V. Hadron quantum numbers

Characteristics of a hadron:

1) Mass

2) Quantum numbers arising from space-time symmetries : total spin *J*, parity *P*, charge conjugation *C*. Common notation:

– J^P (e.g. for proton: $\frac{1}{2}^+$), or

– J^{PC} if a particle is also an eigenstate of *C*-parity (e.g. for π^0 : 0^{-+})

3) Internal quantum numbers: charge Q and baryon number B (always conserved),

$S, C, \tilde{B}, T$ (conserved in e.m. and strong interactions)

How do we know which are the quantum numbers of a newly discovered hadron?

How do we know that mesons consist of a quark-antiquark pair, and baryons - of three quarks?

Some *a priori* knowledge is needed:

Particle	**Mass (GeV/c^2)**	**Quark composition**	**Q**	**B**	***S***	***C***	$\tilde{B}$
p	0.938	uud	1	1	0	0	0
n	0.940	udd	0	1	0	0	0
K^-	0.494	$s\bar{u}$	-1	0	-1	0	0
D^-	1.869	$d\bar{c}$	-1	0	0	-1	0
B^-	5.279	$b\bar{u}$	-1	0	0	0	-1

Considering the lightest 3 quarks (u, d, s), possible 3-quark and 2-quark states will be ($q_{i,j,k}$ are u- or d-quarks):

	sss	ssq_i	sq_iq_j	$q_iq_jq_k$
S	-3	-2	-1	0
Q	-1	0; -1	1; 0; -1	2; 1; 0; -1

	$s\bar{s}$	$s\bar{q}_i$	$\bar{s}q_i$	$q_i\bar{q}_i$	$q_i\bar{q}_j$
S	0	-1	1	0	0
Q	0	0; -1	1; 0	0	-1; 1

❖ Hence restrictions arise: for example, mesons with *S*= -1 and Q=1 are forbidden

→ Particles which fall out of above restrictions are called *exotic* particles (like $dd\bar{u}s$, $uuu\bar{d}s$ etc.)

From observations of strong interaction processes, quantum numbers of many particles can be deduced:

	$p + p$	$\rightarrow$	$p + n$	$+ \pi^+$
Q=	2		1	1
S=	0		0	0
B=	2		2	0

	$p + p$	$\rightarrow$	$p + p$	$+ \pi^0$
Q=	2		2	0
S=	0		0	0
B=	2		2	0

	p	$+ \pi^-$	$\rightarrow$	$\pi^0 + n$
Q=	1	-1		0
S=	0	0		0
B=	1	0		1

Observations of pions confirm these predictions, proving that pions are non-exotic particles.

Assuming that K^- is a strange meson, one can predict quantum numbers of Λ-baryon:

	$K^- + p$	$\rightarrow \pi^0$	$+\Lambda$
Q=	0	0	0
S=	-1	0	-1
B=	1	0	1

And further, for K^+-meson:

	$\pi^- + p$	$\rightarrow K^+$	$+ \pi^- + \Lambda$
Q=	0	1	-1
S=	0	1	-1
B=	1	0	1

→ Before 2003: all of the more than 200 observed hadrons satisfy this kind of predictions

→ July 2003: Pentaquark $uudd\bar{s}$ observed in Japan!

→ Simple quark model -- only quark-antiquark and 3-quark (or 3-antiquark) states exist -- has to be revised!

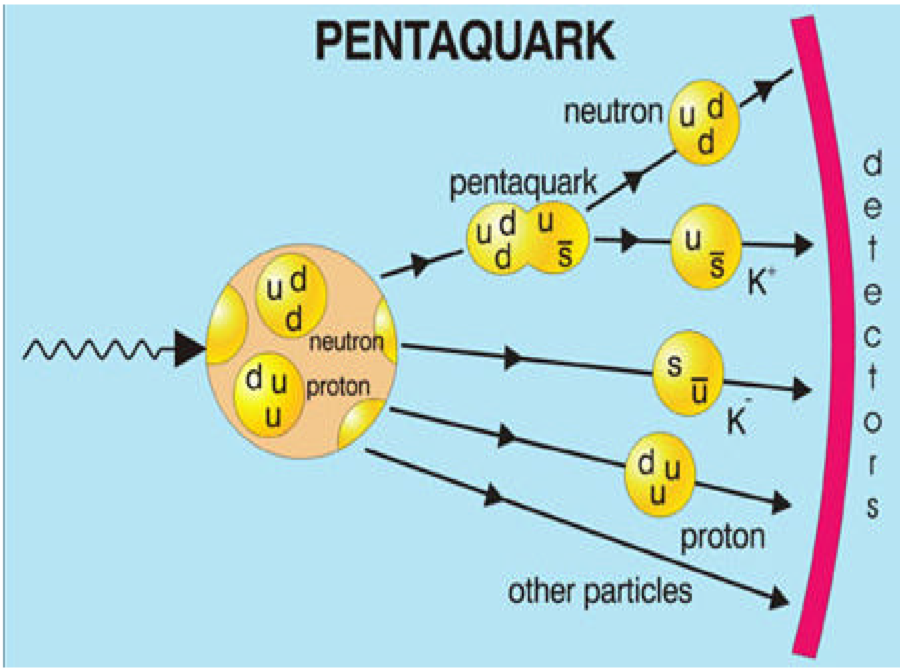

Figure 42: Schematic picture of the pentaquark production.

Pentaquark observation:

- ❖ quark composition uudd$\bar{s}$, mass 1.54 GeV, $B = +1$, $S = +1$, spin = 1/2.
- ❖ Russian theorists predicted in 1998 a pentaquark state with these quantum numbers, having a mass 1.53 GeV!
- ❖ Observation in Japan: 19 events. The signal has been confirmed in other experiments.

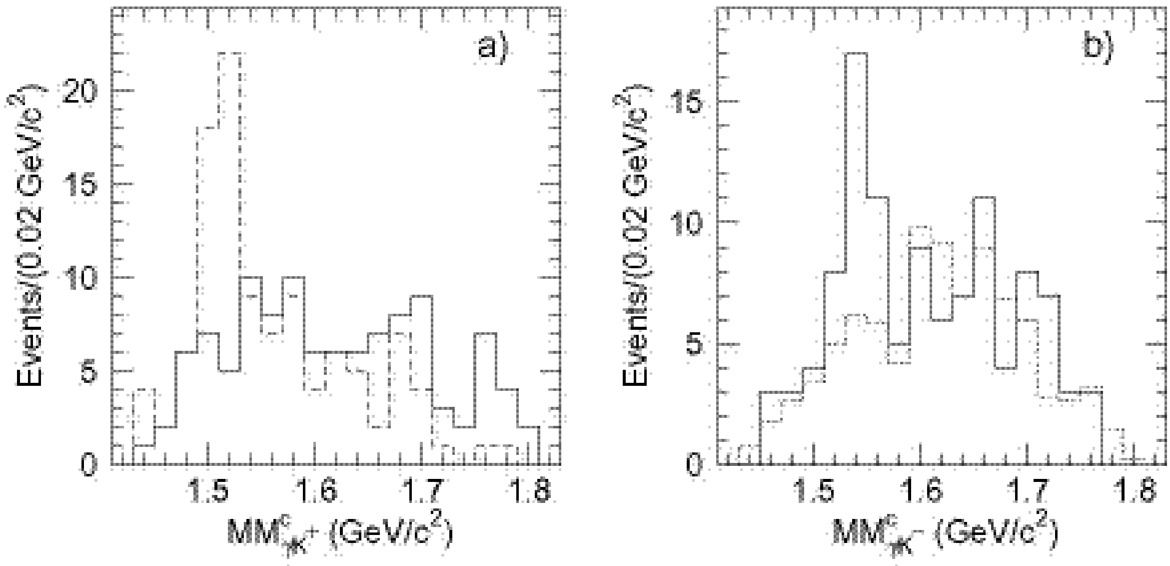

FIG. 3: a) The $MM^c_{\gamma K^+}$ spectrum for K^+K^- productions for the signal sample (solid histogram) and for events from the SC with a proton hit in the SSD (dashed histogram). b) The $MM^c_{\gamma K^-}$ spectrum for the signal sample (solid histogram) and for events from the LH_2 (dotted histogram) normalized by a fit in the region above 1.59 GeV/c^2.

Figure 43: Left: invariant mass distribution of the photon-proton reactions, producing $K^+\Lambda(1520)$. Right: invariant mass distribution of the photon-neutron reactions, producing $Z^+(1540)$.

The reaction for producing the pentaquarks was:

$$\gamma + n \rightarrow Z^{+}(1540) + K^{-} \rightarrow K^{+} + K^{-} + n$$

Laser beam was shot to a target made of ^{12}C (n:p=1:1). A reference target of liquid hydrogen (only protons) was used. In the p-target, the following reaction occurred: $\gamma + p \rightarrow K^{+}\,\Lambda(1520) \rightarrow K^{+} + K^{-} + p$ (visible in the Fig. 43, left)

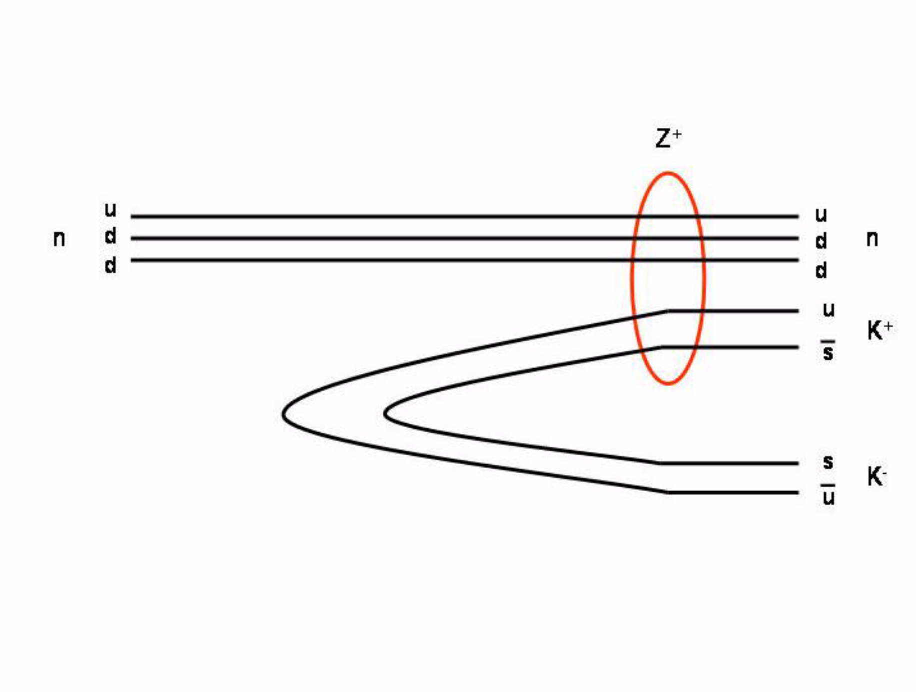

Figure 44: Quark diagram for pentaquark production.

Even more quantum numbers...

It is convenient to introduce some more quantum numbers, which are conserved in strong and e.m. interactions:

– Sum of all internal quantum numbers, except of Q,

hypercharge $Y \equiv B + S + C + \tilde{B} + T$

– Instead of Q :

$$I_3 \equiv Q - Y/2$$

which is to be treated as a projection of a new vector:

– Isospin

$$I \equiv (I_3)_{max}$$

so that I_3 takes 2I+1 values from -I to I

❖ It appears that I_3 is a good quantum number to denote up- and down- quarks, and it is convenient to use notations for particles as $I(J^P)$ or $I(J^{PC})$

	B	S	C	$\tilde{B}$	T	Y	Q	I_3	I
u	1/3	0	0	0	0	1/3	2/3	1/2	1/2
d	1/3	0	0	0	0	1/3	-1/3	-1/2	1/2
s	1/3	-1	0	0	0	-2/3	-1/3	0	0
c	1/3	0	1	0	0	4/3	2/3	0	0
b	1/3	0	0	-1	0	-2/3	-1/3	0	0
t	1/3	0	0	0	1	4/3	2/3	0	0

Hypercharge Y, isospin I and its projection I_3 are additive quantum numbers, so that corresponding quantum numbers for hadrons can be deduced from those of quarks:

$$Y^{a+b} = Y^a + Y^b \; ; \; I_3^{a+b} = I_3^a + I_3^b$$

$$I^{a+b} = I^a + I^b, I^a + I^b - 1, \ldots, \left|I^a - I^b\right|$$

- Proton and neutron both have isospin of 1/2, and also very close masses:

p(938) = uud ; n(940) = udd : $I(J)^P = \frac{1}{2}\left(\frac{1}{2}\right)^+$

proton and neutron are said to belong to an

isospin doublet, $\begin{pmatrix} p \\ n \end{pmatrix}_{I_3} = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}_{I_3}$

Other examples of *isospin multiplets*:

$K^+(494) = u\bar{s}$; $K^0(498) = d\bar{s}$: $I(J)^P = \frac{1}{2}(0)^-$

$\pi^+(140) = u\bar{d}$; $\pi^-(140) = d\bar{u}$: $I(J)^P = 1(0)^-$
$\pi^0(135) = (u\bar{u}-d\bar{d})/\sqrt{2}$: $I(J)^{PC} = 1(0)^{-+}$

→ Principle of *isospin symmetry*: it is a good approximation to treat u- and d-quarks as having same masses.

Particles with I=0 are *isosinglets* :

$$\Lambda(1116) = uds,\ I(J)^P = 0\left(\frac{1}{2}\right)^+$$

❖ By introducing isospin, we imply new criteria for non-exotic particles. S, Q and I for baryons are:

	sss	ssq_i	sq_iq_j	$q_iq_jq_k$
S	-3	-2	-1	0
Q	-1	0; -1	1; 0; -1	2; 1; 0; -1
I	0	1/2	0; 1	3/2; 1/2

S, Q and I for mesons are:

	$s\bar{s}$	$s\bar{q}_i$	$\bar{s}q_i$	$q_i\bar{q}_i$	$q_i\bar{q}_j$
S	0	-1	1	0	0
Q	0	0; -1	1; 0	0	-1; 1
I	0	1/2	1/2	0; 1	0; 1

In pre-2003 observed interactions these criteria are satisfied as well, confirming the quark model.

Pentaquark $uudd\bar{s}$: S=+1, B= +1, Q= +1, $Y \equiv B + S =$ +2, $I_3 \equiv Q - Y/2 = 0$ => I=0.

❖ On the other hand, pentaquarks are rare -- we can still predict new multiplet members based on the simple quark model. Suppose we observe production of the Σ^+ baryon in a strong interaction:

	$K^- + p$	$\rightarrow$	π^-	$+\Sigma^+$
Q=	0		-1	1
S=	-1		0	-1
B=	1		0	1

which then decays weakly :

$$\Sigma^+ \rightarrow \pi^+ + n, \text{ OR } \Sigma^+ \rightarrow \pi^0 + p$$

It follows that Σ^+ baryon quantum numbers are: B=1, Q=1, S= -1 and hence Y=0 and I_3=1.

→ Since $I_3>0 \Rightarrow I \neq 0$, so there are more multiplet members! If a baryon has I_3=1, the only possibility for isospin is I=1, and we have an isospin triplet:

$$S^+, S^0, S^-$$

Indeed, all these particles have been observed:

$$K^- + p \to \pi^0 + \Sigma^0$$
$$\hookrightarrow \Lambda + \gamma$$
$$K^- + p \to \pi^+ + \Sigma^-$$
$$\hookrightarrow \pi^- + n$$

Masses and quark composition of Σ-baryons are:

Σ^+(1189) = uus ; Σ^0(1193) = uds ; Σ^-(1197) = dds

It indicates that the d-quark is heavier than the u-quark under the following assumptions:

a) Strong interactions between quarks do not depend on their flavour. Strong i.a. give a contribution M_o to the baryon mass.

b) Electromagnetic i.a. contribute as $\delta \sum e_i e_j$, where e_i are quark charges and δ is a constant.

The simplest attempt to calculate the mass difference of the up and down quarks:

$$M(\Sigma^-) = M_0 + m_s + 2m_d + \delta/3$$
$$M(\Sigma^0) = M_0 + m_s + m_d + m_u - \delta/3$$
$$M(\Sigma^+) = M_0 + m_s + 2m_u$$

$$\Downarrow$$

$$m_d - m_u = [\, M(\Sigma^-) + M(\Sigma^0) - 2M(\Sigma^+) \,] / 3 = 3.7 \text{ MeV}$$

→ NB : this is a very simple model, because under these assumptions $M(\Sigma^0) = M(\Lambda)$, however, their mass difference $M(\Sigma^0) - M(\Lambda) \approx 77$ MeV.

Generally, combining other methods:

$$2 \text{ MeV} \leq m_d - m_u \leq 4 \text{ MeV}$$

which is negligible compared to hadron masses (but not if compared to the estimated u and d masses themselves: m_u = (1.5-4.5) MeV, m_d = (5.0-8.5) MeV).

Resonances

→ Resonances are highly unstable particles which decay by the strong interaction (lifetimes about 10^{-23} s).

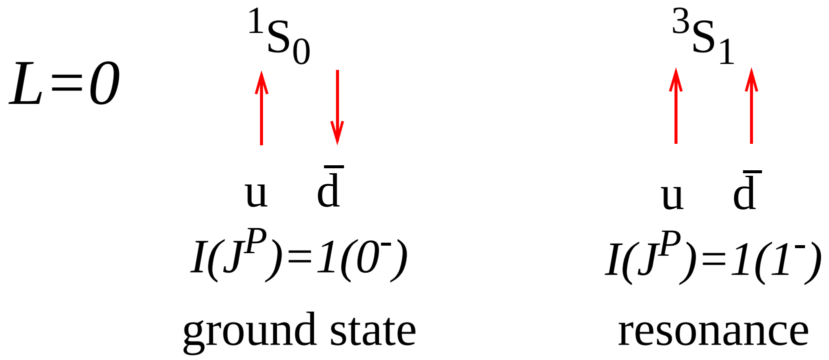

Figure 45: Example of a $q\bar{q}$ system in ground and first excited states

→ If a ground state is a member of an isospin multiplet, then resonant states will form a corresponding multiplet too.

→ Since resonances have very short lifetimes, they can only be detected by registering their decay products:

$$\pi^- + p \rightarrow n + X$$
$$\hookrightarrow A + B$$

→ Invariant mass $W = m_X$ of the resonance X is measured via energies and 3-momenta of its decay products, particles A and B. By using 4-vectors (see eqs. 10 and 11 for scalar product of two 4-vectors), W can be calculated as :

$$p_x^2 = (p_A + p_B)^2 = m_x^2 \equiv W^2$$

$$(p_A + p_B)^2 = (p_A + p_B) \cdot (p_A + p_B) = (E_A + E_B)^2 - (\vec{p}_A + \vec{p}_B)^2$$

$$= E_X^2 - \vec{p}_X^{\,2} = m_X^2 = W^2 \qquad (82)$$

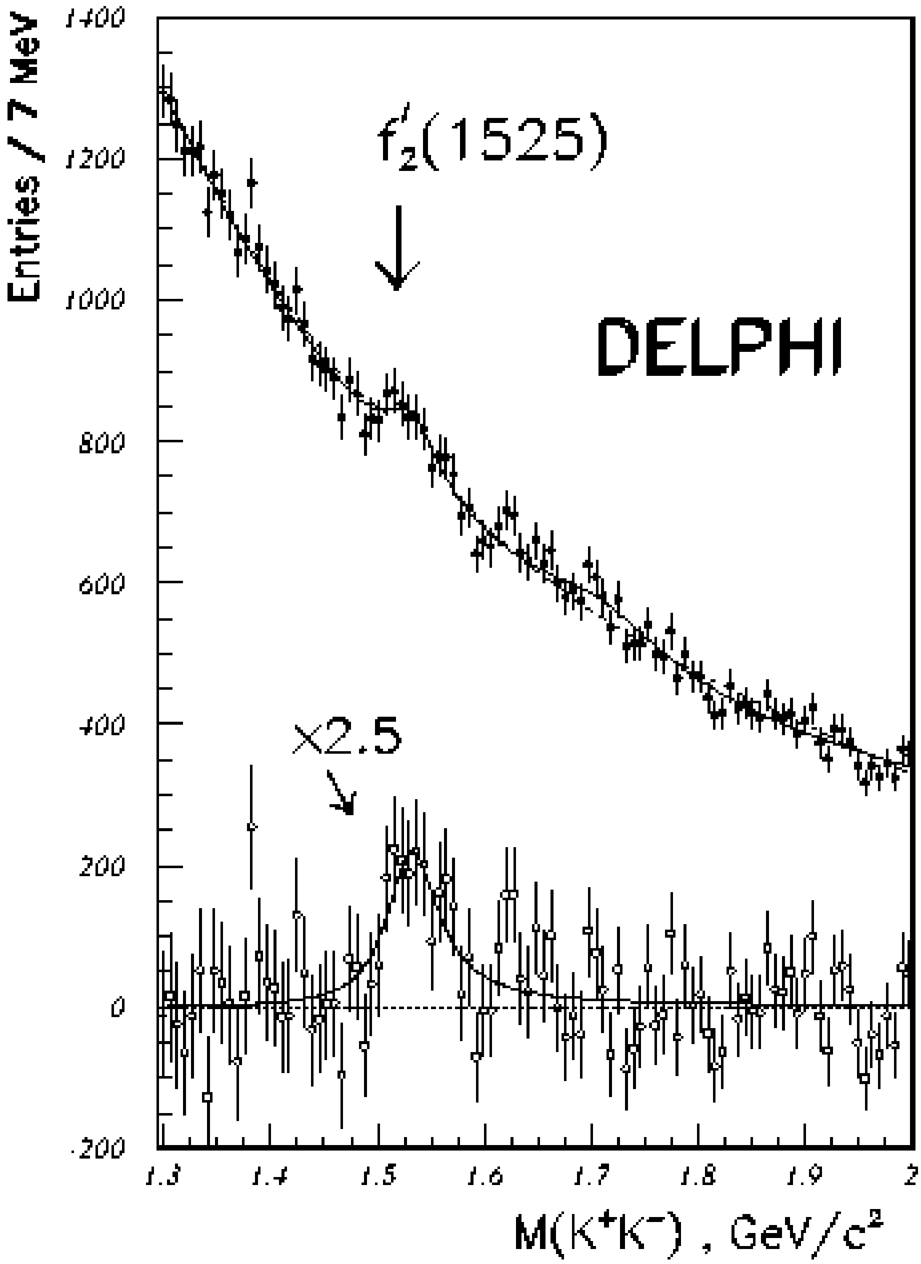

Figure 46: A typical resonance peak in K^+K^- invariant mass distribution

→ Resonance peak shapes are approximated by the Breit-Wigner formula:

$$N(W) = \frac{K}{(W - W_0)^2 + \Gamma^2 / 4} \qquad (83)$$

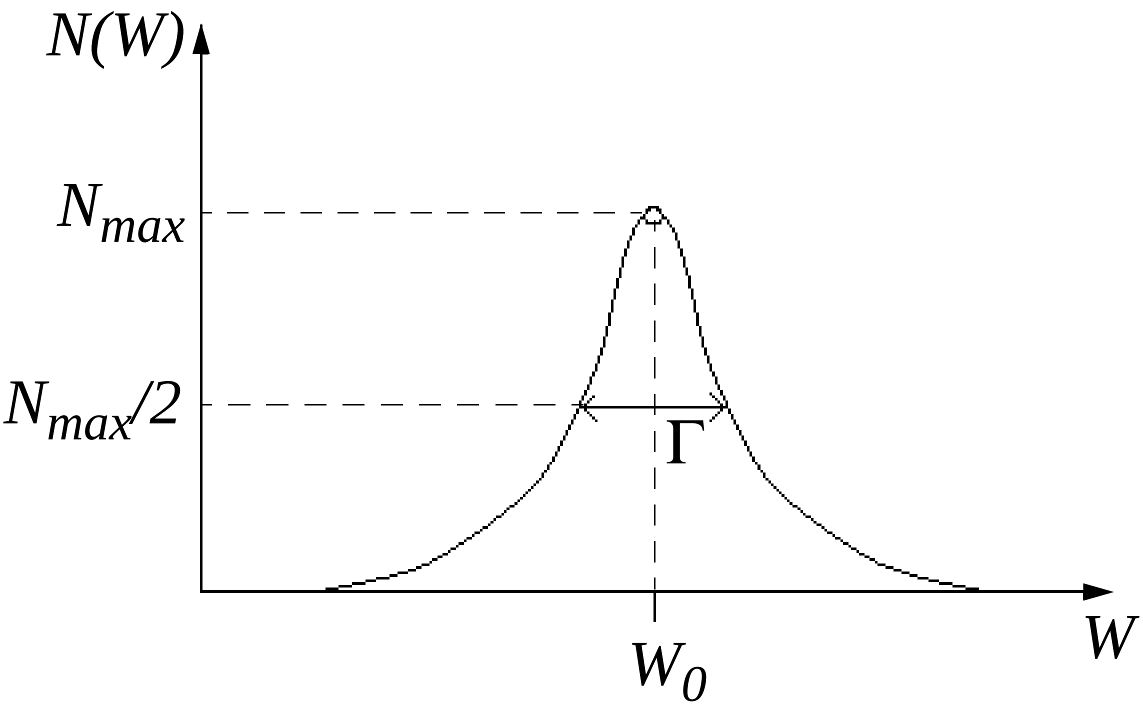

Figure 47: Breit-Wigner shape

❖ Mean value of the Breit-Wigner shape is the mass of the resonance: $M = W_0$

❖ Γ is the width of the resonance, and it is the inverse of the mean lifetime of the particle at rest: $\Gamma \equiv 1/\tau$.

❖ Internal quantum numbers of resonances are derived from their decay products. For example, if a resonance X^0 decays to 2 pions:

$$X^0 \rightarrow \pi^+ + \pi^-$$

then $B = 0; S = C = \tilde{B} = T = 0; Q = 0$

$$\Rightarrow Y=0 \text{ and } I_3=0 \text{ for } X^0.$$

❖ To determine whether I=0 or I=1, searches for isospin multiplet partners have to be done.

Example: $\rho^0(769)$ and $\rho^0(1700)$ both decay to $\pi^+\pi^-$ pair and have isospin partners ρ^+ and ρ^- :

$$\pi^{\pm} + p \rightarrow p + \rho^{\pm}$$
$$\hookrightarrow \pi^{\pm} + \pi^0$$

By measuring the angular distribution of the $\pi^+\pi^-$ pair, the <u>relative</u> orbital angular momentum of the pair, L, can be determined, and hence spin and parity of the resonance X^0 are (S=0):

$$J = L; P = P_{\pi}^2(-1)^L = (-1)^L; C = (-1)^L$$

Some excited states of pion:

resonance	$I(J^{PC})$	L
$\rho^0(769)$	$1(1^{--})$	1
$f_2^0(1275)$	$0(2^{++})$	2
$\rho^0(1700)$	$1(3^{--})$	3

❖ Resonances with B=0 are meson resonances, and with B=1 baryon resonances.

Many baryon resonances can be produced in pion-nucleon scattering:

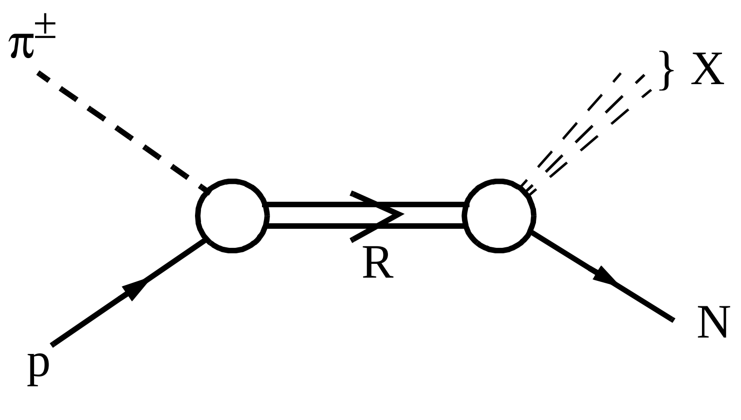

Figure 48: Formation of a resonance R and its subsequent inclusive decay into a nucleon N

❖ Peaks in the observed total cross-section of the $\pi^{\pm}p$-reaction correspond to resonance formation - see Fig. 49.

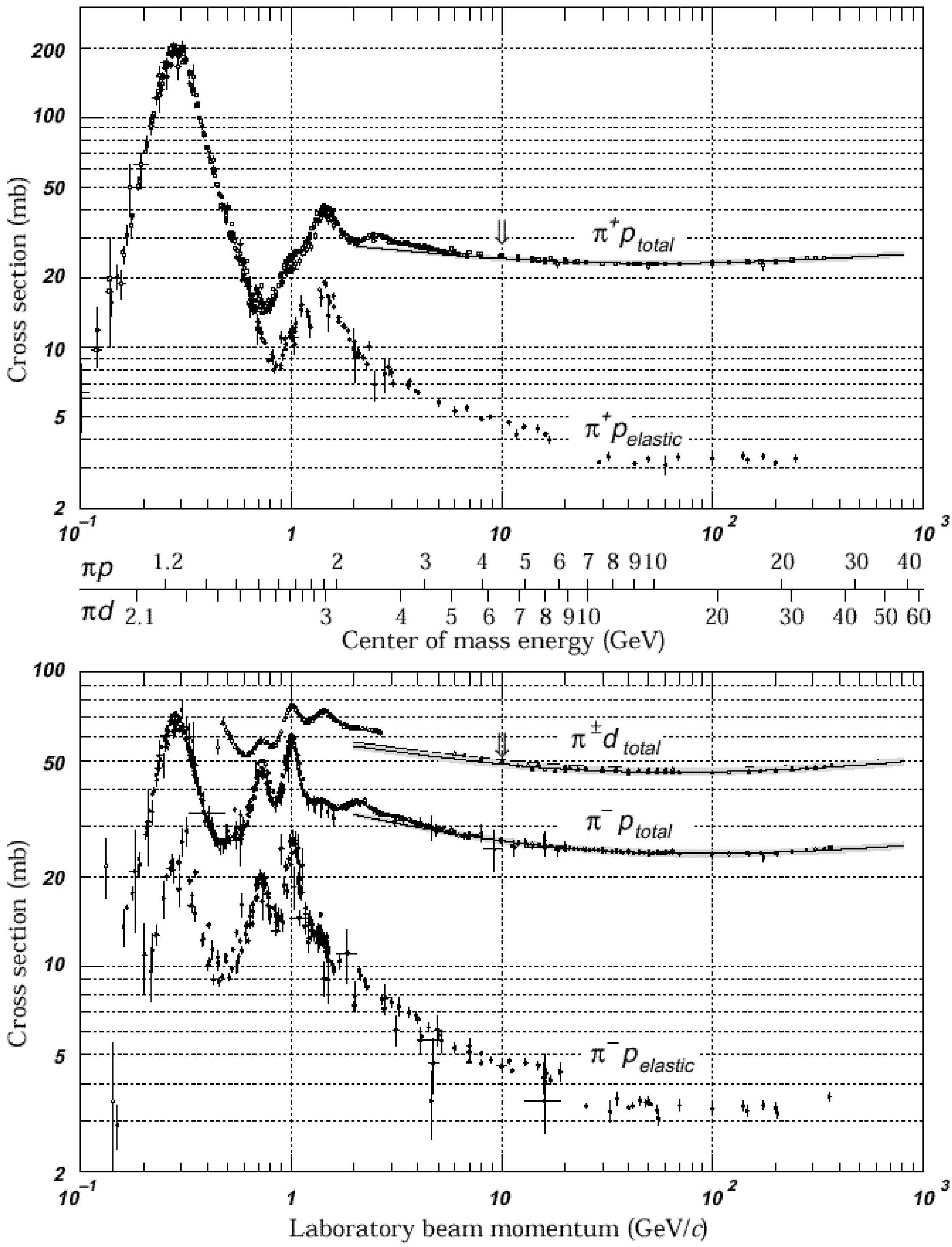

Figure 49: Scattering of π^+ and π^- on proton

All resonances produced in pion-nucleon scattering have the same internal quantum numbers as the initial state: $p+\pi^{+-} \to R$

$$B = 1; S = C = \tilde{B} = T = 0$$

and thus Y=1 and $Q=I_3+1/2$.

Possible isospins are I=1/2 or I=3/2, since for pion I=1 and for nucleon I=1/2

I=1/2 $\Rightarrow$ N-resonances (N^0, N^+)

I=3/2 $\Rightarrow$ Δ-resonances ($\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$)

At Figure 49, peaks at $\approx$1.2 GeV correspond to Δ^{++} and Δ^0 resonances:

$$\pi^+ + p \to \Delta^{++} \to \pi^+ + p$$

$$\pi^- + p \to \Delta^0 \to \pi^- + p$$

$$\pi^0 + n$$

- ❖ Fits with the Breit-Wigner formula show that both Δ^{++} and Δ^0 have approximately same mass of $\approx$1232 MeV and width $\approx$120 MeV.

- ❖ Studies of angular distributions of the decay products show that $I(J^P) = \frac{3}{2}\left(\frac{3}{2}\right)^+$
- ❖ Remaining members of the multiplet are also observed: Δ^+ and Δ^-

→ There is no lighter state with these quantum numbers $\Rightarrow$ Δ is a ground state, although a resonance.

Quark diagrams

→ Quark diagrams are convenient way of illustrating strong interaction processes. Consider an example:

$$\Delta^{++} \rightarrow p + \pi^{+}$$

The only 3-quark state consistent with Δ^{++} quantum numbers is (uuu), while p=(uud) and π^{+}=($u\bar{d}$).

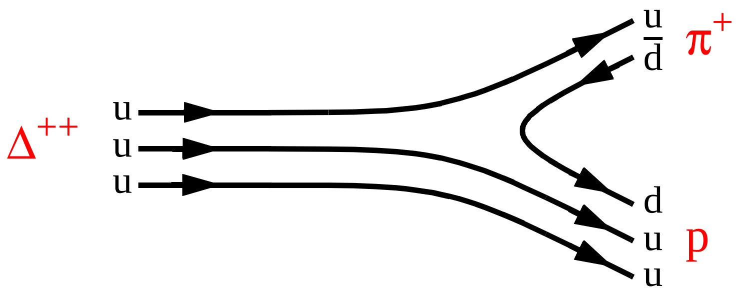

Figure 50: Quark diagram of the reaction $\Delta^{++} \rightarrow p + \pi^{+}$

Analogously to Feinman diagrams:

- ❖ arrow pointing to the right denotes a particle, and to the left – antiparticle
- ❖ time flows from left to right

Allowed resonance formation process:

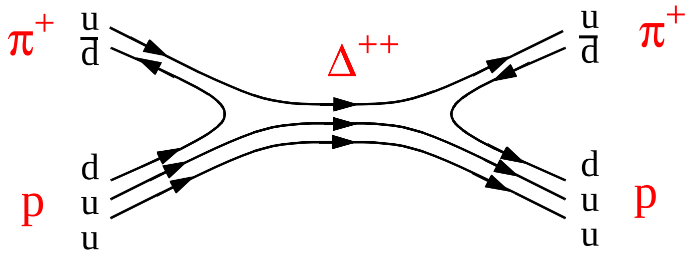

Figure 51: Formation and decay of Δ^{++} resonance in $\pi^{+}p$ elastic scattering

Hypothetical exotic resonance:

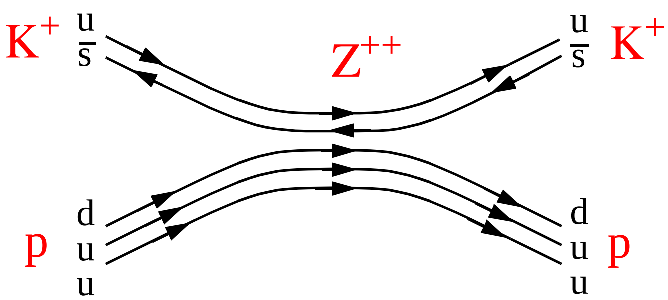

Figure 52: Formation and decay of an exotic resonance Z^{++} in $K^{+}p$ elastic scattering

❖ Quantum numbers of such a particle Z^{++} are exotic, moreover, there are no resonance peaks in the corresponding cross-section:

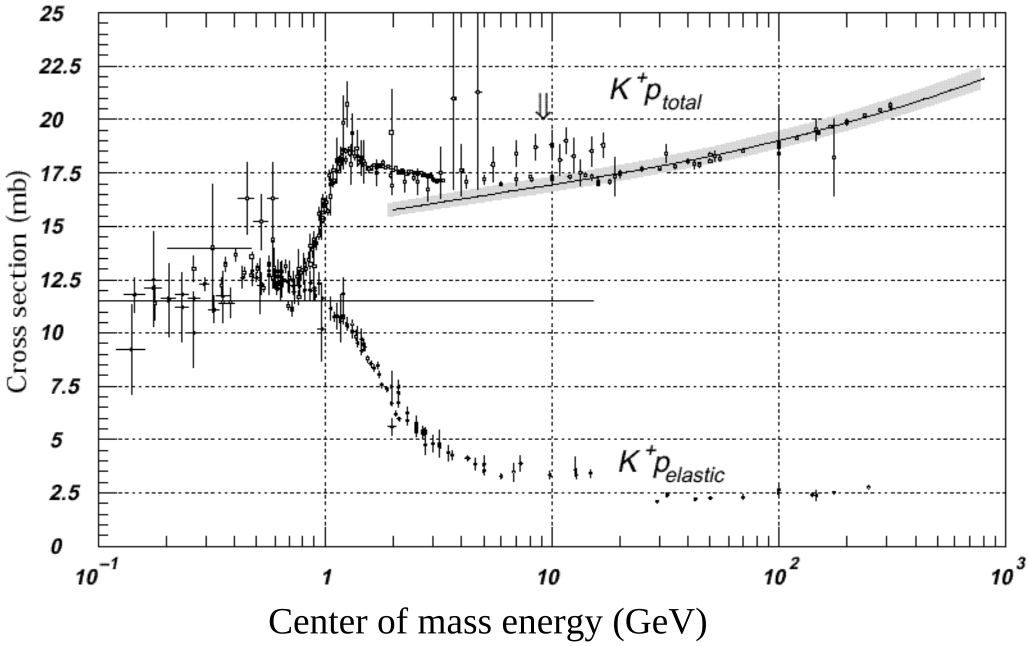

Figure 53: Cross-section for K^+p scattering

SUMMARY

❖ Characteristics of a hadron: Mass; Space-time quantum numbers total spin *J*, parity *P*, charge conjugation *C*; Internal quantum numbers charge Q and baryon number B (always conserved), flavour quantum numbers $S, C, \tilde{B}, T$ (conserved in e.m. and strong interactions).

❖ The possible 3-quark and 2-quark states can be defined using the conservation rules of the quantum numbers. Other states are defined as exotic states - like pentaquarks.

❖ Quantum numbers, which are conserved in strong and e.m. interactions: *hypercharge* $Y \equiv B + S + C + \tilde{B} + T$, *and* $I_3 \equiv Q - Y/2$, which is the third component of *isospin I.*

❖ I_3 is a good quantum number to denote u- and d-quarks. *Isospin symmetry*: approximate symmetry which assumes that u- and d-quarks have the same masses.

❖ Resonances are highly unstable particles which decay by the strong interaction (lifetimes about 10^{-23} s). Resonances can only be detected by registering their decay products: invariant mass *W* = *mass* of the resonance = invariant mass of the decay products. Internal quantum numbers of resonances are derived from their decay products as well.

❖ Resonance peak shapes are approximated by the Breit-Wigner formula which gives *N* (number of events) as the function of *W*. The value $W = W_0$ which gives N_{max} is the mass of the resonance, and Γ, the width of the resonance, is the inverse of the mean lifetime of the particle at rest: $\Gamma \equiv 1/\tau$.

❖ Quark diagrams = “Feynman-like” diagrams for strong interaction processes. Only quark lines are drawn, no gluons, no vertices.