X. Charge conjugation and parity in weak interactions

REMINDER:

Parity

- The parity transformation is the transformation by reflection:

\[ \hat{x}_i \rightarrow \hat{x}'_i = -\hat{x}_i \]

A parity operator \( \hat{P} \) is defined as

\[ \hat{P}\psi((\hat{x}, t)) = p\psi(-\hat{x}, t) \]

where \( p = \pm 1 \)

Charge conjugation

- The charge conjugation replaces particles by their antiparticles, reversing charges and magnetic moments

\[ \hat{C}\Psi_a = c\Psi_{\bar{a}} \]

where \( c = \pm 1 \)

meaning that from the particle in the initial state we go to the antiparticle in the final state.
Symmetries

Space-time symmetries

- Continuous transformations that can be regarded as a series of infinitely small steps.
- Discrete transformations have only two elements i.e. two transformations.

Internal symmetries

- Transformations that do not affect the space- and time-coordinates.

Global symmetries

- The transformation does not depend on $r$ i.e. it is the same everywhere in space.
- The transformation depends on $r$ i.e. it is different in different points in space.

- Baryon number
- Lepton number
- Strangeness number
- Isospin $SU(2)_{flavour}$
- Isospin+Hypercharge $SU(3)_{flavour}$
- Electric charge $U(1)$
- Weak charge+weak isospin $U(1) \times SU(2)$
- Colour $SU(3)$

Lorentz transformation
Translation in space
Translation in time
Rotation around an axis

Parity
Charge conjugation
Time reversal
While parity is conserved in strong and electromagnetic interactions, it is violated in weak processes:

- 1956: Based on the measurements of Kaon decays, Lee & Yang propose that parity is violated in weak processes:

Two known decays of the $K^+$ were:

$K^+ \to \pi^0 + \pi^+$ and $K^+ \to \pi^+ + \pi^+ + \pi^-$

The intrinsic parity of a pion $P_{\pi} = -1$, and for the $\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ states the parities are

$$P_{\pi\pi} = P_{\pi}^2 (-1)^L = 1 \quad \text{since} \ L = L_{12} = 0$$

$$P_{\pi\pi\pi} = P_{\pi}^3 (-1)^L = -1 \quad \text{since} \ L = L_{12} + L_3 = 0$$

Since the two final states have opposite parities, one of the $K^+$ decays must violate parity!
1957: Wu carries out studies of parity violation in $\beta$-decay. The $^{60}\text{Co} \beta$-decay into $^{60}\text{Ni}^* + \text{e}^- + \overline{\text{v}}_e$ was studied.

The $^{60}\text{Co}$ sample was cooled to 0.01 K to prevent thermal disorder.

The sample was placed in a magnetic field $\Rightarrow$ the nuclear spins were aligned along the field direction.

If parity is conserved, processes (a) and (b) must have equal rates.

Electrons were emitted predominantly in the direction opposite the $^{60}\text{Co}$ spin.

Figure 125: Possible $\beta$-decays of $^{60}\text{Co}$: case (a) is preferred.
Another case of both parity and C-parity violation was observed in muon decays:

\[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \]

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

The angular distribution of the electrons (positrons) emitted in \( \mu^- \) (\( \mu^+ \)) decay are given by

\[
\Gamma_{\mu^\pm} (\cos \theta) = \frac{1}{2} \Gamma_{\pm} \left( 1 - \frac{\xi_{\pm}}{3} \cos \theta \right)
\]

(136)

here \( \xi_{\pm} \) are constants – “asymmetry parameters”, and \( \Gamma_{\pm} \) are total decay rates \( \Rightarrow \) inverse lifetimes

\[
\Gamma_{\pm} = \int_{-1}^{1} \Gamma_{\pm} \left( \cos \theta \right) d\cos \theta \equiv \frac{1}{\tau_{\pm}}
\]

(137)
If the process is invariant under charge conjugation (C-invariance) ⇒

\[ \Gamma_+ = \Gamma_- \quad \xi_+ = \xi_- \] (138)

(rates and angular distributions are the same for e\(^-\) and e\(^+\))

If the process is P-invariant, then angular distributions in forward and backward directions are the same:

\[ \Gamma_{\mu \pm} (\cos \theta) = \Gamma_{\mu \pm} (-\cos \theta) \quad \xi_+ = \xi_- = 0 \] (139)

Experimental results:

\[ \Gamma_+ = \Gamma_- \quad \xi_+ = -\xi_- = 1.00 \pm 0.04 \] (140)

Both C- and P-invariance are violated!
However, the combined operation $\text{CP}$ is conserved since that requires

$$\Gamma_{\mu}^+(\cos \theta) = \Gamma_{\mu}^-(\cos \theta)$$

(141)

$$\Gamma_+ = \Gamma_- \quad \xi_+ = -\xi_-$$

(142)

which is in agreement with the experiments.

Figure 127: \(P\)-, \(C\)- and \(CP\)-transformation of an electron

- The combined transformation $\text{CP}$ is a weaker requirement than the individual transformations $P$ and $C$ and it is conserved.
**Helicity**

**Helicity** – the spin is quantized along the particle’s direction of motion instead of along an arbitrary z-direction

\[
\hat{\Lambda} = \frac{\hat{s} \cdot \hat{p}}{\hat{p}} \quad (143)
\]

\[
\hat{\Lambda}\psi = \lambda\psi
\]

The eigenvalues of the helicity operator are \(\lambda=-s,-s+1,\ldots,+s\), ⇒ for spin-1/2 particle it can be either -1/2 or 1/2

A particle with \(\lambda=+1/2\) is called right handed.
A particle with \(\lambda=-1/2\) is called left handed.

A subscript \(R\) or \(L\) is used to denote if a state is right or left handed e.g. \(e^-_R\) and \(\nu_L\)
1958: Goldhaber et al. measured the helicity of the neutrino by studying electron capture in europium:

\[ e^- + ^{152}\text{Eu} \rightarrow ^{152}\text{Sm}^* + v_e \]  \hspace{1cm} (144)

\[ ^{152}\text{Sm} + \gamma \]  \hspace{1cm} (145)

In this reaction the initial state has zero momentum and \(^{152}\text{Sm}^*\) and \(v_e\) recoil in opposite directions.

Events with the \(\gamma\) emitted in the direction of motion of the \(^{152}\text{Sm}^*\) were selected so that the overall observed reaction was:

\[ e^- + ^{152}\text{Eu} (J=0) \rightarrow ^{152}\text{Sm}(J=0) + v_e + \gamma \]  \hspace{1cm} (146)

The spin of the neutrino (+1/2 or -1/2) and the photon (+1 or -1) must add to give the spin of the electron (+1/2 or -1/2).

The helicity (polarization) of the photons was determined by studying their absorption in magnetized iron.
From the helicity of the photons it is possible to determine the helicity of the neutrinos.

Figure 129: From the helicity of the photons it is possible to determine the helicity of the neutrinos.

From the measured photon helicity it was concluded that neutrinos must be left-handed.
**V-A interaction**

- **V-A interaction** theory was introduced by Fermi as an analytic description of spin dependence of charged current interactions.

- It denotes “polar Vector - Axial vector” interaction

  - A **Polar vector** is one which direction is reversed by parity transformation e.g. momentum $\hat{p}$

  - An **Axial vector** is one which direction is not changed by parity transformation e.g. spin $\hat{s}$ or orbital angular momentum $\hat{L} = \hat{r} \times \hat{p}$

  - The weak current has both vector and axial components, hence parity is not conserved in weak interactions

  ➡️ Main conclusion: if $v \approx c$, only left-handed fermions $\nu_L, e_L^-$ etc. are emitted, and right-handed antifermions.

  ➡️ **The very existence of preferred states violates both C- and P- invariance**
Neutrinos (antineutrinos) are always relativistic and hence always left(right)-handed.

For other fermions, the preferred states are left-handed. Right-handed states are not completely forbidden but suppressed by the factor

\[ \left(1 - \frac{v}{c}\right) \approx \frac{m^2}{2E^2} \]  
(147)

Consider the two pion decay modes:

\[ \pi^+ \rightarrow e^+ + \nu_e \]  
(148)

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]  
(149)

Relativistic:

Non-relativistic:

Neutrinos are always lefthanded

Figure 130: Helicities of leptons emitted in a pion decay

- The $\pi^+$ has spin-0 and it is at rest $\Rightarrow$ the spins of the charged lepton and the neutrino must be opposite.
The neutrinos are always left-handed ⇒ the charged leptons have to be left-handed as well.

BUT: the $e^+$ and the $\mu^+$ should be right-handed since they are anti-fermions.

- In these decays the electron will be relativistic but not the muon (due to its large mass).

- It follows that the pion to muon decay should be allowed but the pion to positron decay should be suppressed.

The suppression factor for positrons is expected to be of the order $10^{-5}$.

The measured ratio:

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = (1,230 \pm 0,004) \times 10^{-4} \quad (150)$$
Muons emitted in pion decays are always polarized and this can be used to measure muon decay symmetries by detecting the relativistic electrons in the following decays:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\rightarrow e^- + \bar{\nu}_e + \nu_\mu$$  \hspace{1cm} (151)

The electrons are emitted in decays when both the $\nu_\mu$ and the $\bar{\nu}_e$ are emitted in the direction opposite to the $e^-$: 

(a) Left-handed electrons
Favoured

(b) Right-handed electrons
Surpressed

Figure 131: Muon decays with high energy electron emission.

The electron must have a spin parallel to the muon spin $\Rightarrow$ configuration (a) with left-handed electrons is strongly preferred $\Rightarrow$ this is observed experimentally as a forward-backward asymmetry.
Neutral kaons

It is possible to produce the neutral kaons $K^0=ds$ and $\bar{K}^0=sd$ in $\pi p$-collisions. This is a strong interaction process and strangeness has to be conserved:

$$\pi^- + p \rightarrow K^0 + \Lambda$$

$s$: 0 0 +1 -1

$$\pi^+ + p \rightarrow \bar{K}^0 + p + K^+$

$s$: 0 0 -1 0 +1

The kaons that are produced in this way are pure $K^0=ds$ and $\bar{K}^0=sd$ states.

However, $K^0$ and $\bar{K}^0$ can be converted into each other since strangeness is not conserved in weak interactions:

Figure 132: Example of a process converting $K^0$ to $\bar{K}^0$. 
The observed physical particles are linear combinations of $K^0$ and $\bar{K}^0$, since there is no conserved quantum number to distinguish them. The phenomenon is called $K^0-\bar{K}^0$ mixing.

We know that neither parity nor charge conjugation are conserved in weak decays. The combined operation CP is, however, almost conserved.

In this case the CP operators eigenstates can be written as a mixture of $K^0$ and $\bar{K}^0$:

$$K_1^0 = \frac{1}{\sqrt{2}} \{K^0 + \bar{K}^0\}$$ (152)

$$K_2^0 = \frac{1}{\sqrt{2}} \{K^0 - \bar{K}^0\}$$ (153)

so that

$$\hat{CP}K_1^0 = K_1^0 \quad \text{and} \quad \hat{CP}K_2^0 = -K_2^0$$ (154)

i.e. the CP eigenvalues are $cp=+1$ for $K_1^0$ and $cp=-1$ for $K_2^0$ and the $K_1^0$ can therefore only decay to $cp$-even states while the $K_2^0$ only to $cp$-odd states.
Experimentally observed are two types of neutral kaons: \( K_S^0 \) ("S" for "short", lifetime \( \tau = 0.9 \times 10^{-10}\) s) and \( K_L^0 \) ("long", \( \tau = 500 \times 10^{-10}\) s).

\[ \begin{align*}
\text{Can the } K_S^0 \text{ be identified with the } K_I^0 \\
\text{CP-eigenstate, and the } K_L^0 \text{ with the } K_2^0 \? \\
\text{If CP-invariance holds for neutral kaons, } K_S^0 \\
\text{should decay only into states with } \text{cp}=1 \text{ such as } 2\pi\text{-states, and } K_L^0 \text{ into states with } \text{cp}=-1 \text{ such as } 3\pi\text{-states:}
\end{align*} \]

\[ K_S^0 \rightarrow \pi^+\pi^-, \quad K_S^0 \rightarrow \pi^0\pi^0 \quad (155) \]

- The parity of a two-pion state is \( P = P_\pi^2 (-1)^L = 1 \)
- The C-parity of a \( \pi^0\pi^0 \) state is \( C = (C_{\pi^0})^2 = 1 \),
  and of a \( \pi^+\pi^- \) state: \( C = (-1)^L = 1 \)
- i.e. \( \text{cp} = 1 \) for the \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) states
- i.e. the assumption that \( K_S^0 = K_I^0 \) seem to be correct.
\[ K_L^0 \to \pi^+ \pi^- \pi^0, \quad K_L^0 \to \pi^0 \pi^0 \pi^0 \] (156)

- The parity of the 3-\(\pi\) states are -1
- The C-parity of \(\pi^0 \pi^0 \pi^0\) is \(C = (C_{\pi^0})^3 = 1\)
- The C-parity of \(\pi^+ \pi^- \pi^0\) is \(C = C_{\pi^0}(-1)^L_{\pi\pi} = 1\)
- i.e. the 3-\(\pi\) final states above have cp=-1
- i.e. the assumption that \(K_L^0 = K_2^0\) seem to be correct.

✦ Summary:

The neutral Kaon eigenstates in strong interactions are:

\[
\begin{align*}
K^0 & = d\bar{s} \\
\bar{K}^0 & = s\bar{d}
\end{align*}
\]

The neutral Kaon eigenstates in weak interactions (if CP is conserved) are:

\[
\begin{align*}
K_s^0 & = K_L^0 = \frac{1}{\sqrt{2}} \{K^0 + \bar{K}^0\} \\
K_L^0 & = K_2^0 = \frac{1}{\sqrt{2}} \{K^0 - \bar{K}^0\}
\end{align*}
\]
**CP-violation**

The *CP-violating* decay

\[ K_L^0 \rightarrow \pi^+ \pi^- \quad (157) \]

was first observed in 1964, with a branching ratio of \( B \approx 10^{-3} \).

Figure 133: Sketch of the experiment that discovered CP-violation in weak decays.
In general, the physical states $K^0_S$ and $K^0_L$ don’t have to correspond to pure CP-eigenstates $K^0_I$ and $K^0_2$. Instead

\[
K^0_S = \frac{1}{\sqrt{1 + |\varepsilon|^2}} \{ K^0_I + \varepsilon K^0_2 \}
\]

\[
K^0_L = \frac{1}{\sqrt{1 + |\varepsilon|^2}} \{ \varepsilon K^0_I + K^0_2 \}
\]

where $\varepsilon$ is a small complex parameter: $|\varepsilon| = 2 \times 10^{-3}$

$K^0_S$ contains mostly $K^0_I$ but has also a small $K^0_2$ component while $K^0_L$ consists mostly of $K^0_2$ with a small component of $K^0_I$.

Mixing occur also for neutral B-mesons ($B^0 = \overline{d}b$, $\overline{B}^0 = \overline{b}d$, $B_s = \overline{s}b$ and $\overline{B}_s = \overline{b}s$) and for neutral D-mesons ($D^0 = \overline{c}u$ and $\overline{D}^0 = \overline{u}c$).

There can be different mechanisms for CP-violation, especially in the $B^0-\overline{B}^0$ systems. Several dedicated experiments have been built to study this system.
Summary

• Parity and charge conjugation

  a) Parity is violated in weak processes.

  b) Parity violation was first observed in $^{60}\text{Co}$-decays.

  c) Muon decays can be used to show that both parity and charged conjugation is violated while the combined CP operation is conserved.

• Helicity

  d) Helicity is the spin quantized along the direction of motion.

  e) Neutrinos are left-handed and antineutrinos right-handed.

  f) This was first observed in reactions between electrons and $^{152}\text{Eu}$ atoms.
• **V-A interactions**

  g) While neutrinos are always left-handed other fermions are exclusively left-handed only when they are relativistic.

• **Neutral kaons**

  h) The neutral kaons that are observed experimentally ($K^0_S$ and $K^0_L$) are due to $K^0$-$\bar{K}^0$ mixing

  i) CP-violating decays of neutral kaons have been observed with a small branching ratios.