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On the m_t-dependence of the Bose-Einstein effect

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Abstract

A study of the Bose-Einstein correlation function for JETSET 7.4 generated data is performed. Two-particle correlation function is studied in case of like-charged particles, like-charged pions, pions coming from the string decay and pions from the resonance decays.

The dependence of the shape of the output Bose-Einstein correlation function on the input parameters and the dependence of λ and of the three components of the radius on the transverse mass m_t are studied. The results obtained are compared to the DELPHI data and also to the data from heavy ion collision experiments.

We found that for radii larger than 1 fm the JETSET simulation of the Bose-Einstein effect corresponds to the input correlation function. For lower radii, secondary effects of the simulation algorithm play a large and eventually dominant role. For e^+e^- annihilation one expects a Bose-Einstein radius of around half a fermi, which is outside the range of applicability of this commonly used Bose-Einstein algorithm, but in the current JETSET implementation it reproduces the observed m_t dependence seen in data.

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1 Introduction

The study of Bose-Einstein correlation function for identical particles is of particular interest since the process of hadron production, or fragmentation, in high energy physics is poorly comprehended. At this moment, no appropriate theory can describe it, only phenomenological models being available for the hadronization process and the Bose-Einstein phenomenon in particular. To account for and to describe it properly in event generators, the Bose-Einstein phenomenon must be well understood, therefore, more profound studies are required. Recently it was shown that the measurement of W mass at LEP2 is likely to be affected by Bose-Einstein correlation between pions from different W's [1, 2, 3]. Consequently, a significant interest was shown for the study of Bose-Einstein correlations in e^+e^- annihilation in the last years [4, 5, 6, 7].

High energy heavy ion collision experiments developed precise methods for the boson interferometry studies to obtain information about the space-time development of the particle emitting source. Analysis performed for the three components of the four-momenta difference of two identical bosons shows a transverse mass dependence for the correlation radii, which can be described by hydrodynamical models of the particle source expansion [8, 9].

Recent studies of Bose-Einstein correlations between like-charged bosons in e^+e^- annihilation at the centre-of-mass energy of $E_{cm} = M(Z^0) = 91.2$ GeV using the DELPHI experiment data [13] revealed a transverse mass dependence of the correlation function parameters, similar to that found in heavy ion experiments. Analogous analysis of the $e^+e^- \to Z^0$ events, generated with the JETSET particle generator [12], also showed a tendency for correlation radii to decrease and correlation amplitude to increase with increasing transverse mass. This phenomenon is very similar to the one seen in heavy ion physics explainable by hydrodynamical models. However, the fact that JETSET reproduces the effect quite satisfactory without any explicit input of transverse mass dependence, implies that profound studies of the origin of this effect using this particle generator must be performed in order to explain the observation.

Since JETSET is the simulation program the most commonly used by the high energy experimental groups, the analysis of the method of implementing the Bose-Einstein correlations, its performances, its drawbacks and the possible improvements was studied in [10, 11]. The study of the Bose-Einstein correlation function using JETSET particle generator, which is presented here, is necessary in order to distinguish between the effects given by: all the particles, all pions, direct pions (pions that appeared directly from the decay of string) and pions that are produced in resonance decays. Comparing data obtained by Monte Carlo simulation (JETSET) with experimental data we can learn more about the behaviour of the correlation function and from this about the hadronization process.

In this work, a study of the Bose-Einstein correlation function in electron-positron annihilation process $e^+e^- \to \gamma^* Z^0 \to hadrons$ is performed: two-particle correlation function is studied in case of like-charged particles, like-charged pions, direct like-charged pions and like-charged pions coming from resonance decays, in different transverse mass ranges. The results obtained are compared with the ones reported in previous works [13, 14].

The paper is organised as follows: in the Section 2 the theory of the Bose-Einstein correlations is presented, the Section 3 is dedicated to the description of the JETSET simulation program and the way in which the Bose-Einstein algorithm is included in it. Implementation issues and practical hints are presented in sections 4 and 5. Section 6 presents our results and a comparison with previous experimental results. Finally, in section 7 the reader finds the conclusions.

2 Bose-Einstein correlation

During the 50-ies, in particle physics experiments and in the astronomical observations, it was discovered that bosons emitted from the same source showed the tendency to have close energy-momentum characteristics [15]. This phenomenon of increasing probability for emission of identical bosons from close regions of space and time is called Bose-Einstein correlation (we will not discuss correlations between fermions in this paper).

The Bose-Einstein effect originates in the quantum mechanical interference of the boson wave functions and is a consequence of their symmetry under particle exchange, that influences the wave functions to yield an effective clustering of the particles in the phase space, which explains their preference of occupying the same quantum states. From the characteristics of the resulting interference pattern it is possible, at least in principle, to determine the space-time dimensions of the source.

The string model describes e^+e^- annihilation data with high accuracy and provides an appropriate framework in which to consider the Bose-Einstein enhancement. At present, the only working method to introduce the interference effects in Monte Carlo generators for e^+e^- annihilation is that of shifting final state momenta to reproduce the assumed distribution of some variables [12].

2.1 The Bose-Einstein correlation function

Considering the production of two identical bosons with four-momenta p_1 and p_2, denoting $P(p_1, p_2)$ the probability density of two particles to be produced, and $P(p_1)$ and $P(p_2)$ – the probability densities for a single particle to be produced with momentum p_1 or p_2, the correlation function C_2 of two identical bosons is defined as [16]:

$$C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} \ . \tag{1}$$

From the experimental point of view, $P(p_1, p_2)$ is a double differential cross section. In practice, it is difficult to construct the product $P(p_1)P(p_2)$, therefore it is often replaced by $P_0(p_1, p_2)$, which is equal to $P(p_1)P(p_2)$ in the absence of correlation. One of the major problems in this kind of studies is to build $P_0(p_1, p_2)$, usually called the "reference sample".

The correlation function (1) can be parameterized as:

$$C_2(Q) = 1 + \lambda e^{-Q^2R^2} \ , \tag{2}$$

where $Q = p_1 - p_2$ is the invariant four-momenta difference, and λ and R are free parameters. The interpretation of the λ parameter is that it is related to the fraction of identical bosons which do interfere. Parameter R is usually interpreted as the geometrical radius of the presumably spherical boson emitting source.

Near $Q = 0$, the effect of Coulomb repulsion between two identical charged bosons will lead to the suppression of the probability of finding two like-charged particles with small relative momentum. The effect is dominant in case of pions for $Q < 10$ MeV. For $Q > 50$ MeV the Coulomb repulsion is negligible, since it has only a small effect on the studied distributions and being not included in the simulation program. In this paper we shall study the correlation function for $Q < 5$ GeV.

The customary parameterization for the correlation function in case of electron-positron annihilation is [7]:

$$C_2(Q) = N[1 + \lambda e^{-Q^2R^2}][1 + \delta Q] \ , \tag{3}$$

where N is the overall normalisation and the additional linear term $[1 + \delta Q]$ is used to describe the observed slopes in the correlation function at large Q values.

Several previous studies regarding the two-particle correlation function [13, 14, 17, 18] showed a strong dependence for all the components of the correlation function on the transverse mass of the pair of bosons m_t:

$$m_t = \frac{m_{t,1} + m_{t,2}}{2} = \frac{\sqrt{m_1^2 + p_{t,1}^2} + \sqrt{m_2^2 + p_{t,2}^2}}{2} \ , \tag{4}$$

for experimental data, as well as for simulated JETSET events (m_1, m_2 represent particle masses).

It was observed that λ is increasing with growing m_t in case of e^+e^- annihilation and almost constant for heavy ion physics. The increase of the λ parameter in e^+e^- annihilation is usually explained by vanishing of resonance decay products contribution with increasing m_t. This is a purely kinematical effect given by the resonance decay. Meanwhile, parameter R decreases with growing m_t in both e^+e^- annihilation and heavy ion experiments.

A popular explanation of the observed m_t dependence of radius is that this is also due to resonance decays. Resonances are propagating out of the primary pion source and the pions produced in their decays have comparatively low momenta. Therefore, the effective size of the source increases towards low m_t values. Effects as final state interactions and resonance production can affect the shape of the correlation function. The above explanation cannot possibly be valid for JETSET generated events, since the resonance propagation is not included in this generator. We are going to study why the m_t dependence happens in the JETSET simulation.

3 JETSET and Bose-Einstein correlation

JETSET is a simulation program able to generate hard processes, in particular, the electron-positron annihilation $e^+e^- \to \gamma^*/Z^0 \to q\bar{q}$. (Here '*' is used to denote that the photon is off mass shell.) The quark q in the reaction may have any flavour, except of t. The flavour in each event is picked at random, according to the relative couplings evaluated at the hadronic center of mass energy.

JETSET is intimately connected with the string fragmentation model in the form of so-called Lund model. The JETSET program has a probabilistic and iterative nature, with the fragmentation process being described in terms of one or few simple underlying branches, as, for example, $string \rightarrow hadron + remainder \;\; string$ and so on. At each branching, probabilistic rules are given for production of new flavours and for sharing of the energy and momentum between the products.

To understand the fragmentation model, we can use as an example the simplest possible system: a 2-jet colour singlet $q\bar{q}$ event, as produced in e^+e^- annihilation, where we have a linear confinement picture. The energy stored in the colour dipole field between a charge and an anticharge increases linearly with the separation between charges, if the Coulomb term is neglected. The assumption of the linear confinement provides the starting point for the string model. As q and $\bar{q}$ partons move apart from their common production vertex, a colour flux tube or maybe a colour vortex line is considered to be stretched between the $q\bar{q}$. The potential energy stored inside the string is increased and the string may break, producing a new $q'\bar{q}'$ pair, so that the system splits into two colour singlet systems $q'\bar{q}$ and $q\bar{q}'$. If the invariant mass of these pairs is large enough, further breaks may occur. In the Lund string model, the string break-up process is assumed to proceed until only the on-mass-shell hadrons remain, each hadron corresponding to a small piece of string with a quark in one end and an antiquark in the other.

To simulate the Bose-Einstein effect, an algorithm, which does not represent a true model, is used, for which very specific assumptions and choices are made. In this scheme, the fragmentation is allowed to proceed independently of Bose-Einstein effect, and so are decays of the short-lived particles like ρ. The Q_{ij} value associated to a pair i, j is defined as:

$$Q_{ij} = \sqrt{(p_i + p_j)^2 - 4m^2} \;, \tag{5}$$

where m is the common particle mass and p_i, p_j are particle momenta.

A shifted smaller Q'_{ij} is then to be found, such that the ratio $C_2(Q)$ of "shifted" to the "unshifted" Q distribution is given by the requested parameterization. The shape can be chosen to be either exponential or Gaussian (we have used only the Gaussian form):

$$C_2(Q) = 1 + \lambda e^{-(QR)^r}, \quad r = 1 \;\; or \;\; 2 \;. \tag{6}$$

If the inclusive distribution of the Q_{ij} values is assumed to be given just by the simple, spherical phase space, Q'_{ij} can be found as a solution of the equation:

$$\int_0^{Q_{ij}} \frac{Q^2 dQ}{\sqrt{Q^2 + 4m^2}} = \int_0^{Q'_{ij}} C_2(Q) \frac{Q^2 dQ}{\sqrt{Q^2 + 4m^2}} \;. \tag{7}$$

That gives, indeed, as a new distribution, Q'_{ij} – the product of the old distribution and $C_2(Q)$:

$$Q'_{ij} = Q_{ij} C_2(Q) \;. \tag{8}$$

This algorithm is implemented in the LUBOEI subroutine, which is included into the JETSET generator. The procedure is an imitation of the interference effect, reproducing the assumed shape of the distribution in Q. This does not guarantee that other features of the interference effect will be well described. The Bose-Einstein effect is here interpreted almost as a classical force, acting on the final state particles, rather than being a quantum mechanical phenomenon of the production amplitude. So far, this Bose-Einstein algorithm should be used which caution and only as a first approximation to what Bose-Einstein effect can mean. The astonishing fact is that this parameterization reproduces the m_t dependence seen in experimental data.

4 Computational program. Main assumptions

We used the JETSET in order to generate two-parton $q\bar{q}$ events at a center of mass energy 91.2 GeV. The LUBOEI subroutine, included in the JETSET, was used to simulate Bose-Einstein correlations. Due to the low efficiency of neutral particles reconstruction in e^+e^- experiments, the study of the Bose-Einstein effect was performed only for charged particles. We studied the correlation function for all the charged particles, for all pions, for pions that were produced directly from the hadronization of quarks and gluons and separately for pions that appeared in decays of ρ mesons which were produced in the hadronization.

The correlation function was calculated as a ratio between the Q distribution obtained with Bose-Einstein effect included in the simulation program, and the reference sample, representing the same distribution with Bose-Einstein effect eliminated by the "mixing" procedure (the way in which the mixing is performed will be described in Section 4.2). Since the mixing procedure for the reference sample also destroys some other kinds of correlations, we normalise the correlation functions dividing them by similar distributions obtained by generating events without Bose-Einstein symmetrization:

$$C_2 = \frac{N_{BEon}/N_{BEon}^{MIX}}{N_{BEoff}/N_{BEoff}^{MIX}} , \tag{9}$$

where N is the number of pairs of like-charge particles, like-charge pions, like-charge direct pions or pions from resonances in a bin of Q, or Q_{long}, or $Q_{t,side}$, or $Q_{t,out}$ distributions. In this way we get $C_2(Q)$ almost constant for $Q > 1$ GeV, as we show below.

4.1 The reference system

We used in the analysis the so-called Longitudinal Centre-of-Mass System (LCMS) [8] (see Fig. 1). For each pair of particles, LCMS is defined as the system in which the sum of the two particle momenta is perpendicular to the reference axis given by the corresponding jet direction (as we said before, we analysed only events where Z^0 decays into $q\bar{q}$ pair only). The four-momentum difference of the pair of particles Q is resolved in Q_{long} parallel with the jet axis, $Q_{t,out}$ along to the sum of the particles momenta and $Q_{t,side}$ perpendicular to both directions mentioned before:

$$Q^2 = Q_{t,out}^2 + Q_{t,side}^2 + Q_{long}^2 - \Delta E^2 . \tag{10}$$

Some advantages offered by this reference system are that the projection of the total momentum of the pair to the longitudinal and side direction is zero and that only $Q_{t,out}$ is sensitive to the lifetime of the source.

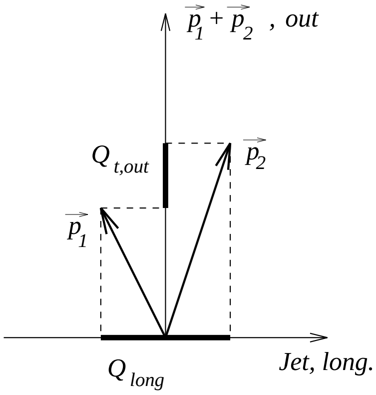

Fig. 1. The LCMS reference system

To transfer the particle momenta to the LCMS, they must first be rotated so that z (longitudinal) axis coincides with the jet axis and next boosted with the longitudinal velocity :

$$\beta_z = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2} , \tag{11}$$

where $p_{z,1}$, $p_{z,2}$ are z-axis projections of momenta and E_1, E_2 are energies of the two particles. The energy difference in the longitudinal center of mass system will be:

$$\Delta E = \beta_t Q_{t,out} , \tag{12}$$

where β_t is the transverse velocity of the pair :

$$\beta_t = \frac{p_{t,1} + p_{t,2}}{E_1 + E_2} . \tag{13}$$

Here $p_{t,1}$ and $p_{t,2}$ are transverse momenta of particles.

From Eq.(2), Eq.(10), Eq.(12) we obtain for the correlation function the following expression:

$$C_2(Q) = 1 + \lambda e^{-R^2 Q^2_{t,out}(1-\beta_t^2) - R^2 Q^2_{t,side} - R^2 Q^2_{long}} . \tag{14}$$

Using the three components of the radius related to this reference system, the correlation function can be parameterized as:

$$C_2 = 1 + \lambda e^{-R^2_{t,out} Q^2_{t,out} - R^2_{t,side} Q^2_{t,side} - R^2_{long} Q^2_{long}} , \tag{15}$$

where $R_{t,out}$, $R_{t,side}$, R_{long} are the corresponding components of the radii, with the following relations:

$$R_{t,side} \sim R_{long} \sim R_{t,out}\gamma \sim R , \tag{16}$$

where $\gamma = 1/\sqrt{1-\beta_t^2}$.

4.2 The reference sample

An extremely important task is to prepare the reference sample to obtain a proper correlation function. That means, we have to find a sample of particles which are not subject to Bose-Einstein correlation but do obey the same kinematics as a regular e^+e^- event.

There are several procedures to prepare the reference sample. For this study, as in [13, 14], we combined particles from different events, assuming that the selection criteria of the two-jet events provide us with a set of kinematically similar events. The mixing procedure can be described in following steps:

- After the thrust[1] axis calculation each event is rotated to a new coordinate system, which has the z axis along the thrust axis.
- Tracks from each rotated event are stored in a reference buffer. Events in the buffer are continuously updated to prevent any regularities in the particles spectra[2] .
- The reference sample is built using randomly picked tracks from the reference buffer. First, a random event of the stored ones is selected, then a track from this event is also randomly picked.

The mixing procedure does not conserve energy and momentum, and destroys not only the Bose-Einstein correlation but even some other kinds of correlations, making the normalisation difficult.

4.3 Normalisation procedure

To eliminate the side effects introduced by the mixing procedure, we accumulate four kinds of distributions for Q as well as for Q_{long}, $Q_{t,side}$, $Q_{t,out}$:

- Bose-Einstein correlation effect included, but the mixing procedure not performed: N_{BEon}, see Fig. 2a (closed circles).
- Bose-Einstein correlation effect included, and the mixing procedure performed: N^{MIX}_{BEon}, see Fig. 2a (open circles).
- Bose-Einstein effect not included, and the mixing procedure performed: N^{MIX}_{BEoff}, see Fig. 2b (open circles).

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[1]The quantity thrust T is defined by:

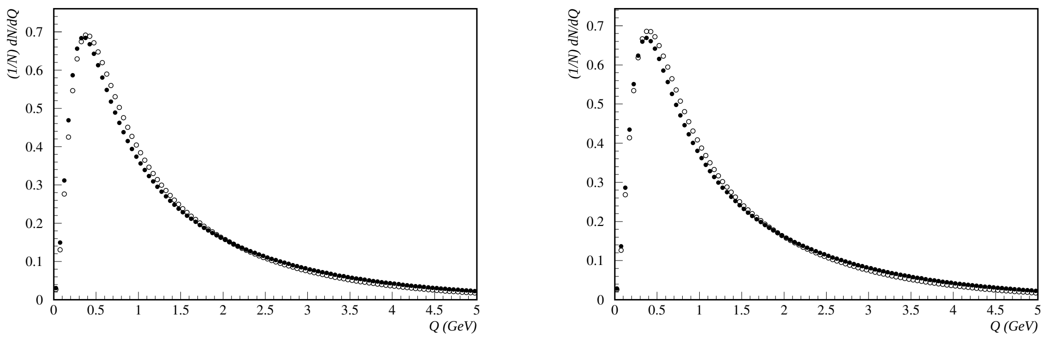
$$T = \max_{|\vec{n}|=1} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \ ,$$

and the thrust axis $\vec{v}_1$ is given by the $\vec{n}$ vector for which maximum is attained. The allowed range is $1/2 \leq T \leq 1$, with a 2-jet event corresponding to $T \approx 1$ and an isotropic event to $T \approx 1/2$.

[2]For direct pions, to save time and computing resources, only events that have at least two direct pions with the same charged were stored.

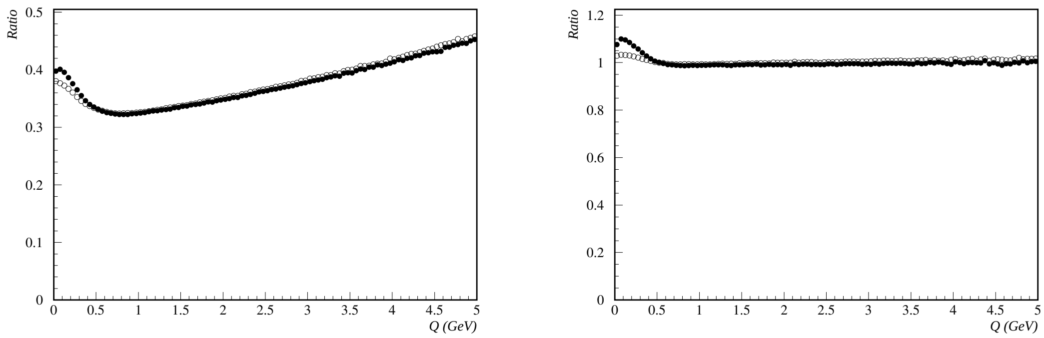
- Bose-Einstein effect not included, and the mixing procedure not performed: N_{BEoff}, see Fig. 2b (closed circles).

The corrected correlation function is given by Eq.(9).



a) b)

Fig. 2. The stored Q histograms (normalised to unit), for all charged particles, 5 millions JETSET events: a) Bose-Einstein correlation effect included, but the mixing procedure not performed: N_{BEon} (closed circles), Bose-Einstein correlation effect included, and the mixing procedure performed: N^{MIX}_{BEon} (open circles); b) Bose-Einstein effect not included, and the mixing procedure performed: N^{MIX}_{BEoff} (open circles), Bose-Einstein effect not included, and the mixing procedure not performed: N_{BEoff} (closed circles)



a) b)

Fig. 3. a) The ratios N_{BEon}/N^{MIX}_{BEon} (closed circles) and $N_{BEoff}/N^{MIX}_{BEoff}$ (open circles); b) The ratios N_{BEon}/N_{BEoff} (closed circles) and $N^{MIX}_{BEon}/N^{MIX}_{BEoff}$ (open circles).

Fig. 3a shows that the ratios N_{BEon}/N^{MIX}_{BEon} and $N_{BEoff}/N^{MIX}_{BEoff}$ are very similar in shape. Even in the $N_{BEoff}/N^{MIX}_{BEoff}$ case an enhancement appears due to the other kinds of correlation. In Fig. 3b, ratios N_{BEon}/N_{BEoff} and $N^{MIX}_{BEon}/N^{MIX}_{BEoff}$ show that even the mixing procedure does not eliminate correlations totally.

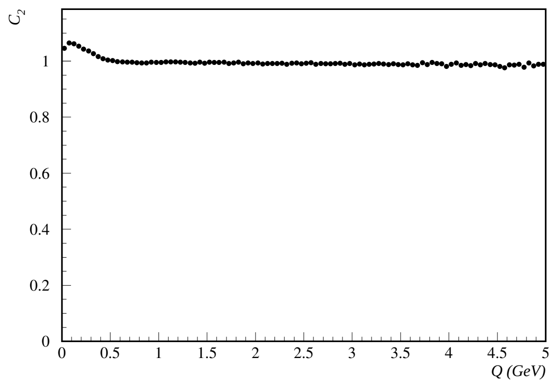


Fig. 4. The final normalised correlation function, as in Eq.(9).

In Fig. 4, the final normalised correlation function displays the pure Bose-Einstein effect in a clear fashion.

5 Three-dimensional correlation function

To be able to test the string model and to compare with the previous experimental data, we generate two-jet events only, requiring also the thrust value to be bigger than 0.95. In order to purify the data samples, to reject the background and to save computing resources the following cuts were introduced:

- The correlation function was constructed only for pairs of particles, each having momentum below 5 GeV/c to avoid the limits of phase space were dynamical correlations are strong.
- To reduce correlations due to the local transverse momentum compensation, pairs were rejected if their opening angle in transverse plane exceeded 120°. This cut was introduced for the compatibility with experimental data and slightly reduces background at high Q values.

The three-dimensional correlation function was obtained as a normalised triple differential cross section (using the same procedure as for the one-dimensional distributions) :

$$\frac{d^3\sigma}{dQ_{t,out}\,dQ_{t,side}\,dQ_{long}}\,, \tag{17}$$

that was measured in four m_t intervals: 0.1 GeV $\leq m_t <$ 0.25 GeV, 0.25 GeV $\leq m_t <$ 0.35 GeV, 0.35 GeV $\leq m_t <$ 0.45 GeV, 0.45 GeV $\leq m_t <$ 0.6 GeV. Knowing three projections of Q, it is possible to construct a three-dimensional correlation function $C_2 \equiv C_2(Q_{t,out}, Q_{t,side}, Q_{long})$ in each of m_t intervals. Fit of these distributions was used to show the dependence of the correlation function parameters on the m_t.

Using the common assumption about the Gaussian shape of the correlation function in all three dimensions, it is convenient to parameterize this three-dimensional function, following Eq.(14), as:

$$C_2 = N(1 + \lambda e^{-R^2_{t,out}Q^2_{t,out} - R^2_{t,side}Q^2_{t,side} - R^2_{long}Q^2_{long}}) \ . \qquad (18)$$

Comparing above equation to the one-dimensional parameterization (3), one can note omission of the linear parameter δ. By fitting the one-dimensional distributions $C_2(Q)$, $C_2(Q_{t,out})$, $C_2(Q_{t,side})$, $C_2(Q_{long})$ using Eq.(3), we found very low values for δ parameters, which can be seen also in Fig. 4 : no linear dependence of C_2 on Q is observed. Due to this fact we fitted the three-dimensional distributions using Eq.(18), without a linear factor. From this fit we obtained the correlation radii $R_{t,out}$, $R_{t,side}$, R_{long}, and λ.

Since the number of pairs of particles that accomplished all our cuts is small (especially in the case of mixed samples for pions from resonances and direct pions, where the multiplicity is low) it was necessary to analyse large samples (see Table 1).

	Number of events	Average multiplicity
All charged particles	5 000 000	14
All pions	10 000 000	12
Direct pions	30 000 000	2
Pions from all resonances	20 000 000	6
Pions from direct ρ	40 000 000	3

Table 1: *Number of events analysed. Samples of pions coming from direct ρ were run for five different values of input radii for the JETSET LUBOEI subroutine*

6 Results and interpretation

The string model provides an appropriate framework, in which the Bose-Einstein enhancement can be studied. The associated calculations are quite complex and time-consuming, especially when more than two particles are considered to be affected by Bose-Einstein symmetrization. For this reason, it is convenient to use the simple semiclassical model in event generators, as it is implemented within the JETSET generator. In the current standard algorithm, identical particles are pulled closer together in such a way as to enhance the two-particle correlation at small Q values. Several shapes of the enhancement can be used in the algorithm, in particular, for the analysis discussed here the, Gaussian shape of the correlation function was chosen with input values for parameters λ and R tuned to the DELPHI data: λ =1 and R^{-1} =0.394 GeV [19].

Correlation function Eq.(9), obtained with the help of JETSET, was fitted in the interval of 0.05 GeV $< Q <$ 1 GeV, using formula Eq.(3) in order to find the correlation parameters. The output fit parameters, λ and R, obtained through the fit of $C_2(Q)$ in cases of all the charged particles, all pions, direct pions and pions from resonances, are given in Table 2.

	λ	R (fm)
Input values	1	0.5
All charged particles	0.078±0.002	0.593±0.014
All charged pions	0.103±0.002	0.591±0.009
Direct pions	0.933±0.014	0.624±0.007
Pions from all resonances	0.548±0.004	0.686±0.004
Pions from direct ρ	0.862±0.011	0.651±0.006

Table 2: *Results of the one-dimensional fit of* $C_2(Q)$ *for* 0.05 *GeV* $< Q < 1$ *GeV*

Table 2 shows that the output values are quite different not only from the input values, but also from case to case. As we said before, the λ parameter represents a measure of the fraction of particles sensitive to the effect. In our case it is easy to understand that for all particles and for all pions, where the particles have different origins, λ has low values. The value of this parameter in case of pions coming from all resonances is also far away from the input value, even if it is considerably higher compared to the one for all charged particles and charged pions. This is due to the fact that there are some pions coming from long lived resonances and thus are not subject to Bose-Einstein symmetrization. In JETSET, the corresponding mechanism of not taking them into account is implemented in the LUBOEI subroutine, which simulates the Bose-Einstein correlations. In case of direct pions and pions coming from ρ decay, where the pions source has a high purity, the fitted values of λ are close to unit.

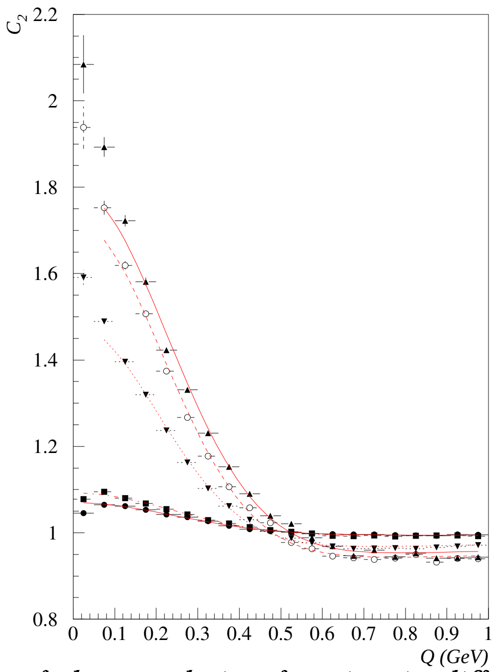


Fig. 5. The shape of the correlation function in different cases: all charged particles (closed circles), all pions (squares), direct pions (up triangles), pions coming from all the resonances decay (down triangles), pions from direct ρ *(open circles).*

For the radii, we obtained fitted values which are 20% to 40% higher than the input. The R value is highest in case of pions coming from all resonances. This can be explained as a consequence of the LUBOEI algorithm, which preserves multiplicity, hence it produces a correlation function $C_2(Q)$ which at high Q eventually becomes smaller than unit, contradicting our parameterizations (3) and (15). Thus, the shape of the resulting correlation function is not particularly close to a Gaussian for all the studied cases (see Fig. 5), even if the input correlation function was Gaussian, hence the values of the fitted parameters are not very well determined and should be discussed with great care.

In the case of the three-dimensional correlation function in LCMS, the Gaussian approximation (15) is very close to the observed correlation shape. This is why we should concentrate the discussion on the parameter values, extracted by using the three-dimensional fit.

6.1 The m_t dependence

The results of the fit of the three-dimensional correlation functions using Eq.(18) in different m_t intervals are represented in Fig. 6.

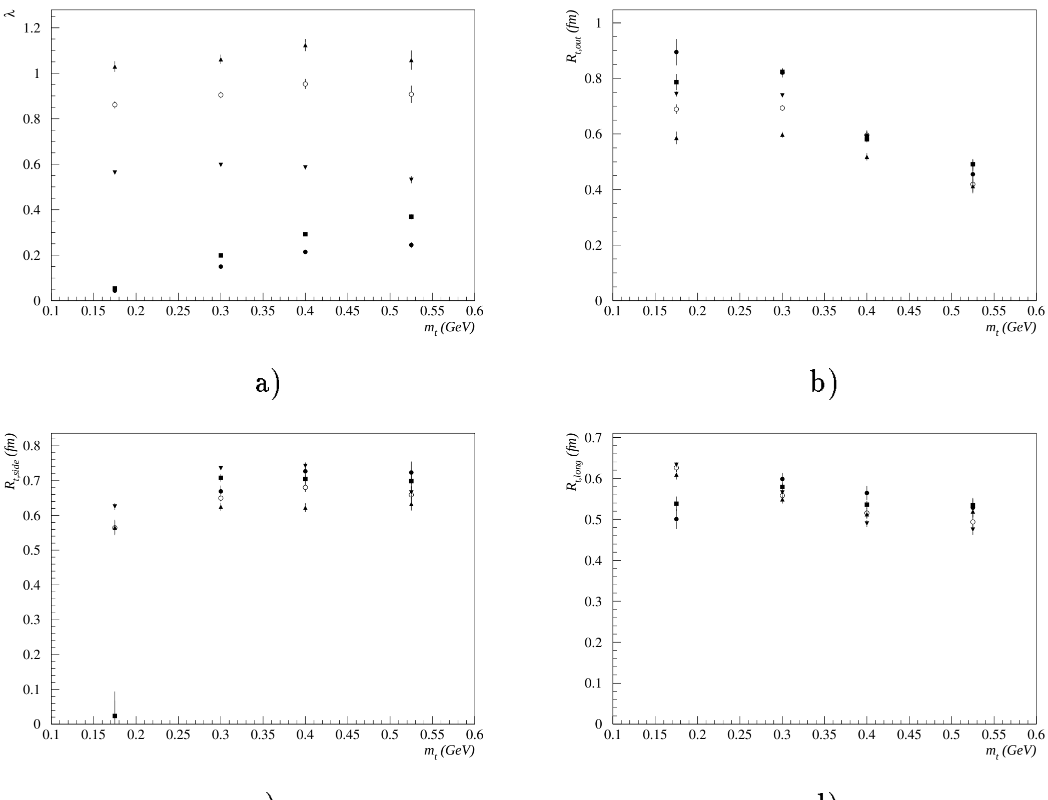


Fig. 6. m_t dependence of a) λ, b) $R_{t,out}$, c) $R_{t,side}$, d) R_{long}. We represent with closed circles the results for all charged particles, with squares the ones for all pions, for direct pions using up triangles, for pions coming from all the resonances decay with down triangles and for pions from direct ρ with open circles.

Fig. 6a shows that λ has a similar behaviour in case of all charged particles and all pions (the vast majority of charged particles in our case are pions). The values are lower in case of all particles than in case of all pions, because hadrons of different kind, that are not subject to Bose-Einstein symmetrization, can be paired.

In case of direct pions and pions only from resonances, λ as a function of m_t is almost constant, as one should expect. Studying the sample of pions coming from the decay of all the resonances (with a width bigger than 20 MeV), we observed that λ is $\sim$50 % smaller than in case of direct pions, although it manifests a similar behaviour. This could be explained by the fact that in the first case we include also pions coming from the decay of B- and D-mesons decay products, which cannot interfere with other pions.

Since the direct pions and the pions coming from direct ρ are all subject to Bose-Einstein correlation, λ values are close to the unit in these cases, being slightly higher for direct pions.

$R_{t,out}$ behaves similarly for all the samples, decreasing with the m_t increase, although the values are different from sample to sample. $R_{t,side}$ is almost constant, having very close values for all the samples (around 0.7 fm). The dependence of R_{long} on m_t is not so strong as the one for $R_{t,out}$, with the values being similar for all the samples of particles. We observe that qualitatively we got the dependence as in Eq.(16), having a rather steep slope for $R_{t,out}$ and an almost constant value for $R_{t,side}$ and R_{long}.

6.2 Comparison to data

Analysis of experimental data on the electron-positron annihilation at the center-of-mass energy 91.2 GeV using the DELPHI results [13] showed the presence of the transverse mass dependence for the Bose-Einstein correlation function (18) parameters. It also has been shown that the JETSET generated events with the DELPHI tuning and detector simulation applied, give a behaviour of the correlation function parameters very similar to the experimental data (see Fig. 7).

Results obtained in the present article by analysing two-jet JETSET generated events with default parameters (apart of the Bose-Einstein specific ones) and without applying a detector simulation, are compared in Fig. 8 to the mentioned above results of analysis of the DELPHI-tuned JETSET. In both cases, all charged particles were involved in the analysis. In spite of many essential differences between the two samples, like presence of three-jet events, different input JETSET parameters and detector-induced effects in the DELPHI-tuned sample, the qualitative behaviour of the Bose-Einstein correlation function parameters is very similar. The λ parameter grows with increasing transverse mass, while $R_{t,out}$ and R_{long} exhibit tendency to decrease and $R_{t,side}$ stays almost constant. The absolute values of the parameters are somewhat different, as should be expected from the basically different simulation conditions.

The similarity in the parameters behaviour for the experimental data and different Monte Carlo samples suggests that the observed phenomenon is well reproduced by JETSET generator.

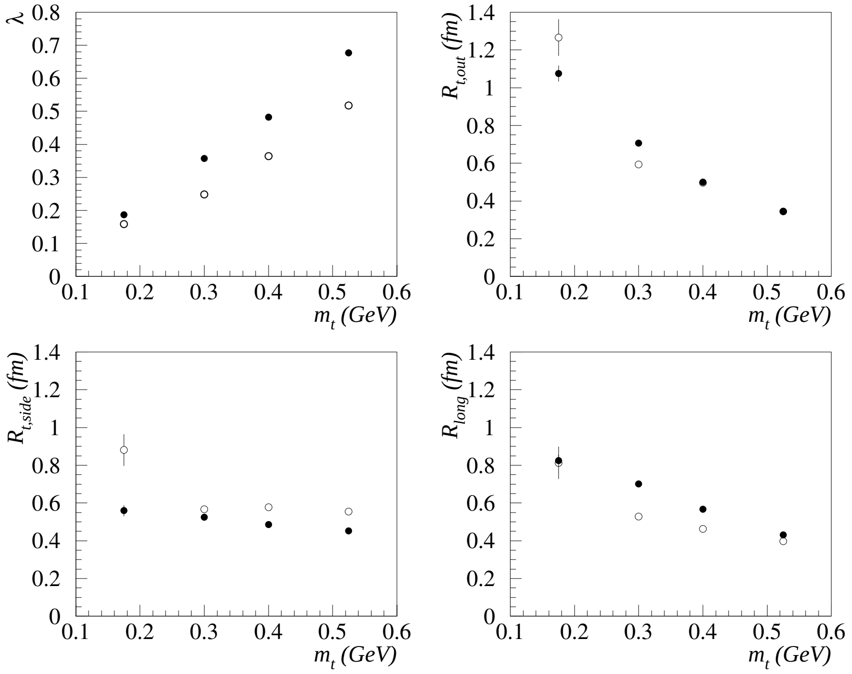


Fig. 7. Transverse mass dependence for the fit parameters of the correlation function $C_2(Q_{t,out}, Q_{t,side}, Q_{long})$ using Eq.(18). Closed circles represent the DELPHI data, while open circles – JETSET with DELPHI tuning and detector simulation applied.

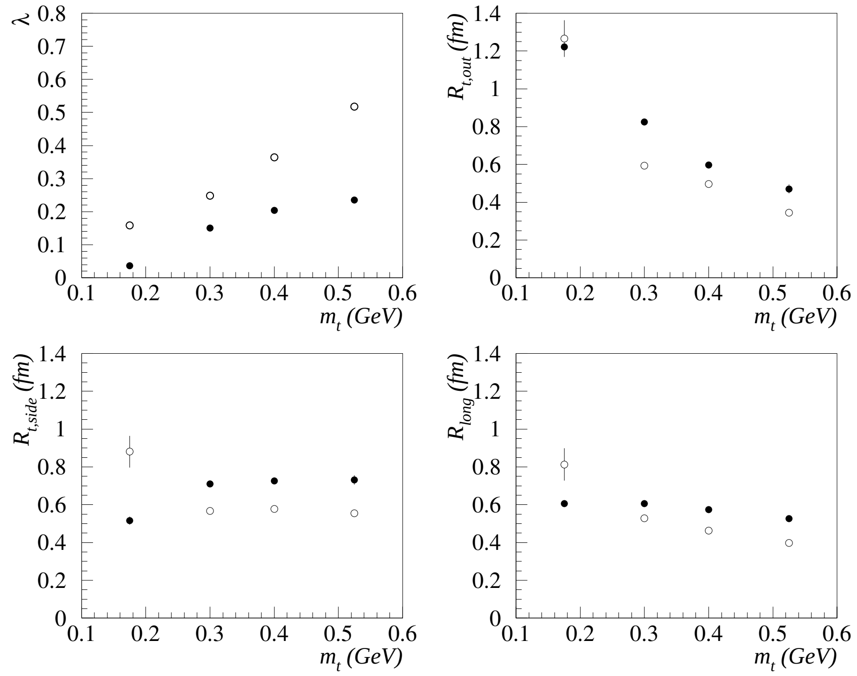


Fig. 8. Transverse mass dependence for the fit parameters of the correlation function $C_2(Q_{t,out}, Q_{t,side}, Q_{long})$ using Eq.(18). Closed circles represent results of the two-jet JETSET generated events, while open circles represent JETSET with DELPHI tuning and detector simulation applied (as in Fig. 7).

In heavy ion interactions at high energy, an m_t dependence of the radii has been found, but the behaviour is different to that found here [17, 18]. In these reactions all the radii show a similar dependence being proportional to $1/\sqrt{m_t}$, – the same approximate dependence that in case of e^+e^- annihilation can be seen for $R_{t,out}$, but cannot be seen for the other components of the radius. Even the λ dependence is quite different in heavy ion reactions (no dependence there). Taken together, the Bose-Einstein effects show quite different behaviour in e^+e^- annihilation and heavy ion reactions.

6.3 The dependence on input radius

As was mentioned before, for the DELPHI tuning of input parameters, i.e. λ =1 and R =0.5 fm [19], we obtained fitted values of the R parameter for all the samples approximately the same, but different from the input ones. For this reason, we studied the dependence of the shape of the output correlation function on the input radii. We used samples of pions coming from the direct ρ decays, since this sample is almost pure and the multiplicity is bigger than for direct pions. It is easy to observe (Fig. 9) that when the input radius decreases, the shape of the output function (full curve) is more and more different from the shape of the input function (dashed curve).

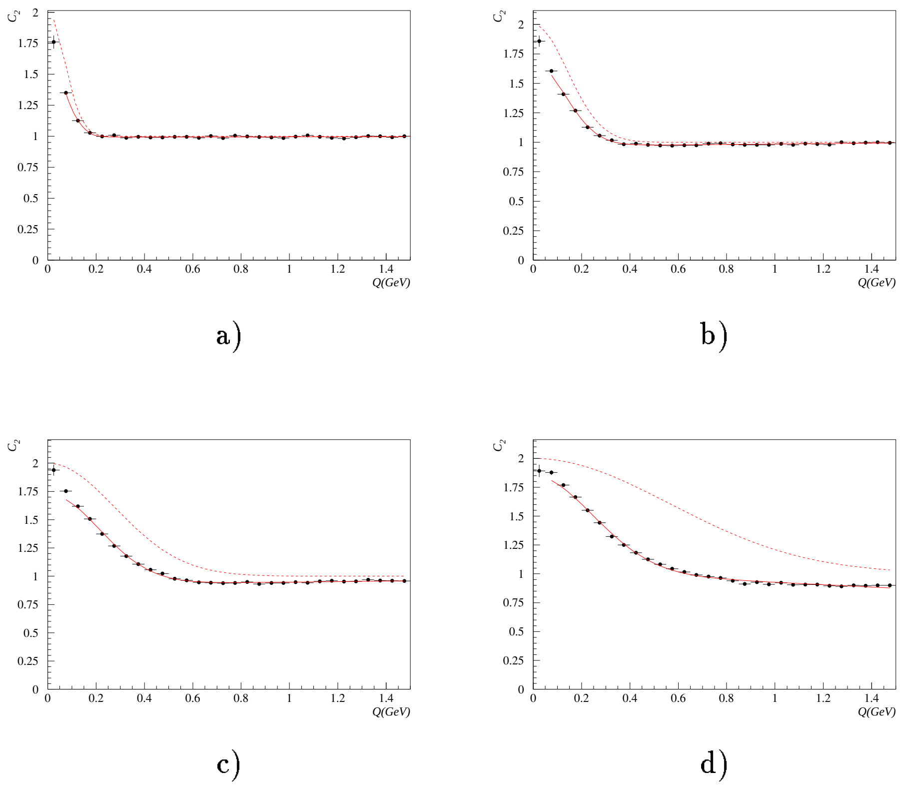


Fig. 9. The shapes of the output correlation function (full curve) and the input correlation function (dashed curve) for different input radii: a) R =2 fm, b) R =1 fm, c) R =0.5 fm, d) R =0.25 fm.

Fig. 10 represents the radii R_{out} obtained through the fit as a function of the input radii R_{in} for LUBOEI. One can observe that for the values of input radii less than 0.5 fm, almost constant values of the output fitted radii (around 0.6 fm) were found. For input R values bigger than 1 fm, the points are lying very close to the diagonal, showing a good agreement between input and output radii.

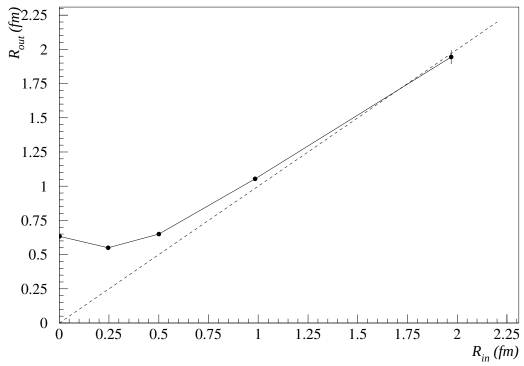
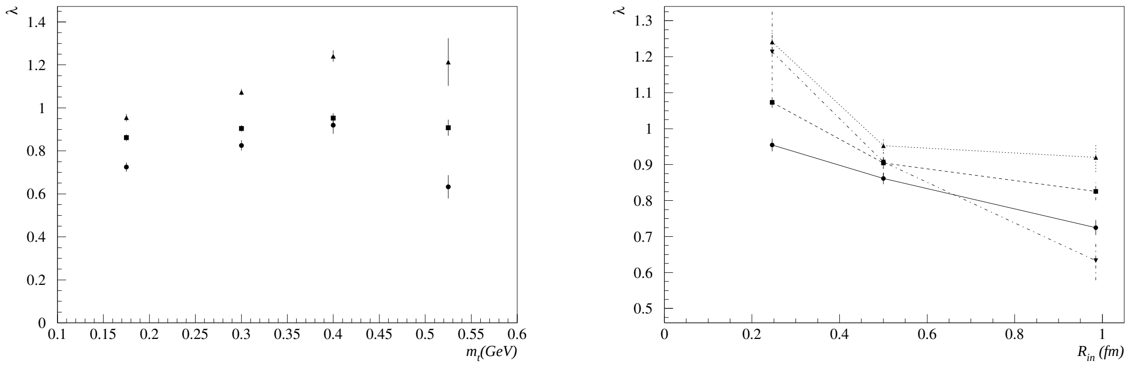


Fig. 10. Fitted values of output radii R_{out} for different input radii R_{in}.

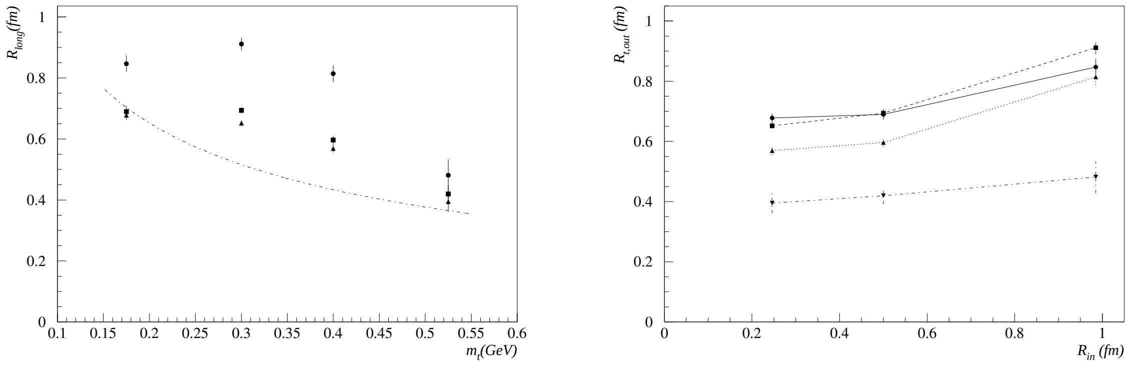
Studying the λ dependence on m_t for different input radii (Fig. 11a), we observed that this parameter is almost constant for all the radii used as input. Same points, but plotted as λ dependence on R_{in} in different m_t inrevals (Fig. 11b) show that λ is slowly decreasing with the input radius increasing.



a) b)

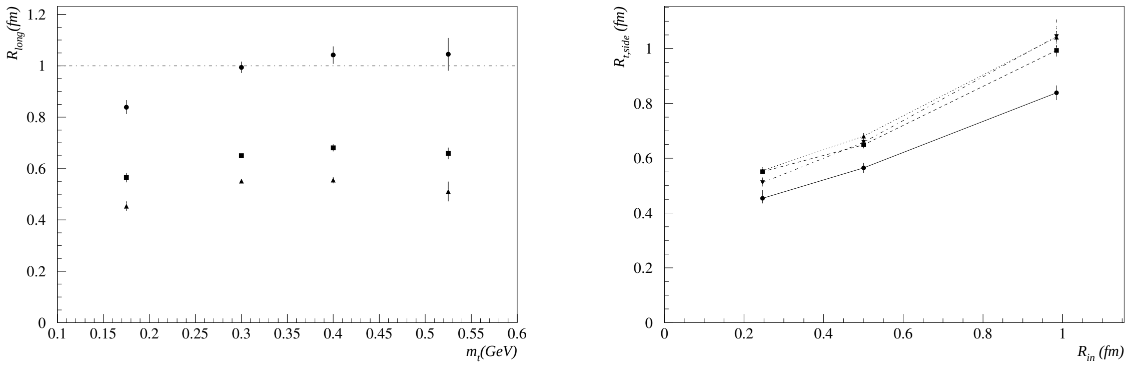
Fig. 11. λ dependence : a) on m_t for different input radii: R =1 fm (closed circles), R =0.5 fm (squares), R =0.25 fm (up triangles); b) on the input radius R_in for different ranges of m_t: 0.1 GeV $< m_t <$ 0.25 GeV (closed circles), 0.25 GeV $< m_t <$ 0.35 GeV (squares), 0.35 GeV $< m_t <$ 0.45 GeV (up triangles), 0.45 GeV $< m_t <$ 0.6 GeV (down triangles).

$R_{t,out}$ has a similar dependence on m_t (Fig. 12a) for all the input radii, with the dependence becoming weaker at small input values for radius. We should note that following Eq.(16), $R_{t,out}$ must depend on m_t in LCMS, due to the longitudinal boost. On the Fig. 12a we plot the R_{in}/γ_t dependence on m_t for $R_{in} = 1\,fm$. JETSET gives $1/\gamma_t$ dependence in LCMS being approximatelly proportional to $1/\sqrt{m_t}$. One can see that $R_{t,out}$ follows similar dependence for all the input radii. At the same time, we cannot see almost any dependence of this parameter on the input radii for all the m_t ranges (Fig. 12b).



a) b)

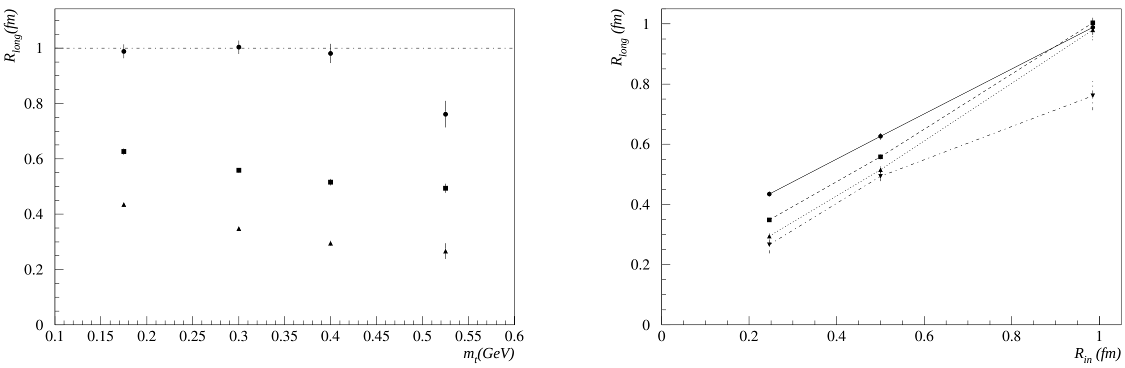
Fig. 12. $R_{t,out}$ dependence a) on m_t for different input radii: $R = 1fm$ (closed circles), $R = 0.5fm$ (squares), $R = 0.25fm$ (up triangles), with dot-dashed line we draw the function $R/\gamma_t(m_t)$ with $R = 1fm$; b) on the input radius and different ranges for m_t: $0.1\ GeV < m_t < 0.25\ GeV$ (closed circles), $0.25\ GeV < m_t < 0.35\ GeV$ (squares), $0.35\ GeV < m_t < 0.45\ GeV$ (up triangles), $0.45\ GeV < m_t < 0.6\ GeV$ (down triangles).



a) b)

Fig. 13. $R_{t,side}$ dependence a) on m_t for different input radii: $R = 1fm$ (closed circles), $R = 0.5fm$ (squares), $R = 0.25fm$ (up triangles); b) on the input radius and different ranges for m_t: $0.1\ GeV < m_t < 0.25\ GeV$ (closed circles), $0.25\ GeV < m_t < 0.35\ GeV$ (squares), $0.35\ GeV < m_t < 0.45\ GeV$ (up triangles), $0.45\ GeV < m_t < 0.6\ GeV$ (down triangles).

In case of the other components of the radius, $R_{t,side}$ and R_{long}, we have a rather weak dependence on m_t for all the input radii (Fig. 13a, Fig. 14a), but a strong dependence, almost direct proportionality, on the input values of radius (Fig. 13b, Fig. 14b).



a) b)

Fig. 14. R_{long} dependence a) on m_t for different input radii: $R = 1fm$ (closed circles), $R = 0.5fm$ (squares), $R = 0.25fm$ (up triangles); b) on the input radius and different ranges for m_t: 0.1 $GeV < m_t < 0.25$ GeV (closed circles), 0.25 $GeV < m_t < 0.35$ GeV (squares), 0.35 $GeV < m_t < 0.45$ GeV (up triangles), 0.45 $GeV < m_t < 0.6$ GeV (down triangles).

To summarise, for the input radius values higher than 1 fm, the input correlation function seems to dominate the constructed correlations. For lower input radii, secondary effects due to the way in which the Bose-Einstein correlation is introduced are important. This is consistent with earlier results from other studies [11, 10].

7 Conclusions

An analysis of the Bose-Einstein correlation function, as it is implemented in the JETSET particle generator, has been performed. Using the specific correlation function parameterization in the Longitudinal Center-of-Mass System it was shown that the plain two-jet JETSET generated events exhibit a similar behaviour to that found previously in the experimental DELPHI data.

In this paper we have shown that secondary effects in the simulation are very important for a radii parameter less than 1 fm and that these secondary effects can give rise to a m_t dependence, which does not stem from the input correlation function. For input radii of 1 fm and higher, the different dependences studied follow the input correlation function behaviour. It is an astonishing fact that in the case of e^+e^- annihilation input parameters of around $\lambda = 1$and $R = 0.5$ fm can reproduce the behaviour of the experimental data and their m_t dependence.

This fact is likely due to the secondary effects of the algorithm, like the way in which the momenta are shifted, and also due to the assumed spherical approximation of the available phase space. It should be remembered that the Bose-Einstein parameters in JETSET for radius less than 1 fm should not be taken as physically meaningful parameters.

8 Acknowledgements

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References

[1] L. Lönnblad, T. Sjöstrand: Phys. Lett. B 351 (1995) 293

[2] P. Abreu et al., DELPHI Coll.: Phys. Lett. B 401 (1997) 181

[3] V. Kartvelishvili, R. Kvatadze, R. Møller: "Estimating the effects of Bose-Einstein correlations on the W mass measurement at LEP2": University of Manchester preprint MC-TH-97/04, MAN/HEP/97/1

[4] D. Decamp et al., ALEPH Coll.: Z. Phys. C 54 (1992) 75

[5] A. Engler, H. R. Hoorani: "Measurement of Bose-Einstein correlations between charged pions using the LEP1 data": L3 Note 2139, submitted to Int. Europhysics Conference on High Energy Physics, Jerusalem 97

[6] P. Abreu et al., DELPHI Coll.: Z. Phys. C 63 (1994) 17

[7] P. Abreu et al., DELPHI Coll.: Phys. Lett. B 286 (1992) 201

[8] T. Csörgő, B. Lörstad: Phys. Rev. C 54 (1996) 1390

[9] A. N. Mahlin and Yu. M. Sinyukov: Z. Phys. C 39 (1988) 39

[10] K. Fiałkowski, R. Wit: Z. Phys. C 74 (1997) 145

[11] S. Haywood: "Where are we going with Bose-Einstein – a mini review": RAL Report RAL-94-074

[12] T. Sjöstrand: Comp. Phys. Comm. 82 (1994) 74;
T. Sjöstrand, "PYTHIA 5.6 and JETSET 7.3": CERN-TH.6488/92 (1992)

[13] B. Lörstad, O. Smirnova: "Transverse Mass Dependence of Bose-Einstein Correlation Radii in e^+e^- Annihilation at LEP Energies": Proceedings of the 7th International Workshop on Multiparticle Production 'Correlations and Fluctuations', June 30 to July 6, 1996, Nijmegen, The Netherlands.

[14] O. Smirnova: "Studies of the Fragmentation Process in Hadronic Decays of Z^0 Boson", Thesis for the degree of Doctor of Philosophy in Physics: Lund University, October 1996, LUNFD6/(NFFL-7127)1996

[15] G. Goldhaber et al.: Phys. Rev. Lett C 3 (1959) 181

[16] B. Lörstad: Int. J. Mod. Phys. A4 No.12 (1989) 2861

[17] H. Beker et al., NA44 Coll.: Phys. Rev. Lett. 74 (1995) 3340

[18] T. Alber et al., NA35 Coll.: Z. Phys. C 66 (1995) 77

[19] P. Abreu et al., DELPHI Coll.: Z. Phys. C 73 (1996) 11