Quantum Chromodynamics (QCD) is the theory of strong interactions, i.e., the force field that binds quarks together in protons and neutrons.
Interactions are carried out by massless spin-1 particles called gauge bosons.

- In quantum electrodynamics (QED), gauge bosons are photons and in QCD they are called gluons.

- Gauge bosons couple to conserved charges:
  - QED: Photons couple to electric charges (Q)
  - QCD: Gluons couple to colour charges ($Y^c$ and $I^c_3$).

- $Y^c$ is called colour hypercharge.
  - $I^c_3$ is called colour isospin charge.

- The strong interaction acts the same on $u,d,s,c,b$ and $t$ quarks because the strong interaction is flavour-independent.
The colour hypercharge ($Y_c$) and colour isospin charge ($I_3^c$) can be used to define three colour and three anti-colour states that the quarks can be in:

$$
\begin{array}{c|cc}
Y_c & I_3^c \\
\hline
r & 1/3 & 1/2 \\
g & 1/3 & -1/2 \\
b & -2/3 & 0 \\
\hline
\end{array}
\quad
\begin{array}{c|cc}
Y_c & I_3^c \\
\hline
\bar{r} & -1/3 & -1/2 \\
\bar{g} & -1/3 & 1/2 \\
\bar{b} & 2/3 & 0 \\
\end{array}
$$

All observed states (all mesons and baryons) have a total colour charge that is zero. This is called colour confinement.

Zero colour charge means that the hadrons have the following colour wave-functions:

$$
qq = \frac{1}{\sqrt{3}} (rr + gg + bb)
$$

$$
q_1q_2q_3 = \frac{1}{\sqrt{6}} (r_1g_2b_3 - g_1r_2b_3 + b_1r_2g_3 - b_1g_2r_3 + g_1b_2r_3 - r_1b_2g_3)
$$
The colour hypercharge ($Y^c$) and colour isospin charge ($I^c_3$) should not be confused with the flavour hypercharge ($Y$) and flavour isospin ($I_3$) that were introduced in the quark model:

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After introducing colour, the total wavefunction of hadrons can now be written as:

$$\Psi_{\text{total}} = \Psi_{\text{space}} \times \Psi_{\text{spin}} \times \Psi_{\text{flavour}} \times \Psi_{\text{colour}}$$
Photons do not carry electric charge but gluons do carry colour charges themselves!

- The gluons can in fact exist in 8 different colour states given by the following colour wave functions:

\[
\begin{align*}
\chi^C_{g_1} &= r \bar{g} & \Gamma^C_3 &= 1 & Y^C &= 0 \\
\chi^C_{g_2} &= \bar{r} g & \Gamma^C_3 &= -1 & Y^C &= 0 \\
\chi^C_{g_3} &= r \bar{b} & \Gamma^C_3 &= \frac{1}{2} & Y^C &= 1 \\
\chi^C_{g_4} &= \bar{r} b & \Gamma^C_3 &= -\frac{1}{2} & Y^C &= -1 \\
\chi^C_{g_5} &= g \bar{b} & \Gamma^C_3 &= -\frac{1}{2} & Y^C &= 1 \\
\chi^C_{g_6} &= \bar{g} b & \Gamma^C_3 &= \frac{1}{2} & Y^C &= -1 \\
\chi^C_{g_7} &= \frac{1}{\sqrt{2}} ( g \bar{g} - r \bar{r} ) & \Gamma^C_3 &= 0 & Y^C &= 0 \\
\chi^C_{g_8} &= \frac{1}{\sqrt{6}} ( g \bar{g} - r \bar{r} - 2 b \bar{b} ) & \Gamma^C_3 &= 0 & Y^C &= 0
\end{align*}
\]

- Gluons do not exist as free particles since they have colour charge.
The colour hypercharge and colour isospin charge are additive quantum numbers like the electric charge. The gluon colour charge in the following process can therefore be easily calculated:

Example:

\[
\begin{align*}
I_3^u &= 1/2 & Y^u &= 1/3 \\
I_3^s &= 0 & Y^s &= -2/3 \\
I_3^r &= 0 & Y^r &= -2/3 \\
I_3^b &= 1/2 & Y^b &= 1/3
\end{align*}
\]

Gluon:

\[
\begin{align*}
I_3^C &= I_3^C(r) - I_3^C(b) = \frac{1}{2} \\
Y^C &= Y^C(r) - Y^C(b) = 1
\end{align*}
\]

\[
\chi_{g^3}^c = r \bar{b}
\]
Gluons can couple to other gluons since they carry colour charge.

- This means that gluons can in principle bind together to form colourless states.

- These gluon states are called glueballs.
The quark-antiquark potential can be described in the following simplified way:

\[
V(r) = \begin{cases} 
\frac{4\alpha_s}{3} \frac{1}{r} & (r < 0, 1 \text{ fm}) \\
\lambda r & (r \geq 1 \text{ fm})
\end{cases}
\]
The strong coupling constant

The strong couplings constant $\alpha_s$ is the analogue in QCD of $\alpha_{\text{em}}$ in QED and it is a measure of the strength of the interaction.

It is not a true constant but a “running constant” since it decreases with increasing $Q^2$.

What is $Q^2$?

Assume that the 4-vectors of the interacting quarks are given by $\bar{P} = (E, \bar{p}) = (E, p_x, p_y, p_z)$.

The 4-vector energy-momentum transfer is then $Q^2 = -\bar{q} \cdot \bar{q}$ (i.e. the “mass” of the gluon) which can be calculated from the 4-vectors of the quarks $\bar{q} = (E_q, \bar{q}) = \bar{P}_1 - \bar{P}_2 = (E_1 - E_2, \bar{P}_1 - \bar{P}_2)$.
In leading order of QCD, $\alpha_s$ is given by:

$$\alpha_s = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2)}$$

where

$N_f$: Number of allowed quark flavours

$\Lambda$: QCD scale parameter that has to be determined experimentally ($\Lambda \approx 0.2 \text{ GeV}$)
At short distances the strong interaction is weaker and at large distance the interaction gets stronger.

The combination of a Coulomb-like potential at small distances and a small $\alpha_s$ at large $Q^2$ (i.e. small distances) means that quarks and gluons act as essentially free particles and interactions can be described by the lowest order diagrams.

At large distances the strong interaction can, however, only be described by higher order diagrams.

Due to the complexity of the higher-order diagrams, the very process of confinement cannot be calculated analytically. Only numerical models can be used!
Electron-positron annihilation

The R-value

At \(e^+e^-\) colliders one has traditionally studied the ratio of the number of events with hadrons to those with muons:

\[
R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}
\]

The cross section for hadron and muon production would be almost the same if it was not for quark flavours and colours i.e.

\[
R = N_c \sum e_q^2
\]

where \(N_c\) is the number of colours (=3) and \(e_q\) the charge of the quarks.
Electron-positron annihilation

\[ R = N_c (e_u^2 + e_d^2 + e_s^2) = 3 \left( \frac{-1}{3} \right)^2 + \left( \frac{-1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 = 2 \quad \text{if } \sqrt{s} < m_\psi \]

\[ R = N_c (e_u^2 + e_d^2 + e_s^2 + e_c^2) = \frac{10}{3} \quad \text{if } \sqrt{s} < m_\gamma \]

\[ R = N_c (e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2) = \frac{11}{3} \quad \text{if } \sqrt{s} > m_\gamma \]

If the radiation of hard gluons is taken into account, an extra factor proportional to \( \alpha_s \) has to be added:

\[ R = 3 \sum_q e_q^2 \left( 1 + \frac{\alpha_s(Q^2)}{\pi} \right) \]
Electron-positron annihilation

- In the lowest order $e^+e^-$ annihilation process, a photon or a $Z^0$ is produced which then converts into a quark-antiquark pair.

- The quark and the antiquark fragment into observable hadrons.

**PETRA (DESY)**
- hadrons
- $e^-$
- $e^+$
- 7-22 GeV
- Length: 2.3 km
- Experiments: Tasso, Jade, Pluto, Mark J, Cello

**LEP (CERN)**
- hadrons
- $e^-$
- $e^+$
- 45-104 GeV
- Length: 27 km (4184 magnets)
- Experiments: DELPHI, OPAL, ALEPH, L3
Electron-positron annihilation

Jets of particles

- Since the quark and antiquark momenta are equal and counter-parallell, the hadrons are produced in two jets of equal energy going in the opposite direction.

- The direction of the jet reflects the direction of the corresponding quark.

\[ e^+ + e^- \rightarrow \gamma^* \rightarrow \text{hadrons} \]

Diagram for two-jet events.

Two-jet event recorded by the Jade experiment at PETRA.
A study of the angular distribution of jets give information about the spin of the quarks.

The angular distribution of $e^+ + e^- \rightarrow \gamma^* \rightarrow \mu^+ + \mu^-$ is

$$\frac{d\sigma}{d\cos\theta}(e^+ e^- \rightarrow \mu^+ \mu^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

where $\theta$ is the production angle with respect to the direction of the colliding electrons.

The angular distribution of $e^+ + e^- \rightarrow \gamma^* \rightarrow q + \bar{q}$ is

- If the quark spin = 1/2
  $$\frac{d\sigma}{d\cos\theta}(e^+ e^- \rightarrow q\bar{q}) = N_c e_q^2 \pi\alpha^2 \frac{1 + \cos^2\theta}{2Q^2}$$

- If the quark spin = 0
  $$\frac{d\sigma}{d\cos\theta}(e^+ e^- \rightarrow q\bar{q}) = N_c e_q^2 \pi\alpha^2 \frac{1 - \cos^2\theta}{2Q^2}$$

where $e_q$ is the fractional quark charge and $N_c$ is the number of colours (=3).
Electron-positron annihilation

The experimentally measured angular distribution of jets is clearly following \( (1 + \cos^2 \theta) \).

The jets are therefore associated with spin 1/2 particles.

Quarks have spin = 1/2 !

The angular distribution of the quark jets in \( e^+e^- \) annihilations, compared with models with spin=0 and 1/2.
The discovery of the gluon

The accelerator: PETRA at the German laboratory DESY.
The discovery of the gluon

The experiment: TASSO

The central part of the TASSO experiment.

- Muon detector
- Drift chamber
- Magnet coil
- Liquid Argon Calorimeter
- Cherenkov detector
The discovery of the gluon

When the PETRA accelerator started up, one began to see three-jet events in the experiments. The interpretation was that the quark or antiquark emitted a high-momentum gluon that fragmented to a jet.

A Tasso 3-jet event.

A Jade 3-jet event.
The discovery of the gluon

- The probability for a quark to emit a gluon is proportional to $\alpha_s$ and by comparing the rate of two-jet with three-jet events one can determine $\alpha_s$.

- At PETRA one measured: $\alpha_s = 0.15 \pm 0.03$ for $\sqrt{s} = 30-40 \text{ GeV}$

- The process of turning quarks and gluons into hadrons is called hadronization:
Electron-positron annihilation

The spin of the gluon.

- It is possible to determine the spin of the gluon by measuring the angular distribution of jets in three-jet events.

- This is done by measuring:

\[
\cos \phi = \frac{\sin \theta_2 - \sin \theta_3}{\sin \theta_1}
\]

where the angles are defined in the following way.

Conclusion: Gluons have spin = 1!

Diagram: Graph showing the angular distribution of jets with the cosine of the angle \( \phi \) plotted against the number of jets, indicating different spin states of the gluon.
Electron-proton scattering

Electrons are good tools for investigating the properties of hadrons since electrons do not have a substructure. The wavelength of the exchanged photon determines how the proton is being probed.

$\lambda \gg r_p$  Very low electron energies
The scattering is equivalent to that from a “point-like” spin-less object.

$\lambda = r_p$  Low electron energies
The scattering is equivalent to that from an extended charged object.

$\lambda < r_p$  High electron energies
The wavelength is short enough to make it possible to interact with the valence quarks in the proton.

$\lambda \ll r_p$  Very high electron energies
The electron can at these short wavelengths interact with the sea of quarks and gluons.
Elastic scattering means that the same type of particles goes into and comes out of the collision.

\[
\begin{align*}
\vec{P}_1 & \quad \text{lepton} = e \text{ or } \mu \\
\vec{P}_2 & \quad \\
\vec{q} = \vec{P}_1 - \vec{P}_2 & = (\nu, \bar{q}) \\
\vec{P}_3 & \quad \text{hadron} = p \\
\vec{P}_4 & = \vec{P}_3 + \vec{q}
\end{align*}
\]

Elastic electron-proton scattering can be used to measure the size of the proton.
Electron-proton scattering

**Differential cross section**

- The **angular distribution** of the particles emerging from a scattering reaction is given by the differential cross section:

\[
\frac{d\sigma(\theta, \varphi)}{d\Omega} \text{ where } d\Omega = \sin \theta d\theta d\varphi
\]

- The **total cross section** of the reaction is obtained by integrating the differential cross section:

\[
\sigma = \int \int d\sigma(\theta, \varphi) \ d\Omega = \int \int \frac{d\sigma(\theta, \varphi)}{d\Omega} \sin \theta d\theta d\varphi
\]
Electron-proton scattering

→ Elastic scattering on a static point-like charge.

The Mott scattering formula describes the angular distribution of a relativistic electron of momentum $p$ which is scattered by a point-like electric charge $e$. The Rutherford scattering formula describes the same for a non-relativistic electron with a momentum $p \ll m$, i.e., it is obtained from the Mott formula by assuming $p=0$.
Electron-proton scattering

Elastic scattering on an extended charged object.

If the electric charge is not point-like, but spread out with a spherically symmetric density function \( e \rightarrow e \rho(r) \) that is normalized to one \( (\int \rho(r) d^3 \hat{x} = 1) \) then the Rutherford scattering formula has to be modified by an electric form factor \( G_E^2(q^2) \):

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_R G_E^2(q^2)
\]

where

\[
\left( \frac{d\sigma}{d\Omega} \right)_R = \frac{m^2 \alpha^2}{4p^4 \sin^4 \left( \frac{\theta}{2} \right)}
\]
Electron-proton scattering

- The electric form factor is the Fourier transform of the charge distribution with respect to the momentum transfer $q$:

$$G_E(q^2) = \int \rho(r) e^{i\mathbf{q} \cdot \mathbf{r}} d^3\mathbf{r}$$

- The electric form factor has values between 0 and 1:
  - Low momentum transfer: $G_E(0) = 1$ for $q = 0$
  - High momentum transfer: $G_E(q^2) \to 0$ for $q^2 \to \infty$

- Measurements of the cross-section can be used to determine the form-factor and hence the charge distribution. The mean quadratic charge radius is for example given by:

$$r_E^2 = \int r^2 \rho(r) d^3\mathbf{r} = -6 \left. \frac{dG_E(q^2)}{dq^2} \right|_{q^2 = 0}$$
Electron-proton scattering

Scattering of electrons on protons depends not only on the electric formfactor \((G_E)\) but also on a magnetic formfactor \((G_M)\) which is associated with the magnetic moment distribution:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_M \left( G_1(Q^2) \cos^2 \theta \right) + \frac{Q^2}{2M^2} G_2(Q^2) \sin^2 \theta
\]

\[
G_1(Q^2) = \frac{G_E^2}{4M^2} + \frac{Q^2}{4M^2} G_M^2
\]

\[
G_2(Q^2) = G_M^2
\]

The proton mass: \(M\)
Electron-proton scattering

- Measurement of the formfactors are conveniently divided up into three regions of $Q^2$:

  i) low $Q^2$ ($Q << M$):
  
  $G_E$ dominates the cross section and $r_E$ can be precisely measured: 
  $$r_E = 0.85 \pm 0.02 \text{ fm}$$

  ii) Intermediate $Q^2$ ($0.02 < Q^2 < 3 \text{ GeV}$):
  
  Both $G_E$ and $G_M$ give sizable contributions and the formfactors can be described by the parameterization:

  $$G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p} \approx \left( \frac{\beta^2}{\beta^2 + Q^2} \right)^2$$

  iii) High $Q^2$ ($Q^2 > 3 \text{ GeV}$):
  
  $G_M$ dominates the cross section.
Electron-proton scattering

- The form factors are normalized so that
  
  Protons: \( G_E(0) = \) total charge = 1  \( G_M(0) = \) magnetic moment = \( \mu_p = +2.79 \)
  
  Neutrons: \( G_E(0) = \) total charge = 0  \( G_M(0) = \) magnetic moment = \( \mu_n = -1.91 \)

- If the proton is a point particle then \( G_E \) and \( G_M \) do not depend on \( Q^2 \) and they should be constants with \( G_E=1 \) and \( G_M=2.79 \).

Measurements of \( G_E \) and \( G_M \) of the proton gives: 

Conclusion: The proton has an extended charge distribution!
Electron-proton scattering

In inelastic electron-proton scattering, the proton is broken up into new hadrons:

\[ \vec{q} = \vec{P}_1 - \vec{P}_2 = (\nu, \bar{q}) \quad Q^2 = -\vec{q} \cdot \vec{q} \]

A new dimensionless variable called the Bjorken scaling variable \( x \) is introduced which can take values between 0 and 1:

\[ x = \frac{Q^2}{2Mv} \]

where \( M \) is the mass of the proton.
Electron-proton scattering

- The differential cross section for inelastic electron-proton scattering can be written as:

\[
\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \left[ \frac{1}{\nu} F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{2}{M} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right) \right]
\]

where two dimensionless structure functions \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) parameterize the photon-proton interaction in the same way a \( G_1(Q^2) \) and \( G_2(Q^2) \) do it in elastic scattering.

- An important concept is that of Bjorken scaling or scale invariance:

\[ F_{1,2}(x, Q^2) = F_{1,2}(x) \quad \text{when} \quad Q^2 \rightarrow \infty \quad \text{and} \quad x \text{ is fixed and finite.} \]

i.e. the structure functions are almost independent on \( Q^2 \) when \( Q \gg M \). It is called scaling because structure functions at a given \( x \) remain unchanged if all particle masses, energies and momenta are multiplied by a scale factor.
Electron-proton scattering

The discovery of quarks at the SLAC 2 mile LINAC
Electron-proton scattering

The discovery of quarks
The MIT-SLAC experiment

8 GeV electrons were hitting a hydrogen target. The scattered electrons were selected by magnets at different angles and identified by detectors inside the brown shielding.
The discovery of quarks: The measurements

- It is possible to calculate $x$ and $Q^2$ from the energies and scattering angle of the electron:

\[
Q^2 = 4 \cdot E_1 \cdot E_2 \sin^2 \left( \frac{\theta}{2} \right)
\]

\[
x = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)}
\]

- From a cross section measurement it is then possible to extract $F_2$.

- The result that $F_2$ does not depend on $Q^2$ was later interpreted as the first evidence for the existence of quarks.
Electron-proton scattering

Deep inelastic electron-proton scattering

- The scale invariance is explained in the parton model by the scattering on point-like constituents (partons) in the proton.

- These partons are identical to the quarks that were postulated by the quark model.

\[ \mathbf{q} = \mathbf{P}_1 - \mathbf{P}_2 \]

\[ Q^2 = -\mathbf{q} \cdot \mathbf{q} = 4 \cdot E_1 \cdot E_2 \sin^2\left(\frac{\theta}{2}\right) \]

\[ x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)} \]

- The parton model is valid if the proton momentum is sufficiently large so that the fraction of the proton momentum carried by the struck quark is given by the Bjorken x.
Deep inelastic electron-proton scattering

The structure function $F_1$ depends on the spin of the partons (quarks) in the parton model:

$$F_1(x, Q^2) = 0 \quad \text{(spin-0)}$$

The Callan-Gross relation:

$$2xF_1(x, Q^2) = F_2(x, Q^2) \quad \text{(spin-1/2)}$$

Measurements shows that the partons have spin 1/2:
**PETRA**
*(DESY)*

\[ e^- \quad \gamma^* \quad \bar{q} \quad e^+ \quad \text{hadrons} \]

Length: 2.3 km
Experiments: Tasso, Jade, Pluto, Mark J, Cello

**LEP**
*(CERN)*

\[ e^- \quad q \quad Z^0 \quad \bar{q} \quad e^+ \quad \text{hadrons} \]

Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

**LINAC**
*(SLAC)*

\[ e^- \quad \gamma \quad \text{hadrons} \]

Length: 3 km
Experiments: SLAC-MIT

**HERA**
*(DESY)*

\[ e^+ \quad \gamma \quad \text{hadrons} \]

Length: 6 km (1650 magnets)
Experiments: H1, ZEUS
Electron-proton scattering

The HERA accelerator

- The HERA accelerator at the German DESY laboratory is the only large electron-proton collider ever built. It used PETRA as a pre-accelerator.
Electron-proton scattering

- HERA, which was 6 km long, had a ring of superconducting magnets for the protons and a ring of warm magnets for the electrons. The center-of-mass energy of the collision of 28 GeV electrons on 920 GeV protons was 320 GeV. This is equivalent to a fix target accelerator with a 54 TeV electron beam.
Electron-proton scattering

The H1 Experiment

The events at HERA were boosted in the proton direction due to the large difference in electron and proton beam energies.

Tracker:
Drift- and Proportional chambers

Muon detectors

Solenoid magnet

Calorimeter:
Liquid argon/lead - em
Liquid argon/steel - had
A measurement of the cross section + the energy and scattering angle of the electron made it possible to measure $F_2$.

No quark sub-structure was observed down to $10^{-18}$m (1/1000th of a proton).
SUMMARY: Electron-Positron interactions

\[ \frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow l^+l^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta) \]

\[ \frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta) \]
SUMMARY: Elastic electron-proton scattering

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_M \left( \frac{G_E^2}{4M^2} + \frac{Q^2}{4M^2} \frac{G_M^2}{4M^2} \right)
\]

\[
G_1(Q^2) = \frac{G_E^2}{1 + \frac{Q^2}{4M^2}} \quad G_2(Q^2) = \frac{G_M^2}{1 + \frac{Q^2}{4M^2}}
\]
SUMMARY: Inelastic electron-proton scattering

\[
\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \frac{1}{v} \cdot \left[ F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right) \right]
\]

\[Q^2 = -\vec{q} \cdot \vec{q}\]

\[x = \frac{Q^2}{2Mv}\]

Bjorken - x