Additional practise exercises for the exam

A

The critical energy of the electromagnetic shower development in iron is E_C = 24 MeV, and one radiation length is X_0 = 1.76 cm.
Estimate the smallest needed thickness of a calorimeter that uses iron as an absorper, if the initial electrons have energies not exceeding E_0 = 100 MeV.

B

A secondary particle beam can consist of several types of different particles. Separators are used to select the type of particle required. The separator consists of two parallel plates with a high potential between them. The beam passes between the plates and then through a deflecting magnet an slit system. Show that the difference in angular deflection, $\Delta\theta$, of two relativistic partilces with momentum p and masses m_1 and m_2, after traversing an electric field of strength E and length L, is:

$$\Delta\theta = \frac{eEL(m_1^2 - m_2^2)}{2p^3}$$

C

The decay $B^+ \to e^+\nu_e + \pi^0/D^0$ was used to set limits on the V_{cb} coupling. The branching ratio is 0.12, the B^+ lifetime is 1.4×10^{-12} s and the relative decay widths $\frac{\Gamma(b\to u)}{\Gamma(b\to c)}$ is less than 0.08.

1) Draw leading Feynman diagrams for the decays
2) The decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ is described as: $\Gamma_\mu = \frac{1}{\mu\,\text{lifetime}} = km_\mu^5$. Using this as an analogy, use Γ_{B^+} to derive a value for $|V_{cb}|$.
3) What is the largest allowed value of $|V_{ub}|$?

D

Suppose a detector can for each event measure a quantity x with $0 \leq x \leq 1$ for with the probability density functions for signal (s) and background (b) are:

$$f(x|s) = 3(1-x)^2$$

$$f(x|b) = 3x^2$$

1) The background hypothesis is rejected if the observed value of x is smaller than a specified cut value, x_{cut}. Find the value of x_{cut} such that the probability to reject the background hypothesis if it is background is $\alpha = 0.05$.
2) What is then the probability to accept an event with $x < x_{cut}$ given that it is signal?
3) Suppose the expected number of background events is $b_{tot} = 100$ and for a given signal model we expect $s_{tot} = 10$ events. Find the expected number of events s and b of signal and background events that will satisfy $x < x_{cut}$ using the value $x_{cut} = 0.1$.

E

Charged particles transverse a gas volume produce ionization, the mean amount depends on the type of particle. Suppose a test statistics t based on ionization measurements has been constructed such that it follows a Gaussian centered at 0 for electrons and 2 for pions; and with a standard deviation of 1 in both cases. A test of the pion hypothesis is constructed using the critical region t<1.
1) Find the size of the test, α. (evaluate numerically)
2) What is the power of the test with respect to the alternative hypothesis that the particle is an electron?
3) Suppose we want to select electrons (signal) and reject pions (background). Thus if t is found in the critical region the pion hypothesis is rejected and we accept the particle as an electron. What is the signal efficiency, ε_e, the probability to accept a particle given that it is an electron?. What is the background efficiency, ε_π, the probability that a pion will be accepted as an electron?
4) Suppose a sample of particles is known to consist of 99% electrons and 1% pions, what is then the purity of the electron sample selected by t<1?