_0 I$

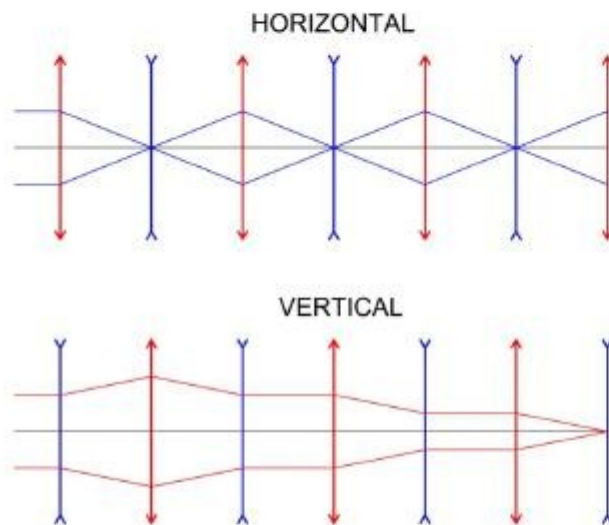8/25/2010   D. Vranic   6

PROBLEM

Quarupole is convergent lens in horizontal, but divergent in vertical direction!

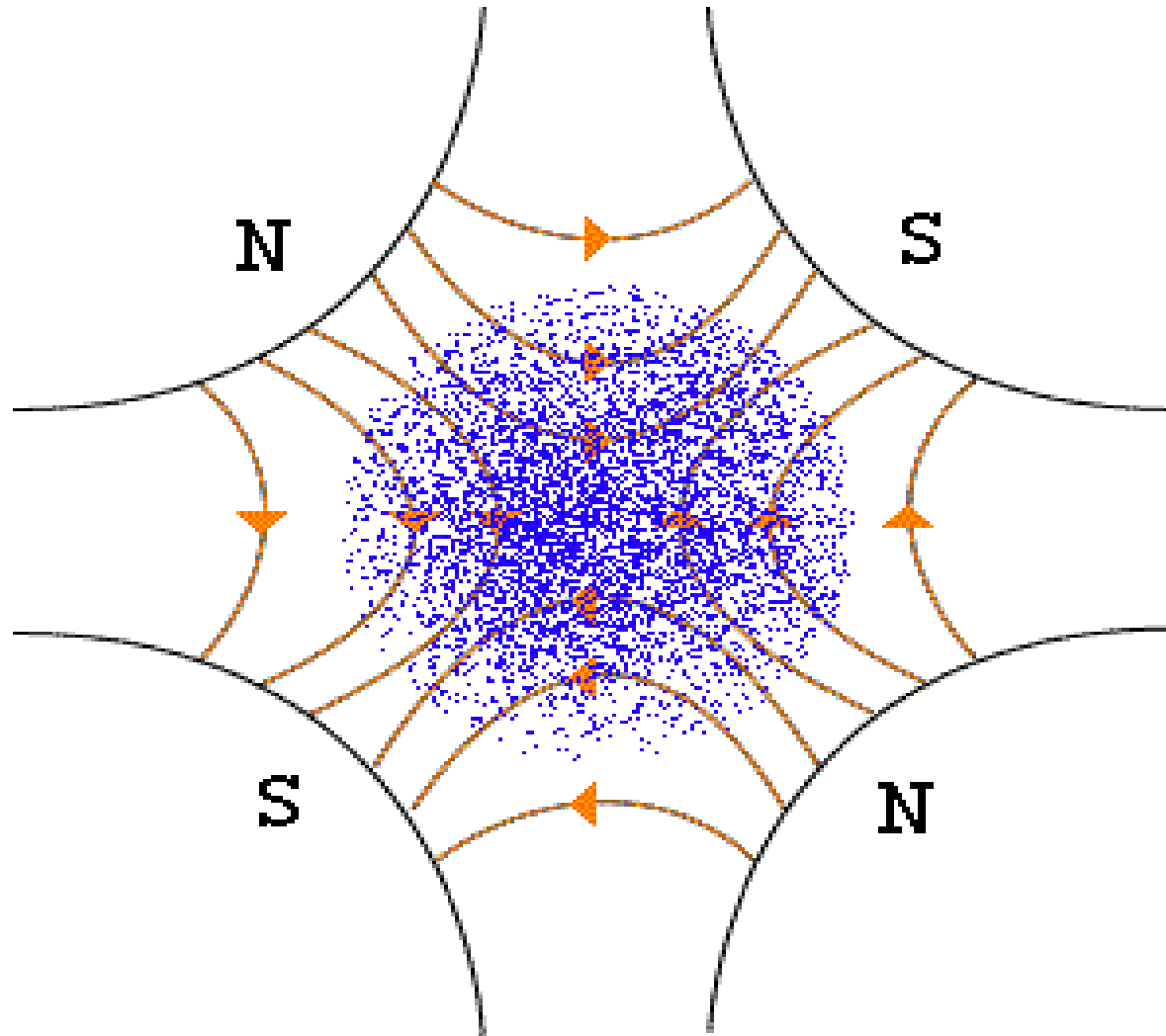There was no solution until 1952, and it is beautiful and simple:

SOLUTION: AG OR STRONG FOCUSING

HORIZONTAL

VERTICAL

FODO LATTICE

F - focusing
D - defocusing
O - drift space or dipoles

If we have alternating convergent and divergent lenses
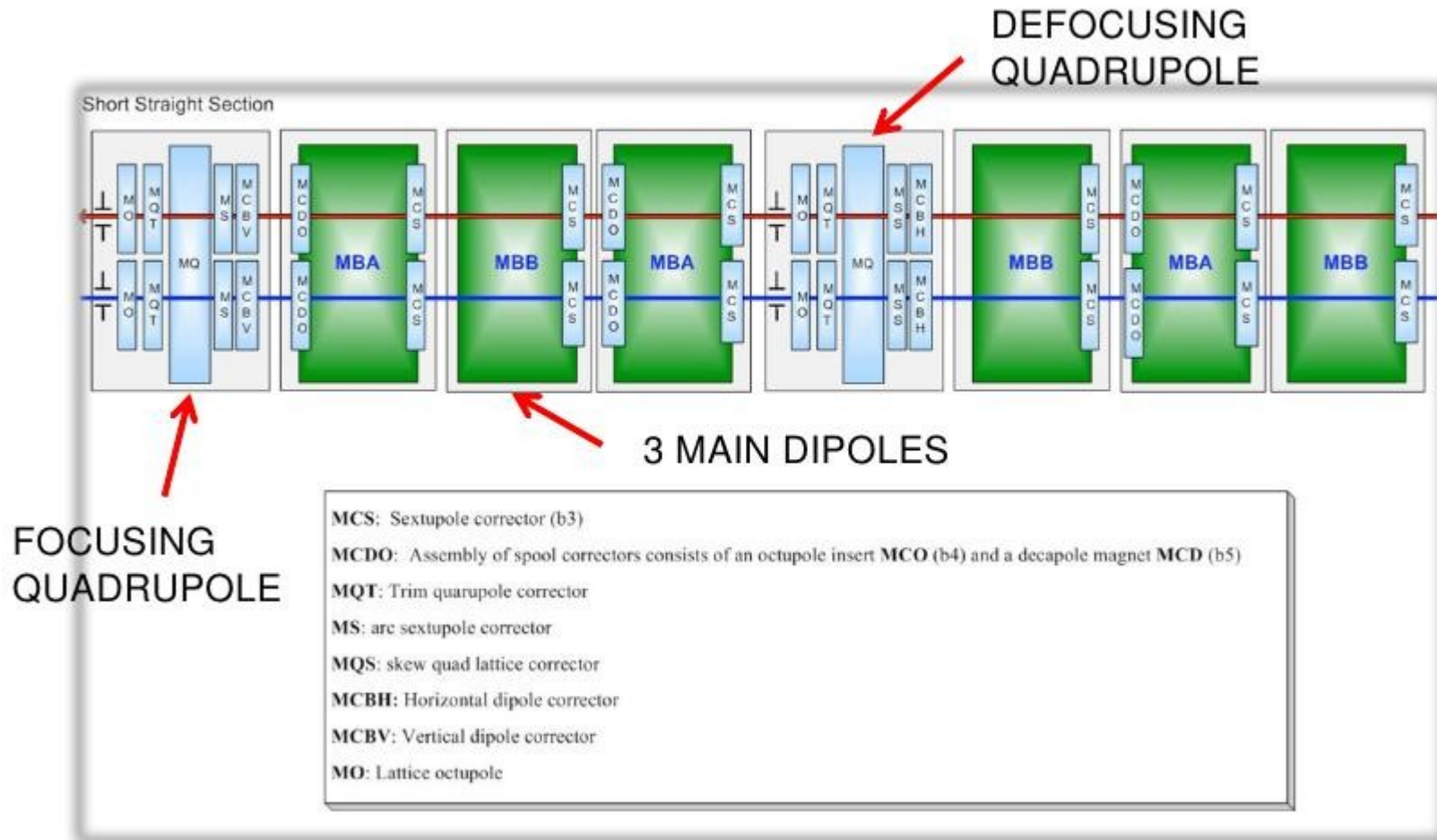with right spacing overall effect is focusing!

Animation taken from The Physics of Accelerators
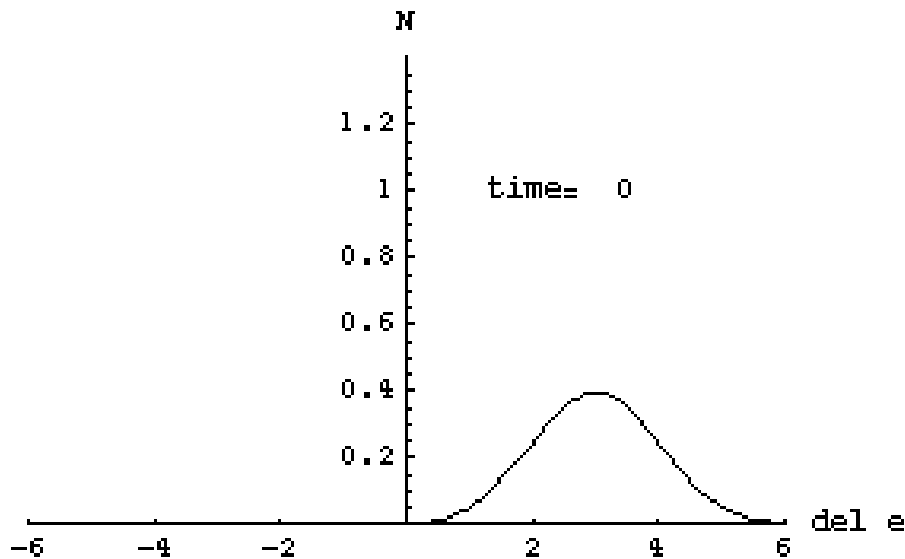Slides by C.R. Prior Rutherford Appleton Laboratory and Trinity College, Oxford

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LHC FODO LATTICE CELL (106.9 m)

The pattern of bending and focusing magnets is called lattice.

DEFOCUSING QUADRUPOLE

FOCUSING QUADRUPOLE

3 MAIN DIPOLES

MCS: Sextupole corrector (b3)

MCDO: Assembly of spool correctors consists of an octupole insert **MCO** (b4) and a decapole magnet **MCD** (b5)

MQT: Trim quarupole corrector

MS: arc sextupole corrector

MQS: skew quad lattice corrector

MCBH: Horizontal dipole corrector

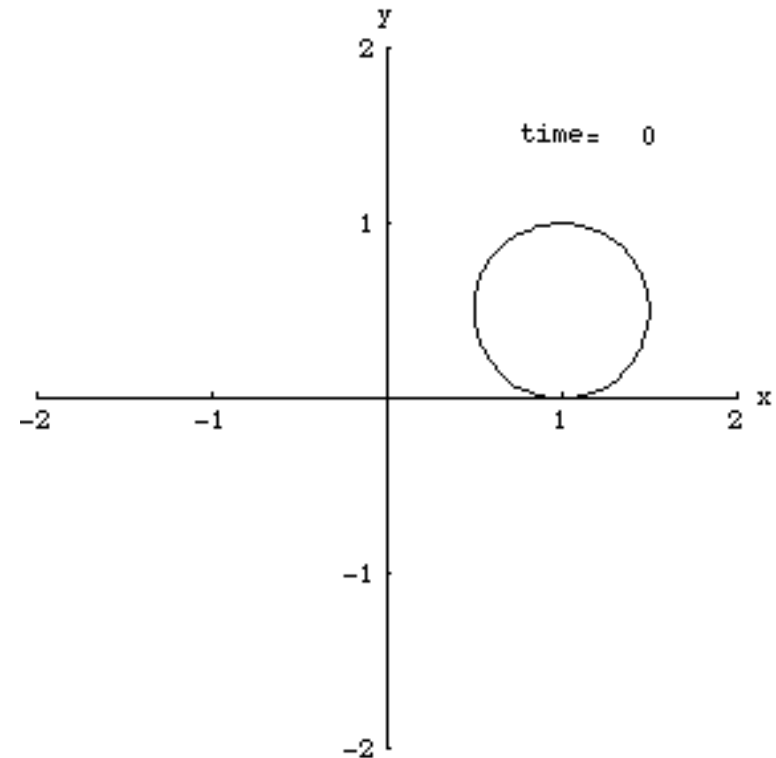**MCBV**: Vertical dipole corrector

MO: Lattice octupole

Energy adjusting by AC (longitudinal) & transverse strong focusing

Longitudinal

Transverse

- "Catching the beam" animations taken from
  - http://www.lns.cornell.edu/~dugan/USPAS/

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Beam dynamics

- Phase space limits

- The beta function

- Focusing the beam at the interaction point

Phase space limit (1/2)
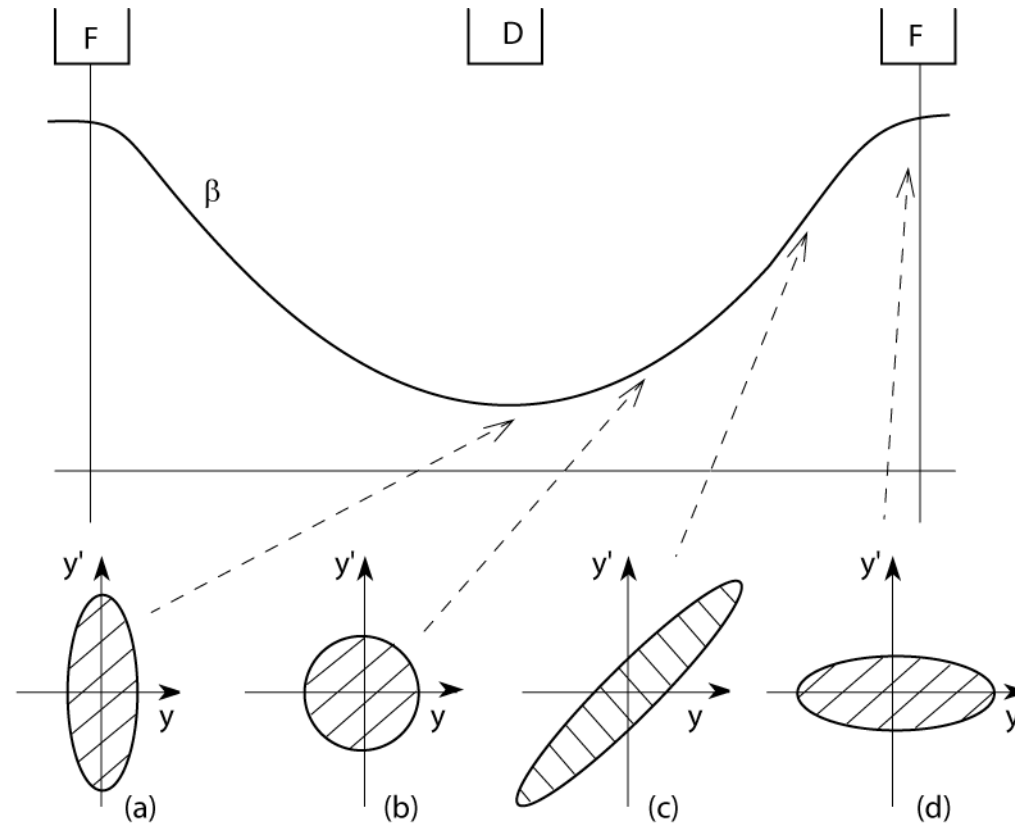
- Liouville's theorem states that for most beams the phase space area cannot change

Phase space limit (2/2)

- In reality the limit is on $p_y * y$:

  - Constant = $p_y * y$ ~ $\gamma m \beta_y * y$ where $\beta_y = dy/dt$

                     ~ $\gamma m \beta * y' * y$ where $y' = dy/dz$

  - So the phasespace limit implies that the area of the phase space ellipse for $y'*y$ (the beam emittance) decreases as $1/p$

  - This is called adiabatic damping. The physical size of the beam decreases as it is accelerated. The width decreases as $1/\sqrt{p}$ [the other $1/\sqrt{p}$ is the decreasing divergence].

The beta function (1/2)

The "gutter" analogy

x
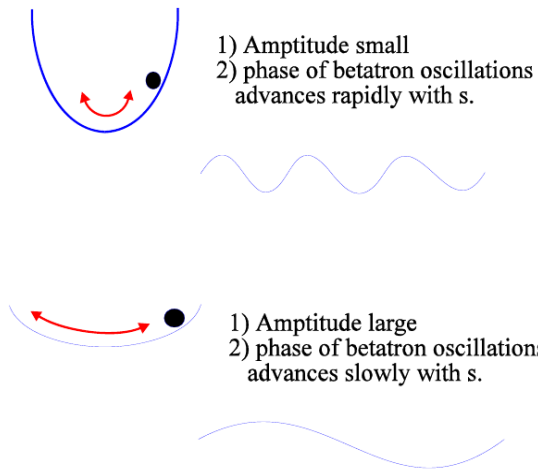
s
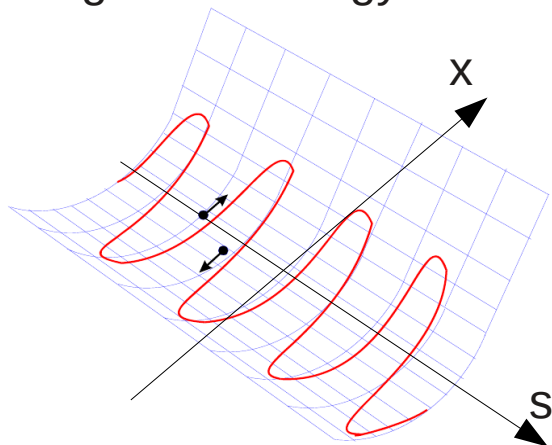
Based on:

ACCELERATORS FOR PEDESTRIANS

Simon Baird

$$x = \sqrt{\varepsilon\,\beta(s)}\cos(\Psi(s) + \varphi)$$

- The idea is to separate the transverse motion into two parts:

  - The initial conditions: ε (emittance) and φ

  - A part depending the focusing and de-focusing by magnets: $\beta(s)$ and $\Psi(s)$

    - The $\beta(s)$ function depends on where in the accelerator we are and <u>not related to the velocity β</u>)

The beta function (2/2)

The "gutter" analogy

1) Amptitude small
2) phase of betatron oscillations advances rapidly with s.

1) Amptitude large
2) phase of betatron oscillations advances slowly with s.

$$x = \sqrt{\varepsilon \beta(s)} \cos(\Psi(s) + \phi)$$

$$x' = \sqrt{\frac{\varepsilon}{\beta(s)}} \sin(\Psi(s) + \phi)$$

- So the beta function is related to how strong focusing we do and is the one we want to optimize in particular at the interaction points

- Note that x*x'~ε so that the phase space area is still the same

Lecture 5 and 6
P. Christiansen (Lund)

Recall lecture 1 and 2

- Intensity or brightness of an accelerator

$$N = \mathcal{L} \cdot \sigma$$

- Events Seen = Luminosity * cross-section

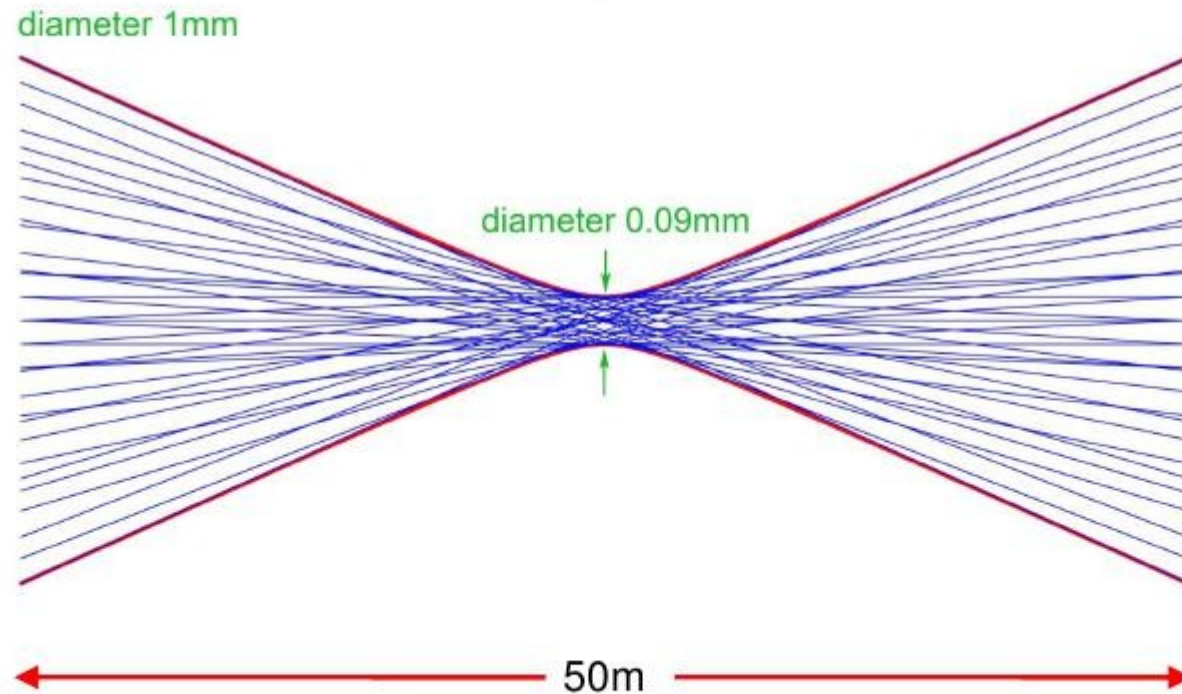   Rare processes (fb) need lots of luminosity (fb^{-1})

- In a storage ring

$$\mathcal{L} = \frac{1}{4\pi} \frac{f \cdot N_1 \cdot N_2}{\sigma_x \cdot \sigma_y}$$

"Current"

"Spot size"

Where *f* is the revelation frequency multiplied by # of colliding bunches
More particles through a smaller area means more collisions
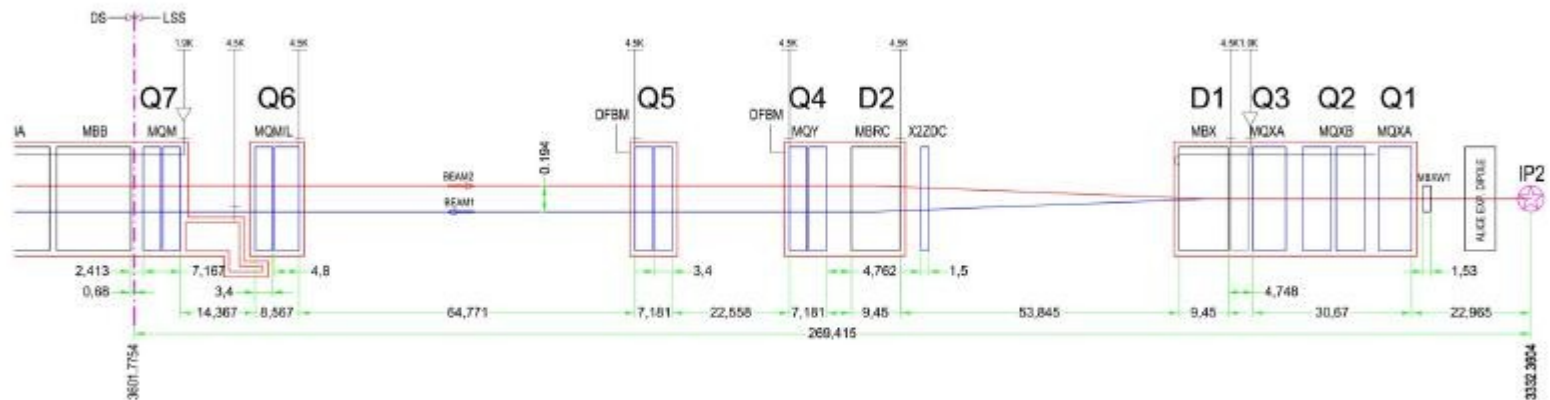
Example of focusing for collisions at P2 (ALICE)

diameter 1mm

diameter 0.09mm

50m

SYMMETRY!

LEFT SIDE   EXAMPLE: Focuses beam!
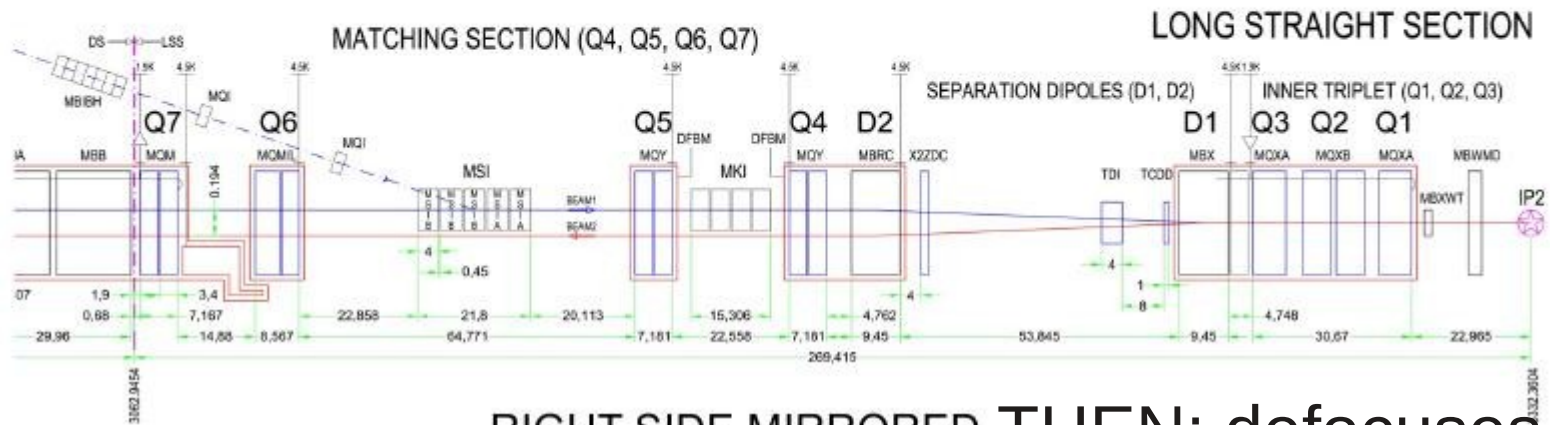
RIGHT SIDE MIRRORED THEN: defocuses beam!

Examples of synchrotrons

Lecture 5 and 6
P. Christiansen (Lund)
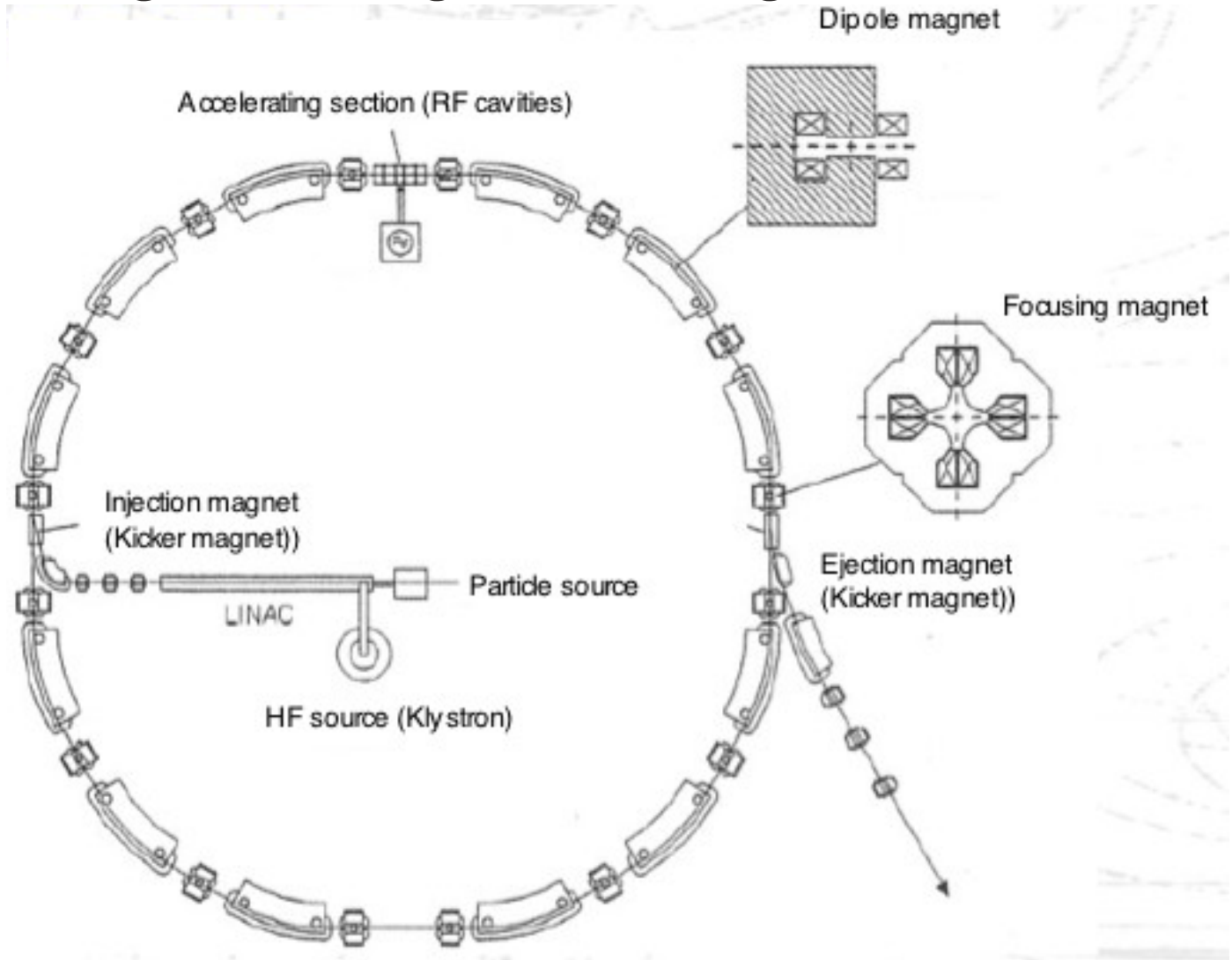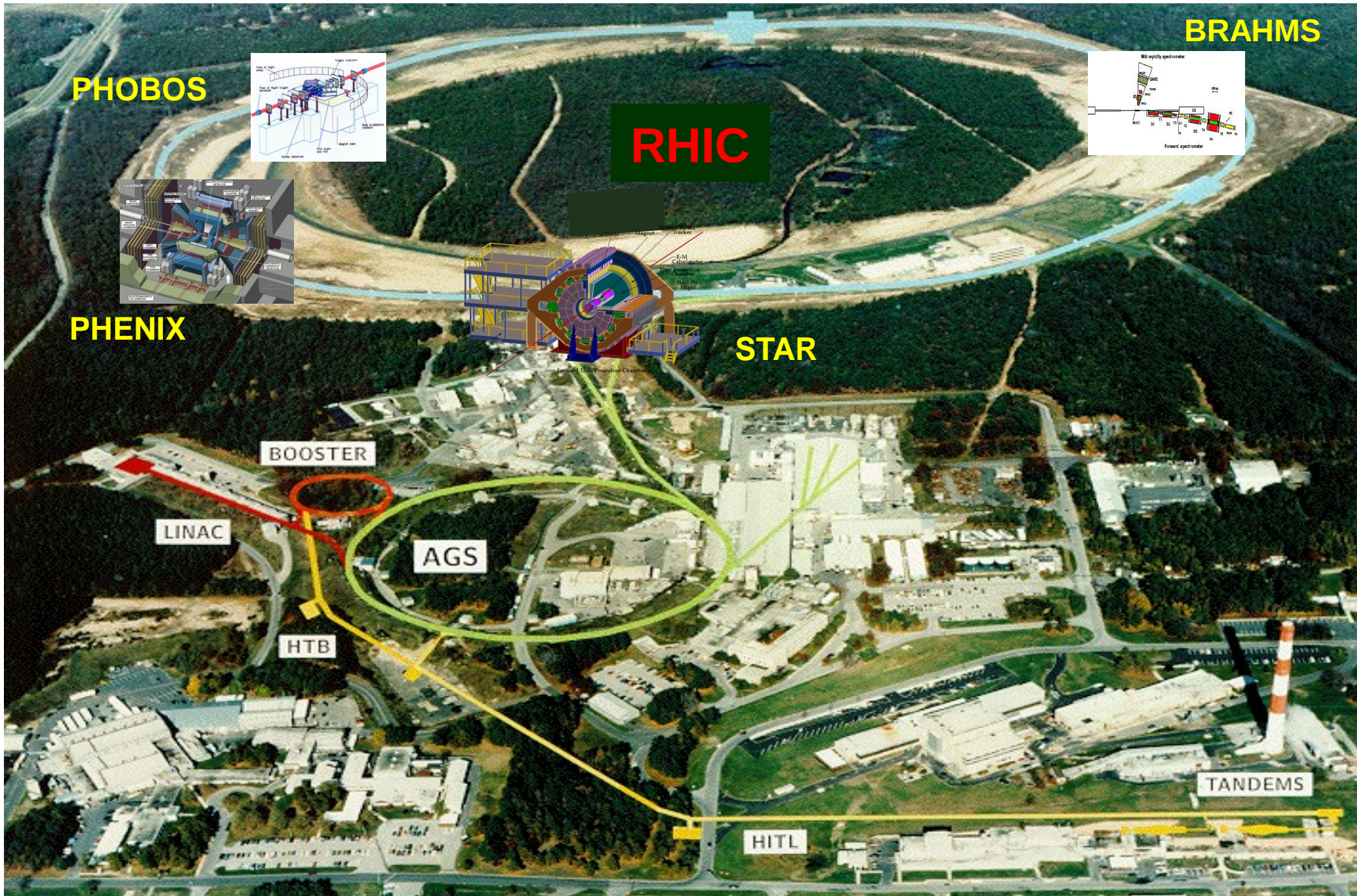
Synchrotrons

Use smaller magnets in a ring + accelerating station

3 GeV protons
BNL 1950s

Basis of all circular
machines built since

Dipole magnet

Accelerating section (RF cavities)

Focusing magnet

Injection magnet
(Kicker magnet))

Particle source

LINAC

Ejection magnet
(Kicker magnet))

HF source (Klystron)

at Brookhaven National

PHOBOS

BRAHMS

RHIC

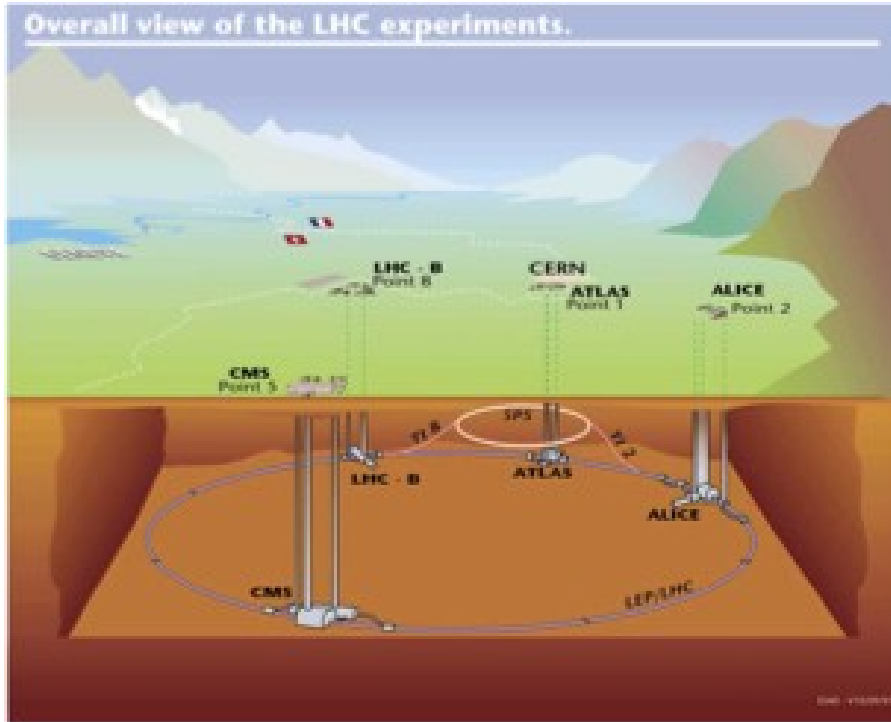PHENIX

STAR

BOOSTER

LINAC

AGS
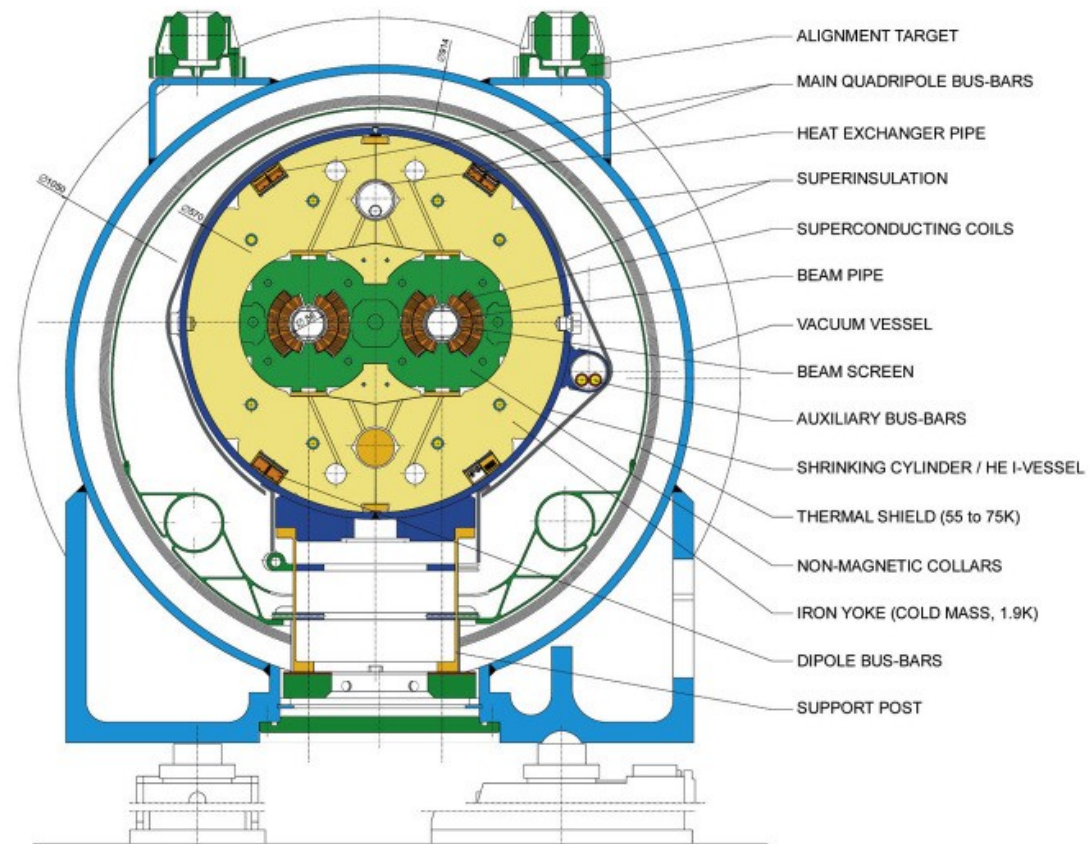
HTB

TANDEMS

HITL

P. Christiansen (Lund)

RHIC ramp with 56 bunches
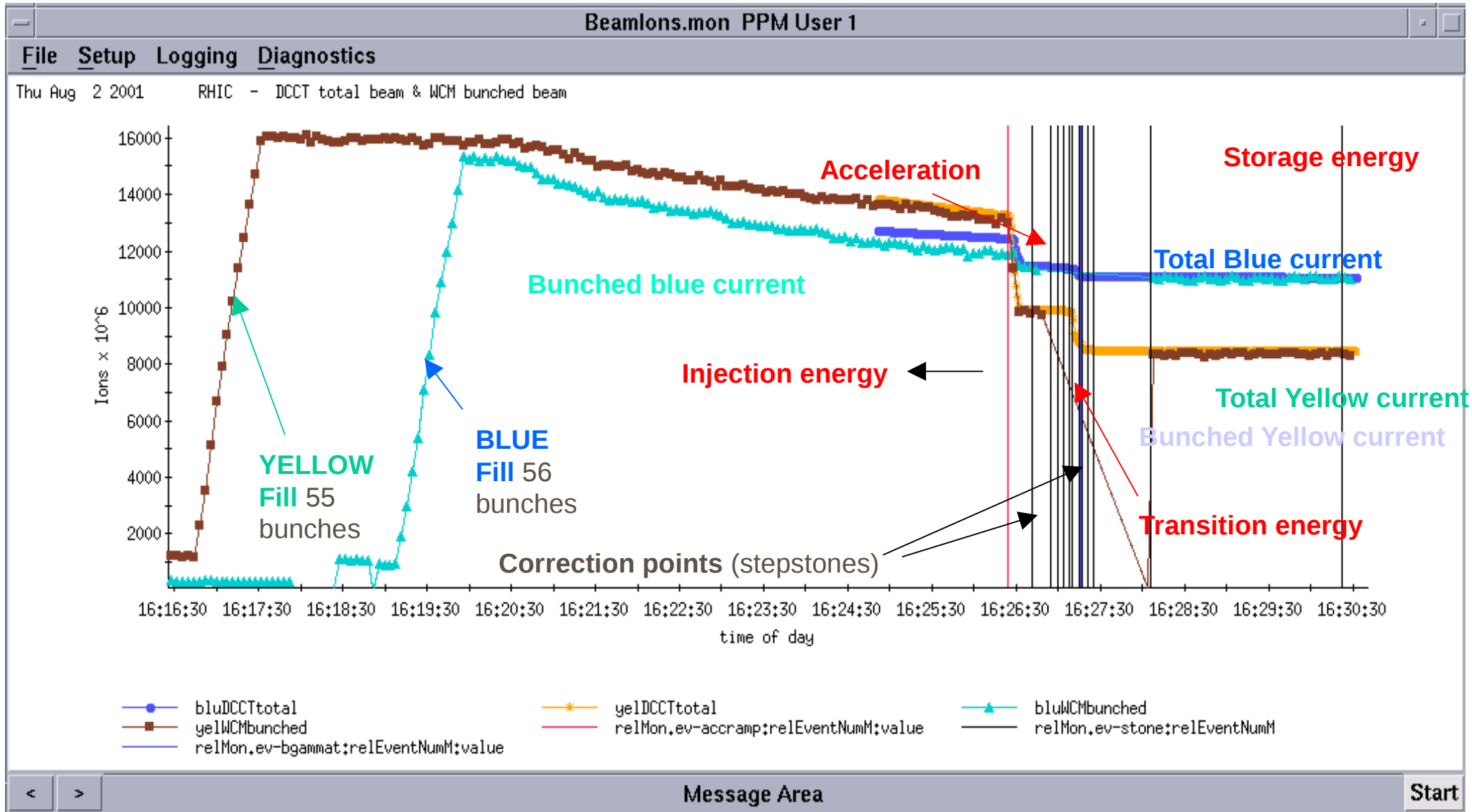
The beam is accelerated from Injection Energy (10 GeV) to Storage Energy (100 GeV). The acceleration process is called "ramp".
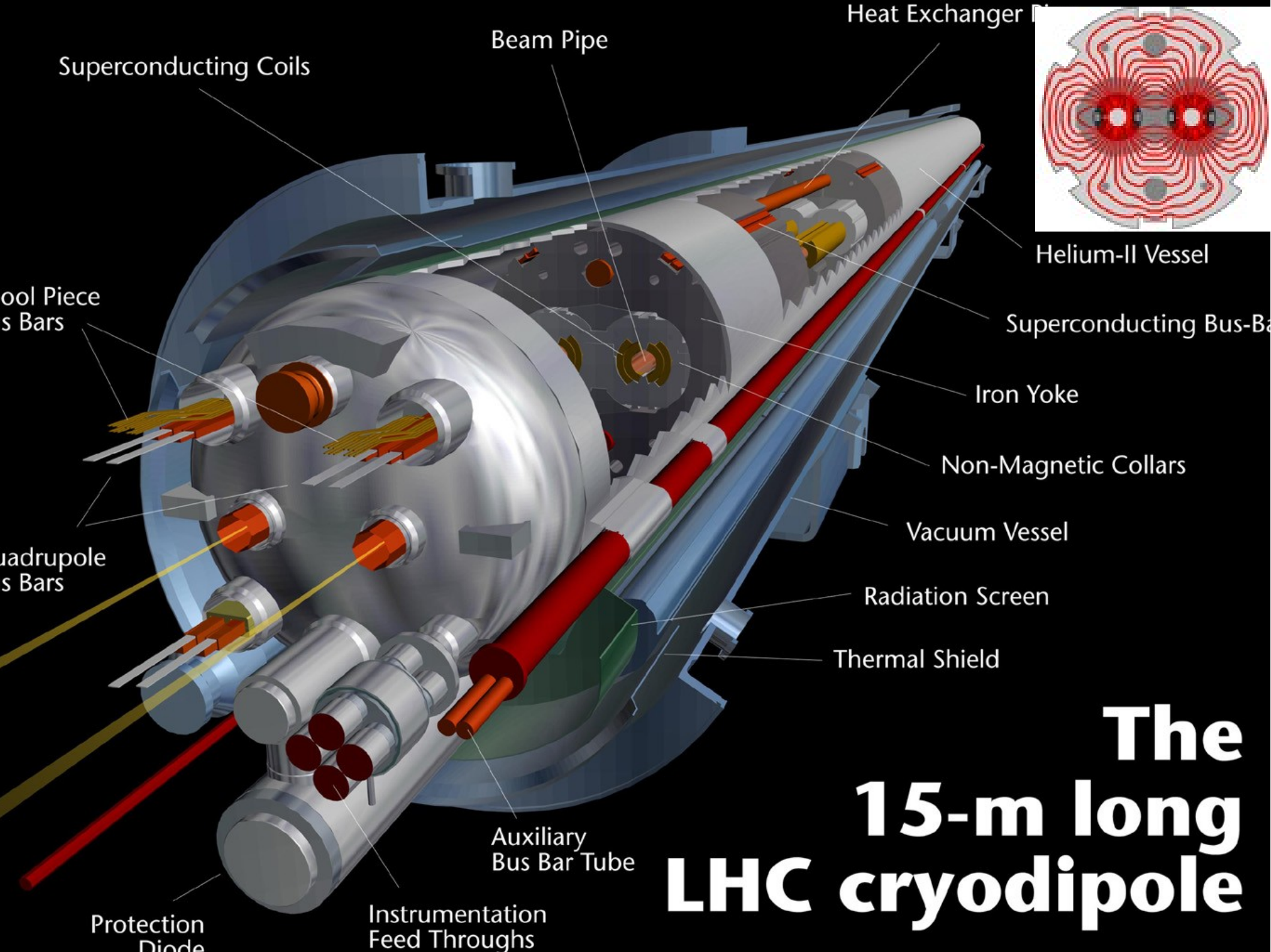
Lecture 5 and 6
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CERN Large Hadron Collider

Overall view of the LHC experiments.

LHC DIPOLE : STANDARD CROSS-SECTION

CERN AC/DI/MM - HE107 - 30 04 1999

- ALIGNMENT TARGET
- MAIN QUADRIPOLE BUS-BARS
- HEAT EXCHANGER PIPE
- SUPERINSULATION
- SUPERCONDUCTING COILS
- BEAM PIPE
- VACUUM VESSEL
- BEAM SCREEN
- AUXILIARY BUS-BARS
- SHRINKING CYLINDER / HE I-VESSEL
- THERMAL SHIELD (55 to 75K)
- NON-MAGNETIC COLLARS
- IRON YOKE (COLD MASS, 1.9K)
- DIPOLE BUS-BARS
- SUPPORT POST

Superconducting Coils

Beam Pipe

Heat Exchanger Pipe
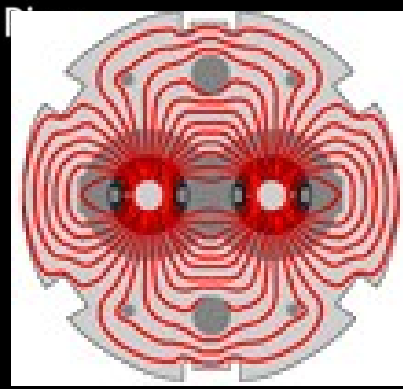
Helium-II Vessel

ool Piece
s Bars

Superconducting Bus-Ba

Iron Yoke

uadrupole
s Bars

Non-Magnetic Collars

Vacuum Vessel
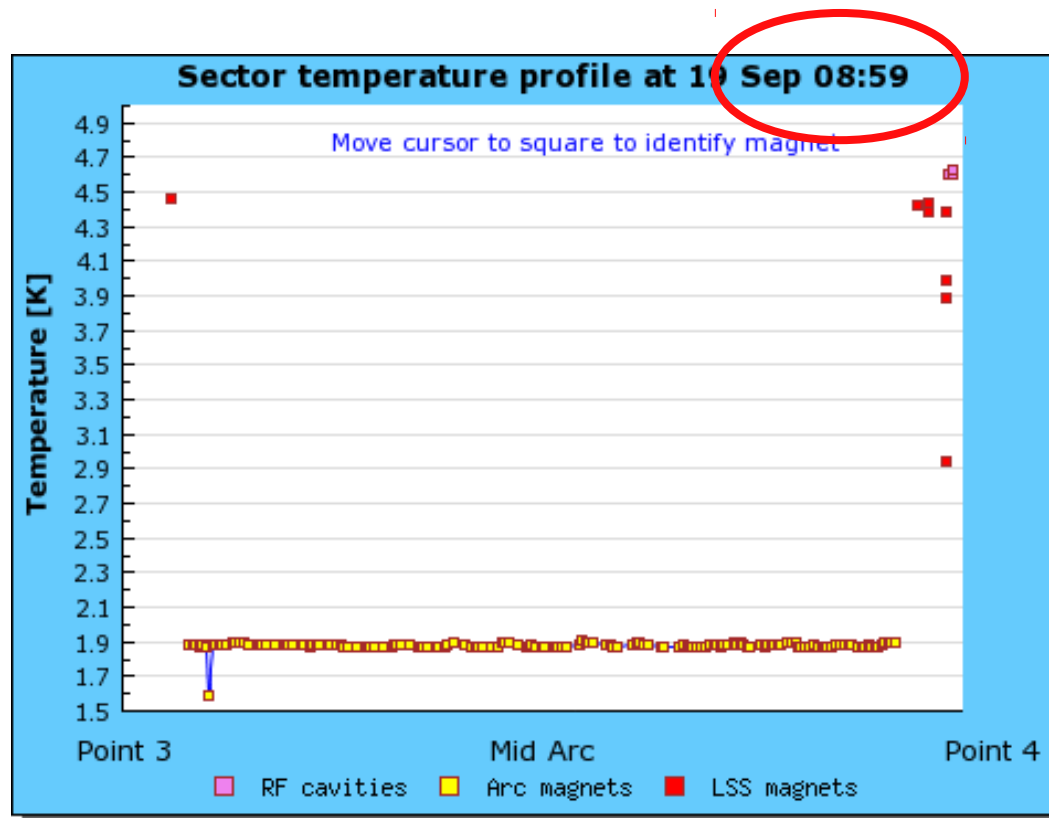
Radiation Screen

Thermal Shield

Protection
Diode

Instrumentation
Feed Throughs

Auxiliary
Bus Bar Tube

**The
15-m long
LHC cryodipole**
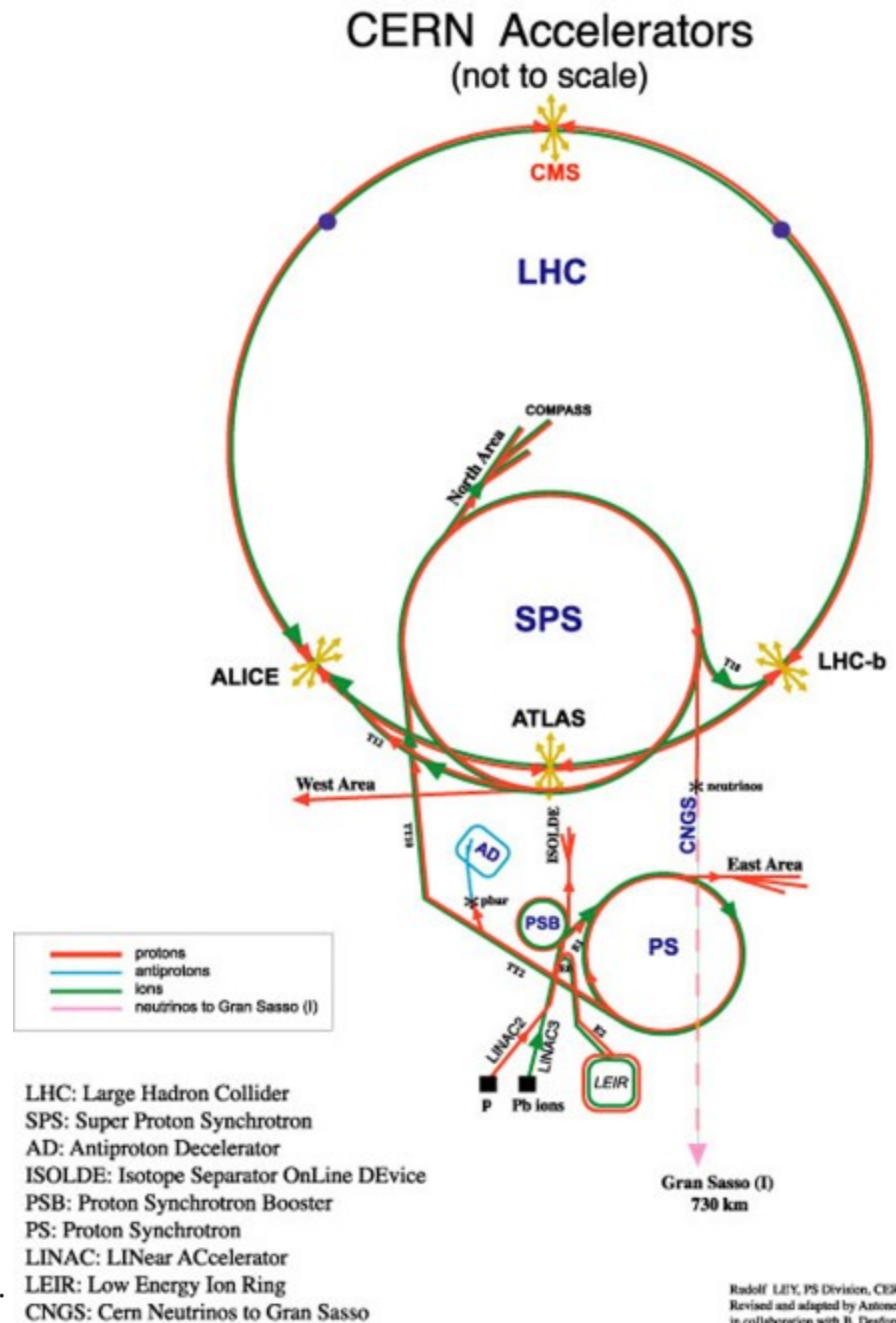
The 19 September 2008 accident

Lecture 5 and 6
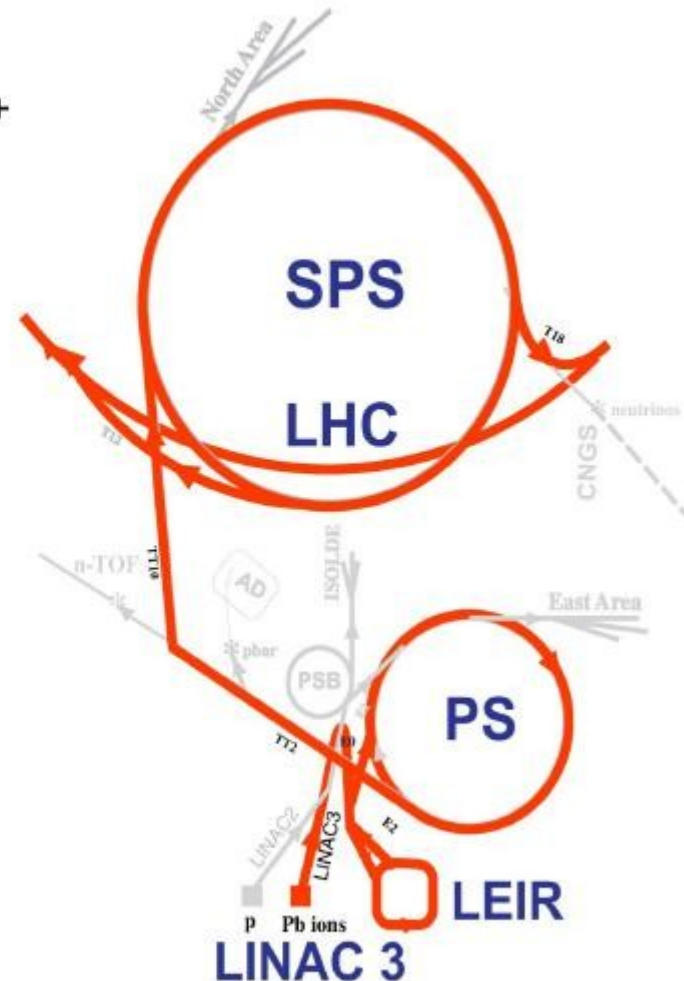P. Christiansen (Lund)

THE 19 SEPTEMBER 2008 INCIDENT

CERN Complex

Old rings still in use
Many different programs

10/4-13

ION BEAM IN THE LHC

- ● ECR ion source
- – Provide highest possible intensity of Pb29+
- ● RFQ + Linac 3
- – Adapt to LEIR injection energy
- – strip to Pb54+
- ● LEIR
- – Accumulate and cool Linac 3 beam
- – Prepare bunch structure for PS
- ● PS
- – Define LHC bunch structure
- – Strip to Pb82+
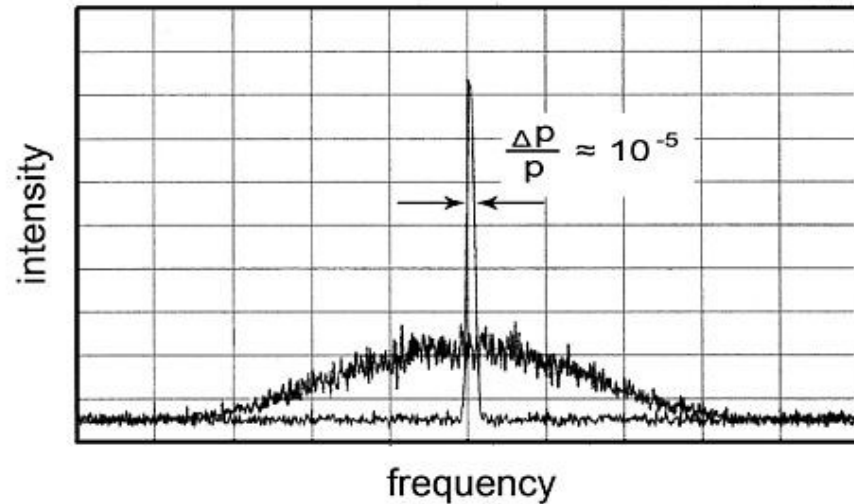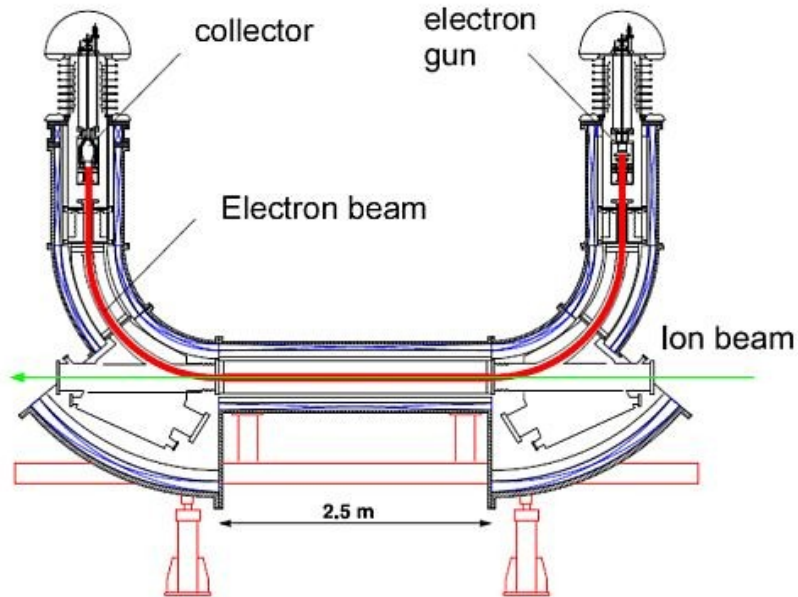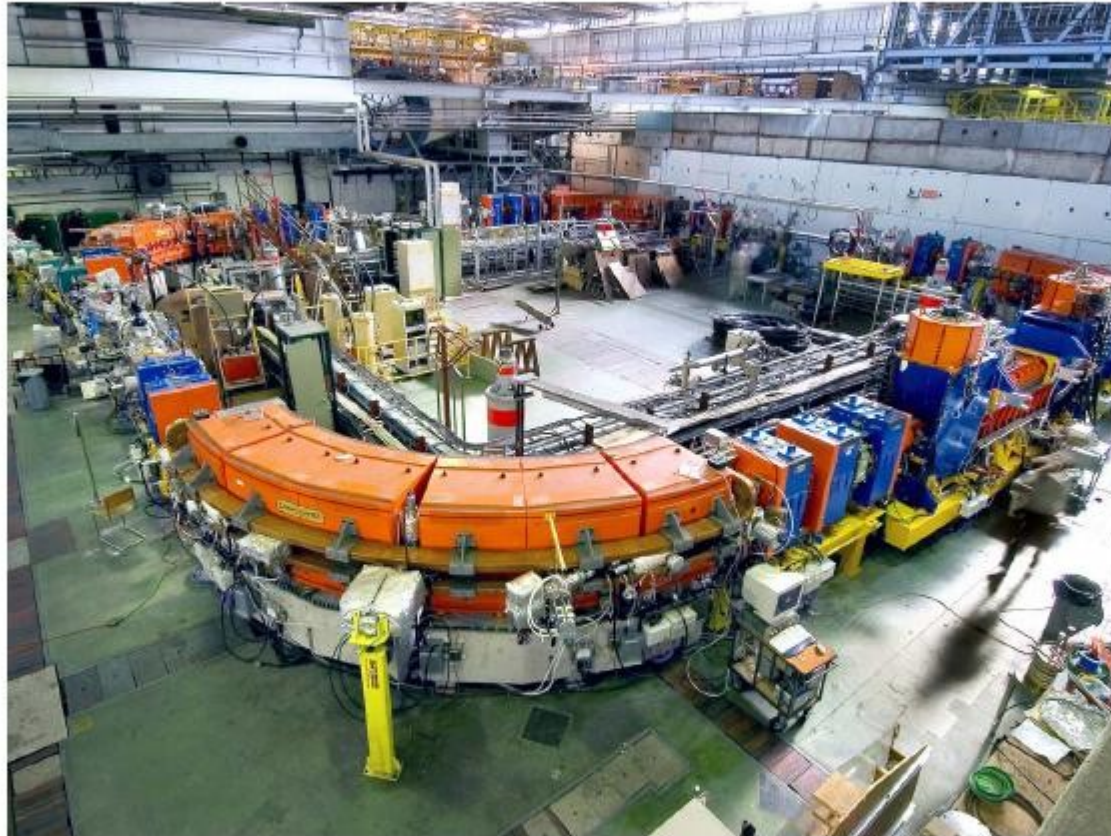- ● SPS
- – Define filling scheme

10/4-13   Lecture 5 and 6   44

P. Christiansen (Lund)

LEIR: Electron cooling example

- From: http://web-docs.gsi.de/

- Elastic collision e+ion will decrease the relative momentum spread in the beam

LEIR

ION PHYSICS: STABLE BEAMS

Energy:	3500 Z GeV	I(B1):	8.32e+11	I(B2):	7.57e+11

FBCT Intensity and Beam Energy — Updated: 16:30:50
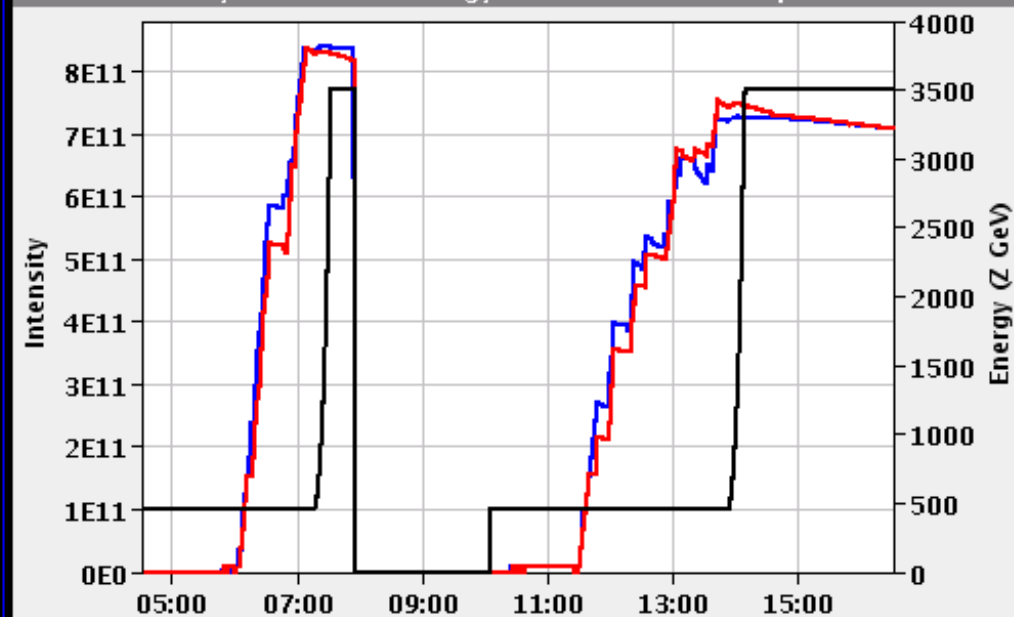
Instantaneous Luminosity — Updated: 16:30:48

— ATLAS — ALICE — CMS
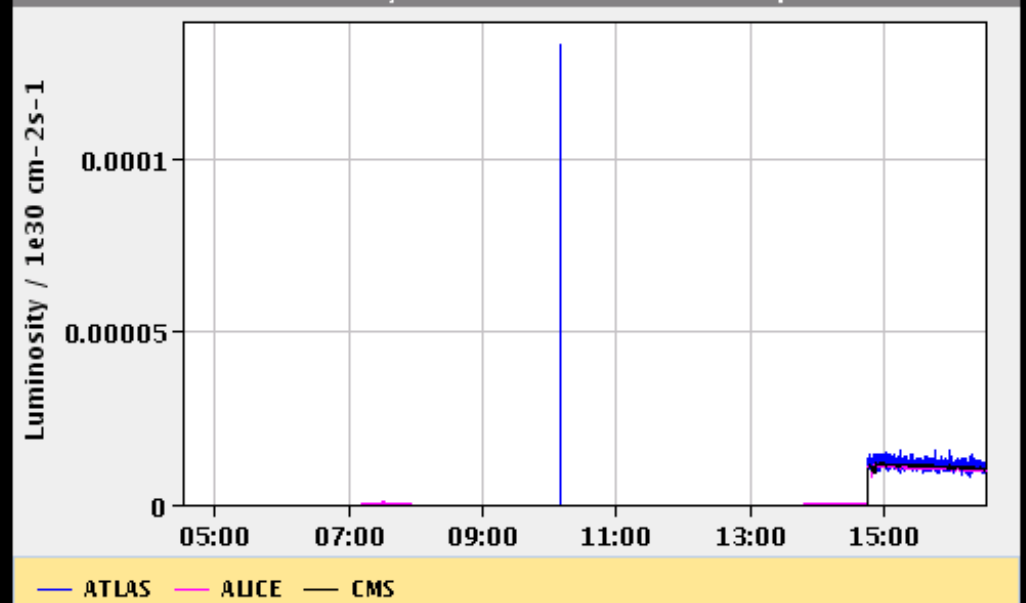
Comments 29-11-2010 14:54:46 :

*** STABLE BEAMS ***

All points optimized

BIS status and SMP flags	B1	B2
Link Status of Beam Permits	true	true
Global Beam Permit	true	true
Setup Beam	false	false
Beam Presence	true	true
Moveable Devices Allowed In	true	true
Stable Beams	true	true

AFS: 500ns_121b_113_114_0_4bpi31inj_IONS

PM Status B1	ENABLED	PM Status B2	ENABLED

DUMP CORE TDE

Beam →

| 1000 | 3000 | 3000 |

7000

Beam View

Dilution Profile

Graphite 1.77 g/cm³
Graphite 1.10 g/cm³

7m long C / C-C TDE in steel shrink-cylinder, followed by 1m Al, 2m Fe ~1000 T of concrete shielding
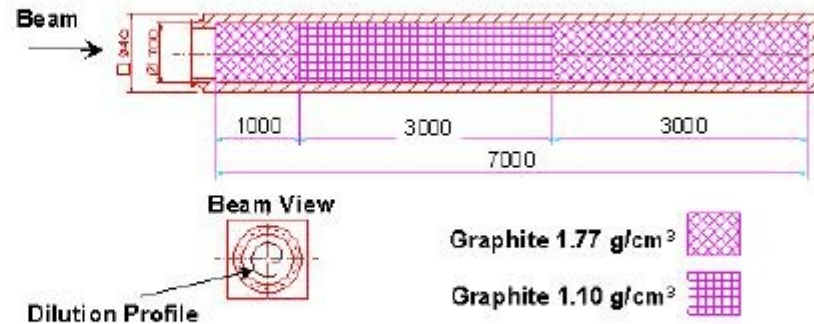
This is the **ONLY element in the LHC that can** withstand the impact of the full 7 TeV beam ! Nevertheless, the dumped beam must be painted to keep the peak energy densities at a tolerable level !

Why graphite? If the material were heavy, all the beam's energy would concentrate in the first half meter of the block.
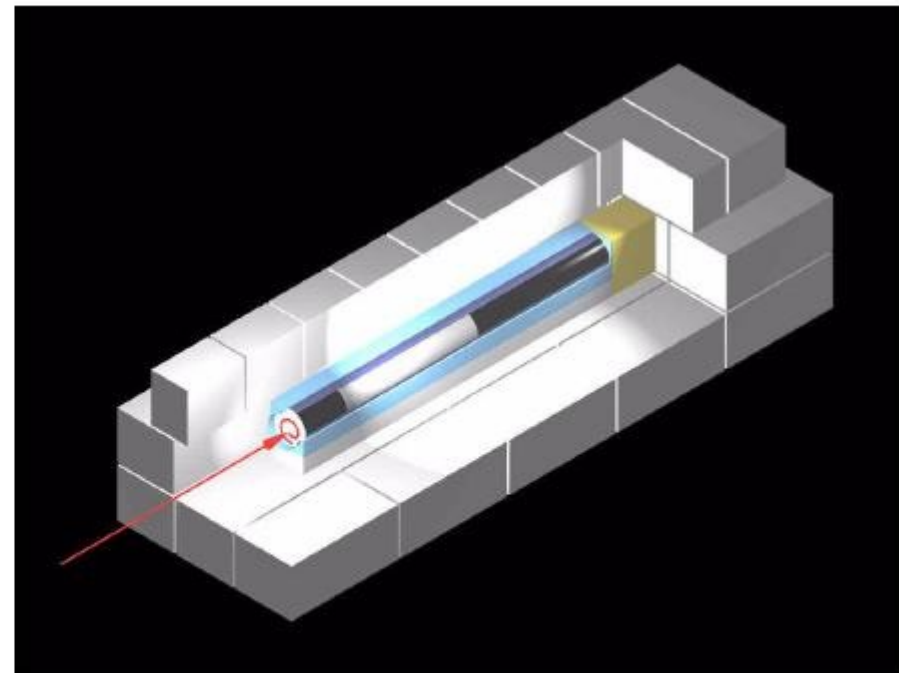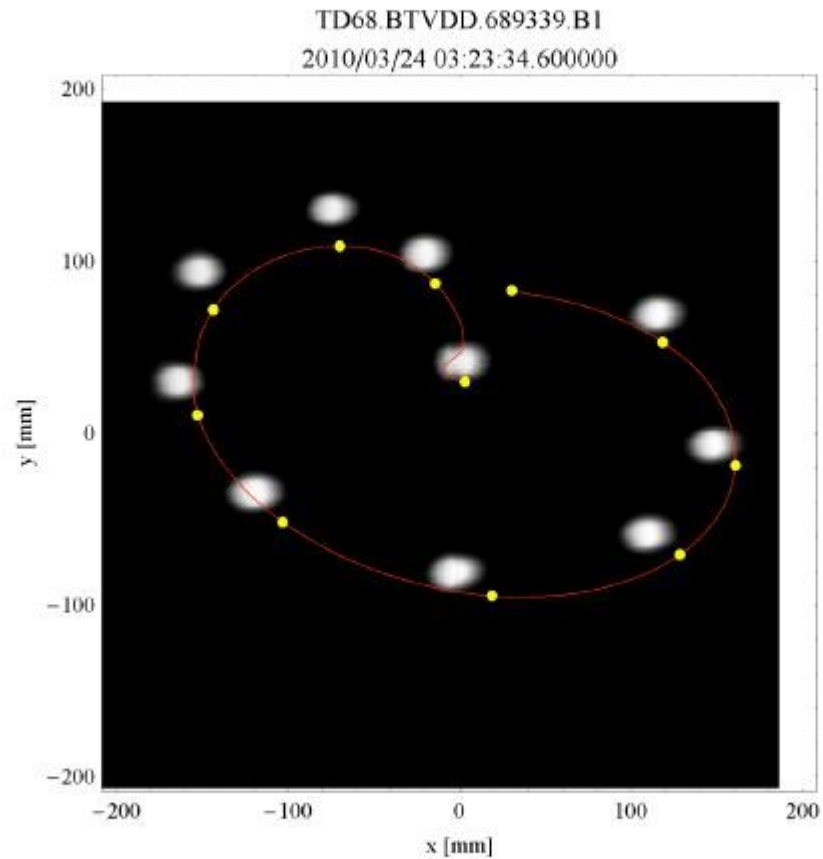

8/25/2010   D. Vranic   11

The beam size has increased to an extent where the sigma is 1.6mm in both planes.

TD68.BTVDD.689339.B1
2010/03/24 03:23:34.600000