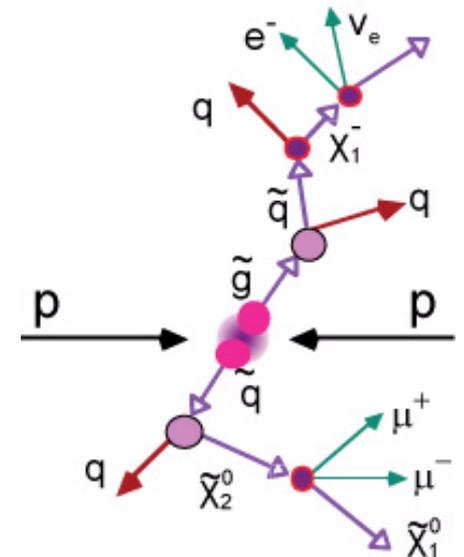
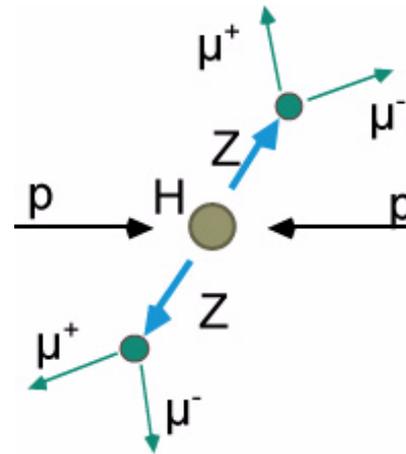
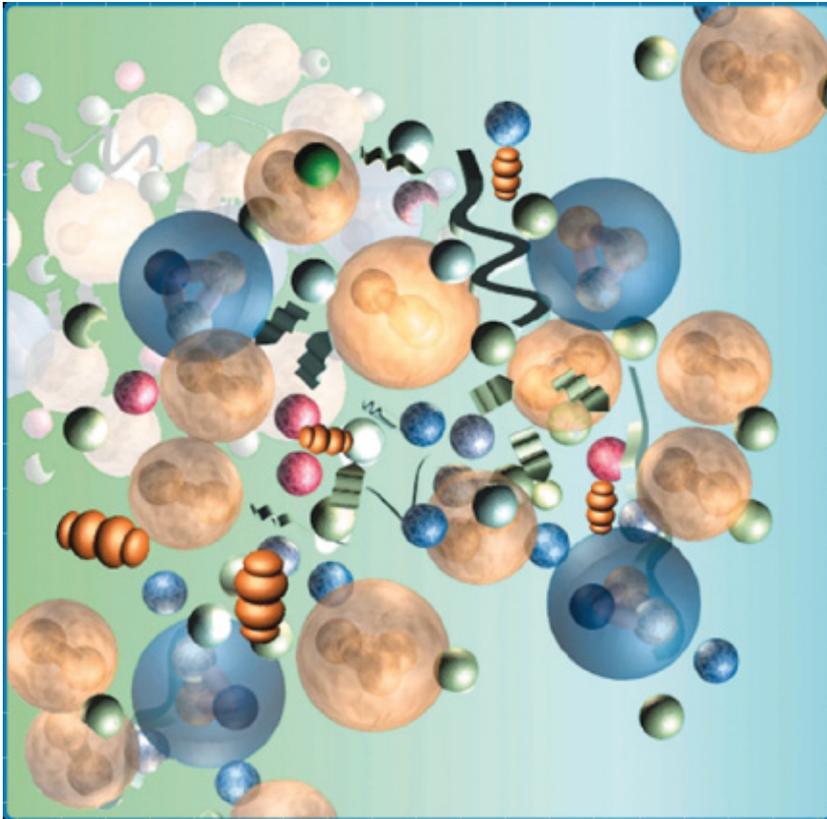


# Modern Experimental Particle Physics

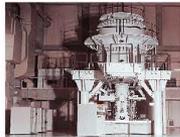


# I. Basic concepts

## Instruments



Accelerators  
LHC, LEP



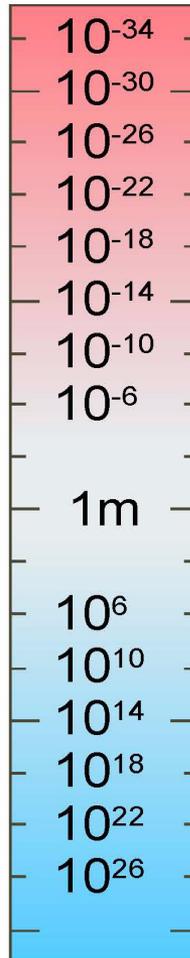
(Particle beams)  
Electron  
Microscope  
Microscope



Telescope

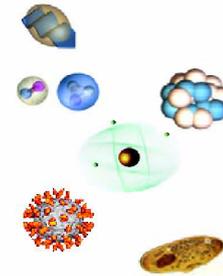


Radio  
Telescope



## Observables

SUSY particle?  
Higgs?  
Z/W (range of nuclear force)  
Proton Nuclei (range of weak force)  
Atom  
Virus  
Cell



Earth radius  
Earth to Sun



Galaxies  
Radius of observable  
Universe



Figure 1: Object sizes and observation instruments

# Main concepts

- ❖ Particle physics studies elementary “building blocks” of *matter* and *interactions* between them.
- ❖ Matter consists of *particles*.
  - ☉ Matter is built of particles called “fermions”: those that have half-integer spin, e.g.  $1/2$ ; obey *Fermi-Dirac statistics*.
- ❖ Particles interact via *forces*.
  - ☉ Interaction is an exchange of a force-carrying particle.
- ❖ Force-carrying particles are called *gauge bosons* (spin-1).

# Forces of nature

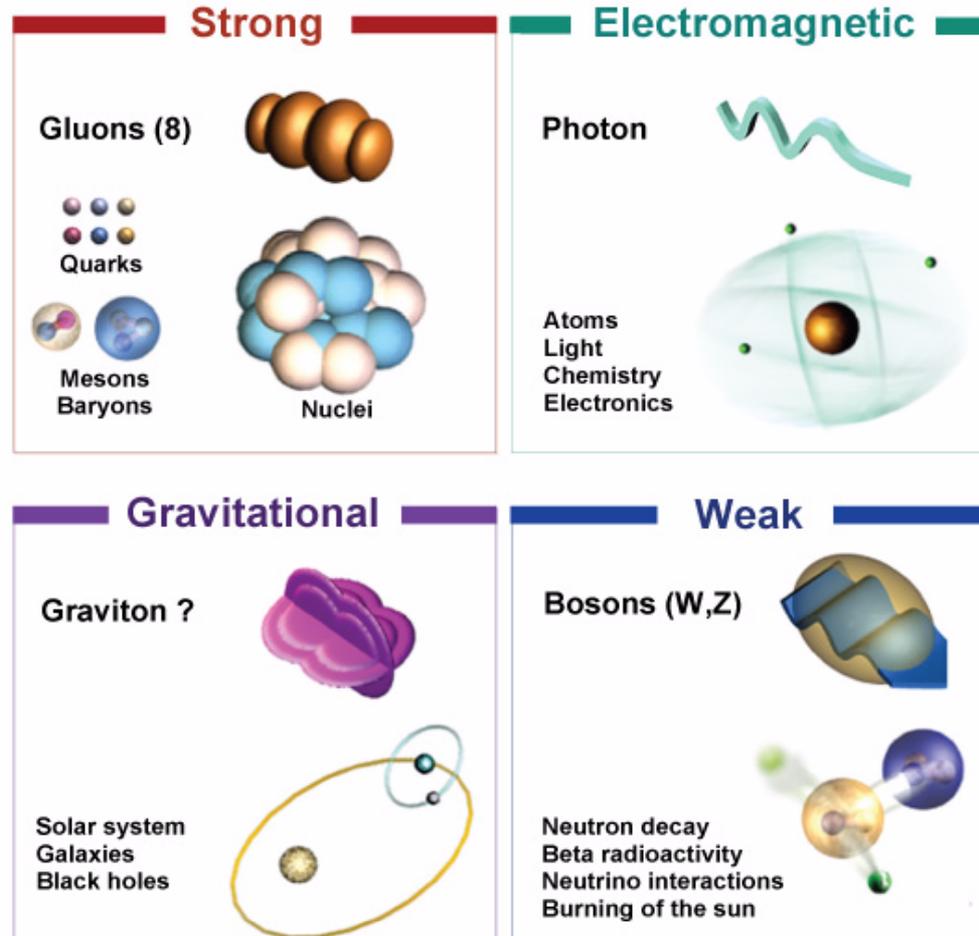


Figure 2: Forces and their carriers

# Summary table of forces:

Force	Acts on/ <i>couples to:</i>	Carrier	Range	Strength	Stable systems	Induced reaction
<b>Gravity</b>	all particles <i>Mass/E-p tensor</i>	graviton G <i>(has not yet been observed)</i>	long $F \propto 1/r^2$	$\sim 10^{-39}$	Solar system	Object falling
<b>Weak force</b>	fermions <i>hypercharge</i>	bosons $W^+, W^-$ and Z	$< 10^{-17}$ m	$10^{-5}$	None	$\beta$ -decay
<b>Electro-magnetism</b>	charged particles <i>electric charge</i>	photon $\gamma$	long $F \propto 1/r^2$	$1/137$	Atoms, molecules	Chemical reactions
<b>Strong force</b>	quarks and gluons <i>colour charge</i>	gluons g (8 different)	$10^{-15}$ m	$1$	Hadrons, nuclei	Nuclear reactions

# The Standard Model

- ❖ Electromagnetic and weak forces can be described by a single theory  
⇒ the “*Electroweak Theory*” was developed in 1960s (Glashow, Weinberg, Salam).
- ❖ Theory of strong interactions appeared in 1970s: “*Quantum Chromodynamics*” (QCD).
- ❖ The “*Standard Model*” (SM) combines all the current knowledge.
  - ☉ Gravitation is VERY weak at particle scale, and it is not included in the SM. Moreover, quantum theory for gravitation does not exist yet.

## Main postulates of SM:

- 1) Basic constituents of matter are *quarks* and *leptons* (spin 1/2)
- 2) They interact by exchanging *gauge bosons* (spin 1)
- 3) Quarks and leptons are subdivided into 3 *generations*

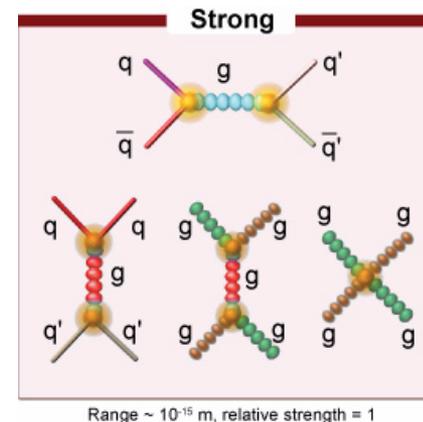
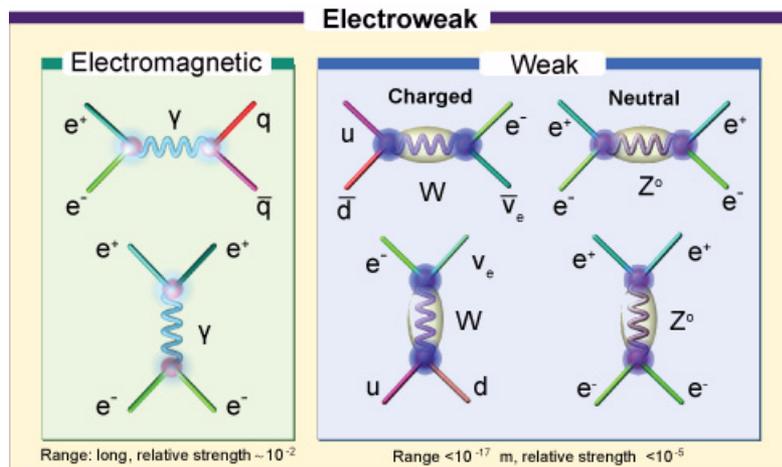
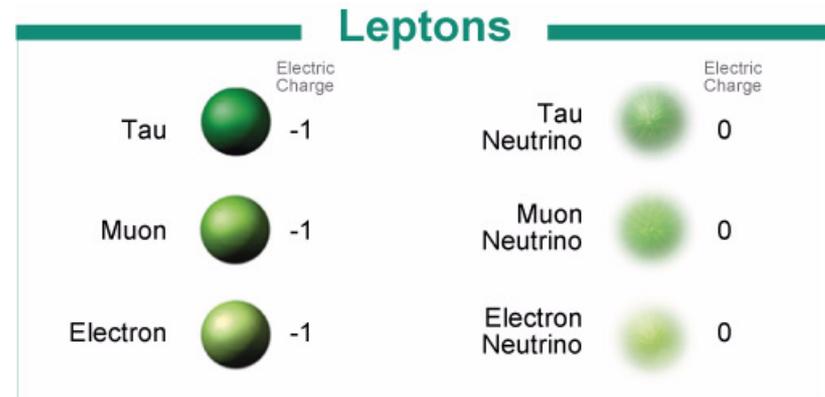


Figure 3: Standard Model: quarks, leptons and bosons

☉ SM does not explain neither appearance of the mass nor the reason for existence of the 3 generations.

# History of the Universe as we know it

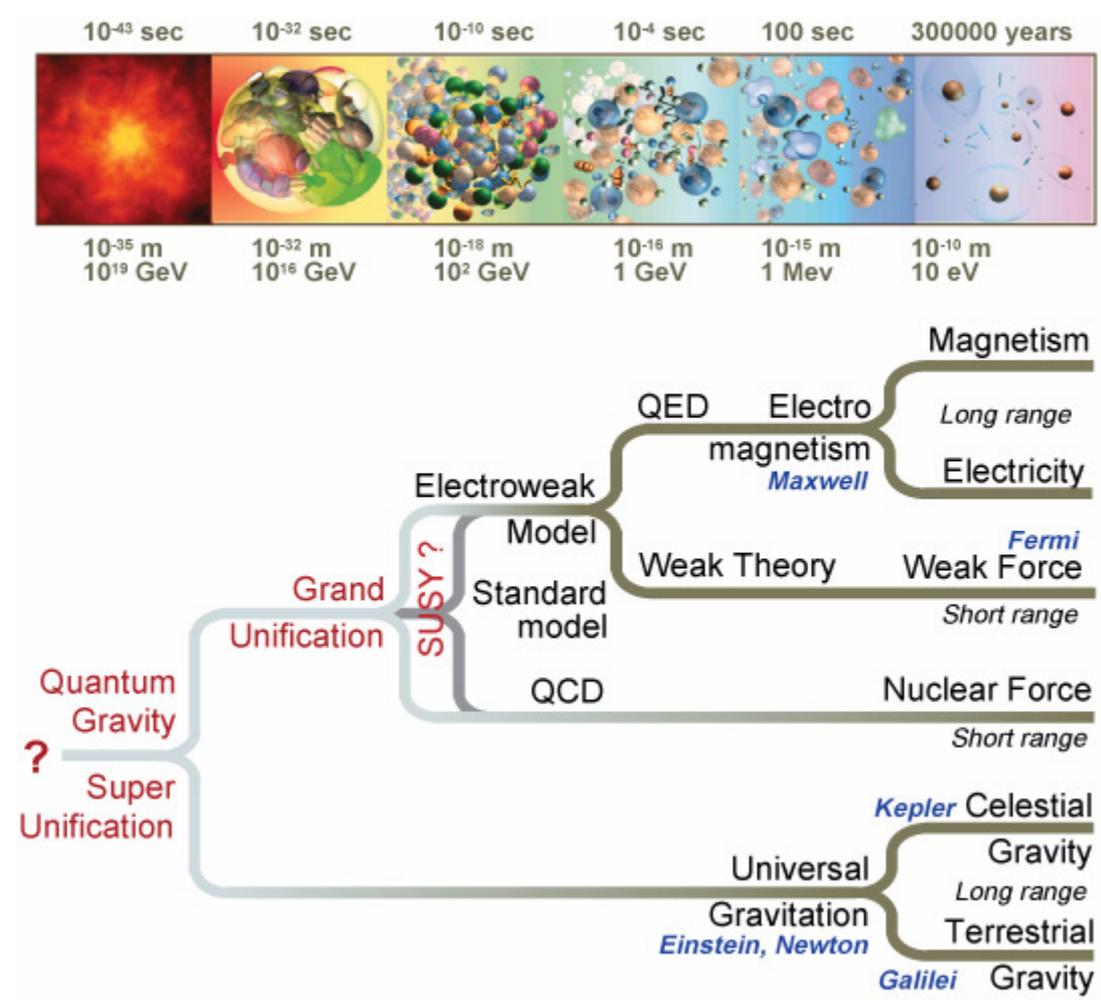


Figure 4: Summary of the current knowledge about our Universe

# Units and dimensions

❖ Particle energy is measured in *electron-volts*:

$$1 \text{ eV} \approx 1.602 \times 10^{-19} \text{ J} \quad (1)$$

☉ 1 eV is energy of an electron upon passing a voltage of 1 Volt.

☉ 1 keV =  $10^3$  eV; 1 MeV =  $10^6$  eV; 1 GeV =  $10^9$  eV

❖ The reduced *Planck constant* and the *speed of light*:

$$\hbar \equiv h / 2\pi = 6.582 \times 10^{-22} \text{ MeV s} \quad (2)$$

$$c = 2.9979 \times 10^8 \text{ m/s} \quad (3)$$

and the “*conversion constant*” is:

$$\hbar c = 197.327 \times 10^{-15} \text{ MeV m} \quad (4)$$

❖ For simplicity, *natural units* are used:

$$\hbar = 1 \quad \text{and} \quad c = 1 \quad (5)$$

thus the unit of mass is  $eV/c^2$ , and the unit of momentum is  $eV/c$

# Four-vector formalism

Relativistic kinematics is formulated with four-vectors:

- ⊙ space-time four-vector:  $x=(t,\vec{x})=(t,x,y,z)$ , where  $t$  is time and  $\vec{x}$  is a coordinate vector ( $c=1$  notation is used)
- ⊙ momentum four-vector:  $p=(E,\vec{p})=(E,p_x,p_y,p_z)$ , where  $E$  is particle energy and  $\vec{p}$  is particle momentum vector

Calculus rules with four-vectors:

❖ 4-vectors are defined as

⊙ *contravariant*:

$$A^\mu = (A^0, \vec{A}), B^\mu = (B^0, \vec{B}), \quad (6)$$

⊙ and *covariant*:

$$A_\mu = (A^0, -\vec{A}), B_\mu = (B^0, -\vec{B}). \quad (7)$$

⊙ *Scalar product* of two four-vectors is defined as:

$$A \cdot B = A^0 B^0 - (\vec{A} \cdot \vec{B}) = A_{\mu} B^{\mu} = A^{\mu} B_{\mu}. \quad (8)$$

⊙ Scalar products of momentum and space-time four-vectors are thus:

$$x \cdot p = x^0 p^0 - (\vec{x} \cdot \vec{p}) = Et - (\vec{x} \cdot \vec{p}) \quad (9)$$

4-vector product of coordinate and momentum represents particle  
wavefunction

$$p \cdot p = p^2 = p^0 p^0 - (\vec{p} \cdot \vec{p}) = E^2 - \vec{p}^2 \equiv m^2 \quad (10)$$

4-momentum squared gives particle's invariant mass

For relativistic particles, we can see that

$$E^2 = p^2 + m^2 \quad (c=1) \quad (11)$$

# Antiparticles

❖ Particles are described by wavefunctions:

$$\Psi(\vec{x}, t) = N e^{i(\vec{p}\vec{x} - Et)} \quad (12)$$

$\vec{x}$  is the coordinate vector,  $\vec{p}$  - momentum vector,  $E$  and  $t$  are energy and time.

Particles obey the classical Schrödinger equation:

$$i \frac{\partial}{\partial t} \Psi(\vec{x}, t) = H \Psi(\vec{x}, t) = \frac{\vec{p}^2}{2m} \Psi(\vec{x}, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\vec{x}, t) \quad (13)$$

$$\text{here } \vec{p} = \frac{\hbar}{2\pi i} \nabla \equiv \frac{\nabla}{i} \quad (14)$$

For relativistic particles,  $E^2 = p^2 + m^2$  (11), and (13) is replaced by the Klein-Gordon equation (15):

⇓

$$-\frac{\partial^2}{\partial t^2}(\Psi) = H^2\Psi(\vec{x}, t) = -\nabla^2\Psi(\vec{x}, t) + m^2\Psi(\vec{x}, t) \quad (15)$$

❖ There exist *negative* energy solutions with  $E_+ < 0$  !

$$\Psi^*(\vec{x}, t) = N^* \cdot e^{i(-\vec{p}\vec{x} + E_+ t)}$$

⊙ There is a problem with the Klein-Gordon equation: it is second order in derivatives. In 1928, Dirac found the first-order form having the same solutions:

$$i\frac{\partial\Psi}{\partial t} = -i\sum_i \alpha_i \frac{\partial\Psi}{\partial x_i} + \beta m\Psi \quad (16)$$

Here  $\alpha_i$  and  $\beta$  are  $4 \times 4$  matrices, and  $\Psi$  are four-component wavefunctions: *spinors* (for particles with spin 1/2).

$$\Psi(\vec{x}, t) = \begin{bmatrix} \Psi_1(\vec{x}, t) \\ \Psi_2(\vec{x}, t) \\ \Psi_3(\vec{x}, t) \\ \Psi_4(\vec{x}, t) \end{bmatrix}$$

*Dirac-Pauli representation* of matrices  $\alpha_i$  and  $\beta$ :

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Here  $I$  is a  $2 \times 2$  unit matrix,  $0$  is a  $2 \times 2$  matrix of zeros, and  $\sigma_i$  are  $2 \times 2$  *Pauli matrices*:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Also possible is *Weyl representation*:

$$\alpha_i = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

# Dirac's picture of vacuum

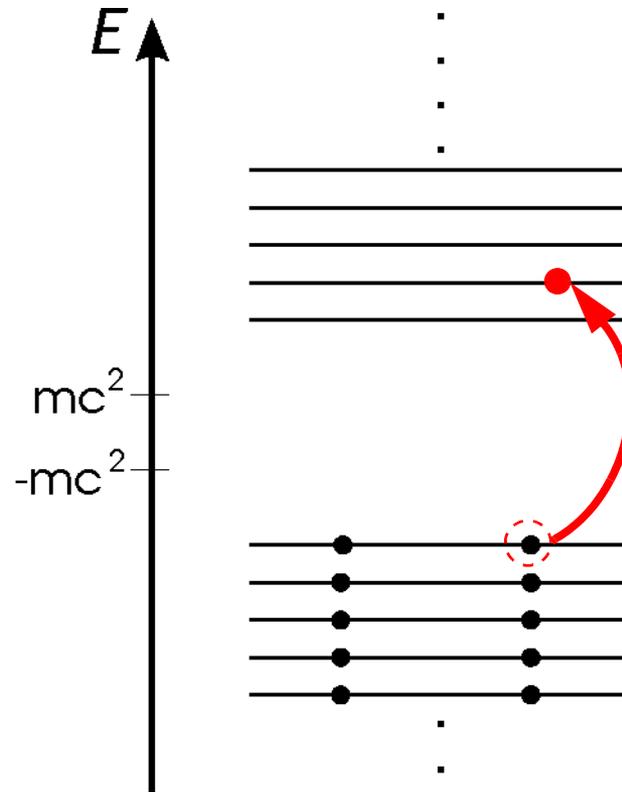


Figure 5: Fermions in Dirac's representation

- ◎ The “hole” created by appearance of an electron with “normal” energy is interpreted as the presence of electron's *antiparticle* with the opposite charge.
- ◎ Every charged particle must have an antiparticle of the same mass and opposite charge, to solve the mystery of “negative” energy.

## Discovery of the positron

- © 1933, C.D.Andersson, Univ. of California (Berkeley) observed with the Wilson cloud chamber 15 tracks in cosmic rays:

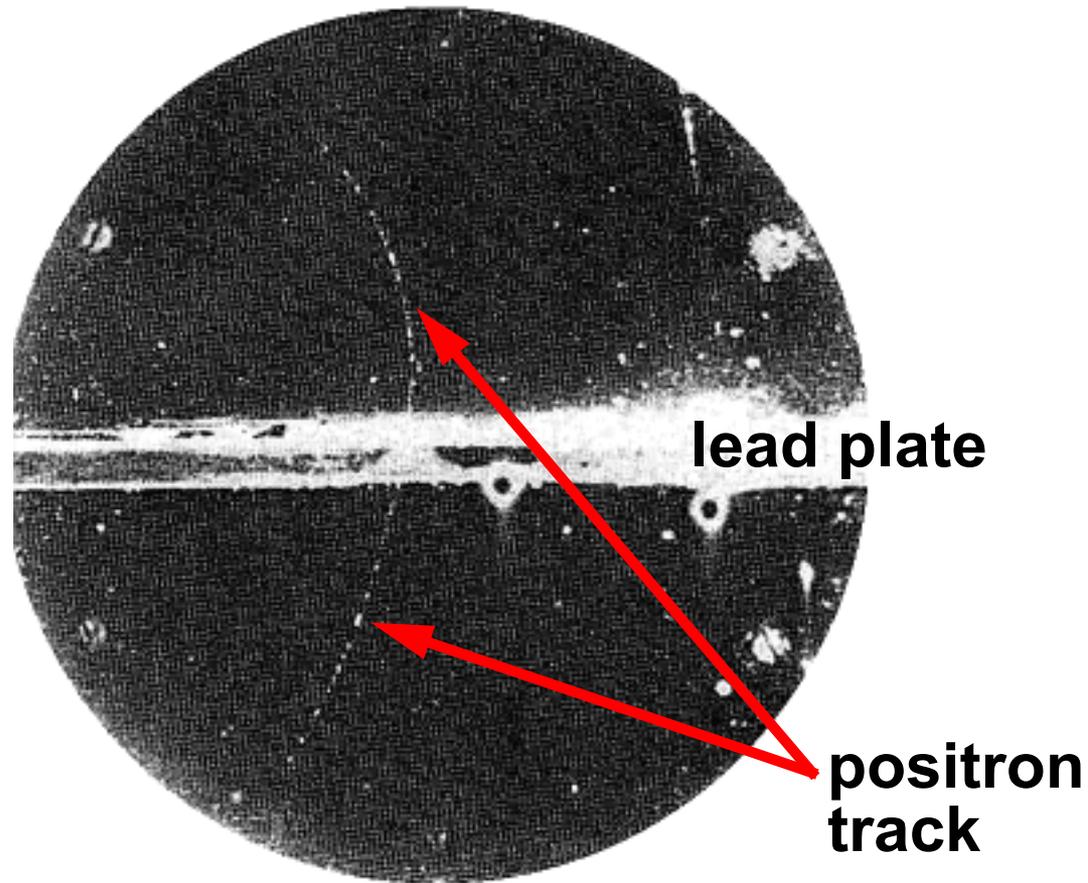


Figure 6: Photo of the track in the Wilson chamber

# Feynman diagrams

In 1940s, R.Feynman developed a diagram technique for representing processes in particle physics.

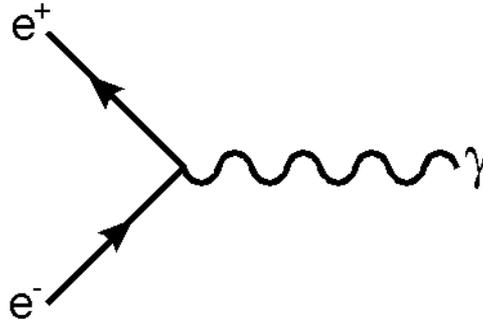


Figure 7: A Feynman diagram example:  $e^+e^- \rightarrow \gamma$

## Main assumptions and requirements:

- ⊙ Time runs from left to right
- ⊙ Arrow directed towards the right indicates a particle, and otherwise - antiparticle
- ⊙ At every vertex, momentum, angular momentum and charge are conserved (but not necessarily energy)
- ⊙ Particles are shown by solid lines, gauge bosons - by helices or dashed lines

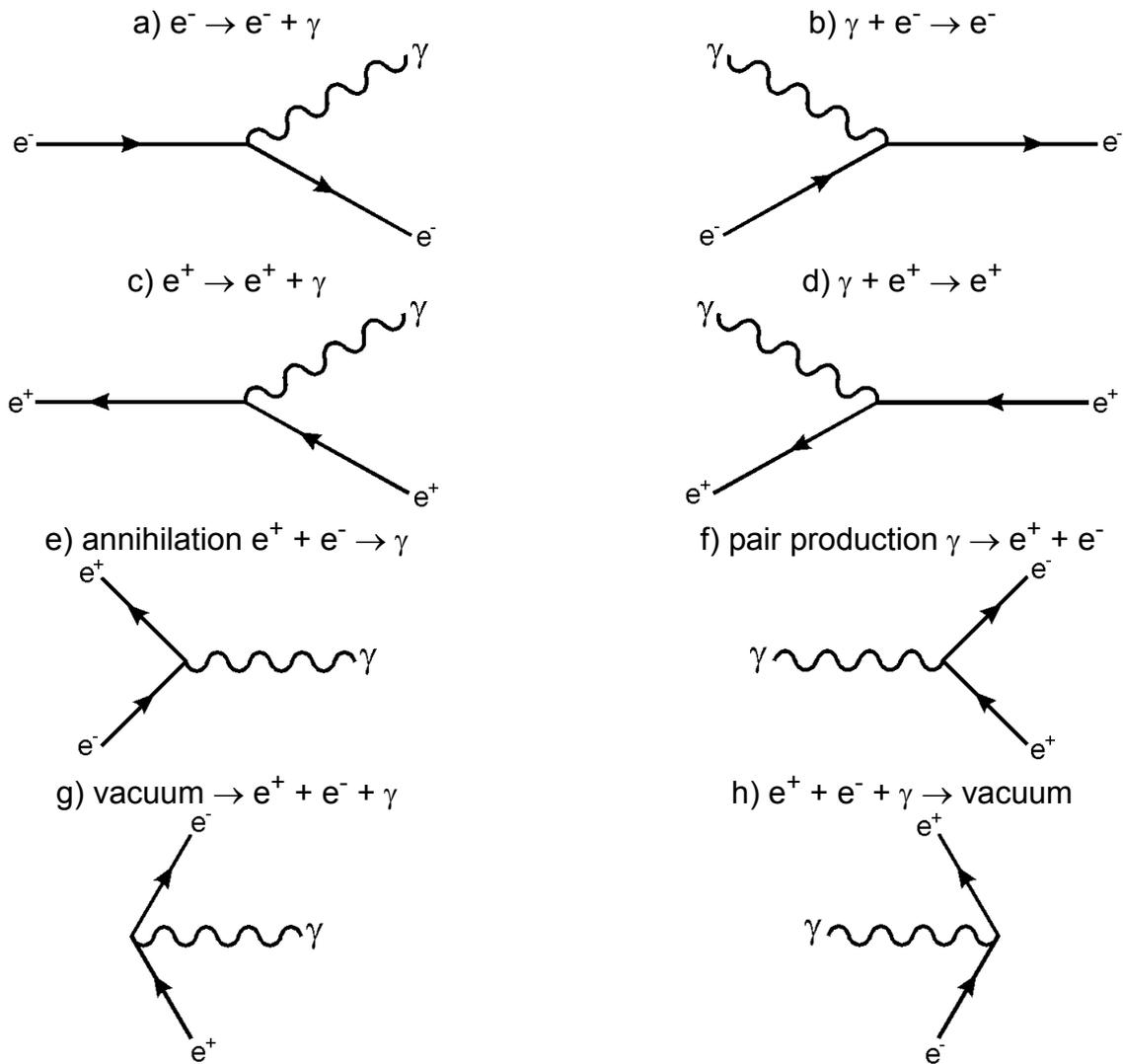


Figure 8: Feynman diagrams for **VIRTUAL** processes involving  $e^+$ ,  $e^-$  and  $\gamma$

☉ A virtual process does not require energy conservation

- ❖ A real process demands energy conservation, hence is a combination of virtual processes.

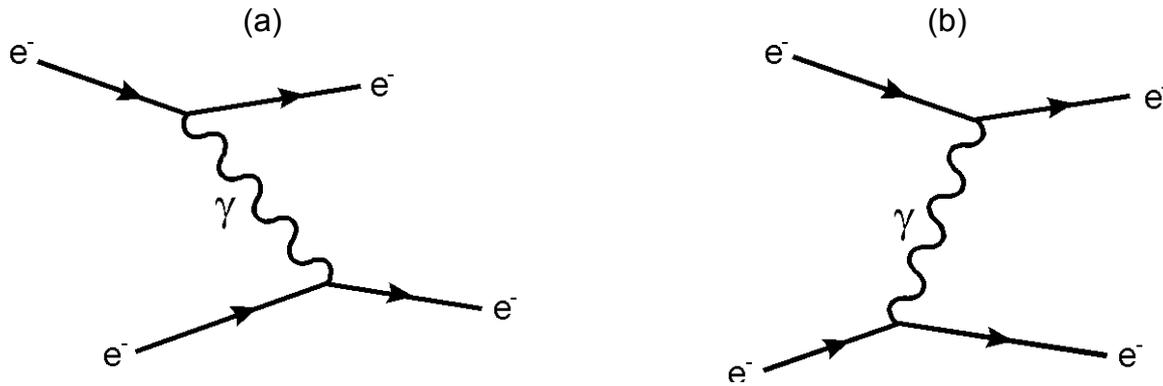


Figure 9: Electron-electron scattering, single photon exchange

- ❖ Any real process receives contributions from *all the possible* virtual processes.

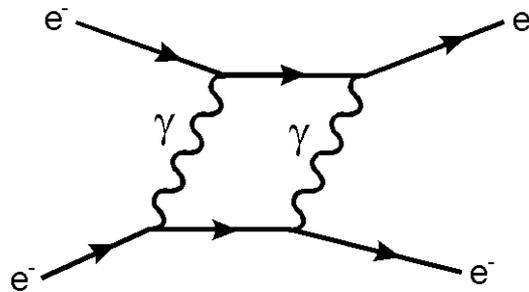


Figure 10: Two-photon exchange contribution

❖ Probability  $P(e^-e^- \rightarrow e^-e^-) = |M(1 \gamma \text{ exchange}) + M(2 \gamma \text{ exchange}) + M(3 \gamma \text{ exchange}) + \dots|^2$  ( $M$  stands for contribution, “*Matrix element*”)

⊙ Number of vertices in a diagram is called its *order*.

⊙ Each vertex has an associated probability proportional to a *coupling constant*, usually denoted as “ $\alpha$ ”. In discussed processes this constant is

$$\alpha_{em} = \frac{e^2}{4\pi\epsilon_0} \approx \frac{1}{137} \ll 1 \quad (17)$$

⊙ Matrix element for a two-vertex process is proportional to  $M = \sqrt{\alpha} \cdot \sqrt{\alpha}$ , where each vertex has a factor  $\sqrt{\alpha}$ . Probability for a process is  $P=|M|^2=\alpha^2$

⊙ For the real processes, a diagram of order  $n$  gives a contribution to probability of order  $\alpha^n$ .

Provided sufficiently small  $\alpha$ , high order contributions are smaller and smaller and the result is convergent:  $P(\text{real}) = |M(\alpha)+M(\alpha^2)+M(\alpha^3)\dots|^2$

Often lowest order calculation is precise enough.

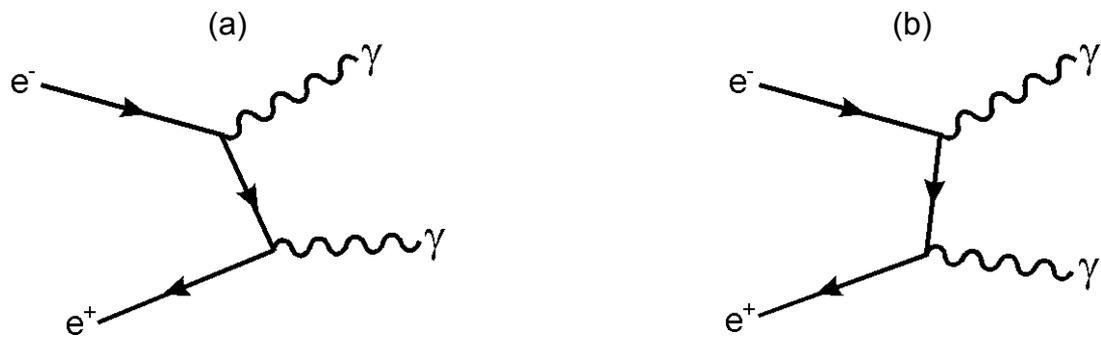


Figure 11: Lowest order contributions to  $e^+e^- \rightarrow \gamma\gamma$ .  $M = \sqrt{\alpha} \cdot \sqrt{\alpha}$ ,  $P=|M|^2=\alpha^2$

⊙ Diagrams which differ only by time-ordering are usually implied by drawing only one of them

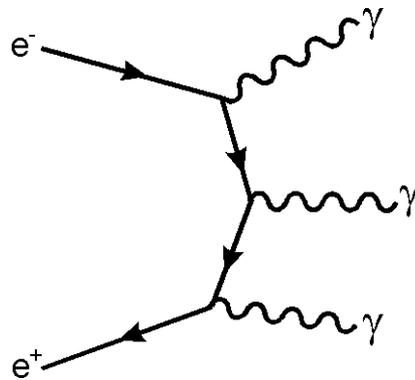


Figure 12: Lowest order of the process  $e^+e^- \rightarrow \gamma\gamma\gamma$ .  $M = \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot \sqrt{\alpha}$ ,  $P=|M|^2=\alpha^3$

This kind of process implies  $3!=6$  different time orderings

❖ Knowing order of diagrams is sufficient to estimate the ratio of appearance rates of processes:

$$R \equiv \frac{\text{Rate}(e^+ e^- \rightarrow \gamma\gamma\gamma)}{\text{Rate}(e^+ e^- \rightarrow \gamma\gamma)} = \frac{O(\alpha^3)}{O(\alpha^2)} = O(\alpha)$$

This ratio can be measured experimentally; it appears to be  $R = 0.9 \times 10^{-3}$ , which is smaller than  $\alpha_{em}$ , being only a first order prediction.

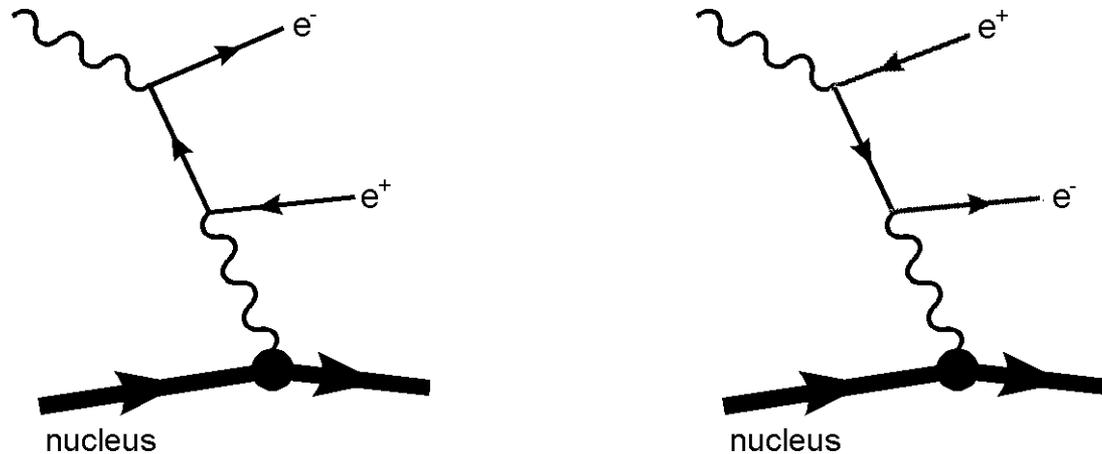


Figure 13: Diagrams that are **not** related by time ordering

🎯 For nuclei, the coupling is proportional to  $Z^2\alpha$ , hence the rate of this process is of order  $Z^2\alpha^3$

## Exchange of a massive boson

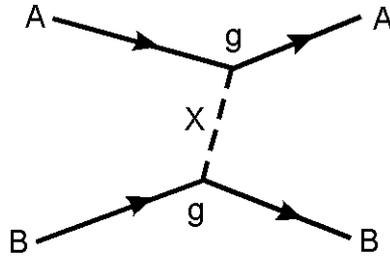


Figure 14: Exchange of a massive particle X

In the rest frame of particle A:  $A(E_0, \vec{p}_0) \rightarrow A(E_A, \vec{p}) + X(E_X, -\vec{p})$

where  $E_0 = M_A$ ,  $\vec{p}_0 = (0, 0, 0)$ ,  $E_A = \sqrt{p^2 + M_A^2}$ ,  $E_X = \sqrt{p^2 + M_X^2}$

From this one can estimate the maximum distance over which X can propagate before being absorbed:  $\Delta E = E_X + E_A - M_A \geq M_X$ , and this

energy violation can exist only for a period of time  $\Delta t \approx \hbar / \Delta E$  (Heisenberg's uncertainty relation), hence the *range of the interaction* is

$$r \approx R \equiv (\hbar / M_X) c = \Delta t c$$

- ❖ For a massless exchanged particle, the interaction has an infinite range (e.g., electromagnetic)
- ❖ In case of a very heavy exchanged particle (e.g., a W boson in weak interaction), the interaction can be approximated by a *zero-range*, or *point interaction*:

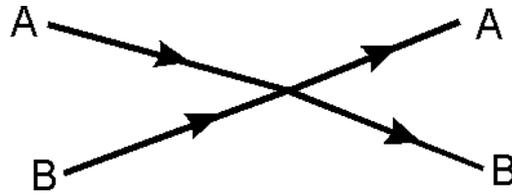


Figure 15: Point interaction as a result of  $M_x \rightarrow \infty$

E.g., for a W boson:  $R_W = \hbar/M_W = \hbar/(80.4 \text{ GeV}/c^2) \approx 2 \times 10^{-18} \text{ m}$

## Yukawa potential (1935)

❖ Considering particle X as an electrostatic spherically symmetric potential  $V(r)$ , the Klein-Gordon equation (15) for it will look like

$$\nabla^2 V(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) = M_X^2 V(r) \quad (18)$$

(In 3D polar coordinates system,  $\frac{\partial^2}{\partial t^2} V(r) = (0)$ ,  $\nabla^2 V(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} V(r) \right)$ ).

Integration of (18) gives the solution of

$$V(r) = -\frac{g^2}{4\pi r} e^{-r/R} \quad (19)$$

Here  $g$  is an integration constant, and it is interpreted as the *coupling strength* for particle X to the particles A and B.

- ❖ In Yukawa theory,  $g$  is analogous to the electric charge in QED, and the analogue of  $\alpha_{em}$  is

$$\alpha_X = \frac{g^2}{4\pi}$$

$\alpha_X$  characterizes the strength of the interaction at distances  $r \leq R$ .

Consider a particle being scattered by the potential (19), thus receiving a momentum transfer  $\vec{q}$

- ❖ Potential (19) has the corresponding amplitude, which is its Fourier-transform (like in optics):

$$f(\vec{q}) = \int V(\vec{x}) e^{i\vec{q}\vec{x}} d^3\vec{x} \quad (20)$$

Using polar coordinates,  $d^3\vec{x} = r^2 \sin\theta dr d\theta d\phi$ , and assuming  $V(\vec{x}) = V(r)$ , the amplitude is

$$f(\vec{q}) = 4\pi g \int_0^{\infty} V(r) \frac{\sin(qr)}{qr} r^2 dr = \frac{-g^2}{q^2 + M_X^2} \quad (21)$$

❖ For the point interaction,  $M_X^2 \gg q^2$ , hence  $f(\vec{q})$  becomes a constant:

$$f(\vec{q}) = -G = \frac{-4\pi\alpha_X}{M_X^2}$$

That means that the point interaction is characterized not only by  $\alpha_X$ , but by  $M_X$  as well

☉ Very useful approximation for weak interactions; in  $\beta$ -decays, this constant is called the “*Fermi coupling constant*”,  $G_F$