IX. Electroweak unification

- Theory of weak interactions only by means of $W^{\pm}$ bosons leads to *divergences*: cross-sections of processes involving two W bosons grow *infinitely* with increasing energy

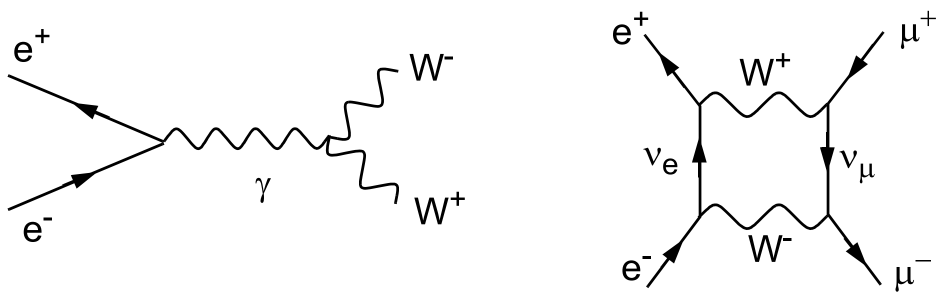


Figure 112: Examples of divergent processes

- A “good” theory (such as QED) must be *renormalizable*: all expressions can be made finite by re-expressing them in a finite number of physical parameters (like e, m_e and $\hbar$ in QED)
  - Electroweak theory is actually renormalizable, though demonstration of it is highly non-trivial

❖ Introduction of Z^0 boson fixes the divergence problem: Z^0 can couple to two W bosons and thus cancel the divergence

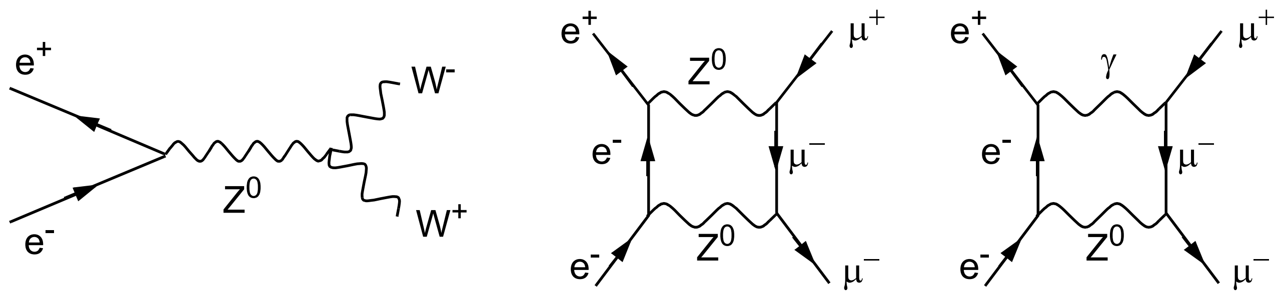


Figure 113: Additional processes to cancel divergence

◎ The divergence can also be cancelled by introducing a “heavy electron”, but experimental evidence unambiguously favors Z^0

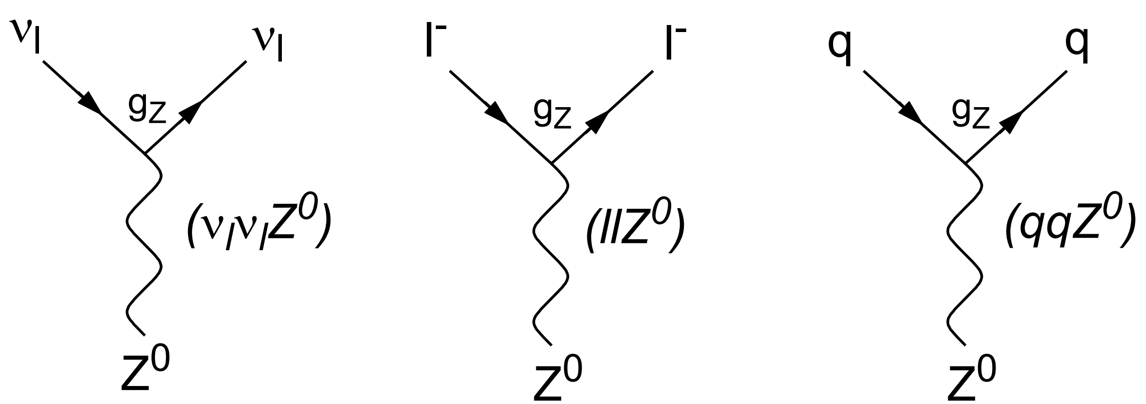


Figure 114: Basic vertices for Z^0-lepton and Z^0-quark couplings

Rules for Z^0 boson vertices:

– Conserved lepton numbers

– Conserved quark flavour (*remember,* in W vertices, quark flavour is not conserved)

- By applying quark-lepton symmetry and assuming there is quark mixing:

$$d'd'Z^0 + s's'Z^0 = (d\cos\theta_C + s\sin\theta_C)(d\cos\theta_C + s\sin\theta_C)Z^0 +$$

$$+(-d\sin\theta_C + s\cos\theta_C)(-d\sin\theta_C + s\cos\theta_C)Z^0 = ddZ^0 + ssZ^0$$

- It is not necessary to apply quark mixing in Z^0 vertices

Experimental test of flavour conservation at Z^0 vertex:

$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$ (a) $K^+ \rightarrow \pi^+ + \nu_l + \bar{\nu}_l$ (b)

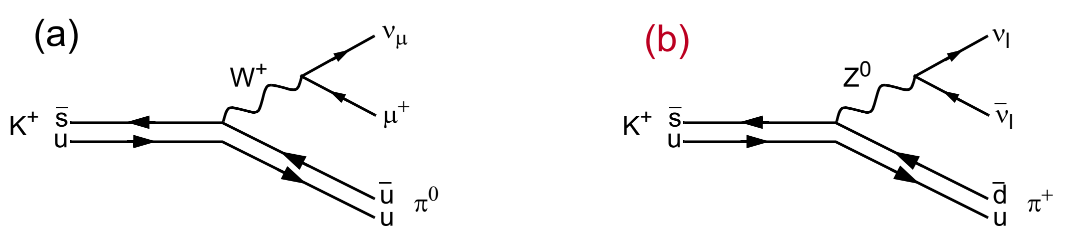


Figure 115: Decay (a) is allowed; decay (b) – forbidden

- Experiment 787 at the Brookhaven National Laboratory (BNL): a dedicated rare kaon decay experiment
  - K^+beam (created by 24 GeV protons from the AGS accelerator) is deposited onto a fixed target

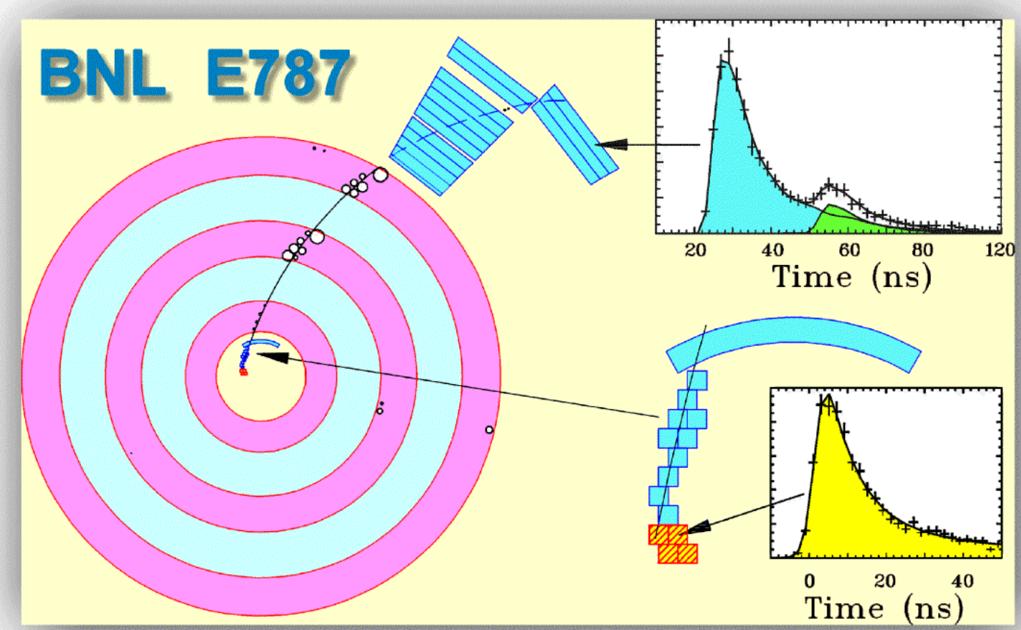


Figure 116: Picture of a rare event in E787 (a single pion track). Only two such events have been observed in 12 years

Measured upper limit on the ratio of the decay rates is:

$$\frac{\sum_{l} \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} = 5 \times 10^{-9}$$

- With this rate, the observed decays can not be due to the flavor-violating Z^0 decays

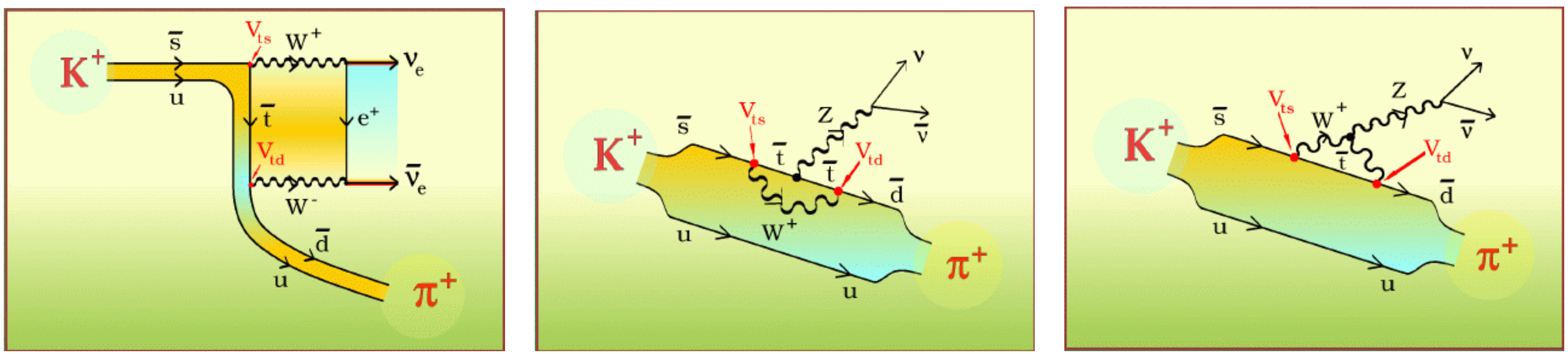


Figure 117: Second-order charged interactions that can explain the observed rare kaon decays.

- Thanks to the t-d vertex in the third diagram, one can estimate the V_{td} element of the CKM matrix:

$$0.007 < |V_{td}| < 0.030$$

Unification condition and boson masses

Comparing vertices involving γ, $W^{\pm}$ and Z^0, one can conclude that they are not independent and can be expressed via the same constant

For a consistent electroweak theory, two conditions are introduced:

❖ The *unification condition* relates coupling constants:

$$\sqrt{\frac{\pi \cdot \alpha_{em}}{2}} = g_W \sin\theta_W = g_Z \cos\theta_W \qquad (162)$$

θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos\theta_W = \frac{M_W}{M_Z} \qquad (163)$$

❖ The *anomaly condition* relates electric charges:

$$\sum_l Q_l + 3\sum_q Q_q = 0 \qquad (164)$$

In the zero-range approximation (see Eq.(146)):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} \Rightarrow M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha}{\sqrt{2}G_F sin^2\theta_W} \quad (165)$$

If we introduce also the neutral current coupling (in low energy zero-range approximation, as usual):

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2} \quad (166)$$

the weak mixing angle can then be expressed as

$$\frac{G_Z}{G_W} = \frac{g_Z^2 M_W^2}{g_W^2 M_Z^2} = sin^2\theta_W \quad (167)$$

From measurements of rates of charged and neutral currents reactions,

$$sin^2\theta_W = 0.227 \pm 0.014$$

which allowed to predict masses of W (using Eq.(165)) and hence Z, as:

$$M_W = 78.3 \pm 2.4\ GeV/c^2;\ M_Z = 89.0 \pm 2.0\ GeV/c^2$$

The most recent result (at Z^0 *peak*):

$$sin^2\theta_W = 0.23122 \pm 0.00015 \tag{168}$$

However, the most precise value for mass ratio is somewhat different:

$$1 - \frac{M_W^2}{M_Z^2} = 0.22318 \pm 0.0052$$

The difference comes from higher-order diagrams:

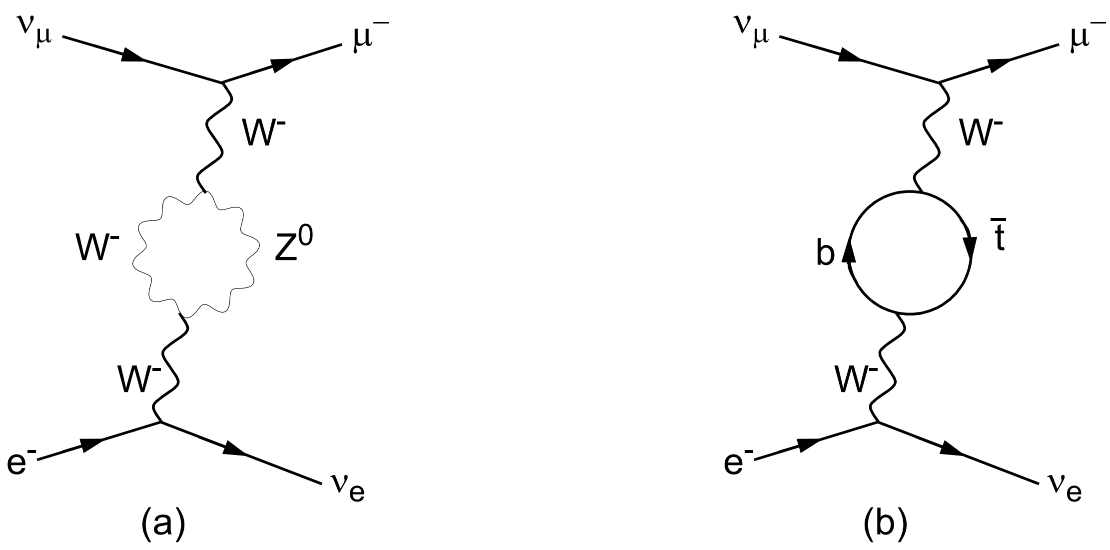


Figure 118: Examples of higher order contributions to inverse muon decay

From higher order corrections, the prediction for the top-quark mass was:

$$m_t = 170 \pm 30\ GeV/c^2 \tag{169}$$

Direct observation gives the value of $m_t = 174.2 \pm 3.3\ GeV/c^2$

- Predictions for W, Z and top masses were the most impressive successes of the electroweak theory

- In any process in which a photon is exchanged, a Z^0 boson can be exchanged as well; in addition, Z^0 couples to neutrinos:

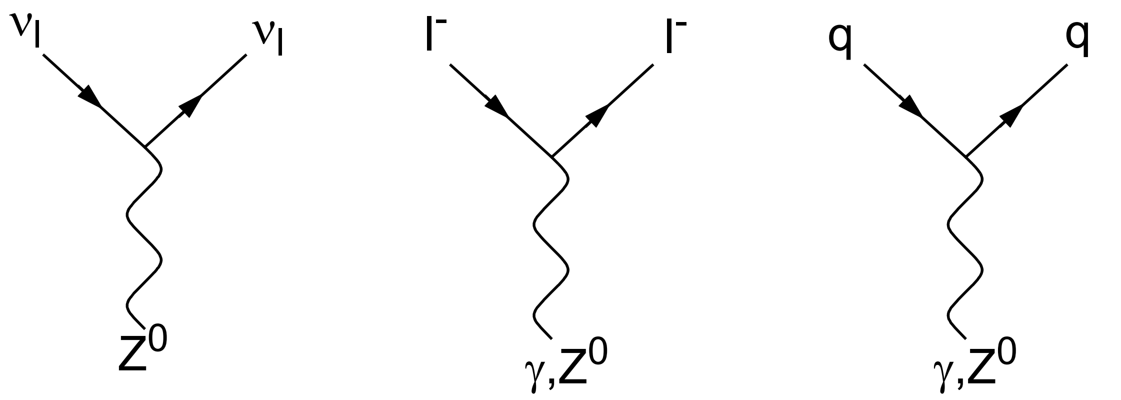


Figure 119: Z^0 and γ couplings to leptons and quarks

Example: reaction $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions:

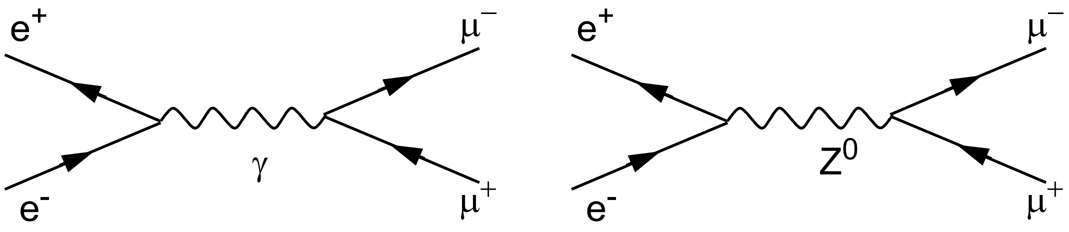


Figure 120: Dominant contributions to the e^+e^- annihilation into muons

With simple dimensional arguments one can estimate the cross section for the photon- and Z-exchange process *at low energy*:

$$\sigma_\gamma \approx \frac{\alpha^2}{E^2} \qquad \sigma_Z \approx G_Z^2 E^2 \tag{170}$$

From Eq.(170), ratio of σ_Z and σ_γ is:

$$\frac{\sigma_Z}{\sigma_\gamma} \approx \frac{E^4}{M_Z^4} \tag{171}$$

At low energies, photon exchange dominates. At high energies ($E=M_Z$), this low-energy approximation fails.

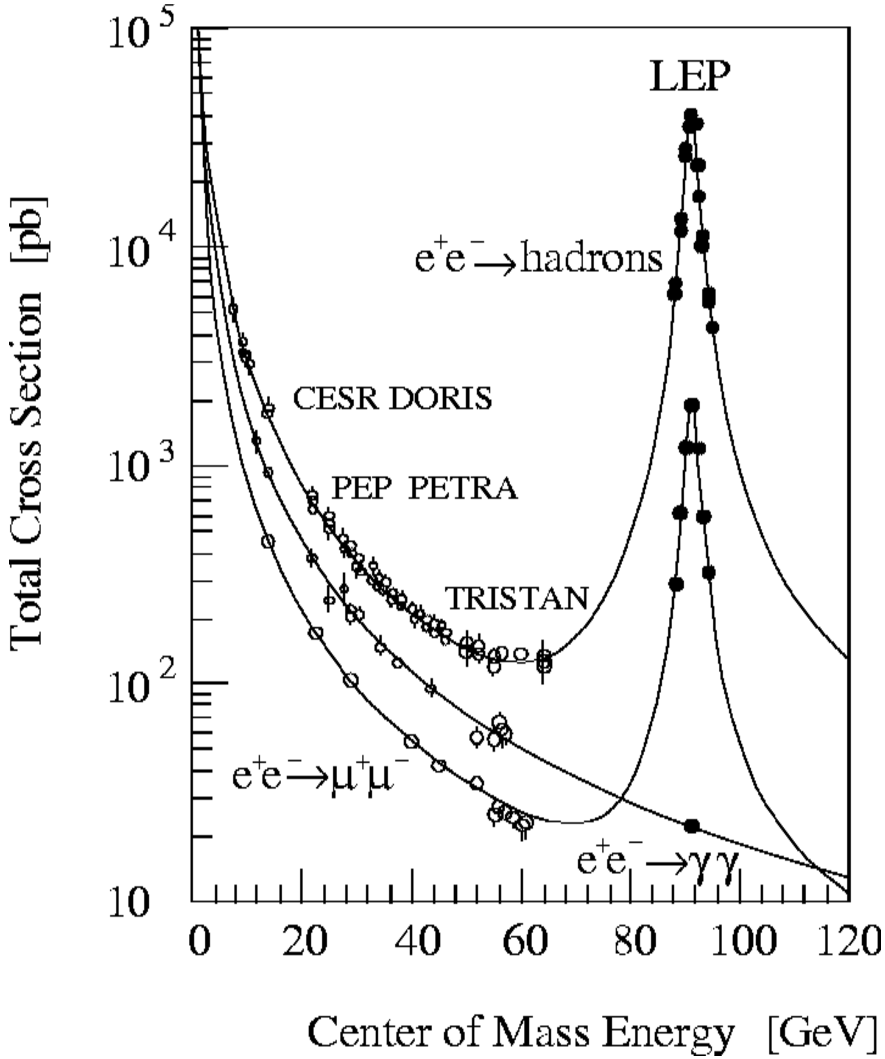


Figure 121: Total cross sections of e^+e^- annihilation

- Z^0 peak is described by the Breit-Wigner formula:

$$\sigma(e^+e^- \to X) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to e^+e^-)\Gamma(Z^0 \to X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right] \tag{172}$$

Here Γ_Z is the total Z^0 decay rate, and $\Gamma_Z(Z^0 \to X)$ are decay rates to other final states.

Height of the peak (at $E_{CM}=M_Z$) is then proportional to the product of branching ratios:

$$B(Z^0 \to e^+e^-)B(Z^0 \to X) \equiv \frac{\Gamma(Z^0 \to e^+e^-)}{\Gamma_Z}\frac{\Gamma(Z^0 \to X)}{\Gamma_Z} \tag{173}$$

Fitted parameters of the Z^0 peak:

$$\begin{aligned} M_Z &= 91.187 \pm 0.007\ GeV/c^2 \\ \Gamma_Z &= 2.490 \pm 0.007\ GeV \end{aligned} \tag{174}$$

- Fitting the peak with Eq.(172), not only M_Z and Γ_Z can be found, but also partial decay rates:

$$\Gamma(Z^0 \rightarrow hadrons) = 1.741 \pm 0.006\ GeV \tag{175}$$

$$\Gamma(Z^0 \rightarrow l^+ l^-) = 0.0838 \pm 0.0003\ GeV \tag{176}$$

- Decays $Z^0 \rightarrow l^+ l^-$ and $Z^0 \rightarrow hadrons$ account for only about 80% of all Z^0 decays

- Remaining decays are those containing only neutrinos in the final state

$$\Gamma_Z = \Gamma(Z^0 \rightarrow hadrons) + 3\Gamma(Z^0 \rightarrow l^+ l^-) + + N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) \tag{177}$$

From Eqs.(174)-(176):

$$N_\nu \Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0.498 \pm 0.009\ GeV$$

Decay rate to neutrino pairs is calculated from diagrams of Figure 119:

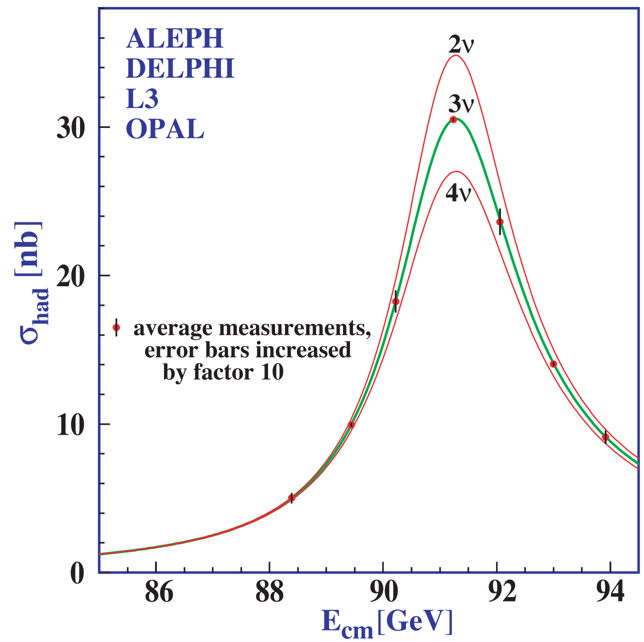
$$\Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0.166\ GeV \qquad (178)$$

which means that $N_\nu \approx 3$. More precisely,

$$N_\nu = 2.994 \pm 0.011 \qquad (179)$$

- There are no explicit restrictions on number of generations in the Standard Model

- However, analysis of Z^0 line shape shows that there are 3 and only 3 kinds of massless neutrinos.
  - If neutrinos are assumed having negligible masses as compared with the Z^0 mass, there must be only THREE generations of leptons and quarks within the Standard Model.



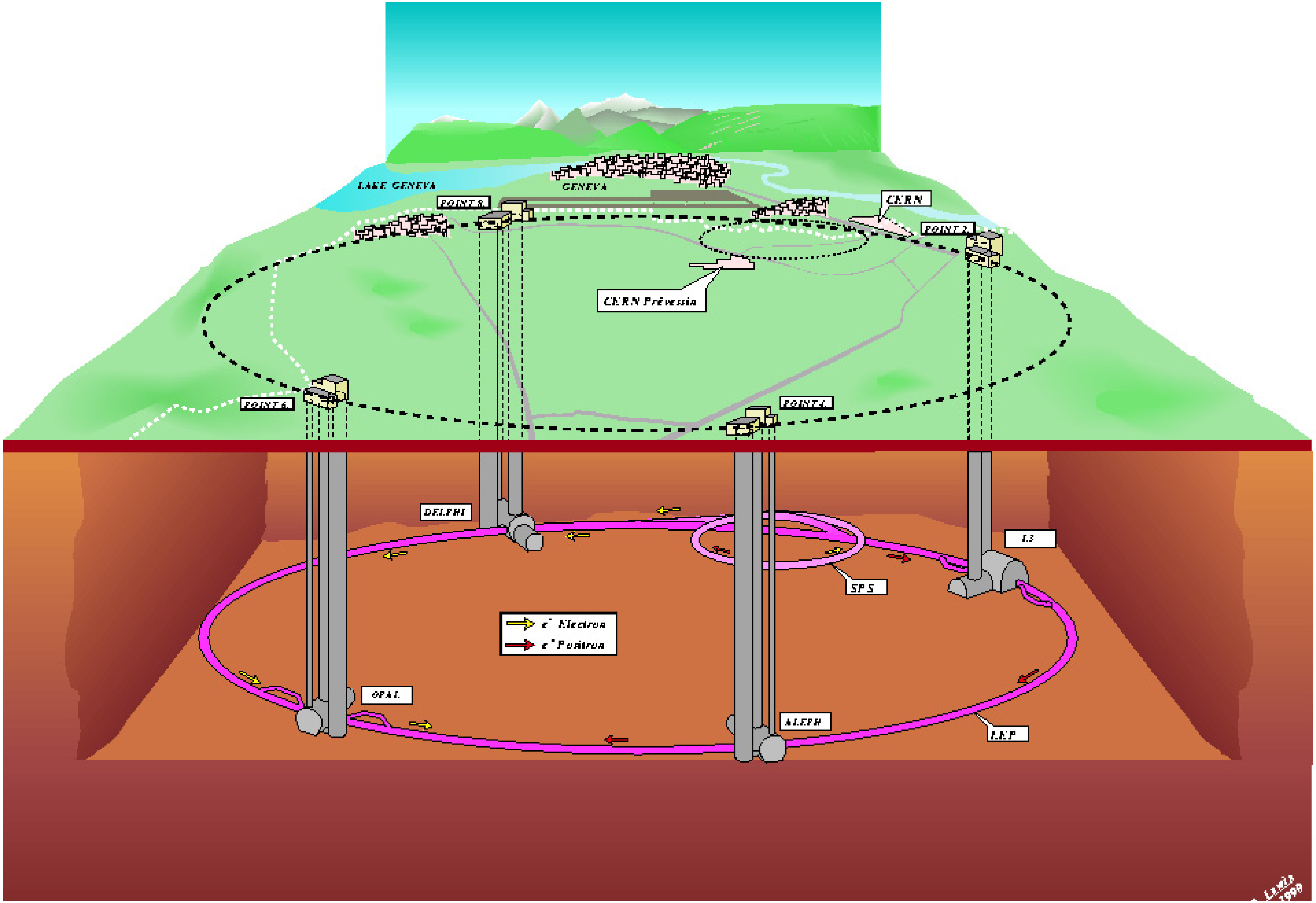


Figure 122: LEP electron-positron collider at CERN (1989-2000)

- Renormalizable theories are *gauge invariant* theories
  - Gauge transformation: certain alteration of a quantum field variables that leave basic properties of the field unchanged; a symmetry transformation
  - There are several forms of gauge invariance corresponding to different interactions

In QED, Schrödinger equation must be invariant under the phase transformation of the wavefunction:

$$\psi(\vec{x}, t) \rightarrow \psi'(\vec{x}, t) = e^{iq\alpha(\vec{x}, t)}\psi(\vec{x}, t) \tag{180}$$

Here $\alpha(\vec{x}, t)$ is an arbitrary continuous function.

If a particle is free, then

$$i\frac{\partial\psi(\vec{x}, t)}{\partial t} = -\frac{1}{2m}\nabla^2\psi(\vec{x}, t) \tag{181}$$

- Transformed wavefunction $\psi'(\vec{x}, t)$ can not be a solution of the Schrödinger equation (181)
- *Gauge principle*: to keep the invariance condition satisfied, a minimal field should be added to the Schrödinger equation, i.e., an <u>interaction</u> should be introduced

In QED, the transition from one electron state to another with different phase, $e^- \to e^-$, demands emission (or absorption) of a photon: $e^- \to e^-\gamma$

More generally, in the electroweak theory, transformations like

$$e^- \to \nu_e \qquad \nu_e \to e^- \qquad e^- \to e^- \qquad \nu_e \to \nu_e$$

lead via the gauge principle to interactions

$$e^- \to \nu_e W^- \quad \nu_e \to e^- W^+ \quad e^- \to e^- W^0 \quad \nu_e \to \nu_e W^0$$

- W^+, W^- and W^0 are corresponding spin-1 gauge bosons.

While W^+ and W^- are the well-known charged currents, W^0 as such has not been observed experimentally

- Electroweak unification regards both Z^0 and γ as mixtures of W^0 and yet another *neutral boson B^0*:

$$\gamma = B^0 \cos\theta_W + W^0 \sin\theta_W$$
$$Z^0 = -B^0 \sin\theta_W + W^0 \cos\theta_W \qquad (182)$$

The corresponding gauge transformation is:

$$\psi_l(\vec{x}, t) \to \psi_l'(\vec{x}, t) = e^{i g_Z y_l \alpha(\vec{x}, t)} \psi_l(\vec{x}, t) \qquad (183)$$

Here l stands for electron or neutrino and y_l are corresponding constants

Introduction of B^0 leads to extra vertices

$$e^- \to e^- B^0 \qquad \nu_e \to \nu_e B^0$$

with new couplings $g_Z y_{e^-}$ and $g_Z y_{\nu_e}$. If the unification condition (162) is satisfied, first mixture in (182) indeed has the coupling of a photon.

- Electroweak theory can be made gauge-invariant by introducing neutral bosons W^0 and B^0.
  - Generally, experimental data agree with gauge invariant electroweak theory predictions.
- However, gauge invariance implies that spin-1 bosons have <u>zero masses</u> if they are the only bosons in the theory (photon and gluon nicely comply with this requirement)

$$\Downarrow$$

one more field should exist

The Higgs Boson

- The scalar *Higgs field* solves the problem by introducing a yet another particle, the *Higgs boson*

– *Higgs boson* H^0 is a spin-0 particle

– Higgs field has a **non-zero** value ϕ_0 in vacuum

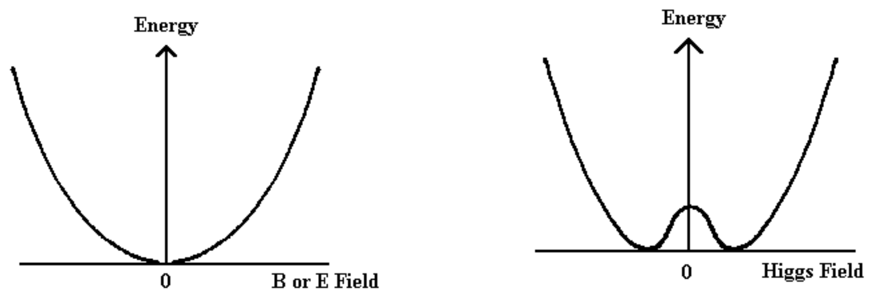


Figure 123: Comparison of the electric and Higgs fields

- The vacuum value ϕ_0 is not gauge invariant $\Rightarrow$ *hidden gauge invariance*, or *spontaneously broken symmetry*.

- Vacuum hence is supposed to be populated with massive Higgs bosons $\Rightarrow$ when a gauge field interacts with the Higgs field it acquires mass (e.g. W and Z bosons become massive)

- In the same manner, fermions acquire masses by interacting with Higgs bosons:

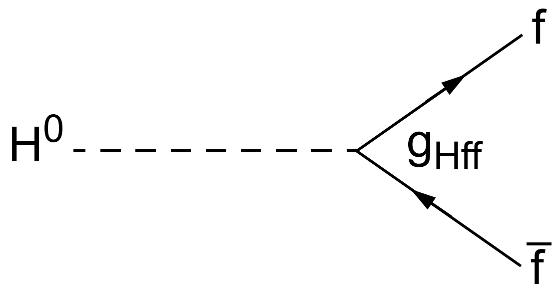


Figure 124: A basic vertex for Higgs-fermion interactions

The coupling constant is related to the fermion mass:

$$g^2_{Hff} = \sqrt{2} G_F m_f^2 \tag{184}$$

- The mass of the Higgs itself is not predicted by the theory, only its couplings to other particles are predicted (Eq.(184))

- Existence of the Higgs has not been (yet) confirmed experimentally
  - There are other ways to satisfy the gauge invariance requirements, but they are less elegant and produced no experimental proof either

Possible signatures of the Higgs:

a) If H^0 is lighter than Z^0 (rather, $M_H \leq 50$ GeV/c^2), then Z^0 can decay by

$$Z^0 \to H^0 + l^+ + l^- \qquad (185)$$

$$Z^0 \to H^0 + \nu_l + \bar{\nu}_l \qquad (186)$$

But the branching ratio is very low:

$$3 \times 10^{-6} \leq \frac{\Gamma(Z^0 \to H^0 l^+ l^-)}{\Gamma_{tot}} \leq 10^{-4}$$

With the LEP statistics, they still should have been detectable; since the reactions (185) and (186) have not been observed, the *lower limit* is $M_H > 58$ GeV/c^2

b) If H^0 is significantly heavier than 60 GeV/c^2, it can be produced in e^+e^- annihilation at higher energies:

$$e^+ + e^- \rightarrow H^0 + Z^0 \tag{187}$$

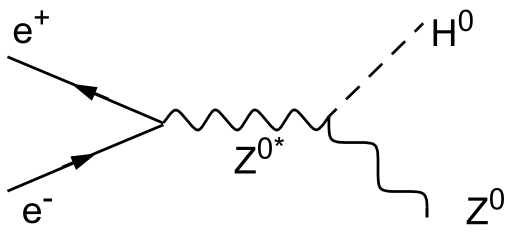


Figure 125: “Higgsstrahlung” in e^+e^- annihilation

- In such a reaction, Higgs with mass up 90 GeV/c^2 could have been detected by observing H^0 decaying into a $b\bar{b}$ pair and Z to a $q\bar{q}$ pair (4 jets, of which 2 are *b-tagged*)
- In the closing days of LEP, ALEPH experiment reported a couple of such candidate Higgs events. Other experiments saw no events of this kind.

Up-to-date limit established by LEP is:

$$M_H > 114.4\ GeV/c^2 \tag{188}$$

c) Higgs with masses up to 1 TeV can be observed at the future proton-proton collider LHC at CERN:

$$p + p \rightarrow H^0 + X \tag{189}$$

where H^0 is produced in electroweak interaction between the quarks

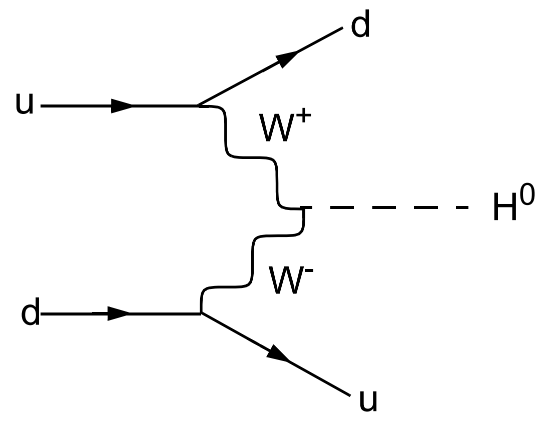


Figure 126: An example of Higgs production process at LHC

Due to the heavy background, good signatures have to be considered:

- If $M_H > 2M_Z$, then the dominant decay modes are:

$$H^0 \rightarrow Z^0 + Z^0 \tag{190}$$

$$H^0 \rightarrow W^- + W^+ \tag{191}$$

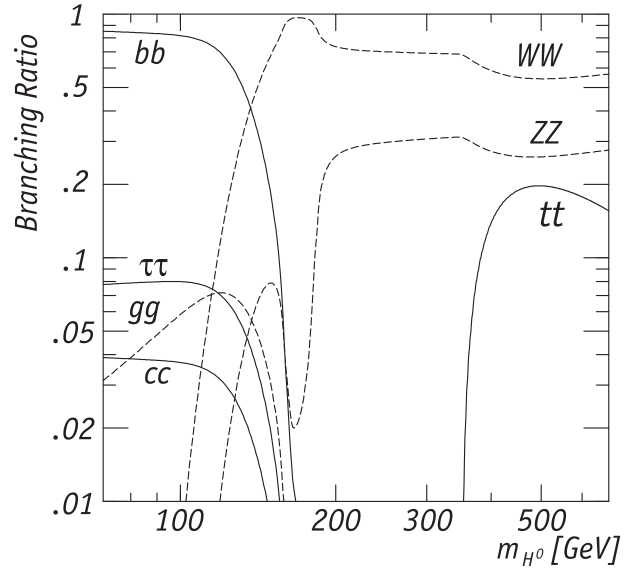


Figure 127: Branching ratios for the main decays of the SM Higgs boson

The most clear signal is when both Z^0 decay into electron or muon pairs:

$$H^0 \rightarrow l^+ + l^- + l^+ + l^- \qquad (192)$$

This will mean 200 GeV/$^2 \leq M_H \leq$ 500 GeV/c^2, but only 4% of all decays

- If $M_H < 2M_W$, the dominant decay mode is

$$H^0 \to b + \bar{b} \qquad (193)$$

but this gives indistinguishable signal at LHC. Another mode is

$$H^0 \to \gamma + \gamma \qquad (194)$$

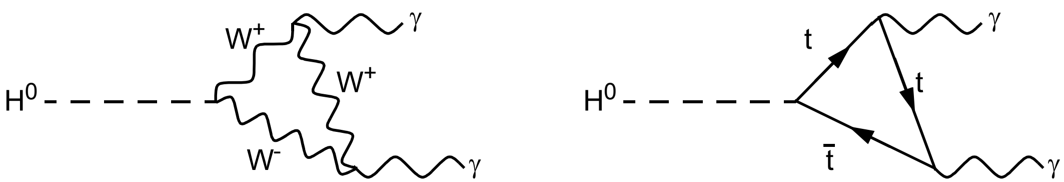


Figure 128: The dominant mechanisms for the decay (194)

Branching ratio of this kind of processes is about 10^{-3}

- The neutral Higgs is the minimal requirement; there might exist more complicated variants, including charged higgs-particles. None have been detected so far.