S11

2nd ECFA/DESY Study on Physics and Detectors for a Linear Electron-Positron Collider

LAL - Orsay - France, April 5-7, 1998

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Parallel groups to plan tasks : Top

Aurelio Juste

TOP PHYSICS @ NEXT LINEAR COLLIDER
(REVIEW OF EXPERIMENTAL STUDIES)

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OUTLINE

1. THRESHOLD SCAN MEASUREMENTS
2. MEASUREMENT OF TOP FORM FACTORS
3. RARE TOP DECAYS
4. DIRECT OBSERVABILITY OF YUKAWA COUPLING

⇒ TOP IN e^+e^-

- **PRODUCTION:**

- MAIN CHANNEL IS $e^+e^- \to \begin{matrix}\gamma^* \\ Z^*\end{matrix} \to t\bar{t}$

$\sqrt{s}$	THRESHOLD	500 GeV	2 TeV
σ (pb)	~0.5	0.65	0.04

↳ ~30 K $t\bar{t}$/50 fb^{-1}

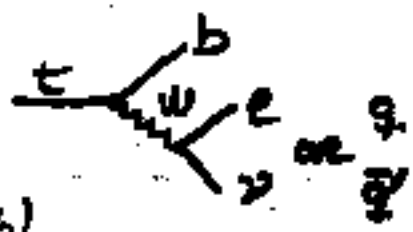
- OTHER CHANNELS ARE NEGLIGIBLE

$e^+e^- \to \nu_e\bar{\nu}_e t\bar{t}$ ($Z\gamma$), $e^+e^- \to \bar{\nu}_e e t \bar{b}$ (WW), $e^+e^- \to e^+e^- t\bar{t}$ (ZZ)

EXCEPT FOR $e^+e^- \to e^+e^-\gamma\gamma$ AT $\sqrt{s} \sim 2$ TeV
↳ $t\bar{t}$

- **DECAY:**

DOMINANTLY THROUGH



$t\bar{t} \to$ 6 JETS	46%	(60%)
4 JETS + $\ell + \nu$	44%	(35%)
2 JETS + $\ell\ell' + \nu\nu'$	10%	(5%)

(IF $\ell = e, \mu$ ONLY)

- **BACKGROUNDS**

	THRESHOLD	500 GeV
$e^+e^- \to f\bar{f}$ (+g+...+γ)	25 pb	15 pb
$\to W^+W^-$	10 pb	7 pb
$\to Z^0Z^0$	0.8 pb	0.5 pb

- EVENTS HAVE
  - HIGHER THRUST
  - LESS JETS
  - ...

} EASY TO DISENTANGLE

A) THRESHOLD SCAN MEASUREMENTS

☹ LARGE t WIDTH ⇒ DECAY TIME OF STATES SHORTER THAN REVOLUTION TIME OF CONSTITU
⇒ NO BOUND $t\bar{t}$ RESONANCE

☺ RAPID t DECAY ⇒ CUTOFF AGAINST LONG-RANGE NON-PERTURBATIVE EFFECTS → RELIABLE THEOR. PREDICTIONS IN PERTURBAT. QCD

- INTERQUARK POTENTIAL ≈ COULOMBIC

$$V(R) \simeq -\frac{4}{3}\frac{\alpha_s(R)}{R}$$

MODIFIED BY CONFINEMENT POTENTIAL $\sim \kappa R$ AT INTERMEDIATE DISTANCES R IN THE TAIL OF TOPONIUM RESONANCE + $t\bar{t}$ YUKAWA POTENTIAL

- BUILT UP PRIMARILY BY SUPERPOSITION OF 1S STATES WHICH CAN CONVINIENTLY BE PERFORMED BY USING NON-RELATIVISTIC GREEN'S FUNCTION

$$G(E; m_t, \Gamma_t, \alpha_s, \lambda_t, M_H)$$

WHICH SATISFIES SCHRÖDINGER'S EQ:

$$\left(H_{LO} - E - \frac{i}{2}\Gamma_t\right) G = 1$$

S-WAVE GREEN'S FUNCT. STRASSLER, PESKIN

SO THAT ONE CAN PREDICT

- TOTAL X-SECTION

$$\sigma \propto \mathrm{Im}\,\langle r=0|G|r=0\rangle$$

MAINLY SENSITIVE TO $\underbrace{m_t, \alpha_s, \lambda_t}_{\text{CORRELATED}}$

- TOP MOMENTUM DISTRIBUTION (FERMI MOTION)

$$\frac{d\sigma}{d|p|} \propto |p|^2 \left|\langle p|G|r=0\rangle\right|^2 = |p|^2 \cdot |\tilde{G}(p)|^2$$

MAINLY SENSITIVE TO m_t, α_s, Γ_t

- TOP ANGULAR DISTRIBUTION (S-P WAVE INTERF.)

$$\frac{d\sigma}{d|p|\,d\cos\theta} \propto |p|^2 \left\{ |\tilde{G}(p)|^2 + C\cdot\cos\theta\,|p|\,\mathrm{Re}\{\tilde{G}(p)\,\tilde{F}^*(p)\} \right\}$$

MAINLY SENSITIVE TO Γ_t, m_t, α_s

LARGE WIDTH RESPONSIBLE OF THE OVERLAP BETWEEN S AND P WAVE STATES

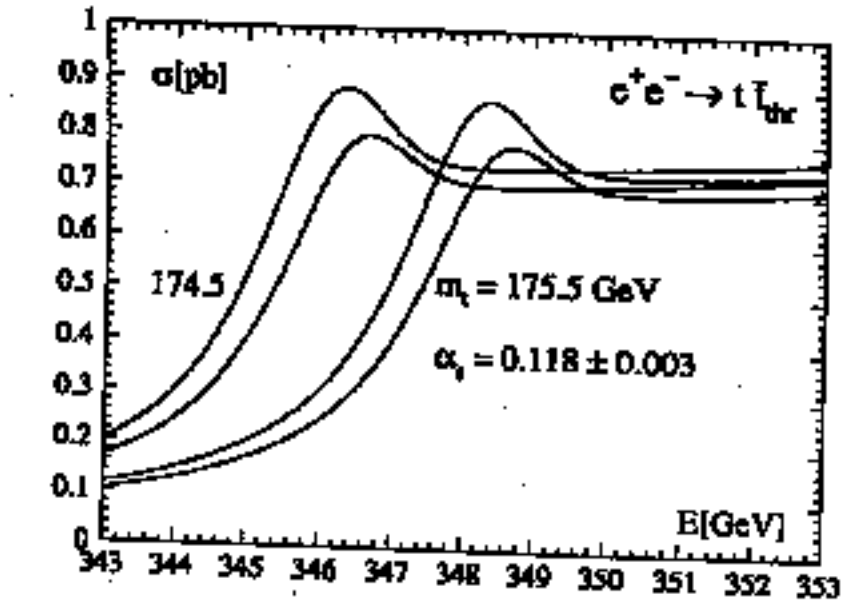
$$\left(H_{0,1} - E - \frac{1}{2}\Gamma_t\right)\cdot F = 1$$

P-WAVE GREEN'S FUNCTION

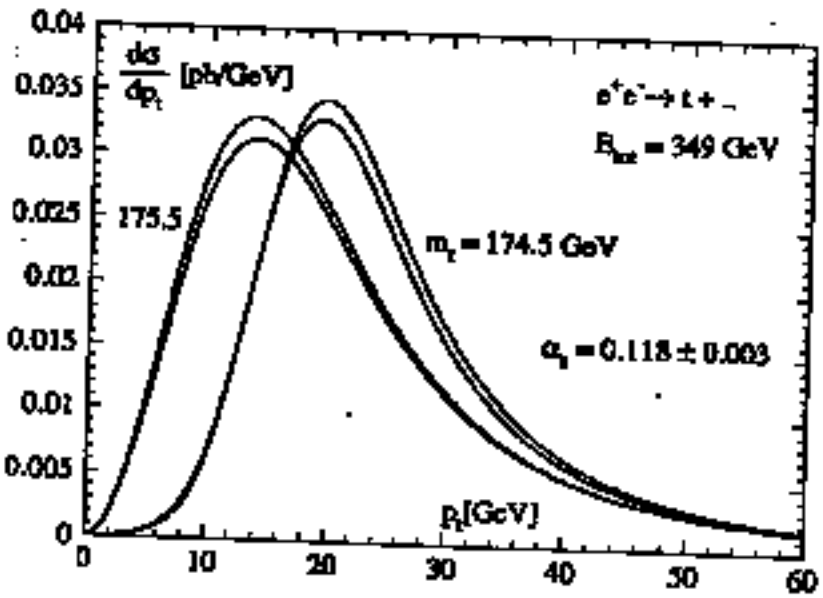
FUJII, SUMINO

ZOLABEK, KÜHN

HAMILTONIAN ET AL.



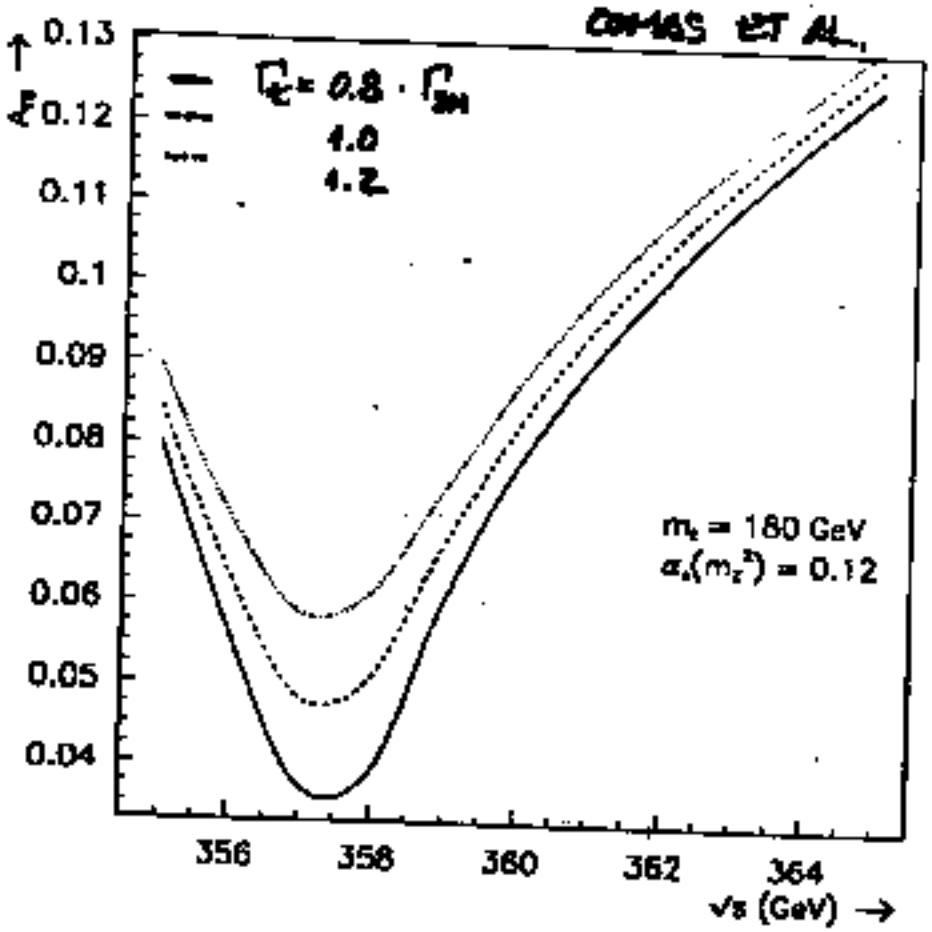
CROSS SECTION



TOP MOMENTUM DISTRIBUTION

Figure 0.6: *Upper part: The cross section for the production of top quarks near the threshold. Demonstrated is the sensitivity of the cross section to the value of the top mass and the QCD coupling (normalized at the Z mass). Lower part: The momentum spectrum of the top quarks near the threshold for a fixed total c.m. energy. The momentum depends strongly on the top mass, yet less on the QCD coupling. Ref.[99].*

FORWARD-BACKWARD CHARGE ASYMMETRY



⇒ ACCELERATOR EFFECTS:

- BEAMSTRAHLUNG AND LINAC ENERGY SPREAD CAUSE A BIG SPREAD IN $\sqrt{s_{eff}}$ ⇒
  - DISTORTIONS IN THRESHOLD
  - STRUCTURE IS WASHED OUT
  - LOSS OF USEFUL L

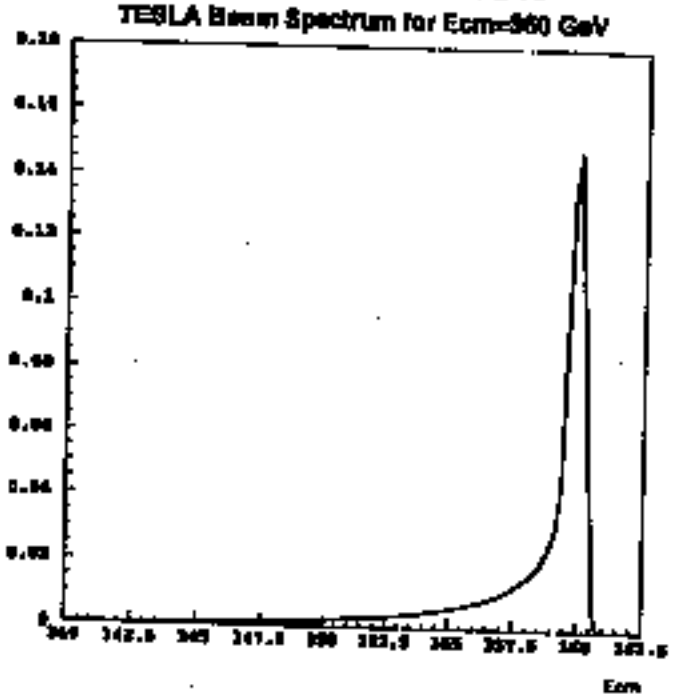
  } DECREASED SENSITIVITY

⇒ MACHINE DESIGN: TRADEOFF BETWEEN

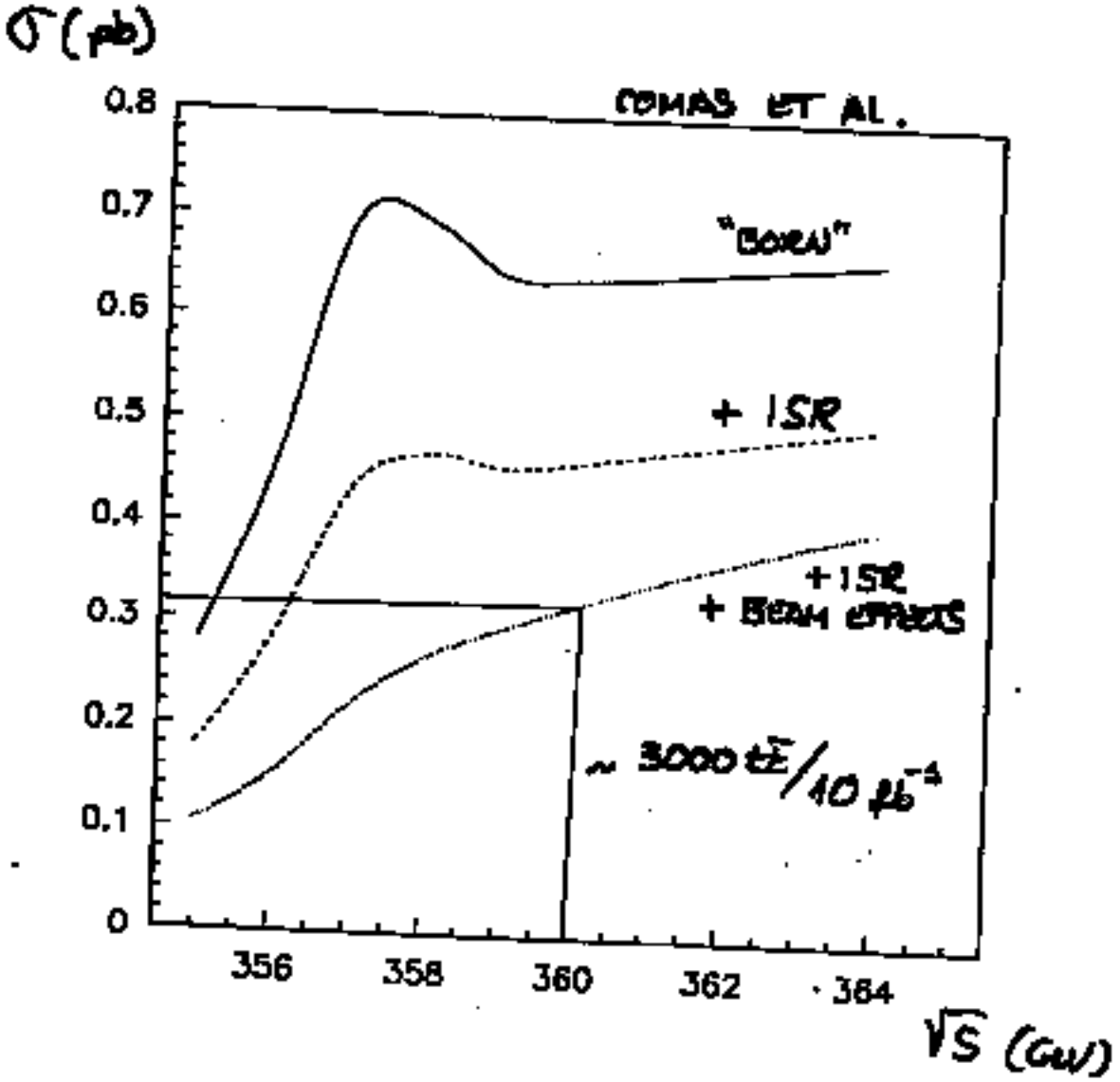
LUMINOSITY ↔ BEAMSTRAHLUNG BEAM SPREAD

- NEED EXCELLENT CONTROL ($\lesssim 0.1\,\%$) OF $dL/d\sqrt{s}$ FROM DATA TO MATCH STATISTICAL UNCERTAINTY OF X-SECT. MEASUREMENT.

SCHULTE



THRESHOLD X-SECTION



$m_t = 180$ GeV

$\alpha_S(M_Z^2) = 0.12$

LATEST TESLA DESIGN

$M_t = 175$ GeV

$\Rightarrow \sim 21000\ t\bar{t}/50\ fb^{-1}$

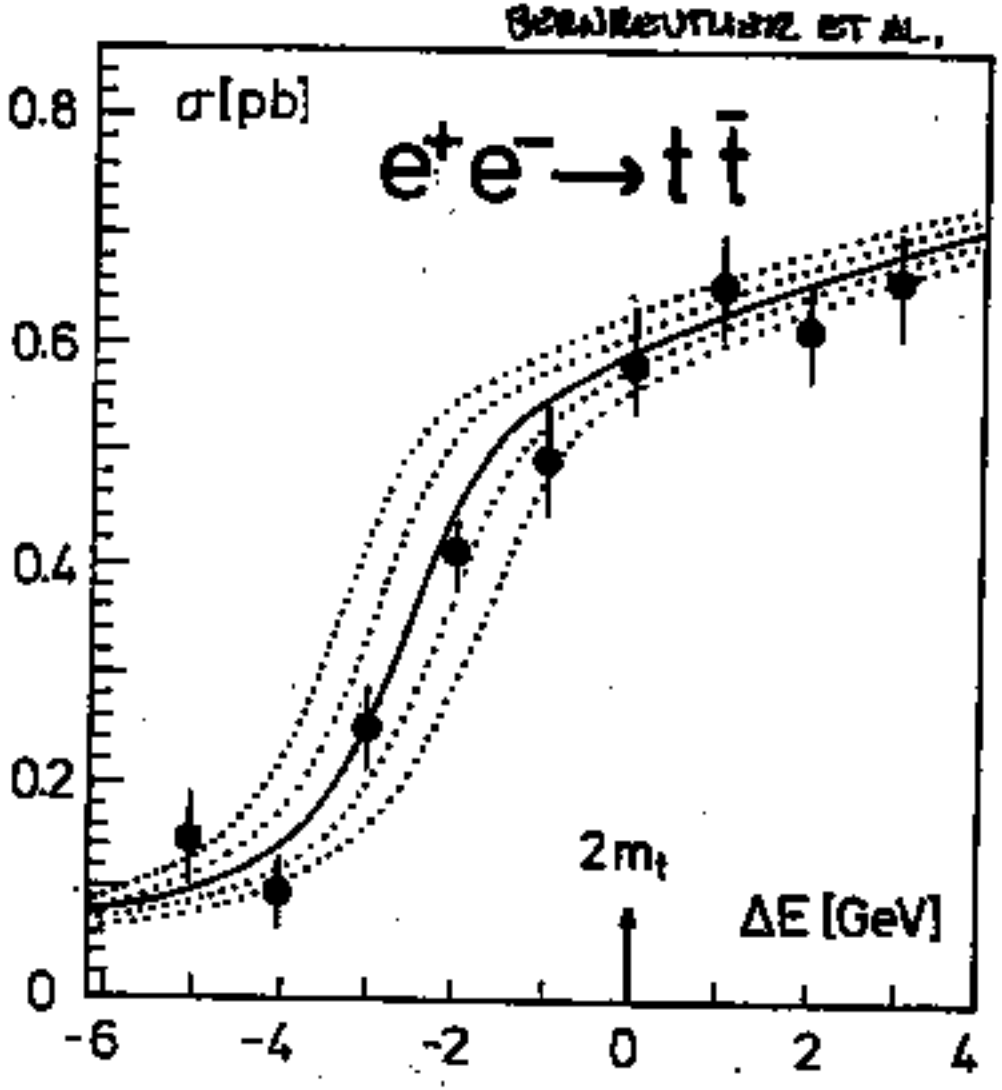
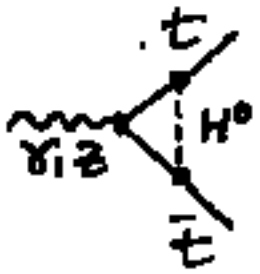


Figure 0.7: *Excitation curve of the top quarks including initial-state radiation and beamstrahlung. The errors of the data points correspond to an integrated luminosity of* $\int \mathcal{L} = 50\ fb^{-1}$ *in toto. The dotted curves indicate shifts of the top mass by 200 and 400 MeV. Ref.[35].*

TOP YUKAWA COUPLING AT THRESHOLD

- THRESHOLD PHYSICS DEPENDS ON λ_t

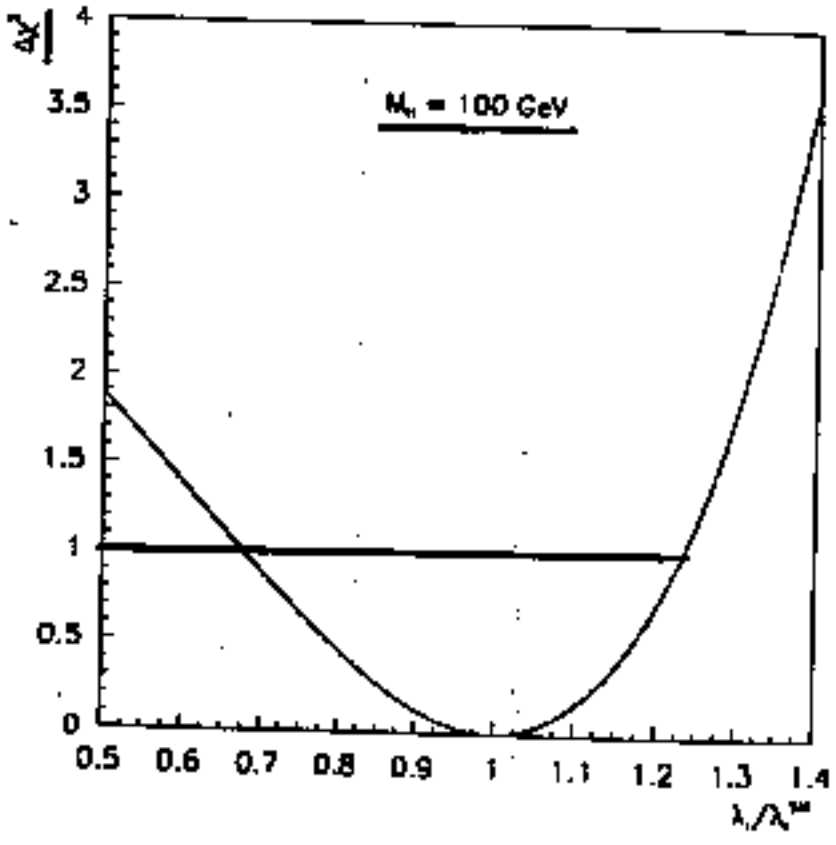


$$V \rightarrow V + V_{Yuk.}(r)$$

$$= -\frac{\lambda_t^2}{4\pi} \cdot \frac{e^{-M_H \cdot r}}{r}$$

+ REMAINING CONSTANT VERTEX CORRS.

- DONE 1-P FIT WITH $9 \times 5\ fb^{-1}$
  FIXING m_t, α_S, $M_H = 100$ GeV



$$\frac{\Delta\lambda_t}{\lambda_t} \lesssim 0.3$$

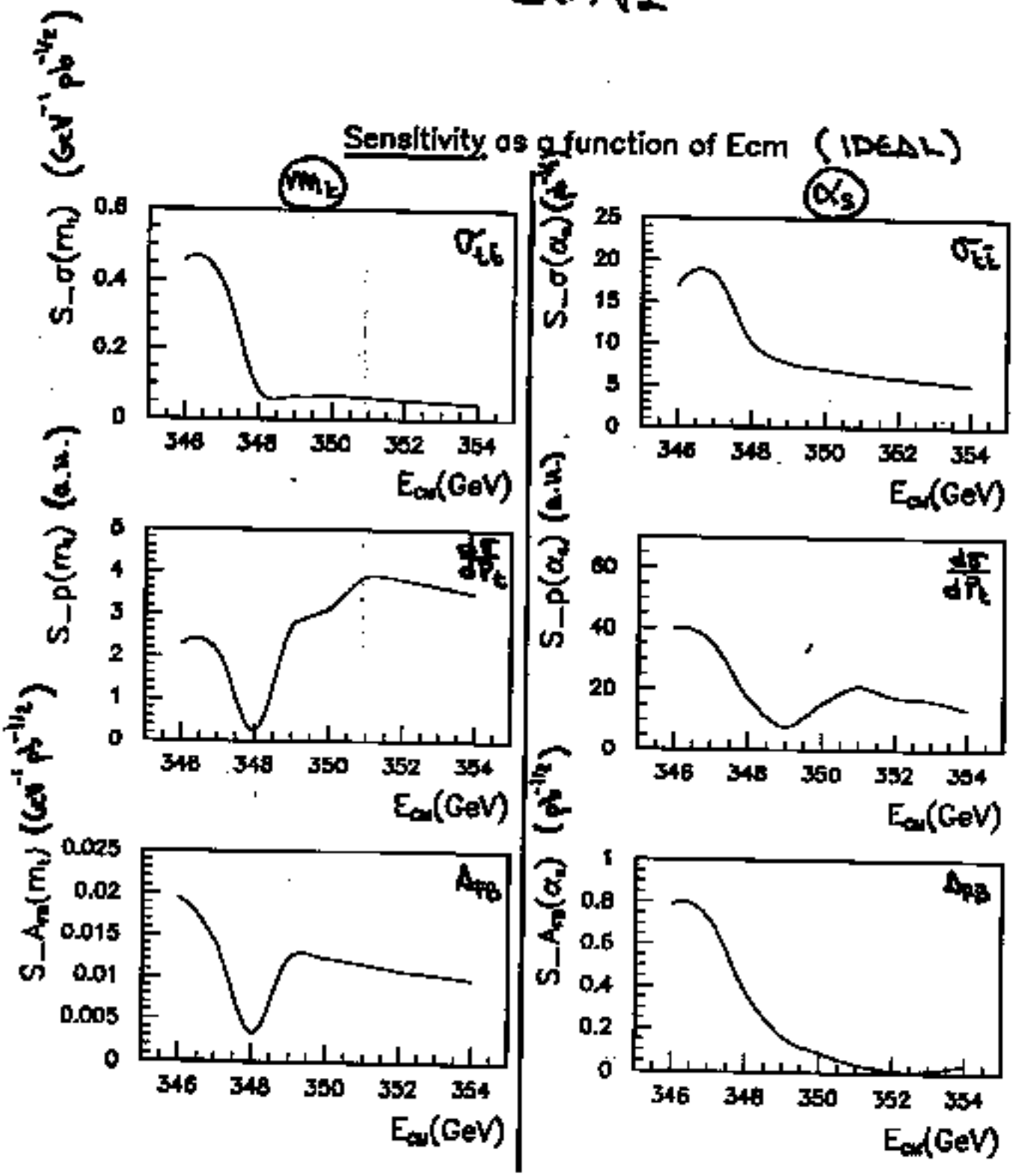
(SENSITIVITY ↓↓ WHEN M_H ↑)

⇨ m_t EXPECTED TO BE KNOWN WITH $\Delta m_t \leq 2$ GeV/c^2 BY THE TIME THE LC WILL START OPERATING.

⇒ CAN WE SPEND THE LUMINOSITY IN A MORE OPTIMAL WAY ?

$$\Delta x \propto \frac{1}{S(x)\sqrt{\mathcal{L}}} \quad ; \quad x : m_t, \alpha_s$$

Sensitivity as a function of Ecm (IDEAL)



RESULTS $\int L dt = 50\ fb^{-1}$

10% to measure being below threshold

SCAN POINTS ($E_{CM} - 2m_t$)	Δm_t (GeV/c²)	$\Delta\alpha_s$	S
9-POINT (-4 GeV to +4 GeV)	0'11	0'0030	+97.5%
5-POINT (-4 GeV to 0 GeV)	0'08	0'0019	+97.5%
* 3-POINT (-4 GeV to -2 GeV)	0'07	0'0016	+99.7%

* WITH EXTERNAL CONSTRAINT: $\Delta\alpha_s = 0'003$!!

⇒ 5-POINT SCAN IMPROVES THE RESULT WITHOUT INCREASING THE CORRELATION BETWEEN m_t AND α_s.

⇒ EXTERNAL INFORMATION (e.g. α_s) VERY VALUABLE !!

B) MEASUREMENTS IN THE CONTINUUM

- t PRODUCED WITH HIGH LONGITUDINAL POLARIZATION

$\oplus$

- RAPID DECAY INTERRUPTS HADRONIZATION IN EARLY STAGE

$\Downarrow$

INITIAL HELICITY CONSERVED AND TRASMITED TO FINAL STATE WITHOUT DEPOLARIZATION

$\Rightarrow$ APPLY LEP-SLC Z POLARIZATION ANALYSIS MACHINERY TO t:

- DETERMINE t HELICITIES FROM DISTRIBUTIONS OF JETS & LEPTONS IN $t \to b\,W^+ \to b\,\ell\bar{\ell}'$

$\Downarrow\Downarrow$

PRECISE DETERMINATION OF t FORMFACTORS

(ONLY QUARK IN WHICH THIS CAN BE [illegible])

$$J^a_\mu \propto \gamma_\mu \left(F^a_{1L} P_L + F^a_{1R} P_R\right) + a=\gamma,Z,W$$

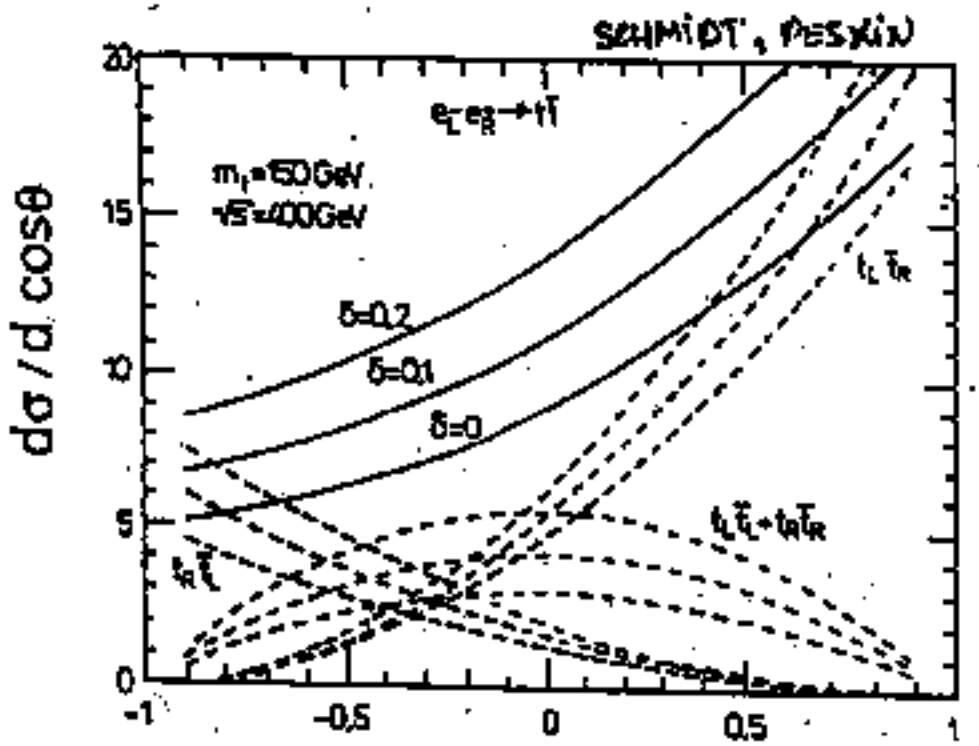
$$+ \frac{i\sigma_{\mu\nu} q^\nu}{2m_t}\left(F^a_{2L} P_L + F^a_{2R} P_R\right)$$

- IN MSM $F^a_{2L} = F^a_{2R} = F^W_{1R} = 0$

PROD. { A) $F^{\gamma,Z}_{2L} + F^{\gamma,Z}_{2R} \neq 0 \Rightarrow$ MAGNETIC QUAD. MOMENT

B) $F^{\gamma(Z)}_{2L} - F^{\gamma(Z)}_{2R} \neq 0 \Rightarrow$ ELECTRIC DIPOL. MOMENT (WEAK)

DECAY { C) $F^W_{1R} \neq 0 \Rightarrow V+A$, W_R



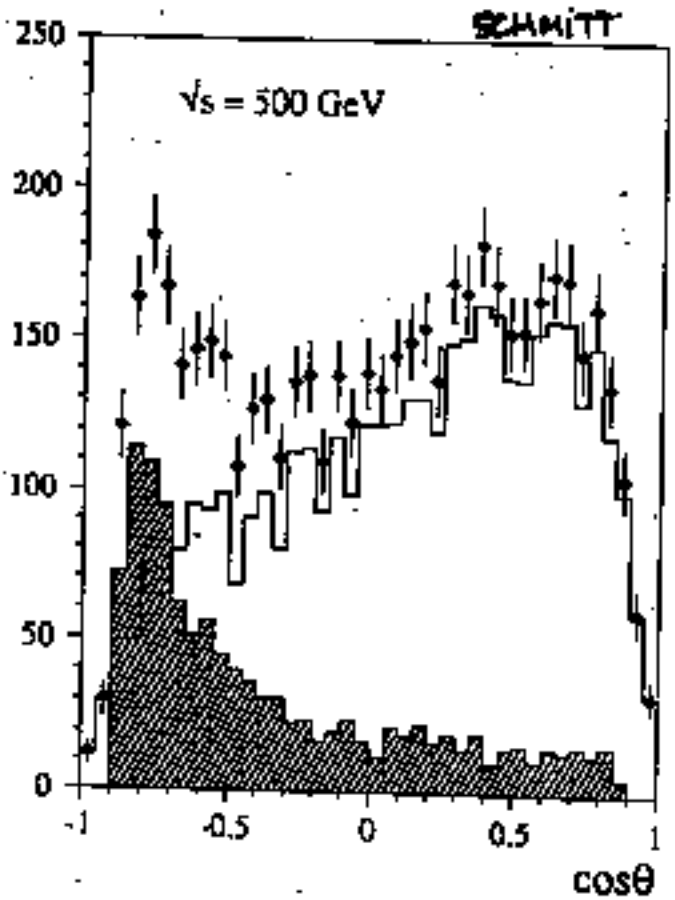


Figure 1: *Simulated thrust axis polar angle distribution, signed by an energetic, isolated lepton. The contributions of $t\bar{t}$ (solid line) and background (shaded histogram) are shown separately. m_t = 150 GeV and $\sqrt{s}$ = 500 GeV.*

⇒ PRODUCTION VERTEX:

A) $(g-2)_{top}$ $\quad jjjj\ell\nu$

- USE $\cos\theta$ $(\theta \equiv \langle t, e^- \rangle)$ ⇒ SENSITIVE TO NON-ZERO ANOMALOUS MAGNETIC MOMENT OF TOP. IN HELICITY FLIP $e^+e^- \to t_L\bar{t}_L, t_R\bar{t}_R$

$$\left.\frac{d\sigma}{d\cos\theta}\right|_{\substack{L\\ R}} \propto \sin^2\theta \left[\underbrace{\frac{m_t}{E}}_{\sim}\left(F_{1L} + F_{1R}\right) + \underbrace{\frac{E}{m_t}}_{\sim} 2\left(F_{2L} + F_{2R}\right)\right]^2$$

F_{1L}, F_{1R}: S.M. ; F_{2L}, F_{2R}: NEW

⇒ $\Delta\delta \lesssim 4\%$ $\quad (\delta \equiv F_{2L}^{\gamma} + F_{2R}^{\gamma})$ $\quad 50\,fb^{-1}$ @ 500 GeV

B) CP-VIOLATING FORM FACTORS

- CP EFFECTS LEAD TO ELECTRIC AND WEAK DIPOLE MOMENTS. FOR INSTANCE, DEFINING

$jj\,\ell^+\ell^-\nu\bar{\nu}$	$jjjj\,\ell\nu$
$\langle A_0 \rangle \equiv \hat{p}_+ \cdot (\hat{q}_+ \times \hat{q}_-)$	$\langle A_1 \rangle \equiv \langle \hat{p}_+ \cdot (\hat{q}_+ \times \hat{q}_x) \rangle - \langle \hat{p}_+ \cdot (\hat{q}_- \times \hat{q}_x) \rangle$

$\hat{p}_+ : e^+$, $\hat{q}_\pm : \ell^\pm$, $\hat{q}_x : jjj$

THEN $\langle A_i \rangle \neq 0 \Rightarrow d_t^{\gamma t} \propto (F_{2R}^{\gamma t} - F_{2L}^{\gamma t}) \neq 0$

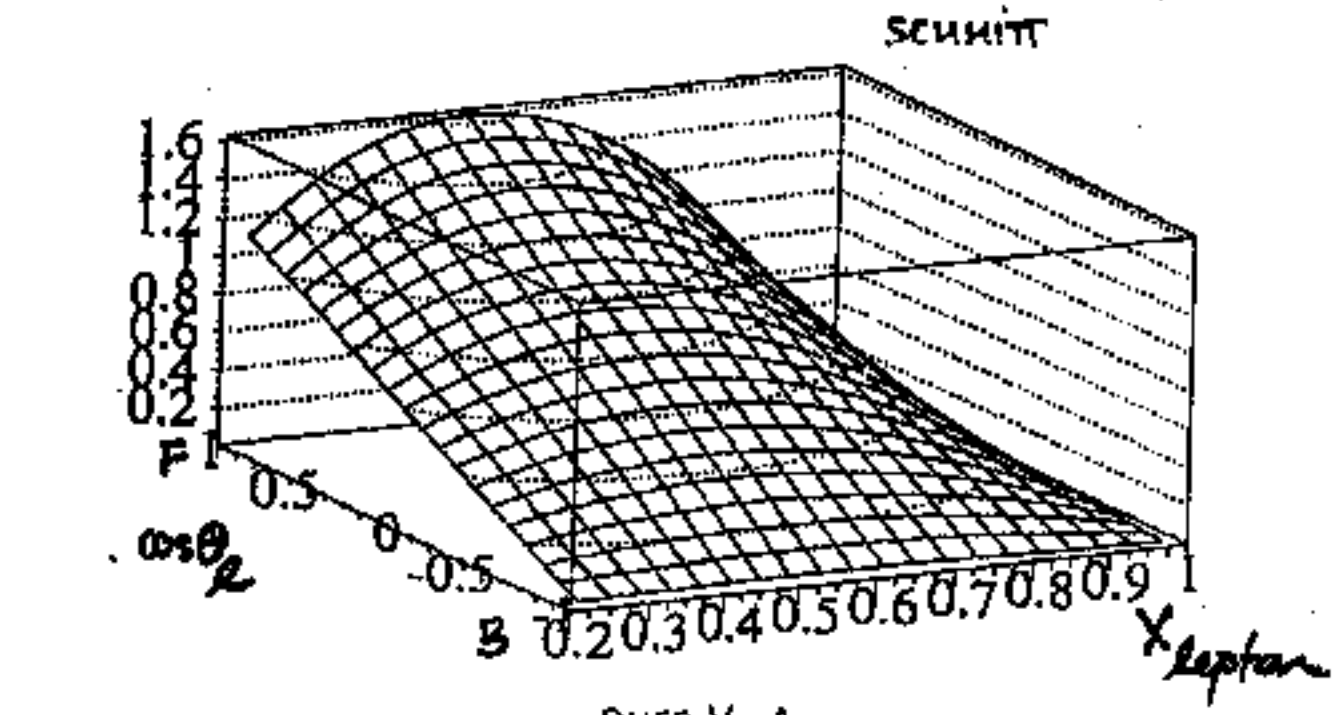
⇒ $\Delta d_t^{\gamma t} \lesssim 5 \times 10^{-18}$ e·cm $\quad 10\,fb^{-1}$ @ 500 GeV

⇒ DECAY VERTEX:

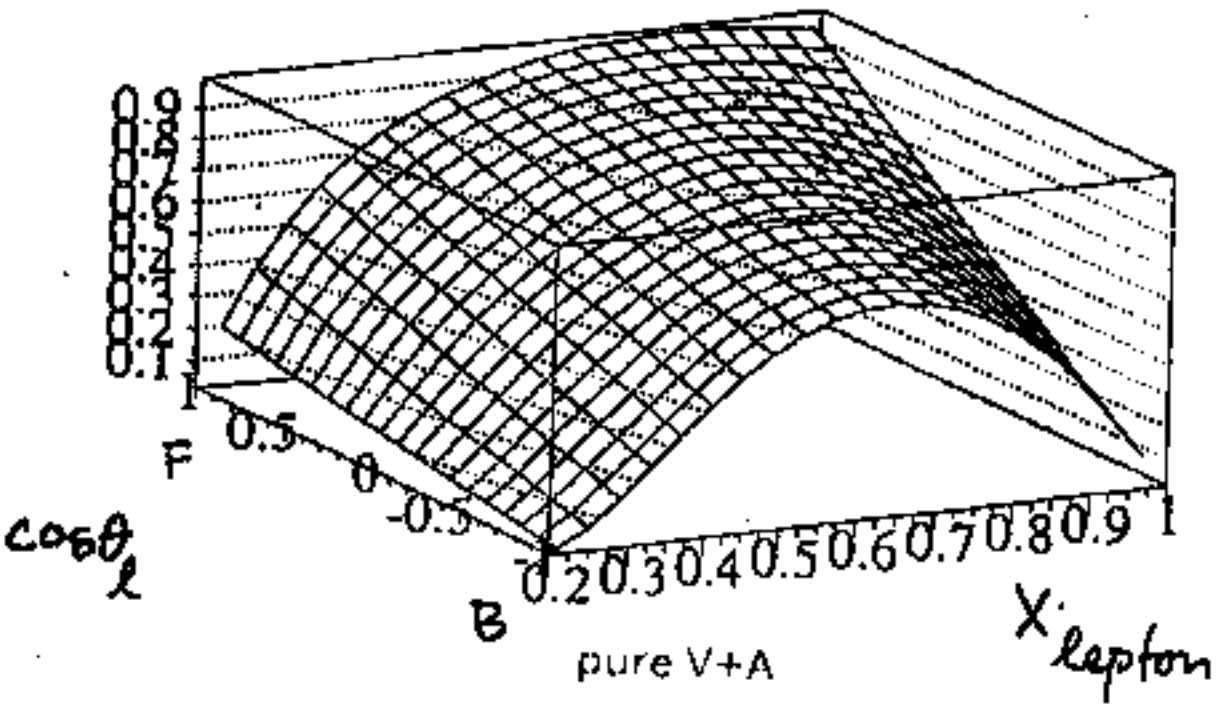
C) V+A STRUCTURE

- EXISTENCE OF $W_R^\pm$ (OR $H^\pm$) WOULD MODIFY THE DECAY FROM PURE V-A ⇒ DISTRIBUTIONS OF x_ℓ, θ_ℓ, x_ν, θ_ν MODIFIED.

⇒ $\Delta\kappa^2 \lesssim 0.02$ $\quad (\kappa^2 \simeq |F_{1R}^W|^2)$ $\quad 50\,fb^{-1}$ @ THRESHOLD



pure V−A



pure V+A

3 RARE TOP DECAYS

⇒ SINCE m_t LARGE → TOP DECAYS ALLOW FOR SEARCHES FOR NEW PARTICLES WITH $m \sim M_W$ ⇒ TEST THE MSSM:

(a) $t \to b H^+$ (b) $t \to \tilde{t} \tilde{\chi}^0$

BR ~ 1–10 % ⇒ ATTRACTIVE

* IF THESE DECAYS ARE ALLOWED PROBABLY THEY WILL BE DISCOVERED IN MORE DIRECT WAYS
⇒ GOAL HERE: MEASURING NON-STANDARD COUPLINGS OF TOP QUARK.

→ PROCESS (a)

DECAY $t \to b H^+$ FOLLOWED BY $H^+ \to \ell^+ \nu_\ell$ ⇒ EXCESS OF $\ell^\pm$ LEPTONS IN TOP DECAYS → APARENT VIOLATION OF UNIVERSALITY ↦

⇒ EXPERIMENTALY:

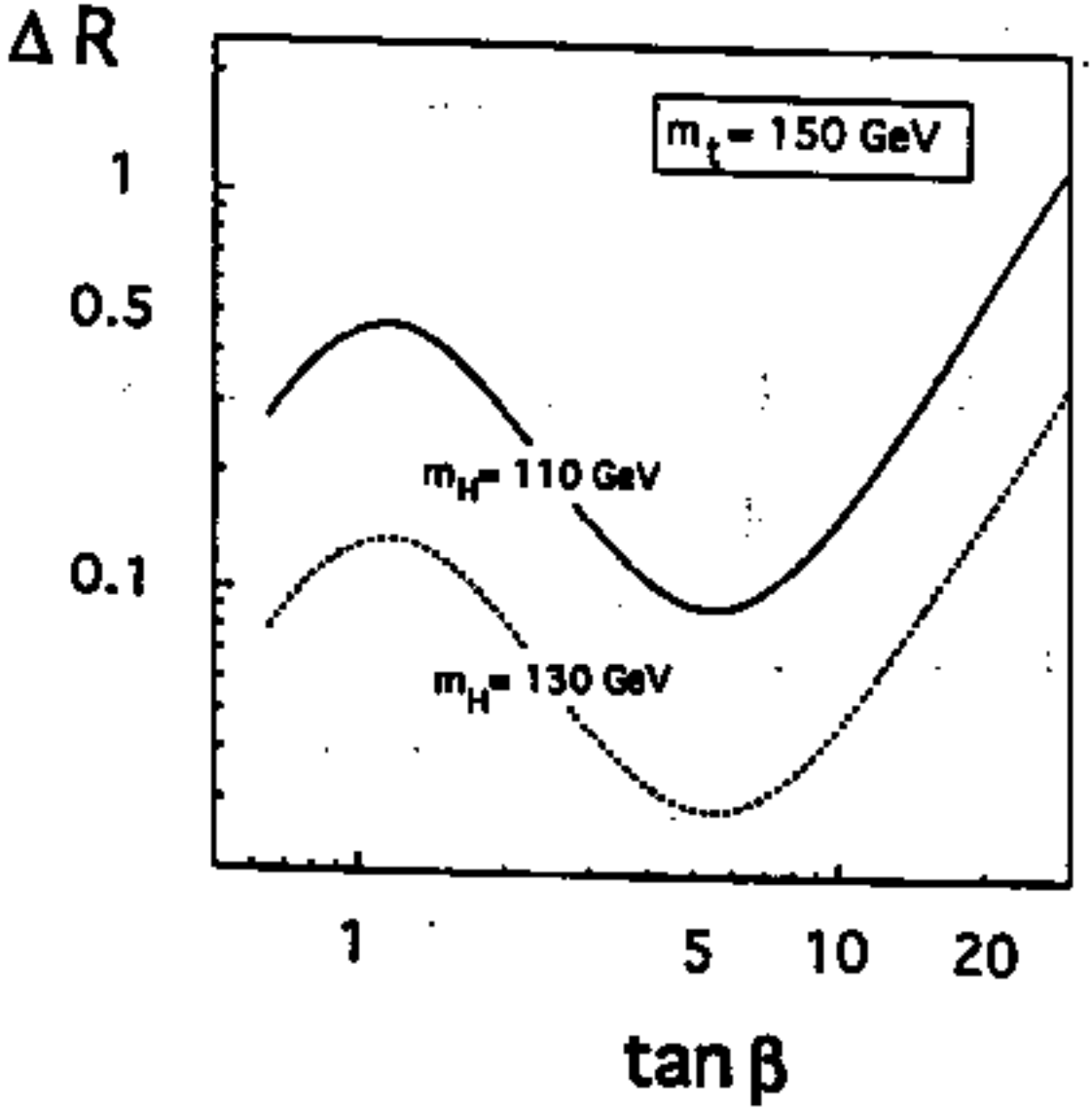
* SMALL EFFECT

* MANY BACKGROUNDS

→ FULL SIMULATION STUDIED:

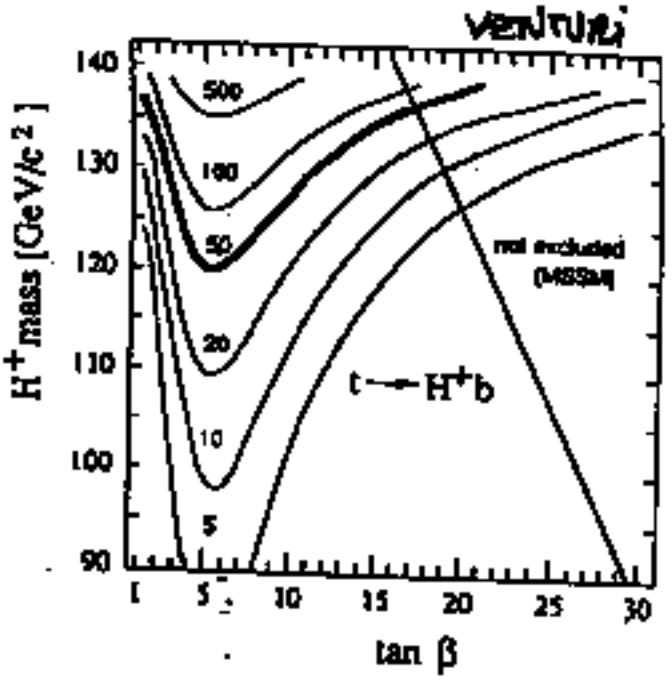
EXCESS OF $2^{\pm}$ FINAL STATES IN TOP DECAYS

$$\Delta R \equiv \frac{\sigma_\tau}{\sigma_e + \sigma_\mu} - \frac{1}{2}$$



$1 < \tan\beta < \frac{m_t}{m_b} \sim 30$

• RESULTS: LUMI (fb^{-1}) FOR 3σ EFFECT



$\sqrt{s} = 400$ GeV

$m_t = 150$ GeV

→ WITH 50 fb^{-1}

$M_{H^+} \gtrsim m_t - 15$ GeV

FOR THE INTERESTING tanβ REGION

→ PROCESS (b)

- DECAY $t \to \tilde{t}\,\tilde{\chi}^0$ BR ~ few %

  $\hookrightarrow c\,\tilde{\chi}^0$ $m_{\tilde{t}} = 80$ GeV $m_{\tilde{\chi}^0} = 55$ GeV

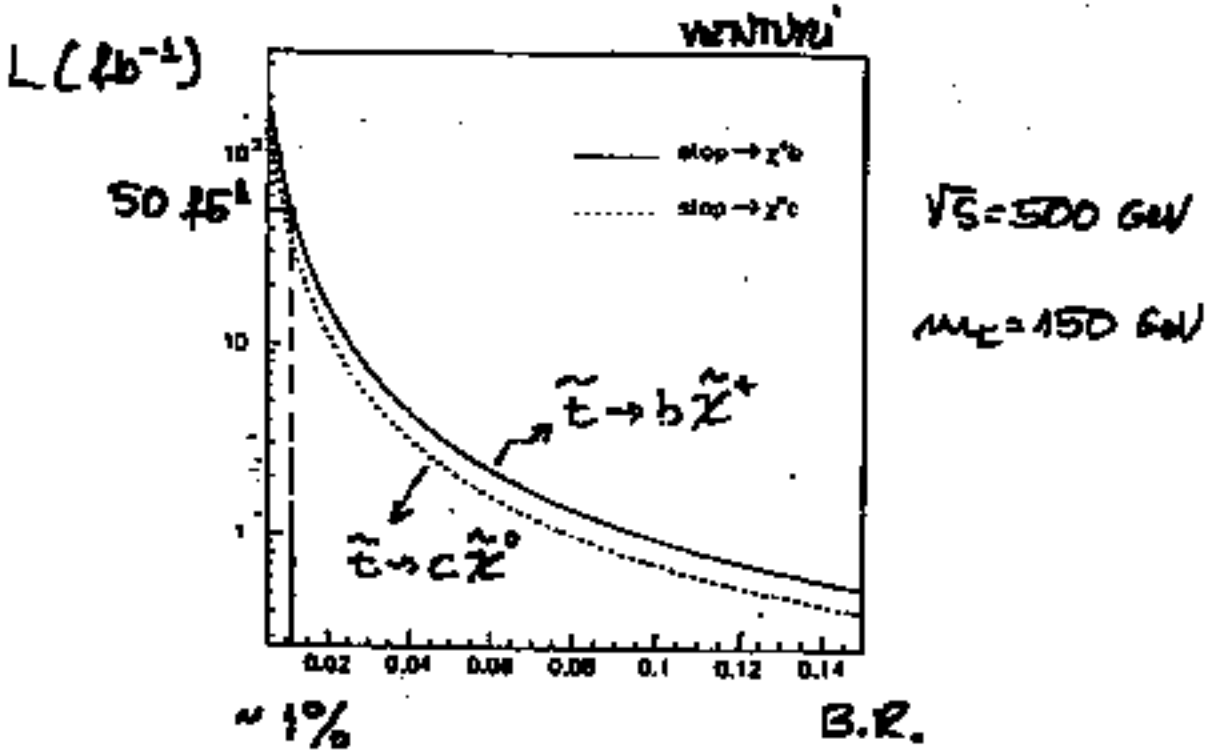
  $\hookrightarrow b\,\tilde{\chi}^+$ $m_{\tilde{t}} = 95$ GeV $m_{\tilde{\chi}^+} = 55$ GeV

  $\hookrightarrow \tilde{\chi}^0 l_1 \bar{l}_2$ $m_{\tilde{\chi}^0} = 35$ GeV

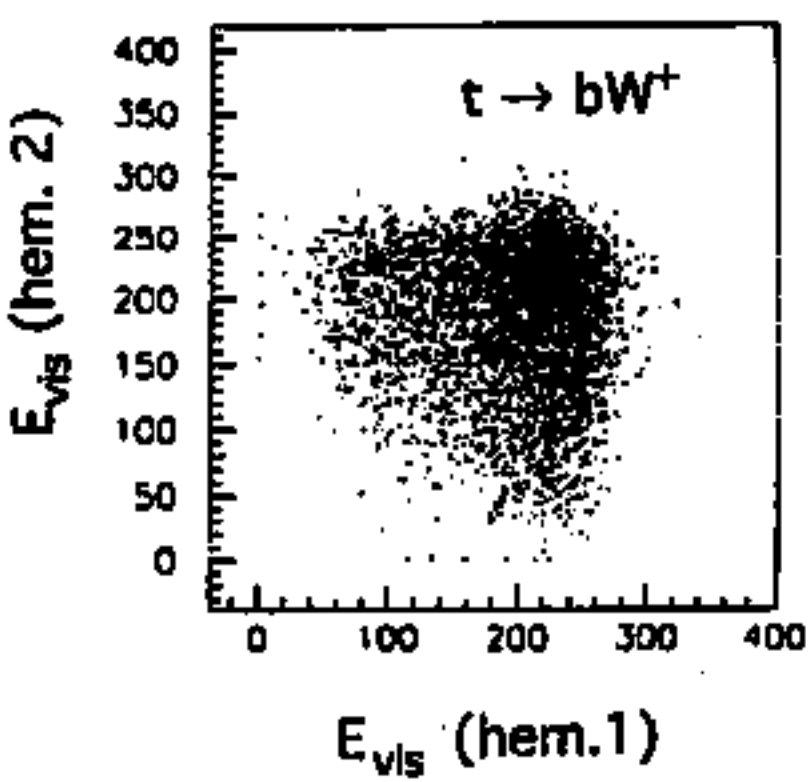
- EXPERIMENTALLY:

  EVENTS WITH STANDARD DECAY IN ONE SIDE USED TO TAG $t\bar{t}$ → LOOK FOR LOW VISIBLE ENERGY IN OPPOSITE HEMISPHERE

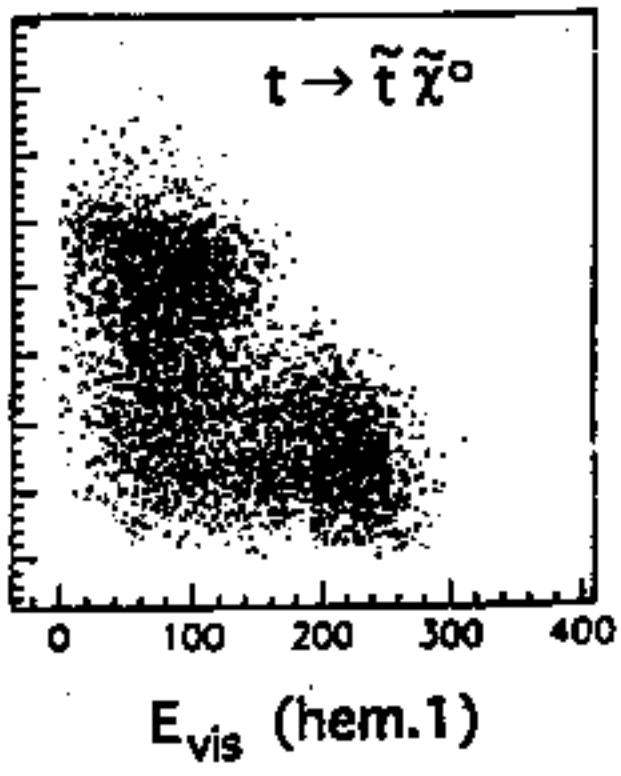
- RESULT: LUMI (fb^{-1}) FOR 3σ EFFECT



VISIBLE ENERGY PER EVENT
HEMISPHERE



BOTH TOP DECAY STANDARD



ONE TOP DECAY STANDARD AND THE OTHER $t \to \tilde{t}\,\tilde{\chi}^0$

4 DIRECT OBSERVABILITY OF TOP YUKAWA COUPLING

⇒ SELECT FINAL STATES WHICH INVOLVE THE $t\bar{t}H^0$ VERTEX → TEST THE S.M. YUKAWA COUPLING

$$\lambda_t^2 = \sqrt{2}\, G_F\, m_t^2 \qquad \lambda_t^2 \sim 0.5 \qquad \lambda_b^2 \sim 4\times10^{-4}$$

⇓ UNIQUE CHANCE TO MEASURE YUKAWA COUPLINGS

⇒ PROCESS:

(a) $m_{H^0} \gtrsim 2 m_t$ ("HEAVY" HIGGS) $\quad e^+e^- \to Z^0\, t\bar{t}$

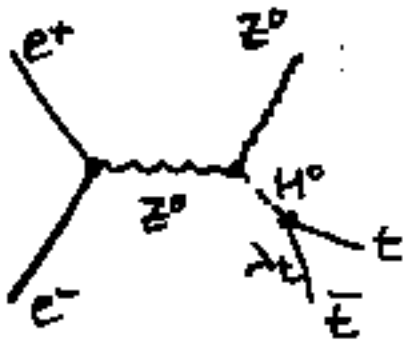
(b) $m_{H^0} < 2 m_t$ ("LIGHT" HIGGS) $\quad e^+e^- \to H^0\, t\bar{t}$

⇒ EXPERIMENTALLY:

* CLEAR SIGNATURE: 8 JETS IN THE FINAL STATE SATISFYING MANY CONSTRAINTS (m_W, m_t, m_{H^0}, m_{Z^0})
* LOW X-SECT (~ FEW fb)

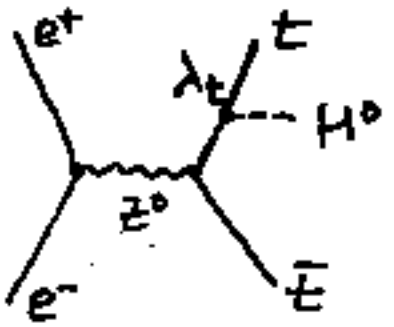
→ PROCESS (a) SIMULATED: TAKE 60 fb^{-1}

25 evt SIGNAL OVER 10 evt NON-RESONANT $Z^0t\bar{t}$ BKGD ⇒ $\frac{\Delta\lambda_t}{\lambda_t} \sim 25\%$



(a)

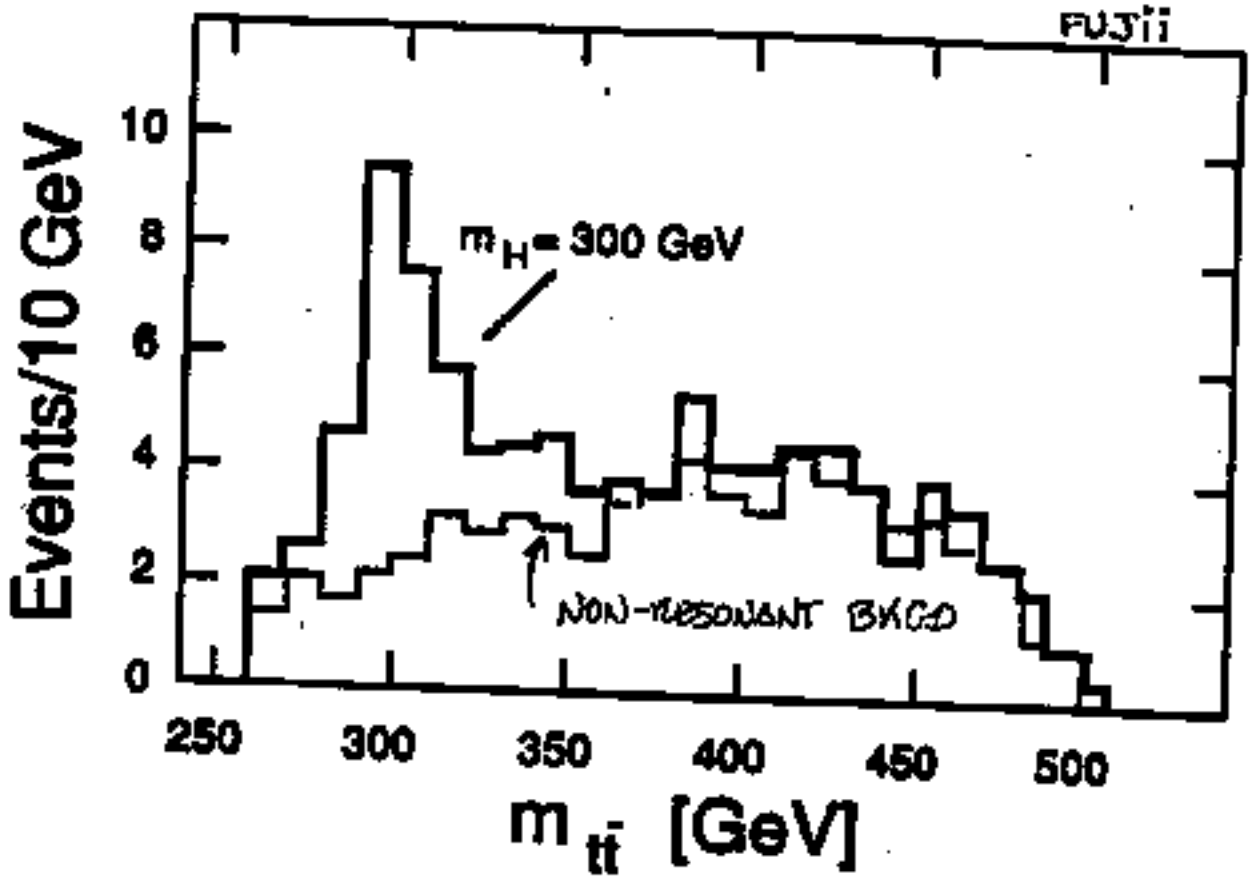
"HEAVY" HIGGS



(b)

"LIGHT" HIGGS

⇒ PROCESS (a)



$\sqrt{s}$ = 600 GeV

L = 60 fb^{-1}

m_t = 130 GeV

→ PROCESS (b):

- ASSUMING "IDEAL" CONDITIONS FOR $L = 50\ fb^{-1}$ (100% ACCEPTANCE & 0% BACKGROUND)

$$\frac{\Delta\lambda_t}{\lambda_t} \lesssim 10\% \quad \text{FOR } M_H \lesssim 240\ GeV$$

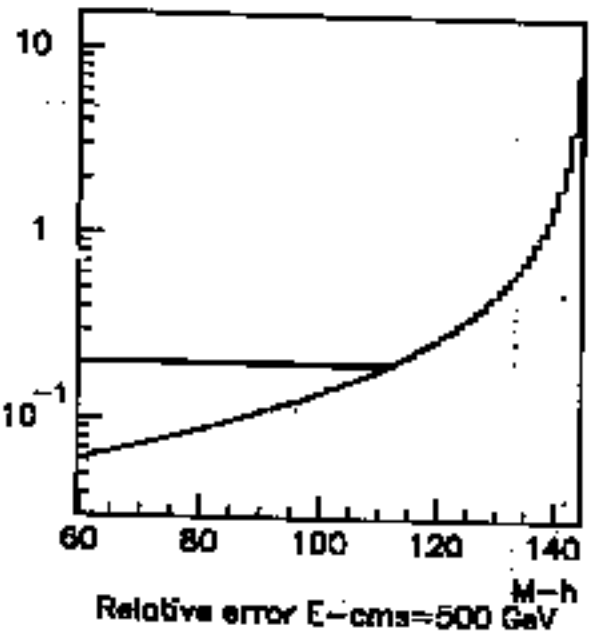
- IN PRACTICE:
  - SIGNAL: $4b + 4\bar{b}$ JETS
    ↑ b-TAGGING CRUTIAL
  - BACKGROUND:
    $e^+e^- \to t\bar{t}(g^*)$, $e^+e^- \to t\bar{t}Z$, $e^+e^- \to t\bar{t}\gamma^*$
    (IRREDUCIBLE IF $M_H \sim M_Z$)

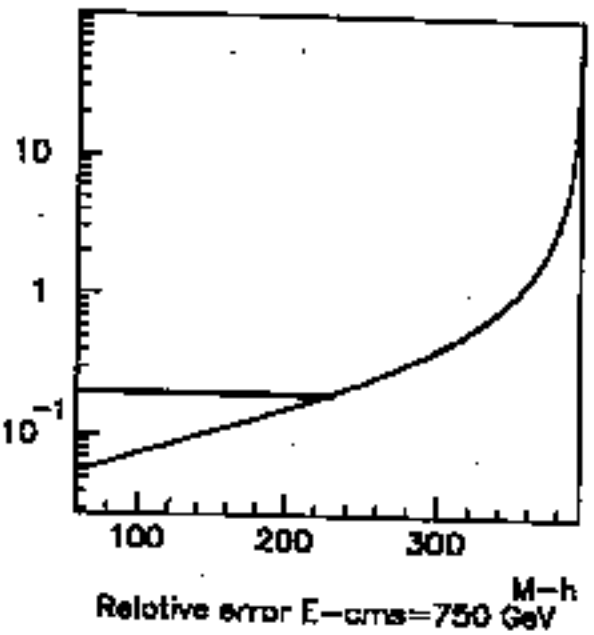
- RESULTS: $L = 50\ fb^{-1}$ $M_H = 100\ GeV$

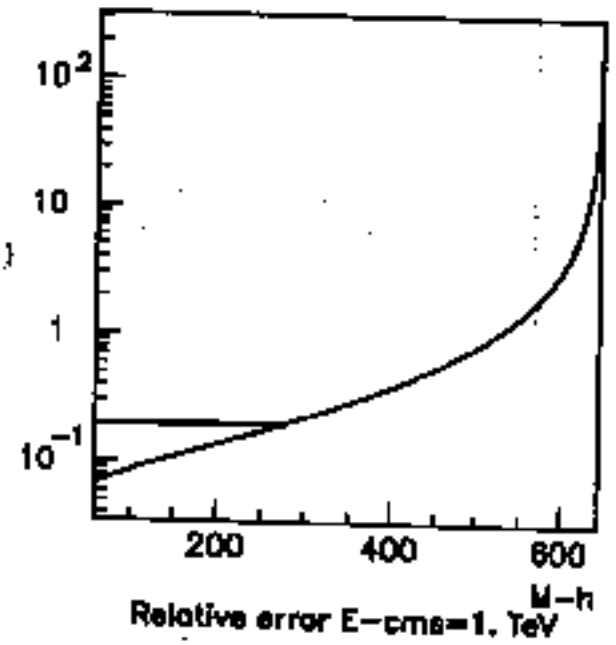
$\sqrt{s}$ (GeV)	500	750	1000
SIGNAL	13	48	36
BKGD	11	29	28
S/N	1.2	1.7	1.3

⇒ MEASUREMENT AT THE LIMIT OF BEING POSSIBLE.

$\frac{\Delta\lambda_t^Z}{\lambda_t^Z}$ FOR $L = 50\ fb^{-1}$







Relative error

$$\frac{\Delta\lambda_t^Z}{\lambda_t^Z}$$

"IDEAL" CONDITIONS

$m_t = 180$ GeV

SUMMARY OF EXPECTED TOP PHYSICS MEASUREMENTS

⇒ @ TOP THRESHOLD

$\Delta m_t \lesssim 120$ MeV
$\Delta \alpha_s \lesssim 0.003$

$\Delta \Gamma_t/\Gamma_t \lesssim 0.05$
$\Delta \lambda_t/\lambda_t \lesssim 0.25$

⇒ TOP FORMFACTORS

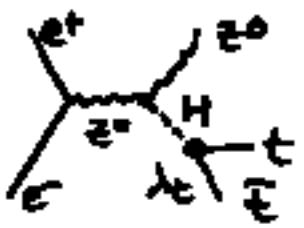
ANOM. MAGNETIC QUAD. MOMENT	$\Delta \delta \lesssim 0.04$
ANOM. ELECTRIC / WEAK DIPOL. MOMENT	$\Delta d^{\gamma/Z} \lesssim 5\times10^{-18}$ e·cm
V+A DECAY STRUCTURE	$\Delta K^2 \lesssim 0.02$

⇒ RARE TOP DECAYS

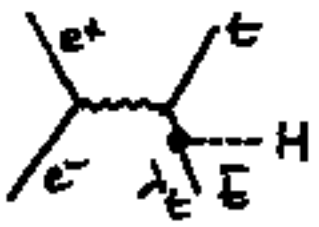
$t \to b H^+$	$m_{H^+} \gtrsim m_t - 15$ GeV	@ 3σ
$t \to \tilde{t}\tilde{\chi}^0$	BR $\lesssim 1\%$	@ 3σ

⇒ DIRECT TOP YUKAWA COUPLING

$m_{H^0} \gtrsim 2m_t$ $\quad \Delta\lambda_t/\lambda_t \sim 0.25$
($\sqrt{s} = 600$ GeV, $\int L dt = 60$ fb^{-1})



$m_{H^0} < 2m_t$ $\quad \Delta\lambda_t/\lambda_t \sim 0.10$
(BUT $\sqrt{s} = 400$ GeV, $\int L dt = 180$ fb^{-1})

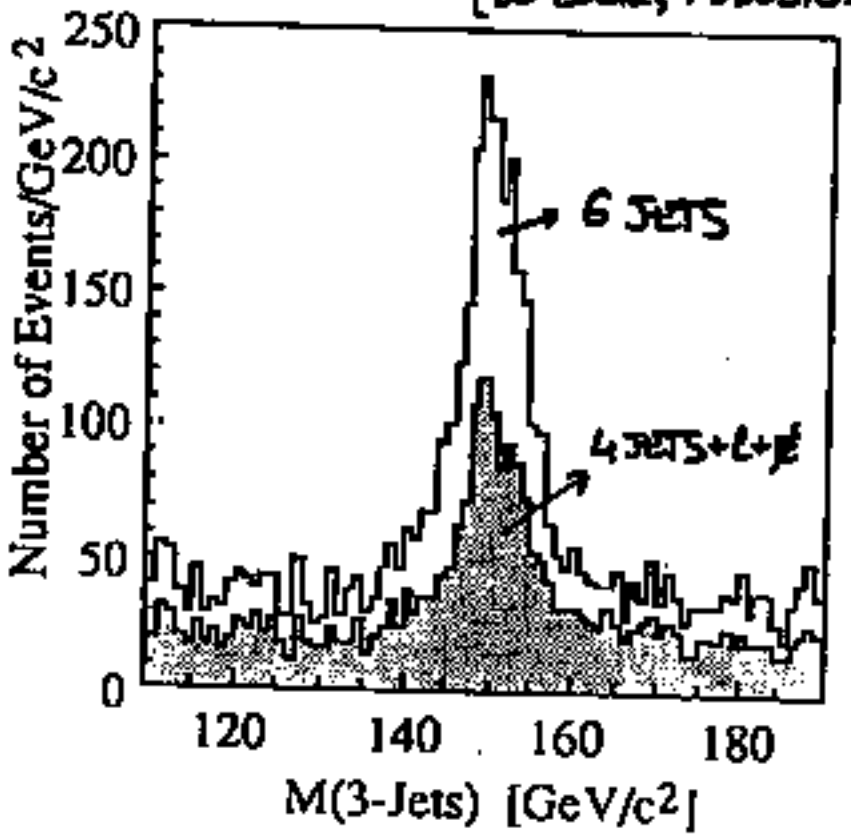


DIRECT m_t MEASUREMENT ABOVE THRESHOLD

- SELECT 6 JET AND 4 JETS + $\ell + \nu$
- USE 3-JET AND JET + ℓ + $\not{E}$ MASSES + CONSTRAINED FIT WITH p_T CONSERVATION
- NEEDS GOOD CONTROL OF I.S.R. + BEAM EFFECTS

$\sqrt{s} = 500$ GeV $m_t = 150$ GeV

(DE BOER, PODOBNIK)



BEFORE CONSTRAINED FIT

- FOR $L = 10\ fb^{-1}$ Δm_t (STAT) ~ 150 MeV

BUT MANY EXPERIMENTAL & HADRON. UNCERTAINTIES

$\Rightarrow \Delta m_t \sim 500$ MeV

S11

2nd ECFA/DESY Study on Physics and Detectors for a Linear Electron-Positron Collider

LAL - Orsay - France, April 5-7, 1998

Date : April 6, 1998 - 11:15

Parallel groups to plan tasks : Top

Thomas Teubner

ECFA DESY Linear Collider Workshop

6. April '98

$t\bar{t}$ at Threshold: Complete NNLO Relativistic Corrections

Thomas Teubner (DESY Hamburg)

→ A. H. Hoang (San Diego) + T.T.: hep-ph/9801397

K. Melnikov + A. Yelkhovsky hep-ph/9802379

Content:
- Introduction / Motivation
- The NNLO corrections:
  Calculation, Results
- Consequences,
  Outlook,
  Discussion

(I.) Introduction / Motivation

....> Yuki's talk

$$\sigma_{tot} \sim \mathrm{Im}\; G(E+i\Gamma_t;\, r=0,\, r'=0)$$

from

$$\left[\left(-\frac{\vec{\nabla}^2}{m_t} + V(r)\right) - (E+i\Gamma_t)\right] G(E+i\Gamma_t;\, r,0) = \delta(r$$

... + $\frac{d\sigma}{dp_t} \sim \Gamma_t\, |\vec{G}(p_t, E+i\Gamma_t)|^2$ + polarization

+ A_{FB} + $\frac{d\sigma}{dp_t\, dE_t\, d\Omega_t}$ + rescattering ...

But: • all studies (<'98) up to $O(\alpha_s, v)$ "only", and

• M. Peter ('97): ...

"The static potential in QCD – a full two-loop calculation"

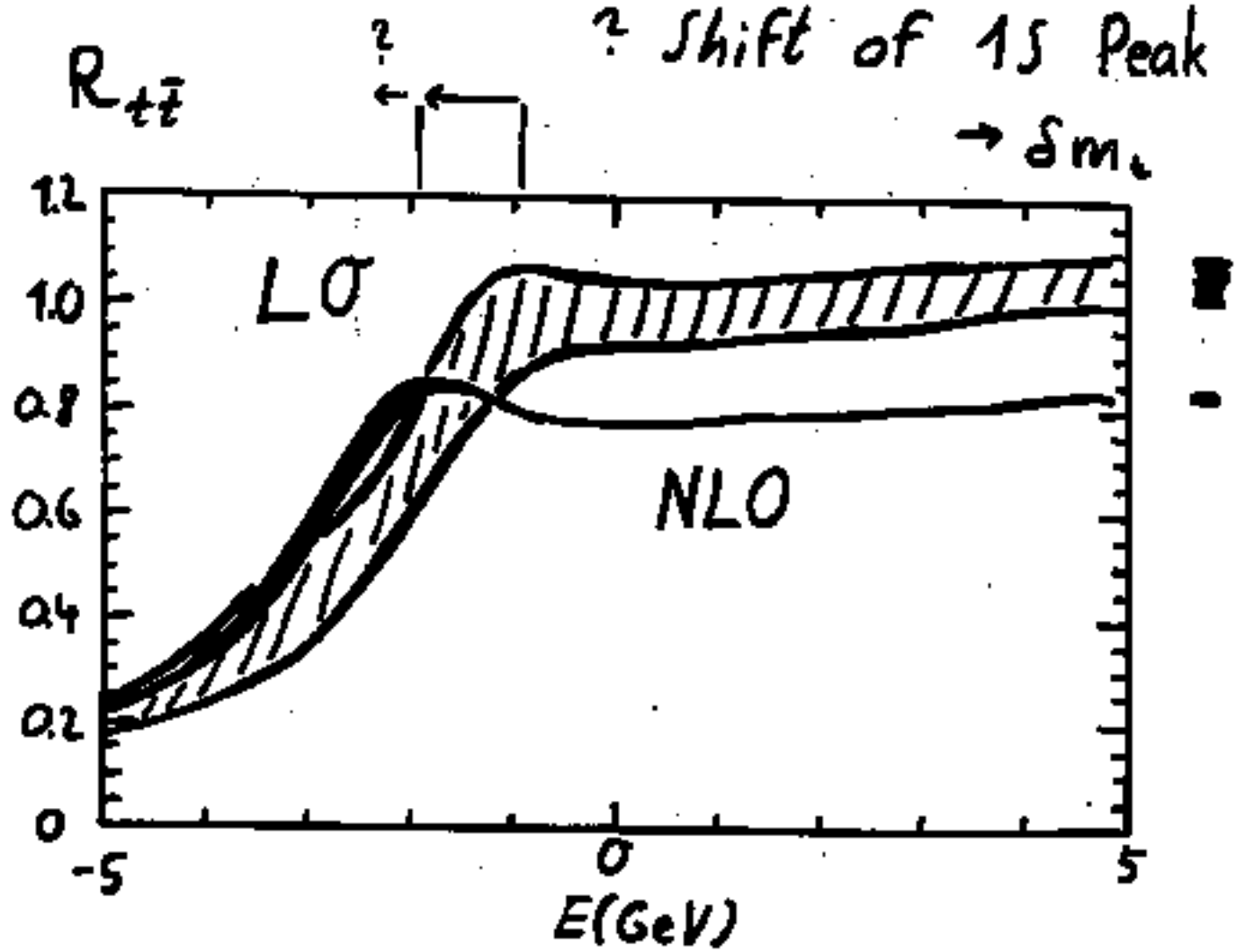
• ? What about the other $O(\alpha_s^2, v\alpha_s, v^2)$ corrections ?

• How to calculate them consistently ?

LO → NLO → NNLO ?!

"bread and butter"
$\delta m_t = 120$ MeV

? Stability
? Normalization
? Shift of 1S Peak → δm_t



$m_t = 175$ GeV , $\Gamma_t = 1.43$ GeV , $\alpha_s(M_Z) = 0.118$

$\mu_{soft} = 50 - 100$ GeV ⇒ band

II. The latest developments: The complete NNLO relativistic corrections

$$\mathcal{O}(v^2,\ v\alpha_s,\ \alpha_s^2)$$

- **Method:** NRQCD (eff. FTh)

$$\mathcal{L}_{NRQCD} = -\frac{1}{2}\,\mathrm{Tr}\, G^{\mu\nu}G_{\mu\nu} + \sum_{q=u,d,s,c,b} \bar{q}\, i \not{D} q + \quad g, q \text{ in the conventional rel. } \mathcal{L}.$$

$$\psi^{+}\left[i D_t + a_1 \frac{\vec{D}^2}{2m_t} + a_2 \frac{\vec{D}^4}{8m_t^3} \right]\psi + \ldots \qquad \psi\text{: Pauli spinor for } t$$

$$+\psi^{+}\left[\frac{a_3 g_s}{2m_t}\vec{\sigma}\cdot\vec{B} + \frac{a_4 g_s}{8m_t^2}(\vec{D}\cdot\vec{E} - \vec{E}\cdot\vec{D}) + \right.$$

$$\left. \frac{a_5 g_s}{8m_t^2}\, i\vec{\sigma}(\vec{D}\times\vec{E} - \vec{E}\times\vec{D}) \right]\psi + \ldots$$

→ purely fixed order perturbatively from QCD, no phenomenological ansatz

→ (semi) analytical calculation

→ use of <u>direct matching</u> of the short-distance coefficients $a_1 \ldots a_5$ to (recently calculated) results of the full QCD.

(***) $$R^{\text{thr}}_{\text{NNLO}}(\bar{E}) = \frac{\pi Q_t^2}{m_t^2} C_1(\mu_{\text{hard}}, \mu_{\text{fac}}) \operatorname{Im}\left[\mathcal{A}_1(\dots)\right] - \frac{4\pi Q_t^2}{3 m_t^2} C_2(\mu_{\text{hard}}, \mu_{\text{fac}}) \operatorname{Im}\left[\mathcal{A}_2(\bar{E}, \mu_{\text{soft}}, \mu_{\text{fac}})\right]$$

↙ short distance coeffs.

non-rel. current correlators (long-distance)

$$\mathcal{A}_1 = \langle 0 | (\tilde{\psi}^\dagger \sigma \tilde{\chi})(\tilde{\chi}^\dagger \sigma \tilde{\psi}) | 0 \rangle$$

$$\mathcal{A}_2 = \frac{1}{2} \langle 0 | (\tilde{\psi}^\dagger \sigma \tilde{\chi})(\tilde{\chi}^\dagger \sigma (-\tfrac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \tilde{\psi}) + \text{h.c.} | 0 \rangle$$

→ 3 different scales: μ_{hard}, μ_{soft}, μ_{fac}

→ @ NNLO NRQCD $\mathcal{A}_{1,2}$ can be calculated from a Schrödinger Eq. for $t\bar{t}$.

$$\left(-\frac{\vec{\nabla}^2}{m_t} - \frac{\vec{\nabla}^4}{4m_t^3} + V_c(\vec{r}) + V_{BF}(\vec{r}) + V_{NA}(\vec{r}) - (E + i\Gamma_t)\right) G(\vec{r}, \vec{r}', E + i\Gamma_t) = \delta^{(3)}(\vec{r} - \vec{r}')$$

→ all interactions in instantaneous potentials!

V_c : static Coulomb potential up to $\mathcal{O}(\alpha_s$

$$V_c(\vec{r}) = -\frac{C_F \alpha_s}{r}\left\{1+\frac{\alpha_s}{4\pi}\left[2\beta_0 \ln(\mu r)+a_1\right]\right.$$

$$\left.+\left(\frac{\alpha_s}{4\pi}\right)^2\left[\beta_0^2\left(4\ln^2(\mu r)+\frac{\pi^2}{3}\right)+2\left(2\beta_0 a_1+\beta_1\right)\ln(\mu r)+a_2\right]\right\}$$

$+\ldots$

M. Peter 97

V_{BF} : Breit-Fermi-Pot. from Positronium

$$V_{BF}(\vec{r}) = \frac{C_F a_s^2 \pi}{m_e^2}\left[1+\frac{8}{3}\vec{S}_e\vec{S}_{\bar{e}}\right]\delta^{(3)}(\vec{r}) +$$

$$\frac{C_F a_s}{2m_e^2 r}\left[\vec{\nabla}^2+\frac{1}{r^2}\vec{r}(\vec{r}\vec{\nabla})\vec{\nabla}\right]+\frac{3C_F a_s}{2m_e^2 r^3}\vec{L}(\vec{S}_e+\vec{S}_{\bar{e}})$$

$$-\frac{3C_F a_s}{m_e^2 r^3}\left[\frac{1}{3}\vec{S}_e\vec{S}_{\bar{e}}-\frac{1}{r^2}(\vec{S}_e\vec{r})(\vec{S}_{\bar{e}}\vec{r})\right]$$

$$V_{NA}(\vec{r}) = -\frac{C_A C_F a_s^3}{2m_e r^2} \qquad a_s = \alpha_s^{(n_f)}(\mu_{soft})$$

the purely non-abelian part.

The Calculation:

- $$\left(-\frac{\vec{\nabla}^2}{M_t} + V_c(\vec{r}) - (E+i\Gamma_t)\right) G_c(\vec{r}, E+i\Gamma_t) = \delta^{(3)}(\vec{r})$$

  numerically (position- or momentum space)

- $\delta G(\vec{r}, E+i\Gamma_t) =$

  $$\int d^3\vec{x}\; G_c^{(0)}(\vec{r},\vec{x},\vec{E}) \left[\frac{\vec{\nabla}^4}{4M_t^3} - V_{BF}(\vec{x}) - V_{NA}(\vec{x})\right] G_c^{(0)}(\vec{x},\vec{r},\vec{E})$$

  with Rayleigh-Schrödinger TIPT (time indep. art. theory)
  analytically. ... $\mathcal{A}_1 = \ldots, \mathcal{A}_2 = \ldots$

  see plots for end-result ...

- $\begin{pmatrix} * \\ * \\ * \end{pmatrix} = R^{thr}_{NNLO}$ is then <u>matched directly</u> Hoang '97
  to the two loop R of full QCD
  above threshold in the (formal) limit $\alpha_s \ll v \ll 1$.
  (for $\Gamma_t = 0$) (keeping $\alpha_s^2, v^2, \alpha_s v$)

  → importance of analyt. results for R

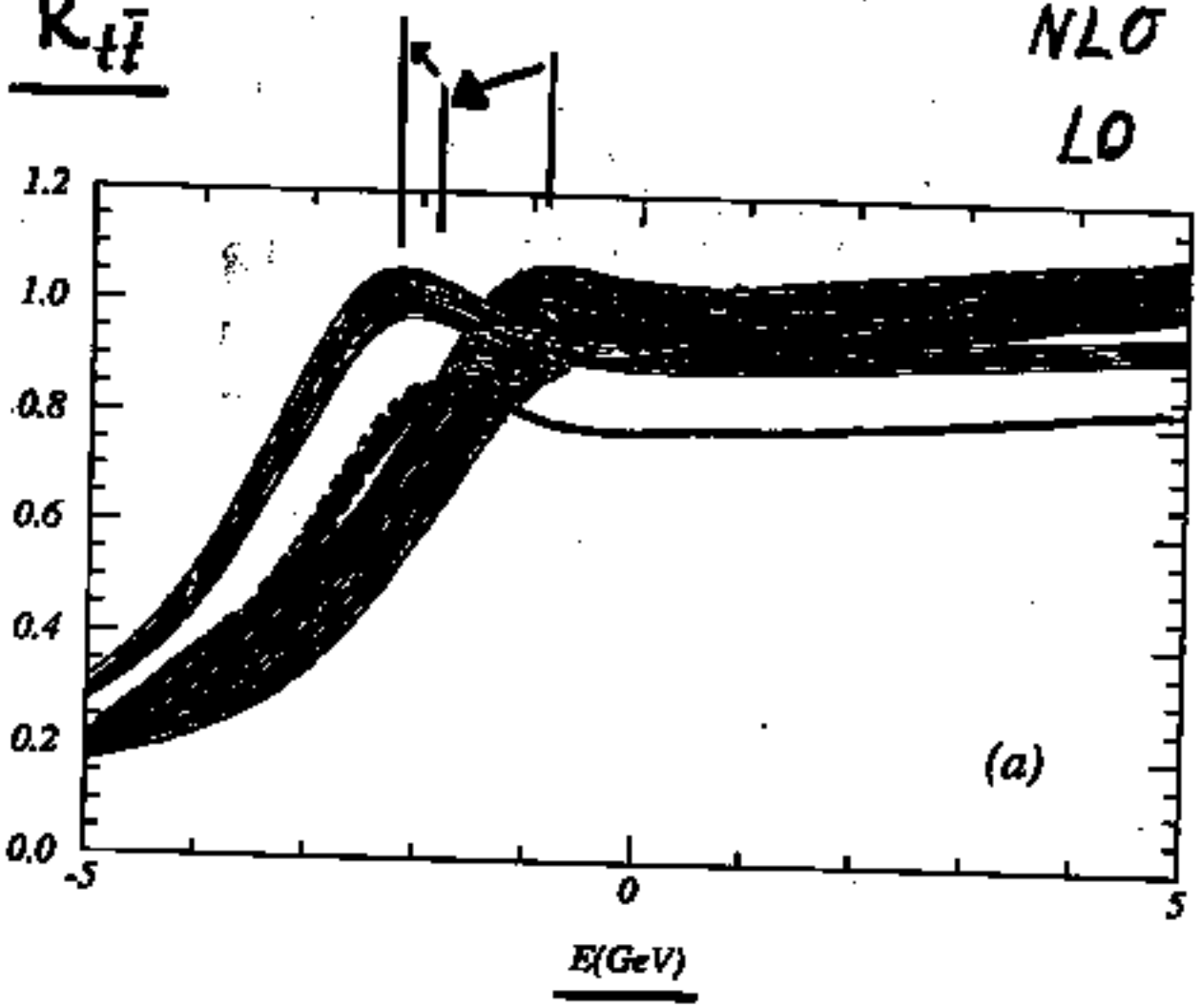
  $\Rightarrow$ $C_1 = \ldots, C_2 = \ldots$ see plots for <u>end-results</u> →

Is the top too light for us ?

→ asymptoticness of the series !

$\sigma(\alpha_s^2)$ calculation by A. H. Hoang + T. Teub.

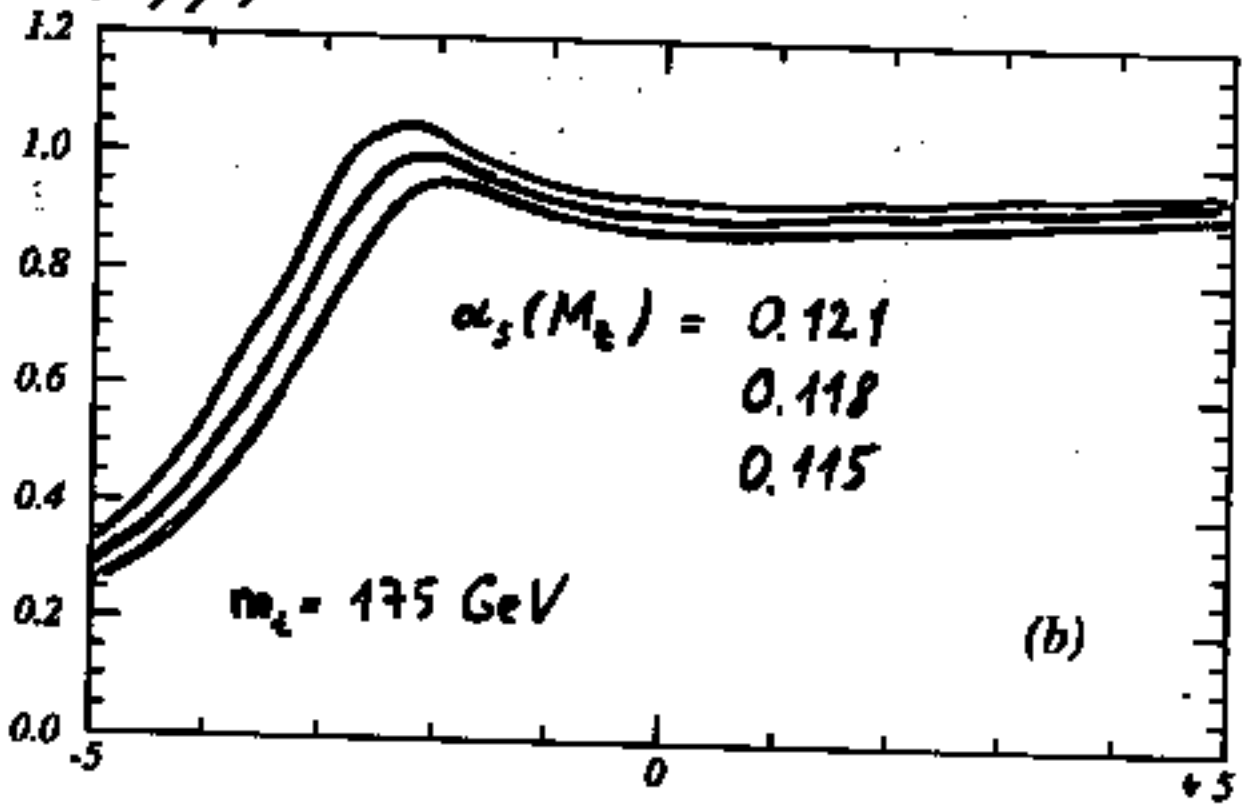
Comparison: full NNLO



$m_t = 175$ GeV, $\Gamma_t = 1.43$ GeV, $\alpha_s^{(n_f=5)}(M_z) = 0.118$

$\mu_{soft} = 50 / 75 / 100$ GeV => bands => ↓ ?

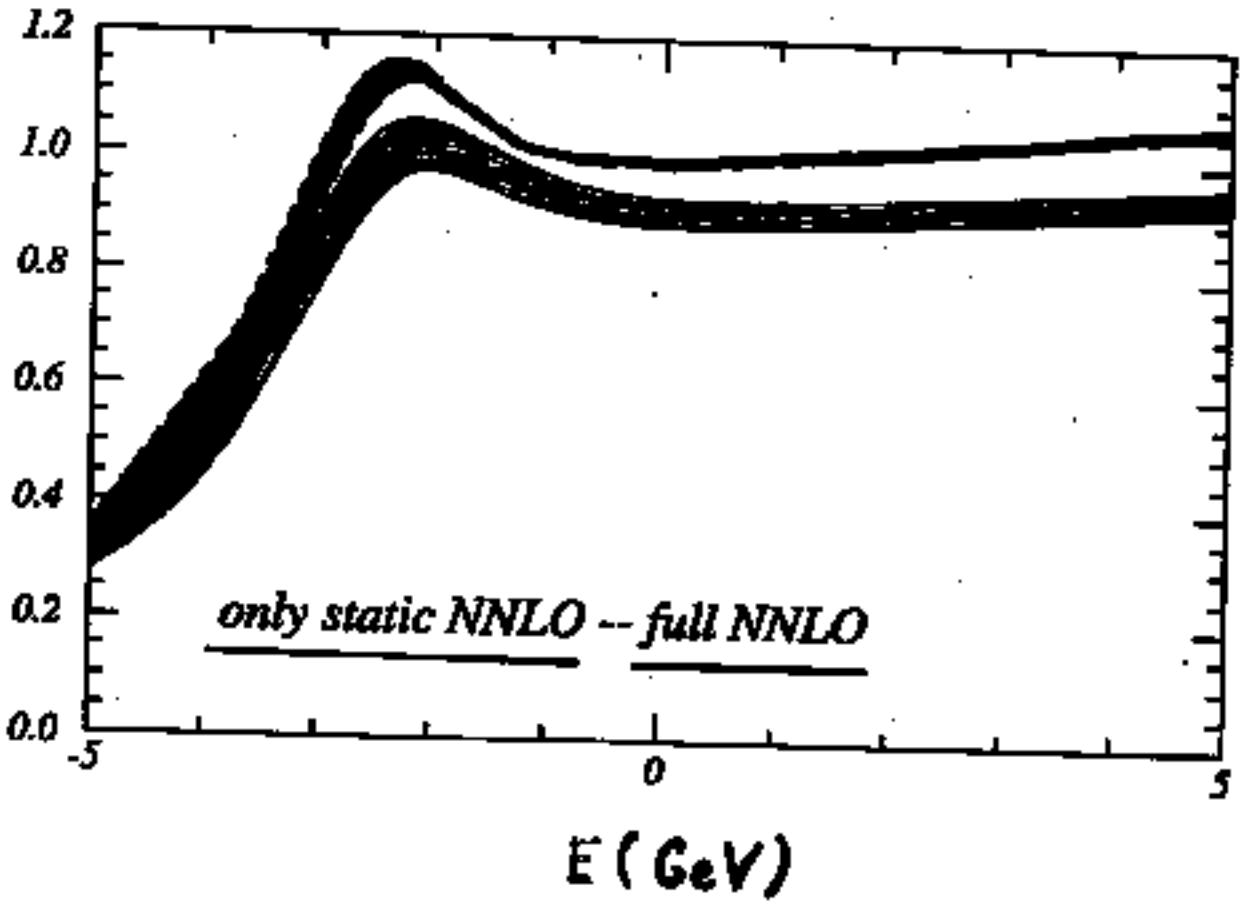
$$R = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



$\underline{E(GeV)}$ = 0 ≙ $\sqrt{s} = 2m_t$

→ Importance of relativistic corrections

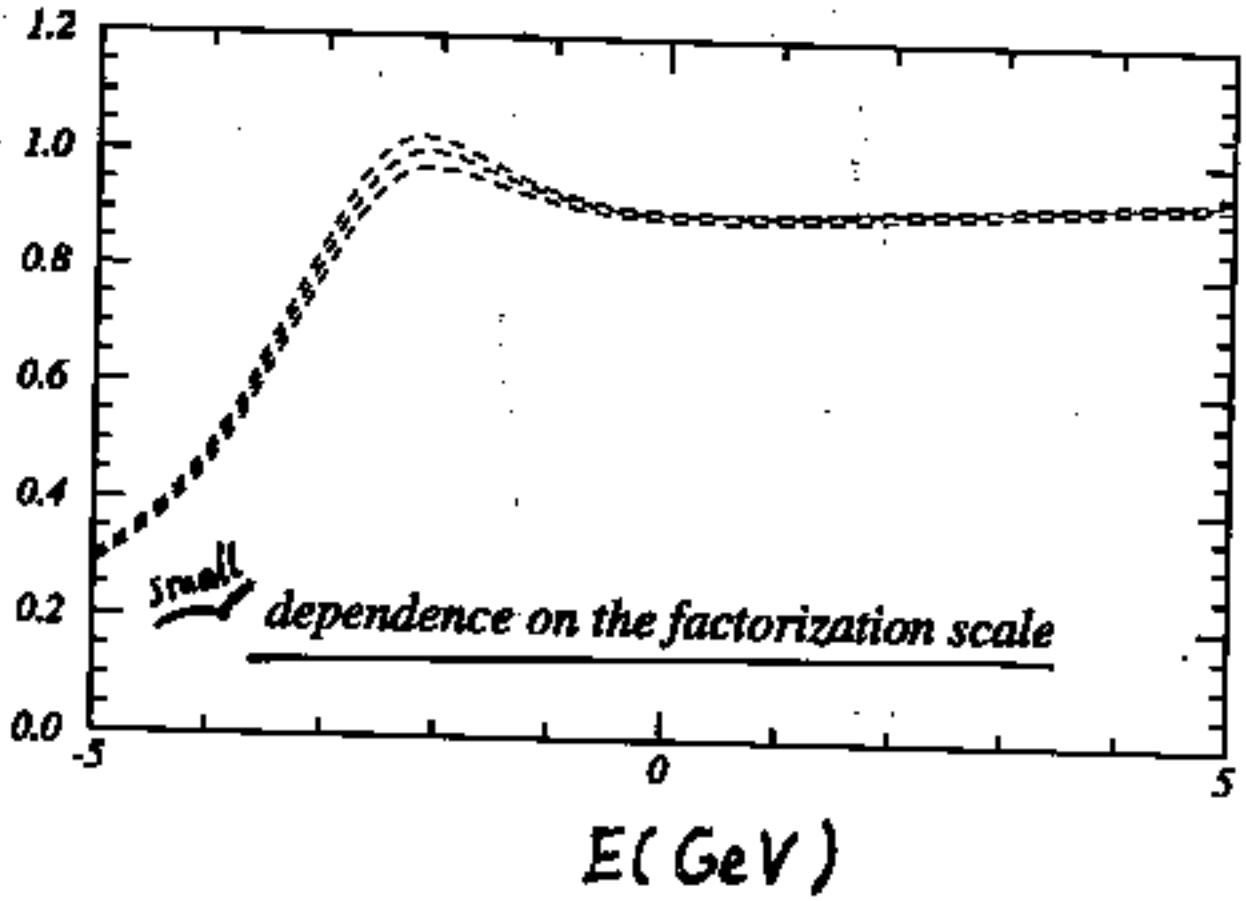
$R_{t\bar{t}}$ @ NNLO .



μ_{soft} = 50 / 70 / 100 GeV => band

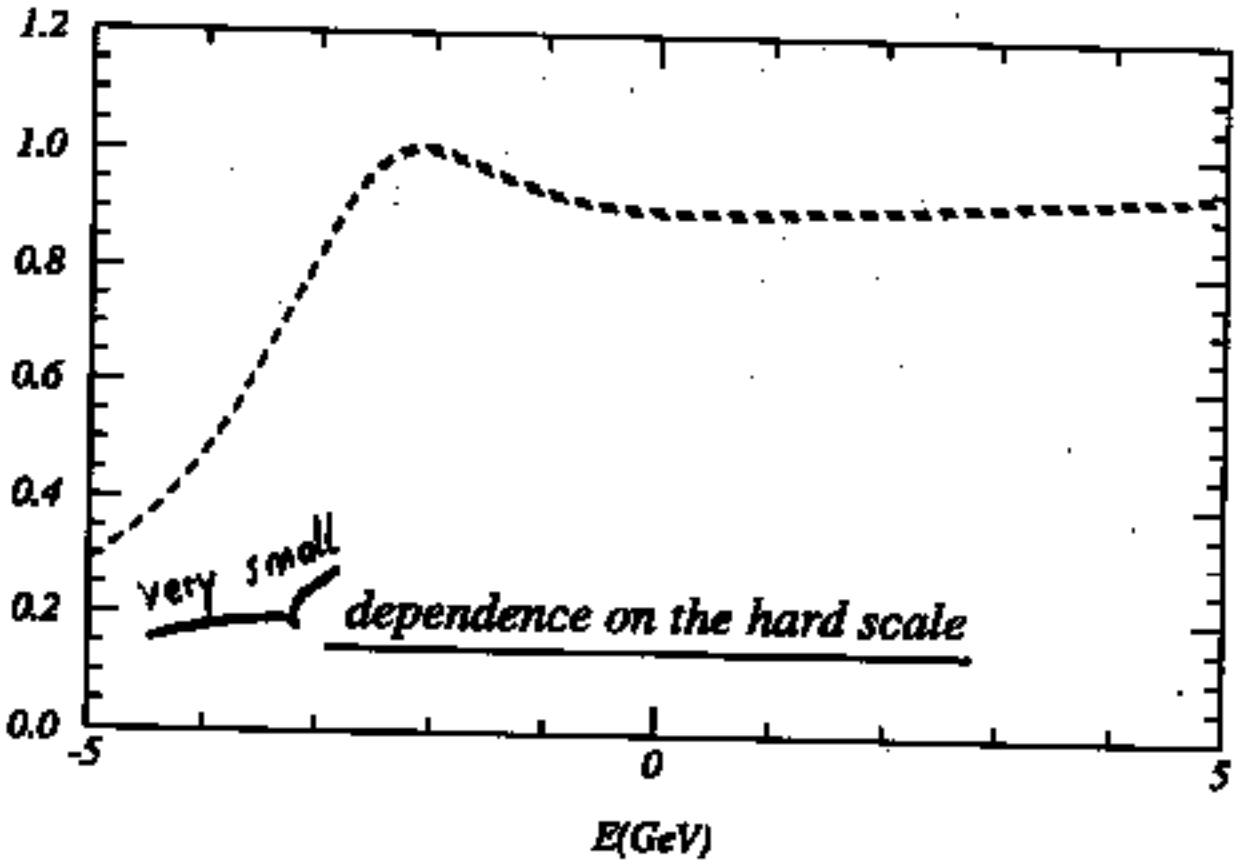
further theoretical uncertainties
.... to be reduced ..

$R_{t\bar{t}}$ @ NNLO



$\mu_{fac} = m_t/2 \ \ldots \ 2m_t$

$R_{t\bar{t}}$ @ NNLO



$\mu_{hard} = m_t/2 \ldots 2 m_t$

- Similar work of
  K. Melnikov + A. Yelkhovsky :
  - no formulation in NRQCD
  - no usage of TIPT, but full numerical solution of the Schrödinger equation

  → very similar results →

  agreement HT ↔ MY [at the level expected, i.e. up to $\Sigma \ln(\mu_{\ldots}$ ..., but...

NNLO FIGURES

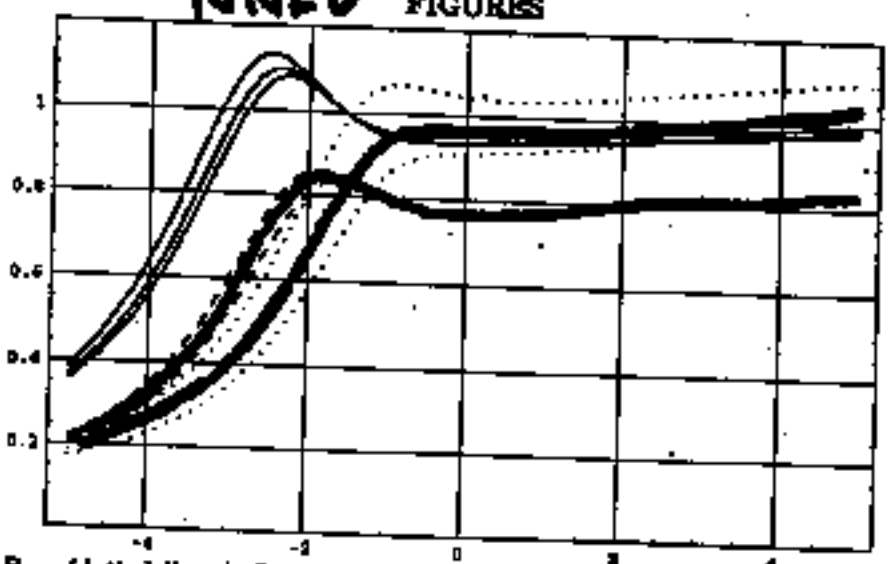


FIG. 1. R_{LO} *(dotted lines),* R_{NLO} *(dashed lines),* R_{NNLO} *(solid lines) as a function of energy* $\sqrt{s} - 2m$, *GeV. In all three cases, three curves correspond to different choices of the soft scale* $\mu = 50$ GeV *(upper curves),* $\mu = 75$ GeV *and* $\mu = 100$ GeV *(lower curves). We also use* $m = 175$ GeV, $\Gamma_t = 1.43$ GeV *and* $\alpha_s(M_Z) = 0.118$ *as the input parameters.*

From: K. Melnikov and A. Yelkhovsky

hep-ph / 9802379

(III.) Consequences / Outlook / Discussion

- Include $O(\alpha_s^2)$ corrections in full (exp.) analysis ⇒ impact on δm_t ?

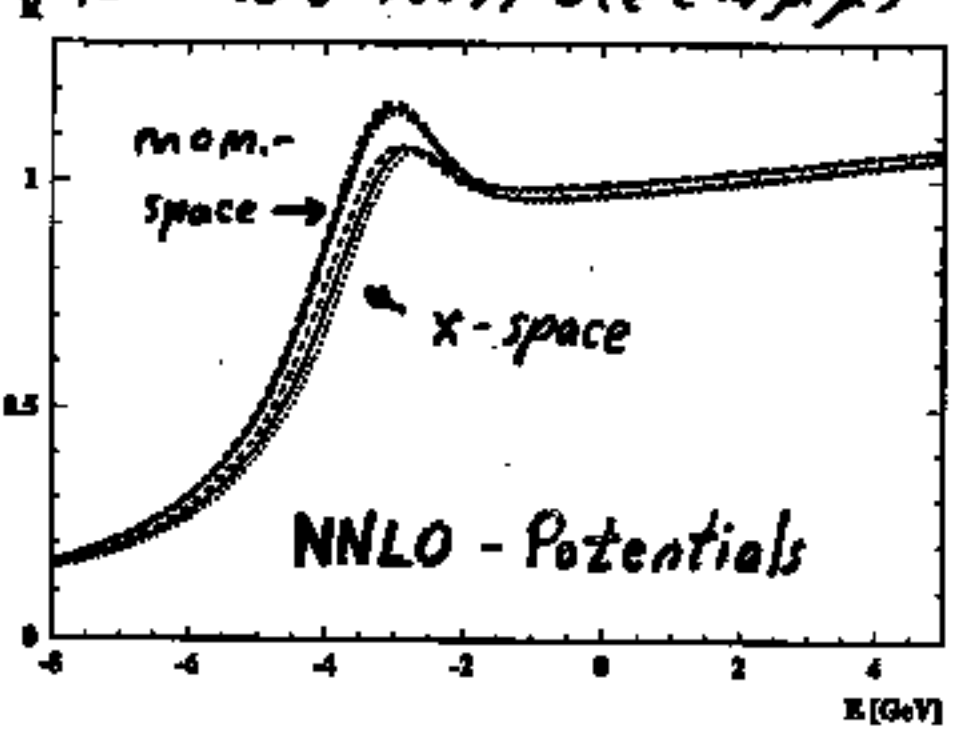
  Indication of unexpected large uncertainty
- In the normalization of R_t ⇒ H-studies difficult ! (higher Lumi will not help ...)

But : ($E \to E + i\Gamma_t$), but

- problem of $\Gamma_t(E,p)$ @ $O(\alpha_s^2)$ — Rescatter @ $O(\alpha_s^2)$ — solvable in time
- NNNLO ? No. (Most probably.)
- Def. of M_t. Use of $M_t(\mu)$
- ... summation of large remaining corrections ? But → Yuki S.'s talk

Comparison of the coordinate with the momentum space technique prediction of

$R = \sigma(e^+e^- \to t\bar{t}) / \sigma(e^+e^- \to \mu^+\mu^-)$



dashed: $\mu^2 = \frac{s}{2}$

solid: $\mu^2 = s$

dotted: $\mu^2 = 2s$

Variation of the ren.-scale

?! Next steps :

- $\frac{d\sigma}{dp}$, P-wave [, A_{FB} , Pol. , $\frac{d\sigma}{dE_1 d\Omega_1 dp_t}$] @ $O(\alpha_s^2)$
- (New) $t\bar{t}$ - Generator .
- Exp. analysis ... Discussion ...
- $\mu^+\mu^- \to t\bar{t}$

S11

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Date : April 6, 1998 - 11:15

Parallel groups to plan tasks : Top

Y. Sumino

Recent Theoretical Progress in $t\bar{t}$ Threshold Analyses

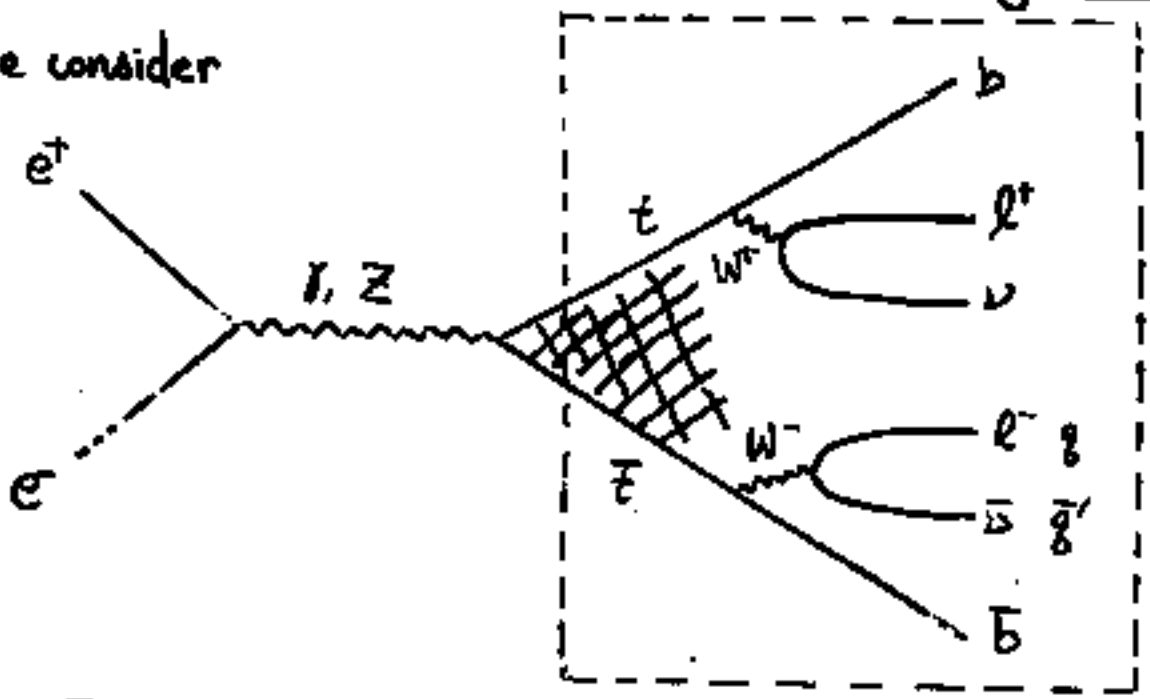
Topics:

- Top Polarization
- Top Decay Processes
- $O(\alpha_s^2)$ Corrections to Total Cross Section

Y. Sumino
(Univ. Karlsruhe/Tohoku Univ.)

§1. $e^+e^- \to t\bar{t} \to b\ell^+\nu\bar{b}W^-$: Production and Decay Chain

We consider



at $\sqrt{s} \approx 2m_t$.

The effects unique to $t\bar{t}$ threshold region:

(1) $t\bar{t}$-boundstate effects
(2) Final-state interactions

} much progress in understanding

§2 Top Polarization

Harlander, Jezabek, Kühn, Teubner
Harlander, Jezabek, Kühn, Peter

Top quarks are highly polarized near threshold.

P_e	P_t
+80%	$\sim +60\%$
0%	$\sim -40\%$
−80%	$\sim -90\%$

(Assumed: $P_{e^+} = 0$)

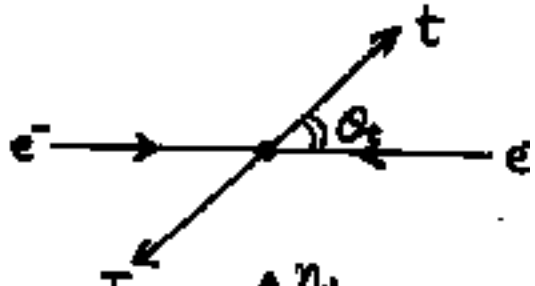
S-wave ($L=0$) dominate.

$\therefore \vec{J}_{\gamma,Z^*} = \vec{S}_t + \vec{S}_{\bar{t}} + \cancel{\vec{X}}$

Interference of S and P-waves contributes ~10% corrections.

Top Polarization Vector

$$\vec{S} = \vec{S}_{//} + \vec{S}_{\perp} + \vec{S}_N$$

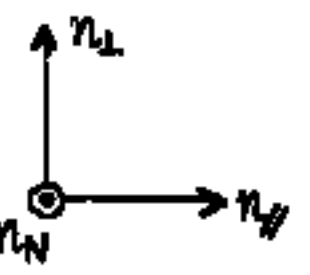


components given by

$$|\vec{S}_{//}| = \underbrace{C^0_{//}}_{\text{leading}} + C^1_{//}\,\mathcal{P}_R(P_t,E)\cos\theta_t$$

$$|\vec{S}_{\perp}| = C_{\perp}\,\mathcal{P}_R(P_t,E)\sin\theta_t \quad \leftarrow \text{S-P interference}$$

$$|\vec{S}_N| = C_N\,\mathcal{P}_I(P_t,E)\sin\theta_t$$



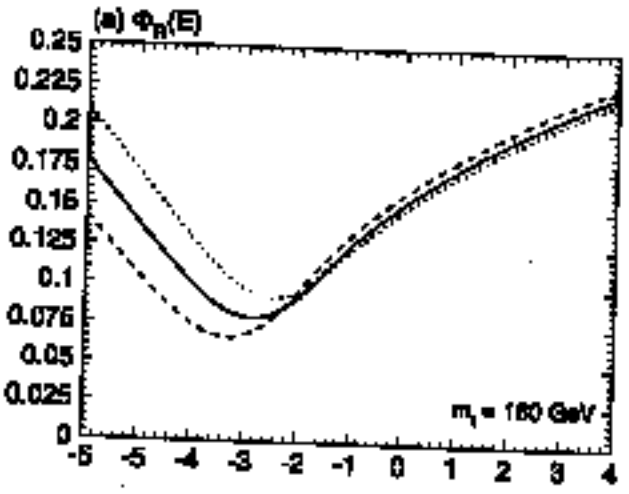
$$E = \sqrt{s} - 2m_t$$

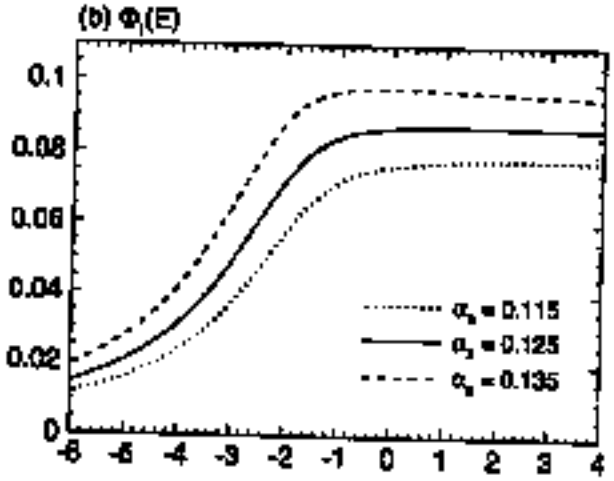
$C^0_{//}, C^1_{//}, C_\perp, C_N$: determined from electroweak parameters & P_e-

$\mathcal{P}_R, \mathcal{P}_I$: " S-P interference (incl. QCD bound-state effects)

At large E,

$$\Phi_R \equiv \langle \mathcal{P}_R \rangle \sim \sqrt{\frac{2E}{m_t}} = \left\langle \frac{P_t}{m_t} \right\rangle, \qquad \Phi_I \equiv \langle \mathcal{P}_I \rangle \sim \alpha_s$$



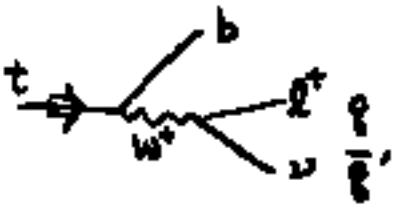


Harlander, Jeżabek, Kühn, Peter

§3. Decay of Top Quarks + Final-State Interactions

Peter + Y.

3-1) Decay of free polarized top quarks



e.g.

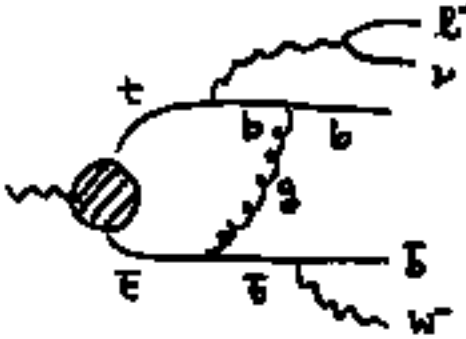
$$\frac{d\Gamma(t_\uparrow \to b\ell^+\nu)}{dE_\ell\, d\Omega_\ell} = h(E_\ell)\,(1 + S\cos\theta_\ell) + \mathcal{O}(\alpha_s)\ \text{corr.}$$

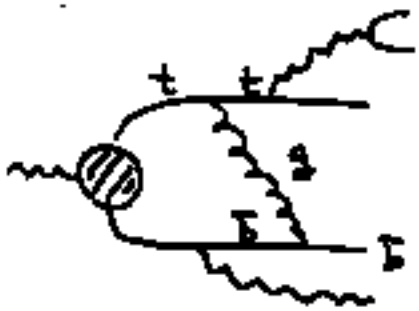
θ_ℓ : angle between $\vec{S}$ & $\vec{p}_\ell$

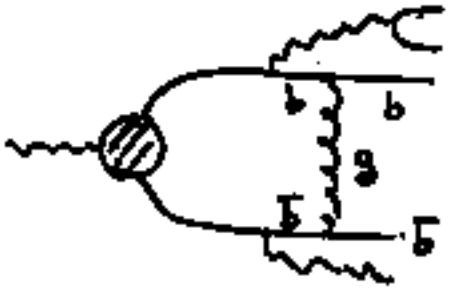
S : top polarization

Kühn, Strong
Jezabek, Kühn
Czarnecki, Jezabek, Kühn

3-2) Final-State Interactions

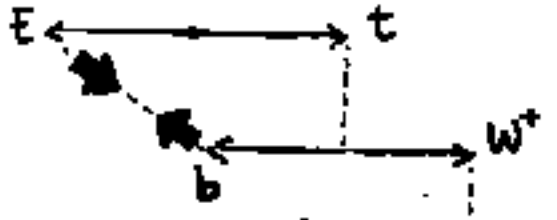






+ Real-gluon emission

e.g. Effect on ℓ^+ energy-angular distribution



$\ell^+ \longleftrightarrow \nu$: E_ℓ small & ℓ^+ in $-\vec{p}_t$-direction

$\nu \longleftrightarrow \ell^+$: E_ℓ large & ℓ^+ in $+\vec{p}_t$-direction

If the probability for b being emitted in $\vec{t}$ direction increases, then <u>above two configurations</u> increases.

energy-angle correlated

3-3) Differential Cross Section $e^+e^- \to t\bar{t} \to b\ell^+\nu\,\bar{b}W^-$

- Without Final-State Interactions (Factorization of production & decay)

$$\frac{d\sigma}{d^3p_t\, dE_\ell\, d\Omega_\ell} = \frac{d\sigma(e^+e^- \to t\bar{t})}{d^3p_t} \times \frac{1}{\Gamma_t}\frac{d\Gamma(t_S \to b\ell^+\nu)}{dE_\ell\, d\Omega_\ell} + O(\alpha_s^2)\ \text{corr.}$$

production cross section of unpolarized top

decay diff. distribution of ℓ^+ from polarized top.

- With Final-State Interactions

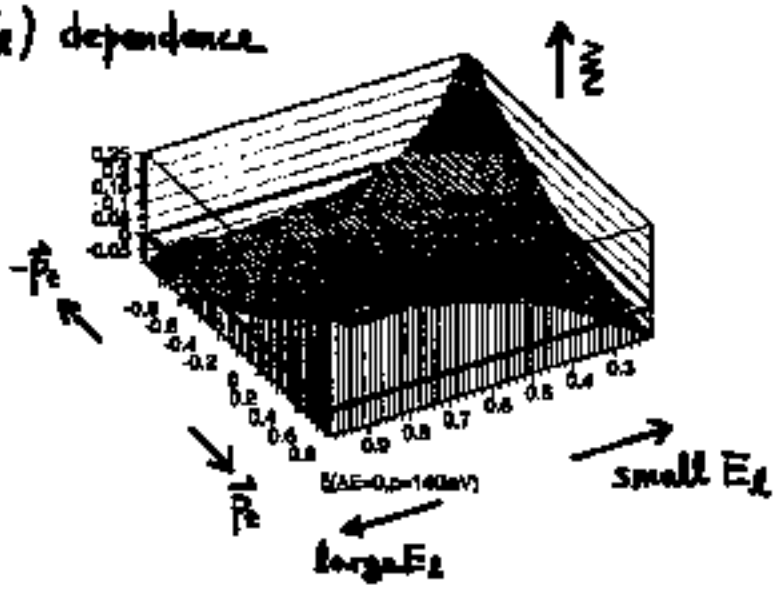
$$\boxed{\begin{aligned}\frac{d\sigma}{d^3p_t\, dE_\ell\, d\Omega_\ell} &= \frac{d\sigma(e^+e^- \to t\bar{t})}{d^3p_t} \times (1 + \underline{\delta_0} + \underline{\delta_1 \cos\theta_t}) \\ &\times \frac{1}{\Gamma_t}\frac{d\Gamma(t_{S+SS} \to b\ell^+\nu)}{dE_\ell\, d\Omega_\ell} \times \Big[1 + \underbrace{\xi(p_t, E, E_\ell, \cos\theta_{\ell E})}_{\text{Non-factorizable}}\Big] + O(\alpha_s^2)\end{aligned}}$$

$$\xi = C_F \cdot 4\pi\alpha_s \int_0^\infty dq \left\{ w_R \cdot \mathrm{Re}\left[\frac{\tilde{G}(q,E)}{\tilde{G}(p_t,E)}\right] - w_I \cdot \mathrm{Im}\left[\frac{\tilde{G}(q,E)}{\tilde{G}(p_t,E)}\right] \right\}$$

one-param. integral

w_R, w_I : fns of $p_t, q, \cos\theta_{\ell E}, E_\ell$

$(\cos\theta_{\ell E}, E_\ell)$ dependence



Peter + Y.S.

Harlander, Jaisbek, Kühn, Peter

4-1) Moments of charged lepton energy-angular distribution

- $\langle n\cdot p_\ell\rangle \equiv \left(\frac{d\sigma}{d^3\vec{p}_t}\right)^{-1} \int dE_\ell d\Omega_\ell \; (n\cdot p_\ell) \; \frac{d\sigma(e^+e^-\to b\ell^+\nu\bar{b}W^-)}{d^3\vec{p}_t\, dE_\ell d\Omega_\ell}$ fixing $\vec{p}_t$
- $\ll n\cdot p_\ell \gg$ fixing $p_t = |\vec{p}_t|$
- $\lll n\cdot p_\ell \ggg$ fully integrated

$\sim \vec{n}\cdot\vec{S} + n^0 = \text{const.}$

Since $$\frac{d\Gamma(t_S\to b\ell^+\nu)}{dE_\ell d\Omega_\ell} = h(E_\ell)\,(1+|\vec{S}|\cos\theta_\ell) + \mathcal{O}(\alpha_s)\ \text{corr.}$$

4-2) Observable Proper to Top Decay Processes — Peter + Y.S.

$$\bar{A}(E_\ell, a) \equiv \frac{\left.\frac{d\sigma(e^+e^-\to b\ell^+\nu\bar{b}W^-)}{dE_\ell\, da}\right|_{E_\ell,\, a=\vec{p}_\ell\cdot\vec{S}}}{\left.\text{“}\right|_{E_\ell',\, a=\vec{p}_\ell'\cdot\vec{S}}} \to \frac{\left[\frac{d\Gamma(t_S\to b\ell^+\nu)}{dE_\ell d\Omega_\ell}\right]_{E_\ell,\, a=\vec{p}_\ell\cdot\vec{S}}}{\left[\text{“}\right]_{E_\ell',\, a=\vec{p}_\ell'\cdot\vec{S}}}$$

$+\mathcal{O}(\alpha_s^2)$ corr.

$$\begin{cases} E_\ell' = 2m_t\left(\frac{m_t^2+M_W^2}{M_W^2} - \frac{2m_t}{E_\ell}\right)^{-1} & \text{: reversal of } W^+ \text{ direction} \\ \vec{p}_\ell' = -\vec{p}_\ell \end{cases}$$

$\bar{A}$ is determined only by the free top decay cross section.
Independent of boundstate effects or final-state interactions.
Same is true even if the top-decay vertices are anomalous.

§5. $O(\alpha_s^2)$ Corrections to Total Cross Sections & QCD Potential

● $O(\alpha_s^2)$ Corrections to σ_{tot} are large $\sim 15\% \sim O(\alpha_s)$ corr.

Hoang & Teubner
Melnikov & Yelkhovsky

• A large part originates from $\delta_2 V_{QQ}(r)$.

Its theoretical uncertainty has been studied within perturbative QCD. Jezabek, Kühn, Peter, Sumino, Teubner

$$V_{QQ}(r) = -C_F \frac{\alpha_V(1/r)}{r} \quad \xleftrightarrow[\text{Fourier tr.}]{} \quad \tilde{V}_{QQ}(q) = -C_F \frac{4\pi \tilde{\alpha}_V(q)}{q^2}$$

$$\alpha_V \text{ or } \tilde{\alpha}_V = \alpha'_{\overline{MS}} + \star\, \alpha_{\overline{MS}}^2 + \star\, \alpha_{\overline{MS}}^3 + O(\alpha_{\overline{MS}}^4)$$

known up to $\alpha_{\overline{MS}}^3$ Peter

Relation between α_V & $\tilde{\alpha}_V$ is known to α_V^5:

$$\alpha_V(1/r) = \tilde{\alpha}_V + 1.225\,\tilde{\alpha}_V^3 + 5.596\,\tilde{\alpha}_V^4 + 22.202\,\tilde{\alpha}_V^5 + O(\tilde{\alpha}_V^6) \quad (n_f = 5)$$

$\uparrow$

$\tilde{\alpha}_V = \tilde{\alpha}_V(1 - e^{\gamma_E}/r)$

1) At $1/r \simeq 100\,\text{GeV}$, $\delta\alpha_V/\alpha_V \simeq 3\%$

$\sigma_{peak} \propto \alpha_V^2 \implies \delta\sigma_{peak}/\sigma_{peak} \gtrsim 6\%$

2) Even if $O(\alpha_{\overline{MS}}^4)$ corr. is calculated, it is likely $\delta\sigma_{peak}/\sigma_{peak}$ will not be reduced due to the asymptoticness of the perturbative expansion.

§6. Summary

1) Top Polarization

- High Polarization
- S-P interference

} (EW-parameter/ boundstate dynamics)

2) Top Decays

- Effect of final-state interactions

$$\frac{d\sigma(e^+e^- \to t\bar{t} \to b\ell^+\nu\, \bar{b}W^-)}{d^3\vec{p}_t\, dE_\ell\, d\Omega_\ell} = \frac{d\sigma(e^+e^- \to t\bar{t})}{d^3\vec{p}_t}\,(1+\delta)\cdot\frac{1}{\Gamma_t}\frac{d\Gamma(t \to b\ell^+\nu)}{dE_\ell\, d\Omega_\ell}\times(1+\tfrac{\alpha_s}{\pi})$$

$$+ \mathcal{O}(\alpha_s^2)\ \text{corr}$$

3) New Observables

- $\langle \eta_{p_\ell} \rangle \sim \vec{n}\cdot\vec{S}$
- $$A \equiv \frac{\left\langle \left[\frac{d\sigma(e^+e^- \to b\ell^+\nu\, \bar{b}W^-)}{d^3\vec{p}_t\, dE_\ell\, d\Omega_\ell} \right]_{E_\ell,\ \vec{p}_t\cdot\vec{S}} \right\rangle}{\left\langle \left[\quad " \quad \right]_{E_\ell',\ -\vec{p}_t\cdot\vec{S}} \right\rangle} \to \frac{\left[\frac{d\Gamma(t \to b\ell^+\nu)}{dE_\ell\, d\Omega_\ell} \right]_{E_\ell,\ \vec{p}_t\cdot\vec{S}}}{\left[\quad " \quad \right]_{E_\ell',\ -\vec{p}_t\cdot\vec{S}}}$$

$$+ \mathcal{O}(\alpha_s^2)\ \text{corr.}$$

⇒ Further studies of top form factors near threshold using highly polarized top samples + large cross section!

4) $\mathcal{O}(\alpha_s^2)$ Corrections to σ_{tot} / QCD potential

- $\delta\sigma_{peak}/\sigma_{peak} \gtrsim 6\%$ from uncalculated $\mathcal{O}(\alpha_s^3)$ corr.
- Problem of asymptoticness of perturbative series.