IX. Electroweak unification

The problem of divergence

❖ A theory of weak interactions only by means of $W^{\pm}$ bosons leads to infinities

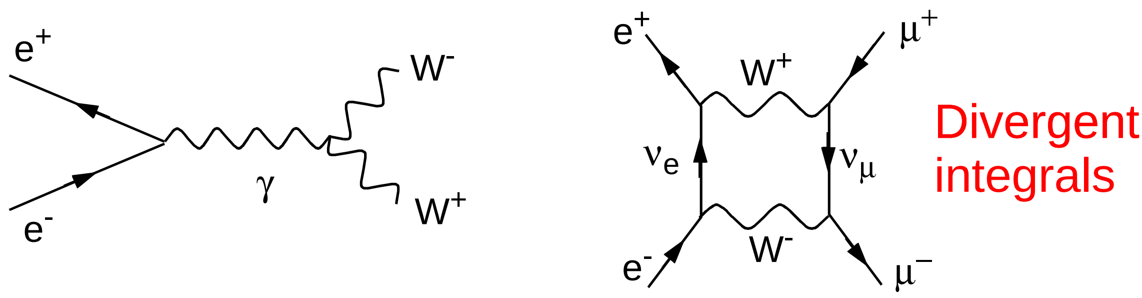


Figure 108: Examples of divergent processes.

→ Introduction of the Z^0 boson fixes the problem because the addition of new diagrams cancel out the divergencies:

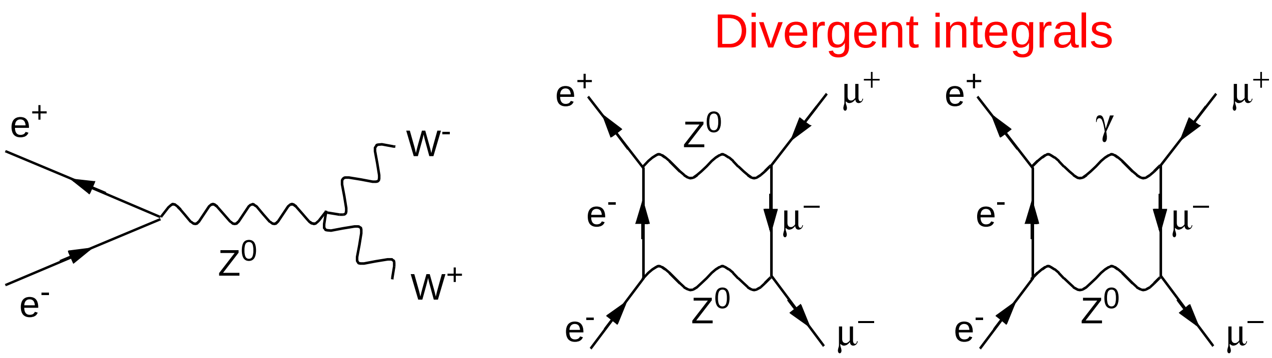


Figure 109: Additional processes which cancel the divergence.

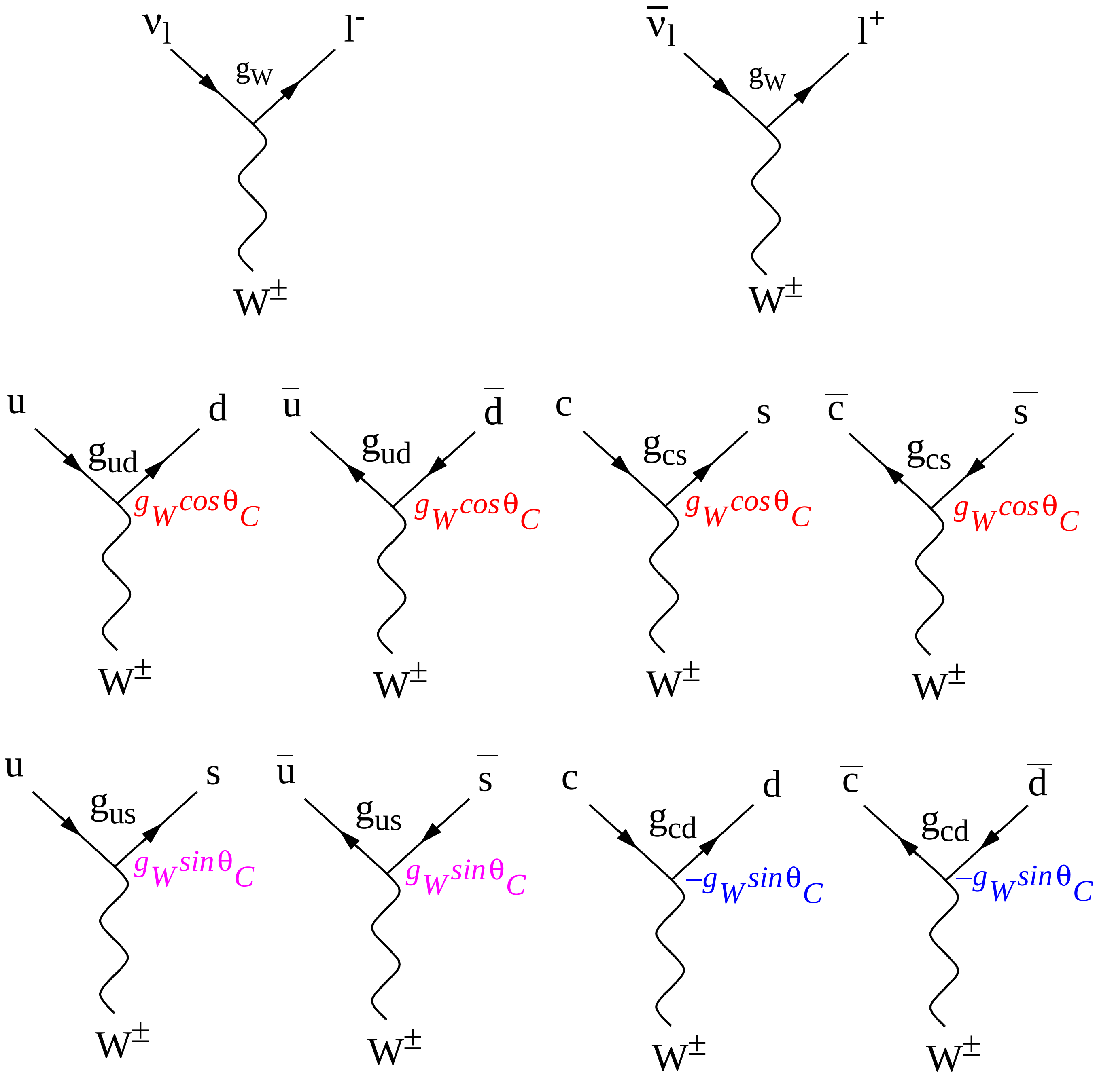


Figure 110: REMINDER: The basic W lepton and quark vertices (if the third generation is not taken into account).

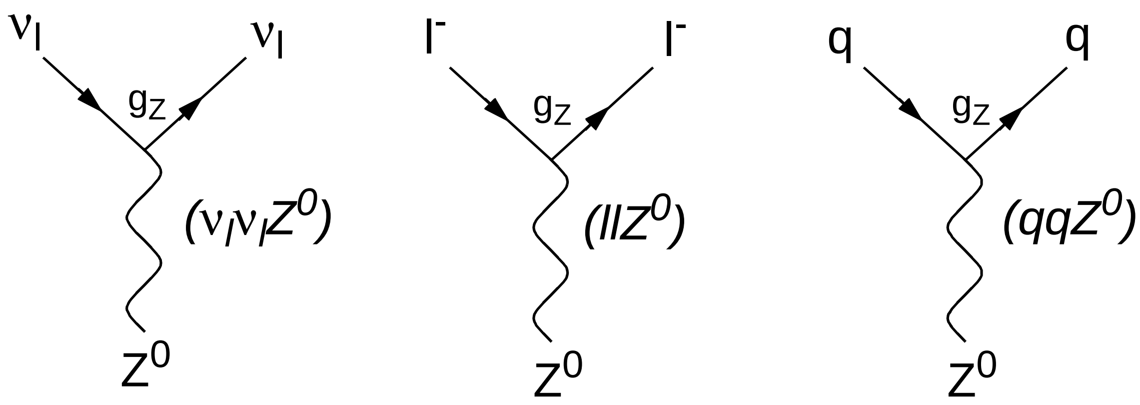


Figure 111: The basic Z^0 lepton and quark vertices.

- Basic vertices with W bosons have:

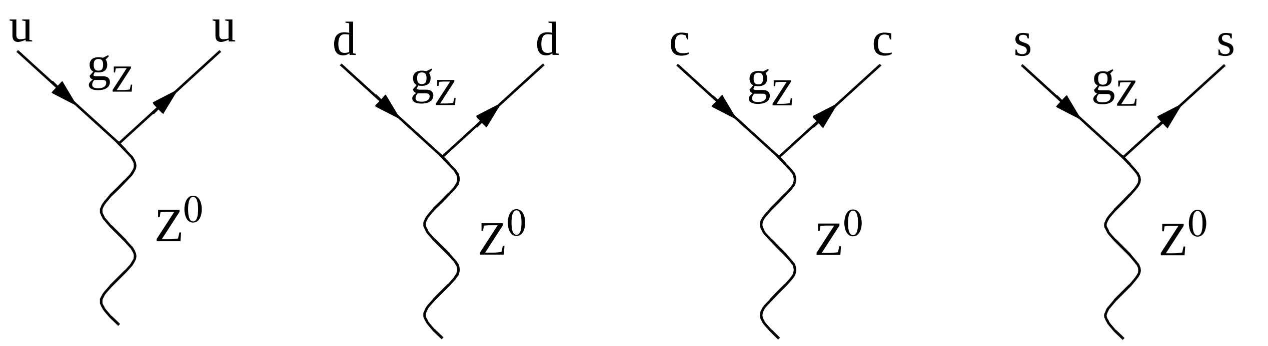
– Conserved lepton numbers

– Not conserved quark flavour (quark mixing)

- Basic vertices with Z^0 bosons have:

– Conserved lepton numbers

– Conserved flavour (no quark mixing)



Test of flavour conservation

Flavour is conserved at a Z^0 vertex (in contrast to a W vertex).This can be verified by experiments.

Consider the following two possible processes that change strangeness:

$$K^+ \to \pi^0 + \mu^+ + \nu_\mu \qquad \text{(a)}$$

and

$$K^+ \to \pi^+ + \nu_l + \bar{\nu}_l \qquad \text{(b)}$$

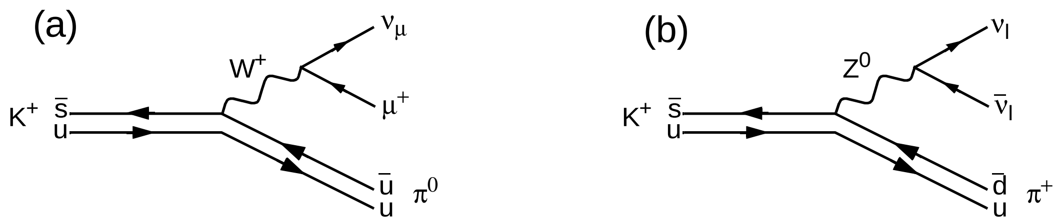


Figure 112: Decay (a) is allowed; decay (b) – forbidden

The measured upper limit on the ratio of the decay rates (b) to (a) is:

$$\frac{\sum\limits_{l} \Gamma(K^+ \to \pi^+ + \nu_l + \nu_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} < 10^{-7}$$

The unification condition and masses

The coupling constants at γ–, $W^{\pm}$– and Z^0-vertices are not independent from each other. In order for all infinities to cancel in electroweak theory, the unification relation and the anomaly condition have to be fulfilled.

→ The *unification condition* establishes a relation between the coupling constants ($\alpha_{em}=e^2/4\pi\varepsilon_0$):

$$\sqrt{\frac{\pi\cdot\alpha}{2}} = g_W \sin\theta_W = g_Z \cos\theta_W \qquad (114)$$

θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos\theta_W = \frac{M_W}{M_Z} \qquad (115)$$

→ The *anomaly condition* relates electric charges: $\sum_l Q_l + 3\sum_q Q_q = 0$

where the factor 3 comes from the number of colors.

Historically, the W and Z masses were predicted from low energy interactions.

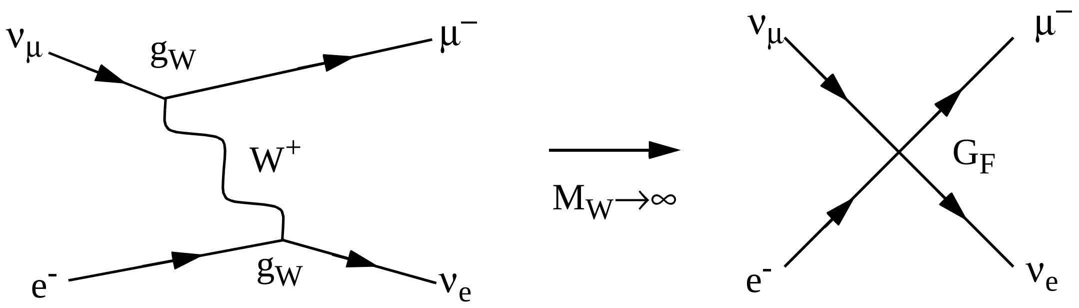


Figure 113: The low energy zero range approximation.

In the zero-range approximation i.e. in the low-energy limit, the charged current reactions are characterized by the Fermi constant (G_F):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2}$$

From this expression, the unification condition and the definition of θ_W one then obtains:

$$M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W}$$

$$M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_F \cos^2\theta_W \sin^2\theta_W}$$

Introducing the neutral current coupling constant (G_Z) (also in the low energy zero-range approximation) one gets

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2}$$

and the weak mixing angle can be expressed as

$$\frac{G_Z}{G_F} = \frac{g_Z^2}{g_W^2}\frac{M_W^2}{M_Z^2} = \sin^2\theta_W$$

From the measurements at low energy of rates of charged and neutral currents reactions it is therefore possible to determine that:

$$\sin^2\theta_W = 0,277 \pm 0,014$$

from this measurement at low energies (below the W and Z masses) it was possible to predict the masses of W and Z:

$$M_W = 78,3 \pm 2,4\ GeV/c^2;\ M_Z = 89,0 \pm 2,0\ GeV/c^2$$

When the W and Z boson were discovered at CERN with the masses predicted from low energy experiments it was a strong confirmation that the electroweak theory was correct.

Today the most precise estimation of the Weinberg angle using many measurements give:

$$\sin^2\theta_W = 0{,}2255 \pm 0{,}0021$$

Putting this value into the previous formulas give
M_W = 78.5 GeV and M_Z = 89.3 GeV
while the direct measurements of the masses give
M_W = 80.4 GeV and M_Z = 91.2 GeV

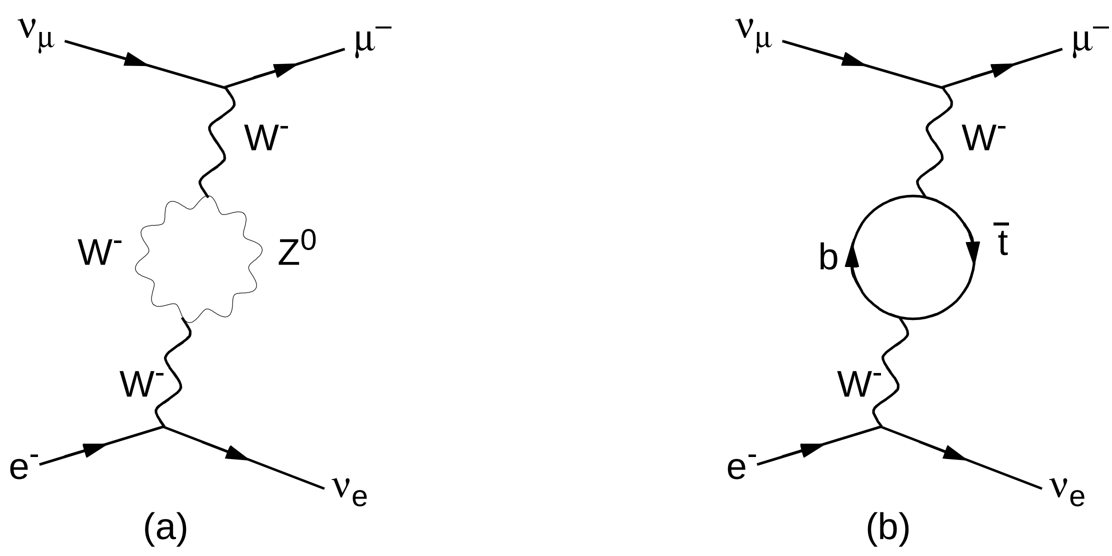


Figure 114: Examples of higher order contributions to inverse muon decay

The difference is due to higher-order diagrams which were not included in the previous low-energy formulas.

Since the top-quark is involved in higher order corrections, the measurement of electroweak processes could be used to predict the top-quark mass before it was discovered:

$$m_t = 170 \pm 30\ GeV/c^2$$

The directly measured mass of the top quark at Fermilab by CDF is today

$$m_t = 176 \pm 5\ GeV/c^2$$

in perfect agreement with the prediction !

Electroweak reactions

❖ In any process in which a photon can be exchanged, a Z^0 boson can be exchanged as well:

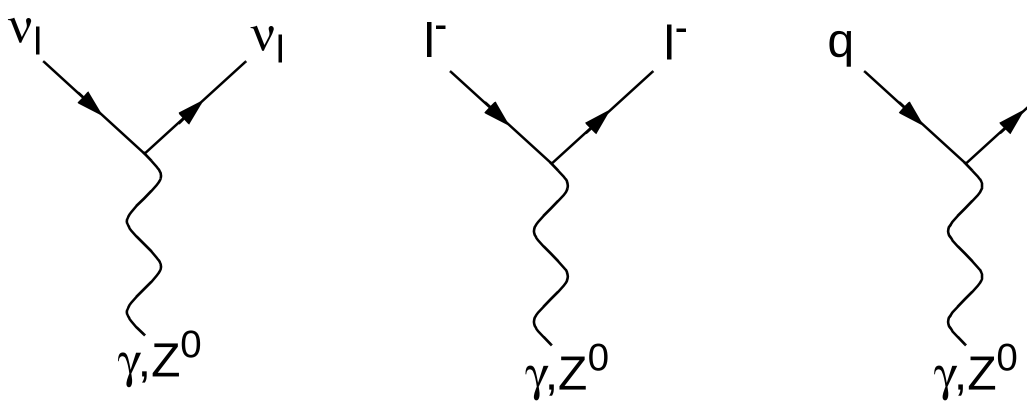


Figure 115: Z^0 and γ couplings to leptons and quarks

Example: The reaction $e^+e^- \to \mu^+\mu^-$ has two dominant contributions:

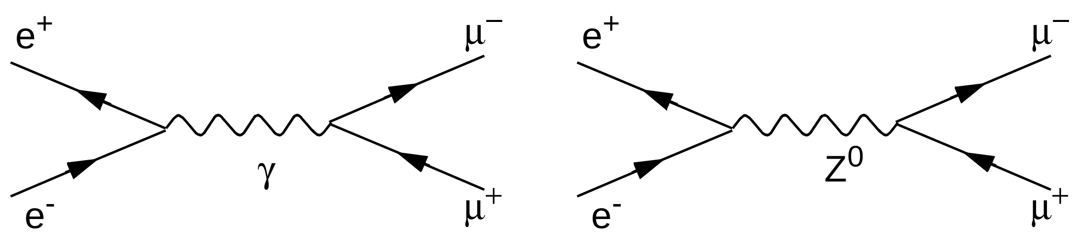


Figure 116: Dominant contributions to the e^+e^- annihilation into muons

With simple dimensional arguments one can estimate the cross section for the photon- and Z-exchange process at low energy:

$$\sigma_\gamma \approx \frac{\alpha^2}{E^2} \qquad \sigma_Z \approx G_Z^2 E^2$$

Where E is the energy of the colliding electron and positron beams.

From these expressions, the ratio of σ_Z and σ_γ is:

$$\frac{\sigma_Z}{\sigma_\gamma} \approx \frac{E^4}{M_Z^4} \tag{116}$$

One can conclude that at low energies the photon exchange process dominates. However, at energies $E_{CM}=M_Z$, this low-energy approximation fails

The Z^0 peak is described by the Breit-Wigner formula:

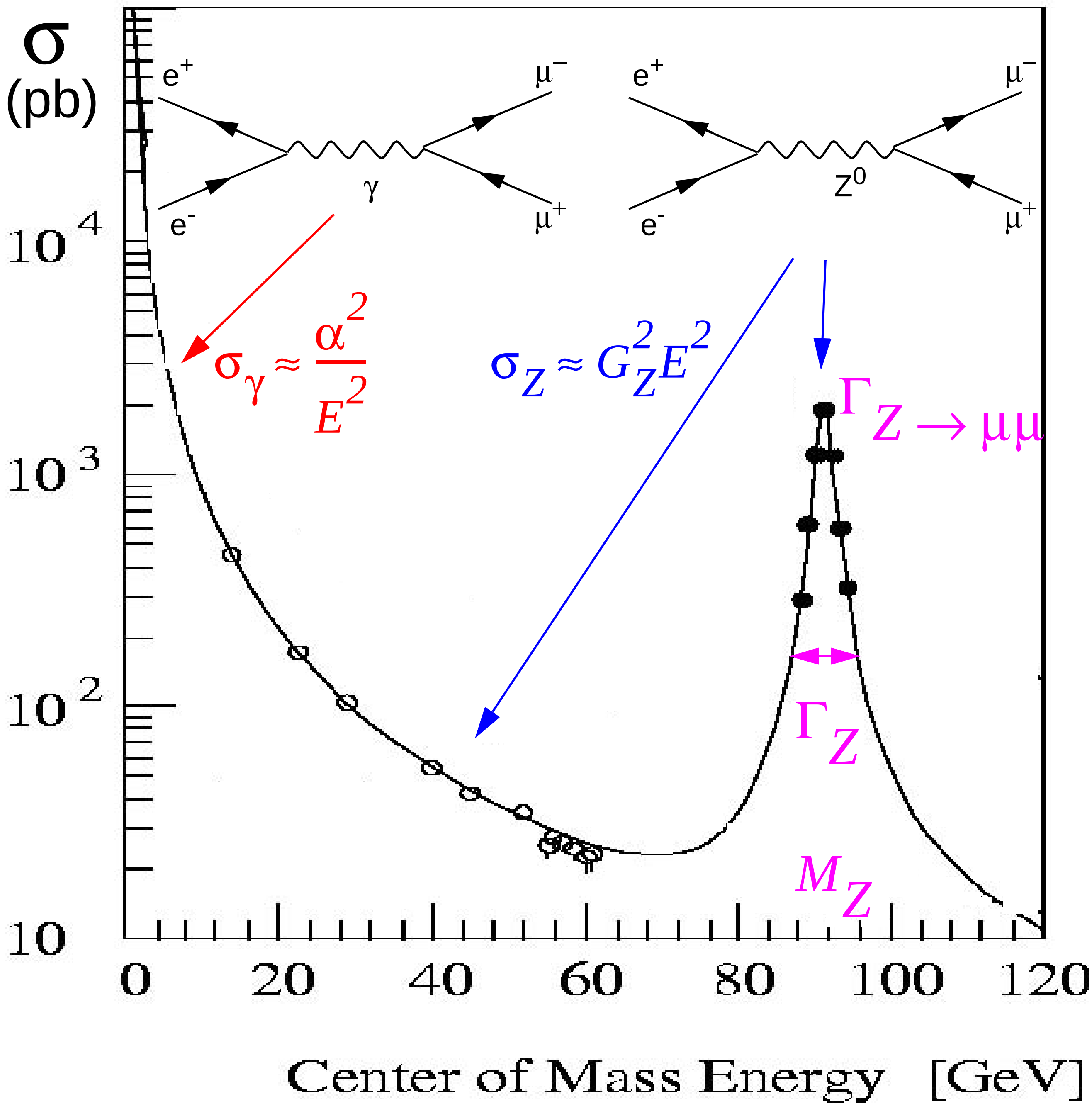
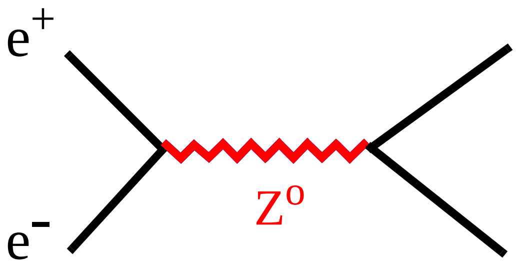


Figure 117: The cross sections of e^+e^- annihilation into μμ

$$\sigma(ee \to \mu\mu) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to ee)\Gamma(Z^0 \to \mu\mu)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

The number of neutrino families

- The Z boson can decay in the following way:



	e^- μ^- τ^-	ν	q
	e^+ μ^+ τ^+	$\bar{\nu}$	$\bar{q}$
Branching ratio (B):	0.10	0.20	0.70
Decay width (Γ):	0.25 GeV	0.50 GeV	1.74 GeV

The lifetime (τ) , the branching ratio (B) and the partial decay width (Γ) are related to each other by

$$\tau = \frac{B}{\Gamma}$$

$$\tau = \frac{B_Z}{\Gamma_Z} = \frac{B_{had} + B_{ll} + B_{\nu\nu}}{\Gamma_{had} + \Gamma_{ll} + \Gamma_{\nu\nu}}$$

$$\tau = \frac{B_{had}}{\Gamma_{had}} = \frac{B_{ll}}{\Gamma_{ll}} = \frac{B_{\nu\nu}}{\Gamma_{\nu\nu}} = 3\times 10^{-25} s$$

Note: $1 GeV^{-1} = 6,582\times 10^{-25} s$

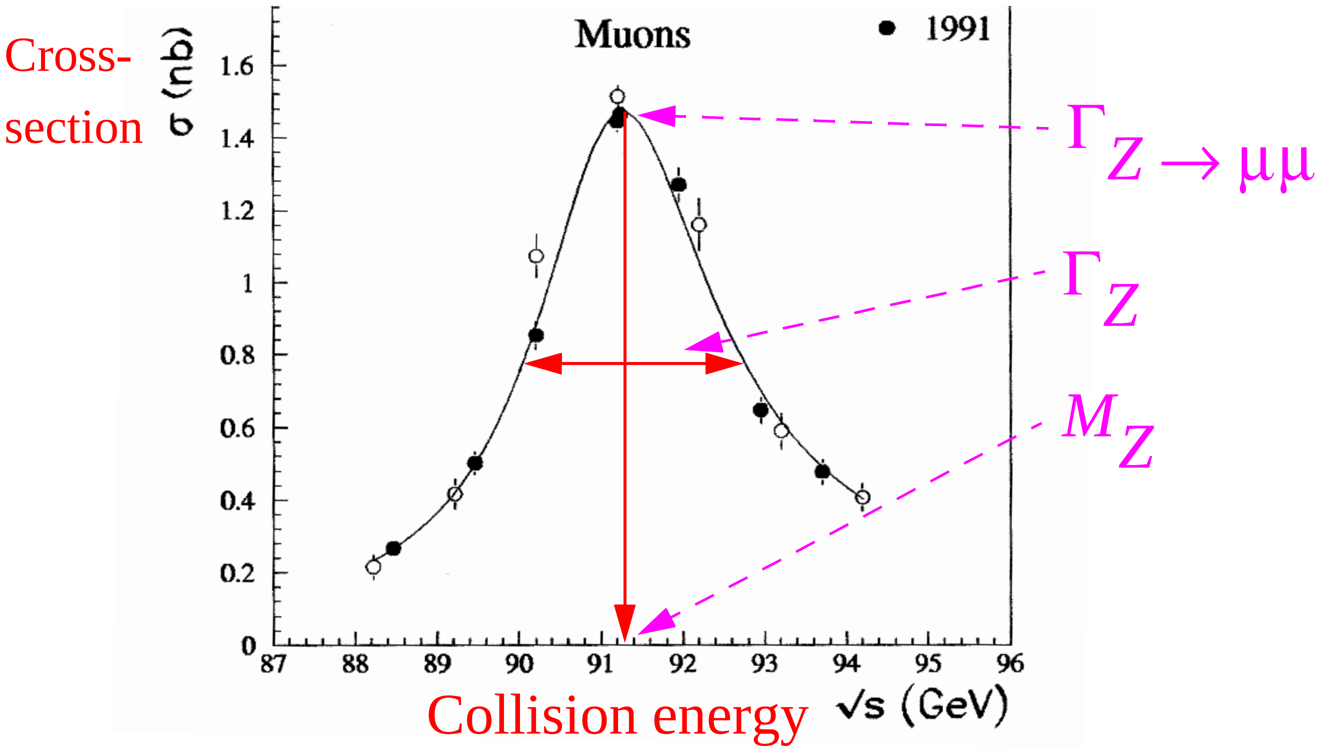


Figure 118: The leptonic decay of the Z^0 into muons.

The peak can be fitted with the Breit-Wigner formula:

$$\sigma(e^+e^- \to X) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to e^+e^-)\Gamma(Z^0 \to X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

Here Γ_Z is the total Z^0 decay rate, and $\Gamma_Z(Z^0 \to X)$ is the decay rates to the final state X. The height of the peak (at E_{CM}=M_Z) is proportional to the product of the branching ratios:

$$B(Z^0 \to e^+e^-)B(Z^0 \to X) \equiv \frac{\Gamma(Z^0 \to e^+e^-)}{\Gamma_Z}\frac{\Gamma(Z^0 \to X)}{\Gamma_Z}$$

The fitted parameters of the Z^0 peak in the leptonic and hadronic decay modes give:

$$M_Z = 91{,}187 \pm 0{,}007\ GeV/c^2$$

$$\Gamma_Z = 2{,}490 \pm 0{,}007\ GeV$$

$$\Gamma(Z^0 \to hadrons) = 1{,}741 \pm 0{,}006\ GeV$$

$$\Gamma(Z^0 \to l^+ l^-) = 0{,}0838 \pm 0{,}0003\ GeV$$

- The decays $Z^0 \to l^+l^-$ and $Z^0 \to hadrons$ account for only about 80% of all Z^0 decays
- The remaining decays are those containing only neutrinos in the final state since

$$\Gamma_Z = \Gamma(Z^0 \to hadrons) + 3\Gamma(Z^0 \to l^+ l^-) + \\ + N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l) \tag{117}$$

From the measurement of all other partial widths one can therefore estimate the partial decay to neutrinos which cannot be measured directly:

$$N_\nu \Gamma(Z^0 \to \nu_l \bar{\nu}_l) = 0{,}498 \pm 0{,}009\ GeV$$

The decay rate to neutrino pairs can also be calculated from the diagrams shown previously and this gives $\Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0{,}166\ GeV$ which togethere with $N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0{,}498\ GeV$ gives

$$N_\nu = 2{,}994 \pm 0{,}011$$

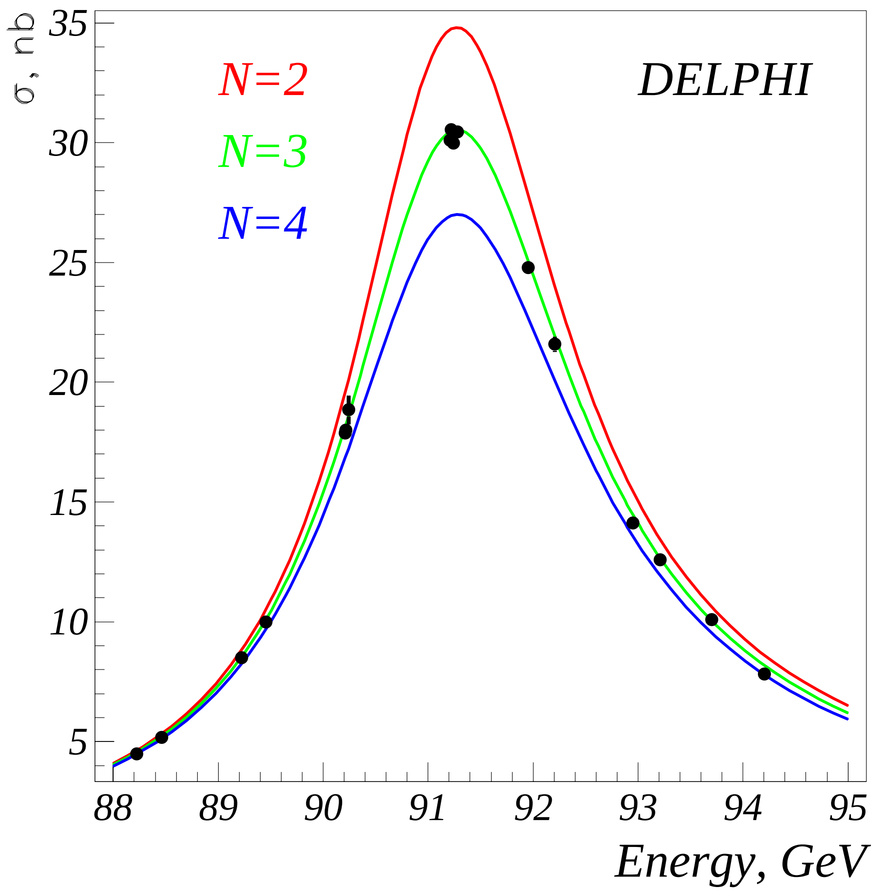


Figure 119: The decay of the Z^0 to hadrons and theoretical predictions based on different assumptions for the number of neutrino families (N)

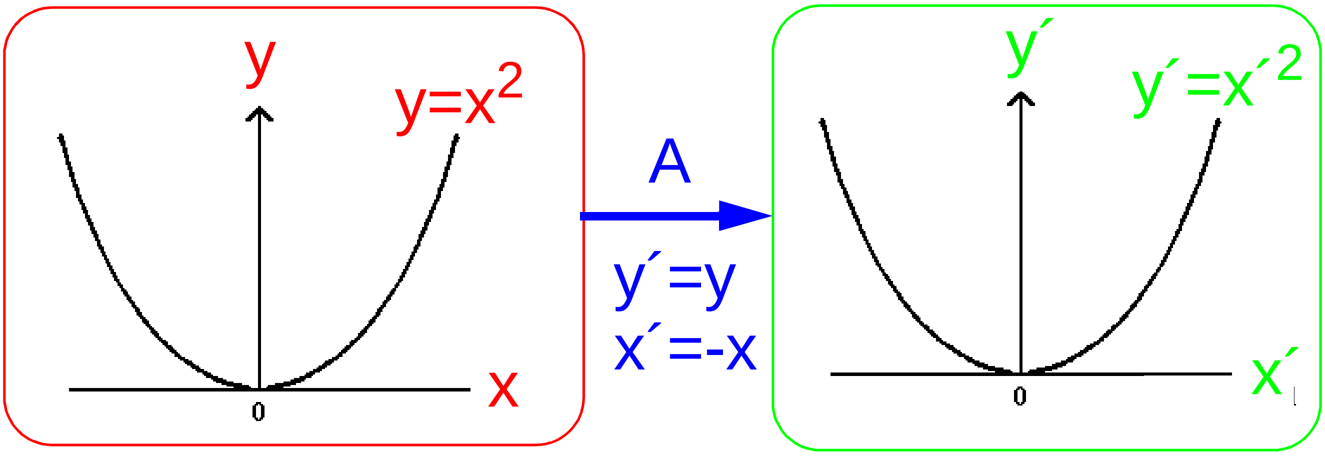
- There are no explicit restrictions on the number of generations in the Standard Model.

- However, the analysis of the Z^0 line shape at LEP shows that there are 3 and only 3 kinds of light neutrinos.

  - If neutrinos are assumed having negligible masses as compared with the Z^0 mass, there must be only THREE generations of leptons and quarks within the Standard Model.

Gauge invariance

Reminder:



The equation $y=x^2$ is symmetric or invariant under the transformation A i.e. it looks the same before and after the transformation.

→ Modern quantum field theories are *gauge invariant* theories i.e. they are theories were the main equations do not change when a gauge transformation is performed.

→ By requiring that the theories are gauge invariant one can in fact deduce the various interactions.

What is a gauge transformation ?

There are several forms of gauge transformations corresponding to different interactions.

Example from non-relativistic electromagnetism:

Assume that we do not know the Schrödinger equation for electromagnetic interactions but we do know that it has to be invariant under a so-called U(1) phase transformation:

$$\psi(\vec{x}, t) \to \psi'(\vec{x}, t) = e^{iq\alpha(\vec{x}, t)}\psi(\vec{x}, t) \tag{118}$$

Here $\alpha(\vec{x}, t)$ is an arbitrary continuous function.

If a non-relativistic particle is free, then the equation of motion is the free particle Schrödinger equation:

$$i\frac{\partial\psi(\vec{x}, t)}{\partial t} = -\frac{1}{2m}\nabla^2\psi(\vec{x}, t) \tag{119}$$

→ The phase transformed wavefunction $\psi'(\vec{x}, t)$ is, however, not a solution of this Schrödinger equation.

→ *Gauge principle*: to keep the invariance condition satisfied, a minimal field should be added to the Schrödinger equation, i.e., an interaction should be introduced

→ This can be done by requiring that the Schrödinger equation should also be invariant under a gauge transformation of the type:

$$\bar{A} \to \bar{A}' = \bar{A} + \nabla\alpha$$

$$V \to V' = V - \frac{\partial\alpha}{\partial t}$$

where $\bar{A}$ and V are the vector and scalar potentials of the electromagnetic field in which a particle with charge q is moving.

In order for the free-particle Schrödinger equation to be invariant under both the U(1) phase transformation and the gauge transformation, the equation has to be changed to:

$$i\frac{\partial\psi(\vec{x}, t)}{\partial t} = \left[\frac{1}{2m}(\bar{p} - q\bar{A}) + qV\right]\Psi(\vec{x}, t)$$

Unification and the gauge principle

In QED, the transition from one electron state to another with a different phase, $e^- \to e^-$, demands emission (or absorption) of a photon: $e^- \to e^- \gamma$

More generally, one can define gauge transformations that not only change the phase but also transforms electrons and neutrinos:

$$e^- \to \nu_e \qquad \nu_e \to e^- \qquad e^- \to e^- \qquad \nu_e \to \nu_e$$

these lead via the gauge principle to interactions

$$e^- \to \nu_e W^- \quad \nu_e \to e^- W^+ \quad e^- \to e^- W^0 \quad \nu_e \to \nu_e W^0$$

where W^+, W^- and W^0 are the corresponding spin-1 gauge bosons.

While W^+ and W^- are the well-known bosons responsible for charged currents, W^0 is not observed experimentally.

→ This problem is solved by the unification of electromagnetism with weak interactions since this result in that both the Z^0 and the γ are mixtures of W^0 and yet another neutral boson B^0:

$$\begin{aligned} \gamma &= B^0 \cos\theta_W + W^0 \sin\theta_W \\ Z^0 &= -B^0 \sin\theta_W + W^0 \cos\theta_W \end{aligned} \tag{120}$$

The gauge transformation which achieve this is called a local gauge transformation of the type.

$$U(1) \otimes SU(2)_L$$

The requirement of gauge invariance under this transformation leads to new vertices:

$$e^- \rightarrow e^- B^0 \qquad \nu_e \rightarrow \nu_e B^0$$

For these vertices the electromagnetic charge has to be replaced with new couplings $g_Z y_{e^-}$ and $g_Z y_{\nu_e}$

One can show that the new couplings can be chosen such that

$$\gamma = B^0 \cos\theta_W + W^0 \sin\theta_W$$

has the coupling of the photon if the unification condition is satisfied i.e. if

$$\frac{e}{2\sqrt{2\varepsilon_0}} = g_W \sin\theta_W = g_Z \cos\theta_W$$

→ Conclusion: Electroweak theory can be made gauge-invariant by introducing neutral bosons W^0 and B^0. The Z^0 and γ states that are observed in experiments are linear combinations of these.

The Higgs boson

❖ Generally, experimental data agree with gauge invariant electroweak theory predictions.

→ However, gauge invariance implies that the gauge bosons have zero masses if they are the only bosons in the theory. Photon in QED and gluons in QCD comply with this but not the Z and W bosons.

$$\Downarrow$$

a new field should be introduced !

❖ The scalar *Higgs field* solves the problem:

– *The Higgs boson* H^0 is a spin-0 particle

– The Higgs field has a **non-zero** value ϕ_0 in vacuum (the field is non-zero in the groundstate).

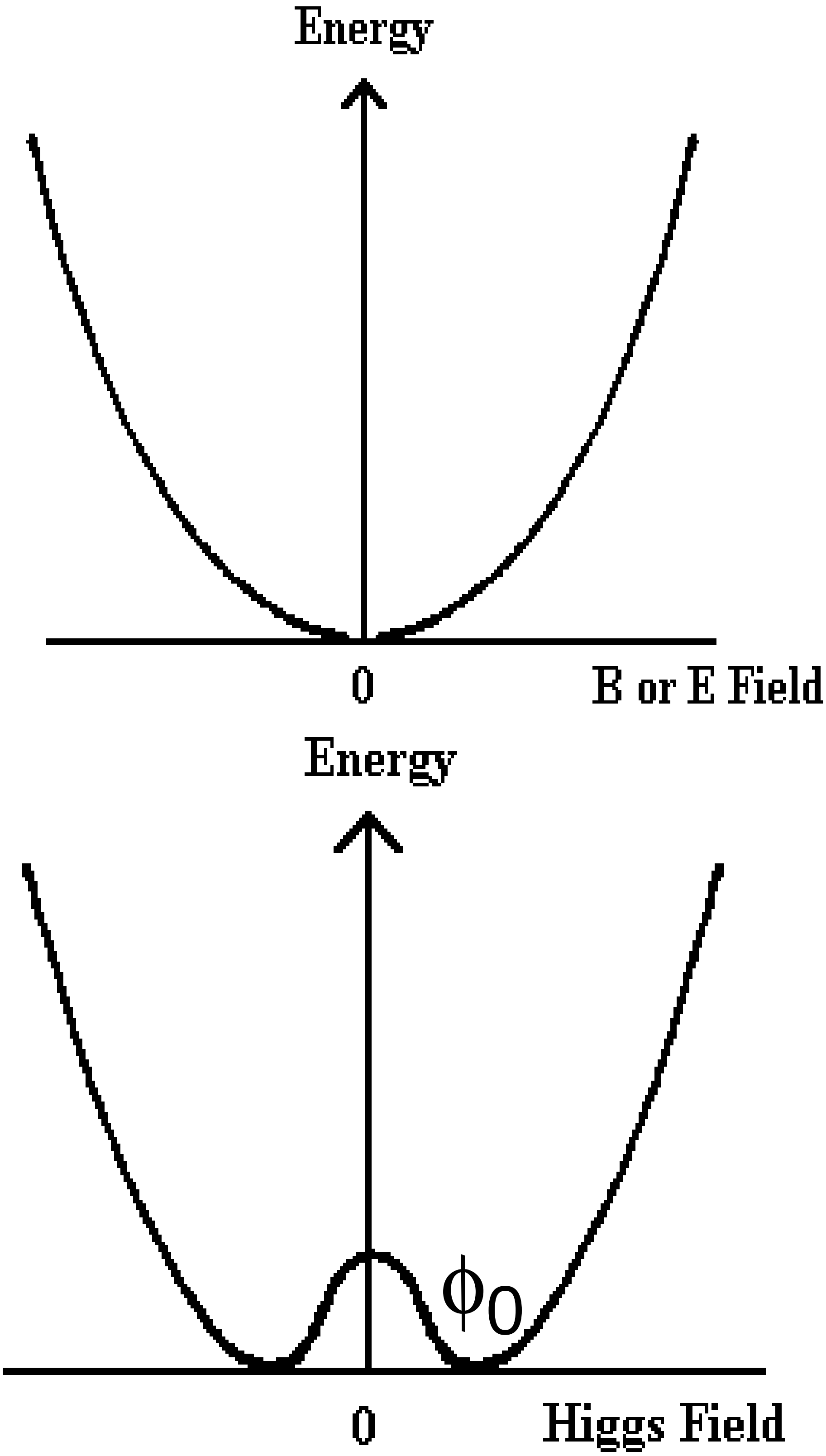


Figure 120: Comparison of the electric and Higgs fields

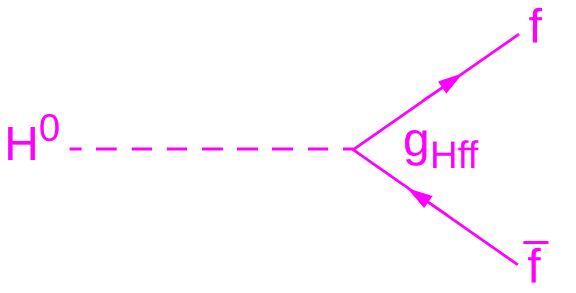
- The interactions of the Higgs field with the gauge bosons is gauge invariant, however, the vacuum value ϕ_0 is not gauge invariant $\Rightarrow$ the interaction has *hidden gauge invariance* (or its symmetry is *spontaneously broken)*.

Since the vacuum expectation value is not zero, the vacuum is supposed to be populated with massive Higgs bosons $\Rightarrow$ when a gauge field interacts with the Higgs field it acquires mass

The W and Z bosons require masses in the ratio given by

$$\cos\theta_W = \frac{M_W}{M_Z}$$

In the same way, fermions acquire masses by interacting with Higgs bosons and the coupling constant is related to the fermion masses:



$$g_{Hff}^2 = \sqrt{2}G_F m_f^2$$

Figure 121: Basic vertex for Higgs-fermion interactions

The search for the Higgs boson

- The mass of the Higgs itself is not predicted by the theory, only the couplings to other particles.
- The existence of the Higgs boson has not been confirmed by experiments.

Searches for the Higgs at LEP.

a) If the H^0 was lighter than the Z^0 ($M_H \leq 60$ GeV), then the Z^0 could decay by

$$Z^0 \to H^0 + l^+ + l^- \tag{121}$$

$$Z^0 \to H^0 + \nu_l + \bar{\nu}_l \tag{122}$$

But the branching ratio is very low:

$$3\times10^{-6} \leq \frac{\Gamma(Z^0 \to H^0 l^+ l^-)}{\Gamma_{tot}} \leq 10^{-4}$$

The measurements at LEP 1 has set a *lower limit* on the Higgs mass which is $M_H > 58$ GeV/c^2

b) If the H^0 is heavier than 60 GeV/c^2, it could have been produced in e$^+$e$^-$ annihilations at LEP 2. The most important process is:

$$e^+ + e^- \rightarrow H^0 + Z^0 \qquad (123)$$

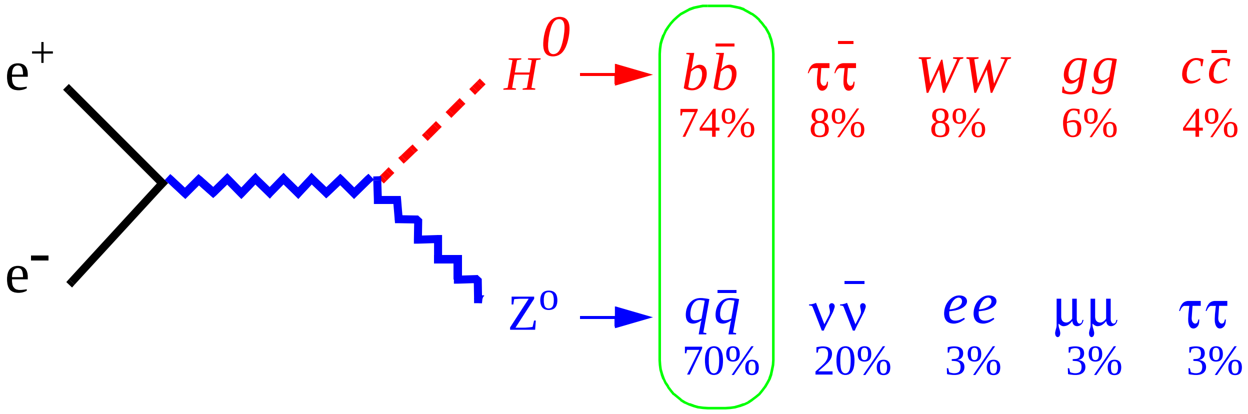


Figure 122: “Higgsstrahlung” in e$^+$e$^-$ annihilation

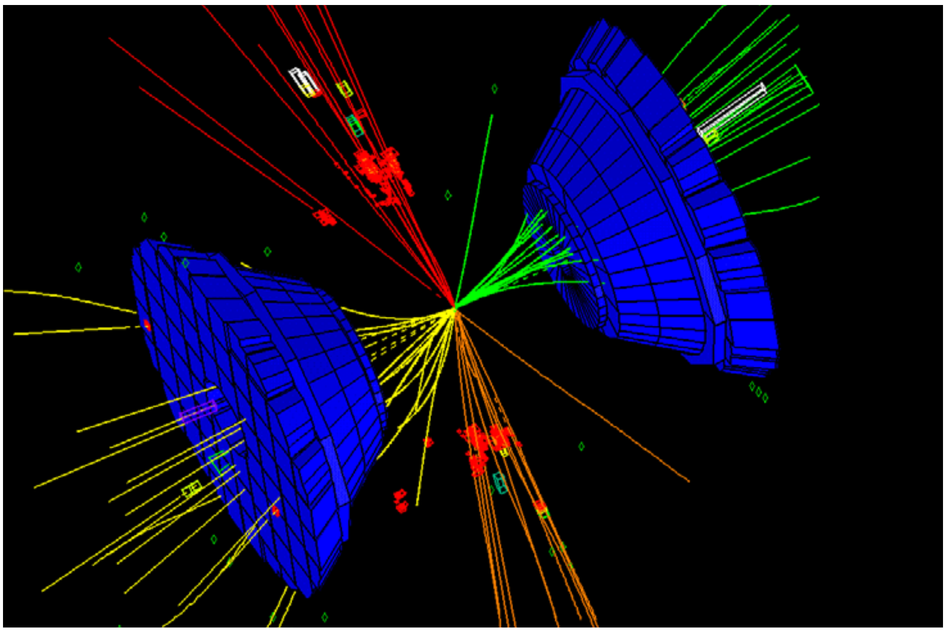


Figure 123: Example of a Higgs candidate event (Delphi).

- During the last year of operation of LEP 2, the ALEPH experiment recorded a couple of events which could be due to the decays of a Higgs with a mass of about 115 GeV/c^2. The other LEP experiments could not confirm the ALEPH results and the DELPHI experiment set a limit of:

$$M_H > 114\ GeV/c^2$$

The measurement of many electroweak parameters at LEP (and other places) makes it possible to make a global fit with the Higgs mass as a free parameter

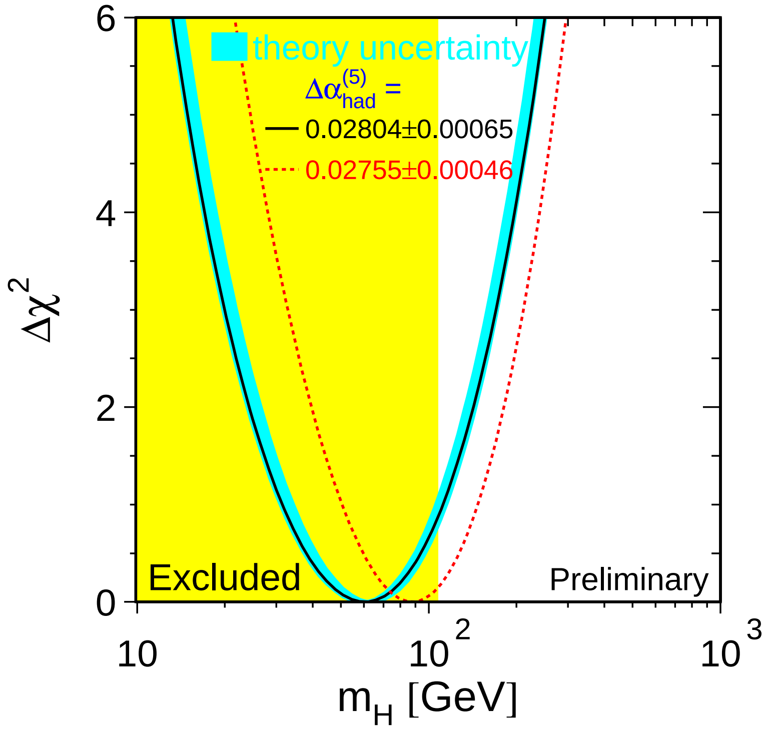


Figure 124: A prediction of the Higgs mass from a global fit to electroweak measurements.

- The result of the fit is a prediction of a low mass for the Higgs boson < 165 GeV.

Searches for the Higgs at LHC.

c) Higgs with masses up to 1 TeV can be observed at the future proton-proton collider LHC at CERN:

$$p + p \rightarrow H^0 + X \qquad (124)$$

where H^0 is produced in electroweak interaction between the quarks

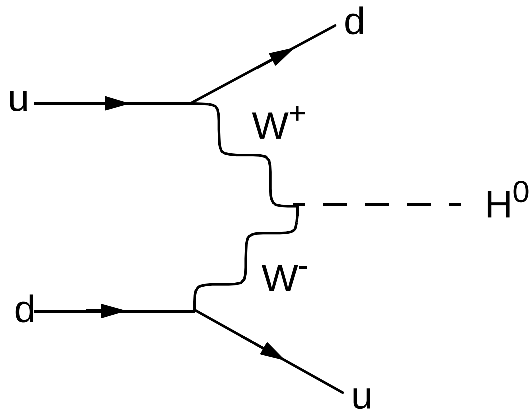


Figure 125: An example of Higgs production process at LHC

At the LHC the background is huge and a good signature have to be found.

– If $M_H < 2M_W$, (160 GeV/c^{2}) the dominant decay mode is

$$H^0 \to b + \bar{b} \tag{125}$$

but these events will be swamped by background. A more promising decay mode is

$$H^0 \to \gamma + \gamma \tag{126}$$

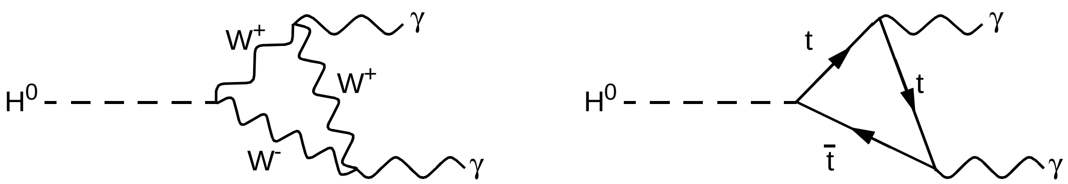


Figure 126: The dominant mechanisms for the decay to photons

The branching ratio of this kind of processes is, however, only 10^{-3}

– If $M_H > 2M_Z$, the dominant decay modes are:

$$H^0 \rightarrow Z^0 + Z^0 \tag{127}$$

$$H^0 \rightarrow W^- + W^+ \tag{128}$$

The most clear signal is when both Z^0s decay into electron or muon pairs:

$$H^0 \rightarrow l^+ + l^- + l^+ + l^- \tag{129}$$

These decays can be found if $200 \leq M_H \leq 600$ GeV, but only 4% of all Higgs particles decay to four electrons or muons.

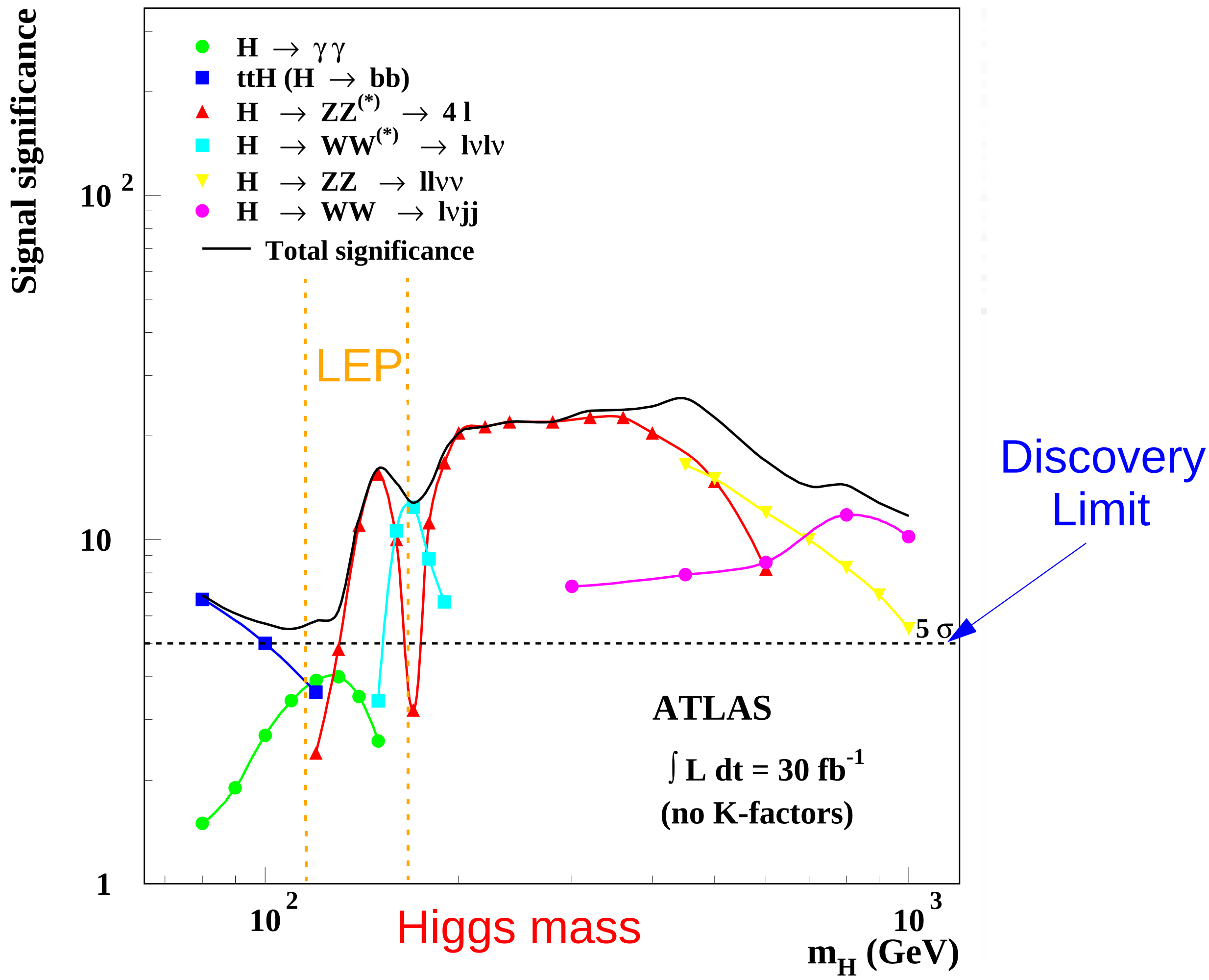


Figure 127: Higgs discovery potenial at the LHC.

Summary

- **The problem of divergence**

  a) By introducing the Z-bosons one can cancel out divergent diagrams from the W-bosons.

  b) There is no quark mixing in Z-vertices.

- **Test of flavour conservation.**

  c) Kaon decay show that flavour is conserved at a Z-vertex (but not a W-vertex).

- **The unification condition and masses.**

  d) The unification condition establishes a relation between the electromagnetic coupling constants.

  e) The ratio of the W- and Z-masses is given by the weak mixing angle (the Weinberg angle).

- **Electroweak reactions**

  f) Fitting the Z-peak gives the mass and width of the Z-boson. From this, it can be determined that the number of light neutrino families is 3.

• Gauge invariance.

g) A gauge transformation is a symmetry transformation.

h) Field theories which do not change under gauge transformation are gauge invariant.

i) Imposing gauge invariance on the weak interaction theory leads to the prediction of three massless W-bosons.

j) The unification of electromagnetism with weak interactions leads to the introduction of the B^0-boson which is connected to the electromagnetic field.

k) The neutral gauge bosons that are observed in experiments (γ and Z^0) are mixtures of the B^0 and W^0 states.

• The Higgs boson.

l) The Higgs field and its gauge boson are introduced to explain the large masses of the W- and Z-bosons.

m) The Higgs field has the unusual feature of having a non-zero expectation value in vacuum.

- **The search for the Higgs boson**

n) The LEP experiments have been the main place for the search for a Higgs up to now.

o) In the future the search will take place at the Tevatron followed by the LHC.