# VI. (V) Hadron quantum numbers

Characteristics of a hadron:

1) Mass
2) Quantum numbers arising from space symmetries : $J$, $P$, $C$. Common notation:
   - $J^P$ (e.g. for proton: $\frac{1}{2}^+$), or
   - $J^{PC}$ if a particle is also an eigenstate of $C$-parity (e.g. for $\pi^0$ : $0^{-+}$)
3) Internal quantum numbers: Q and B (always conserved), $S$, $C$, $\tilde{B}$, $T$ (conserved in electromagnetic and strong interactions)

- How do we know what are quantum numbers of a newly discovered hadron?
- How do we know that mesons consist of a quark-antiquark pair, and baryons – of three quarks?

Some *a priori* knowledge is needed:

| Particle | Mass (Gev/c2) | Quark composition | Q | B | S | C | $\tilde{B}$ |
|---|---|---|---|---|---|---|---|
| p | 0.938 | uud | 1 | 1 | 0 | 0 | 0 |
| n | 0.940 | udd | 0 | 1 | 0 | 0 | 0 |
| $K^-$ | 0.494 | $s\bar{u}$ | -1 | 0 | -1 | 0 | 0 |
| $D^-$ | 1.869 | $d\bar{c}$ | -1 | 0 | 0 | -1 | 0 |
| $B^-$ | 5.279 | $b\bar{u}$ | -1 | 0 | 0 | 0 | -1 |

For the lightest 3 quarks (u, d, s), possible 3-quark and 2-quark states will be ($q_{i,j,k}$ are u- or d- quarks):

| | sss | $ssq_i$ | $sq_iq_j$ | $q_iq_jq_k$ | $s\bar{s}$ | $s\bar{q}_i$ | $\bar{s}q_i$ | $q_i\bar{q}_i$ | $q_i\bar{q}_j$ |
|---|---|---|---|---|---|---|---|---|---|
| S | -3 | -2 | -1 | 0 | 0 | -1 | 1 | 0 | 0 |
| Q | -1 | 0; -1 | 1; 0; -1 | 2; 1; 0; -1 | 0 | 0; -1 | 1; 0 | 0 | -1; 1 |

- Hence restrictions arise: for example, mesons with $S$ = -1 and Q = 1 are *forbidden*

❖ Particles which fall out of above restrictions are called *exotic* particles (like $dd\bar{u}s$ , $uuu\bar{d}s$ etc.)

From observations of **strong interaction** processes, quantum numbers of many particles can be deduced:

| | $p + p$ | $\rightarrow p + n$ | $+\ \pi^{+}$ |
|---|---|---|---|
| Q= | 2 | 1 | 1 |
| S= | 0 | 0 | 0 |
| B= | 2 | 2 | 0 |

| | $p + p$ | $\rightarrow p + p$ | $+\ \pi^{0}$ |
|---|---|---|---|
| Q= | 2 | 2 | 0 |
| S= | 0 | 0 | 0 |
| B= | 2 | 2 | 0 |

| | $p$ | $+\ \pi^{-}$ | $\rightarrow \pi^{0} + n$ |
|---|---|---|---|
| Q= | 1 | -1 | 0 |
| S= | 0 | 0 | 0 |
| B= | 1 | 0 | 1 |

Observations of pions confirm these predictions, ensuring that pions are non-exotic particles.

Assuming that $K^-$ is a strange meson, one can predict quantum numbers of $\Lambda$-baryon:

$$K^- + p \rightarrow \pi^0 + \Lambda$$

| | $K^-$ | $\pi^0$ | $\Lambda$ |
|---|---|---|---|
| Q= | 0 | 0 | 0 |
| S= | -1 | 0 | -1 |
| B= | 1 | 0 | 1 |

And further, for $K^+$-meson:

$$\pi^- + p \rightarrow K^+ + \pi^- + \Lambda$$

| | $\pi^-$ | $K^+$ | $\Lambda$ |
|---|---|---|---|
| Q= | 0 | 1 | -1 |
| S= | 0 | 1 | -1 |
| B= | 1 | 0 | 1 |

- All of the more than 200 hadrons of certain existence satisfy this kind of predictions
  - It so far confirms validity of the quark model, which suggests that only quark-antiquark and 3-quark (or 3-antiquark) states can exist

# Pentaquark observation

- In 1997, a theoretical model predicted *pentaquark* possibility with mass 1.54 GeV
- In 2003, LEPS/SPring-8 experiment in Japan reported an observation of a particle with precisely this mass, and having structure consistent with pentaquark

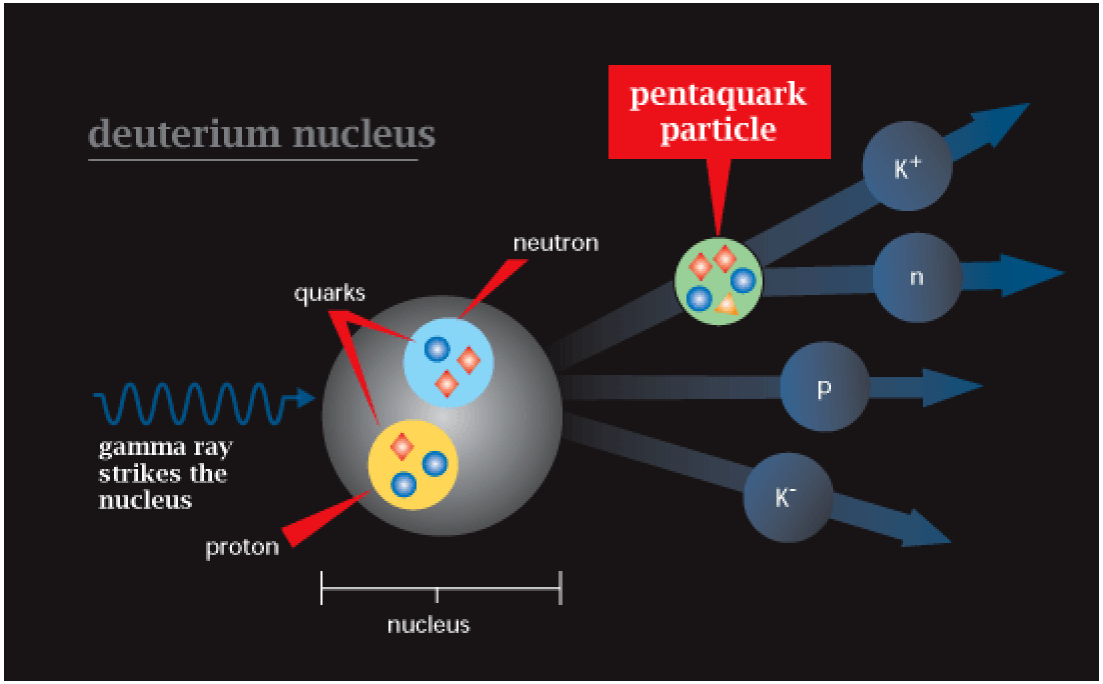


Figure 87: Pentaquark production and observation at JLab

- Reported $\Theta^+$ particle composition: $uudd\bar{s}$, $B = +1$, $S = +1$, spin = 1/2

# LEPS/SPring-8 experimental setup:

$$\gamma + n \rightarrow \Theta^{+}(1540) + K^{-} \rightarrow K^{+} + K^{-} + n$$

- Laser beam was shot to a target made of $^{12}C$ (n:p=1:1)

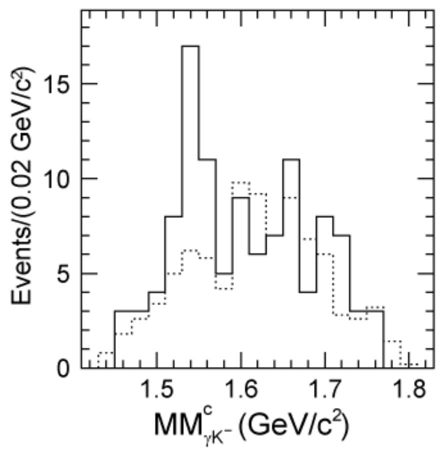


Figure 88: New particle signal (the peak) reported by LEPS

- A reference target of liquid hydrogen (only protons) showed no signal
- Many experiments reported similar observations

## New dedicated precision experiments show no signal

- Main problem: how to estimate the background. Search continues...

# *Even more quantum numbers...*

It is convenient to introduce some more quantum numbers, which are conserved in strong and electromagnetic interactions:

- Sum of all <u>internal</u> quantum numbers, except of Q,

$$\textit{hypercharge}\ \mathrm{Y} \equiv \mathrm{B} + S + C + \tilde{B} + T$$

- Instead of Q :

$$\mathrm{I}_3 \equiv \mathrm{Q} - \mathrm{Y}/2$$

...which is to be treated as a projection of a new vector:

- *Isospin*

$$\mathrm{I} \equiv (\mathrm{I}_3)_{max}$$

so that $\mathrm{I}_3$ takes 2I+1 values from -I to I

- $\mathrm{I}_3$ is a good quantum number to denote up- and down- quarks, and it is convenient to use notations for particles as: $\mathrm{I}(J^P)$ or $\mathrm{I}(J^{PC})$

| | B | S | C | $\tilde{B}$ | T | Y | Q | $I_3$ |
|---|---|---|---|---|---|---|---|---|
| u | 1/3 | 0 | 0 | 0 | 0 | 1/3 | 2/3 | 1/2 |
| d | 1/3 | 0 | 0 | 0 | 0 | 1/3 | -1/3 | -1/2 |
| s | 1/3 | -1 | 0 | 0 | 0 | -2/3 | -1/3 | 0 |
| c | 1/3 | 0 | 1 | 0 | 0 | 4/3 | 2/3 | 0 |
| b | 1/3 | 0 | 0 | -1 | 0 | -2/3 | -1/3 | 0 |
| t | 1/3 | 0 | 0 | 0 | 1 | 4/3 | 2/3 | 0 |

- Hypercharge Y, isospin I and its projection $I_3$ are additive quantum numbers, thus quantum numbers for hadrons can be deduced from those of quarks:

$$Y^{a+b} = Y^a + Y^b \; ; \; I_3^{a+b} = I_3^a + I_3^b$$

$$I^{a+b} = I^a + I^b,\ I^a + I^b - 1,\ \ldots,\ \left|I^a - I^b\right|$$

- Proton and neutron both have isospin of 1/2, and also very close masses:

$$\text{p(938) = uud ; n(940) = udd : } I(J)^P = \frac{1}{2}\left(\frac{1}{2}\right)^+$$

proton and neutron are said to belong to *isospin doublet*

# Other examples of *isospin multiplets*:

$$K^+(494) = u\bar{s}\,;\ K^0(498) = d\bar{s} : I(J)^P = \tfrac{1}{2}(0)^-$$

$$\pi^+(140) = u\bar{d}\,;\ \pi^-(140) = d\bar{u} : I(J)^P = 1(0)^- \text{ and}$$

$$\pi^0(135) = (u\bar{u}-d\bar{d})/\sqrt{2} : I(J)^{PC} = 1(0)^{-+}$$

- Principle of *isospin symmetry*: it is a good approximation to treat u- and d-quarks as having same masses

Particles with I=0 are called *isosinglets* :

$$\Lambda(1116) = uds,\ I(J)^P = 0\left(\frac{1}{2}\right)^+$$

- By introducing isospin, we get more criteria for non-exotic particles:

| | sss | $ssq_i$ | $sq_iq_j$ | $q_iq_jq_k$ | $s\bar{s}$ | $s\bar{q}_i$ | $\bar{s}q_i$ | $q_i\bar{q}_i$ | $q_i\bar{q}_j$ |
|---|---|---|---|---|---|---|---|---|---|
| S | -3 | -2 | -1 | 0 | 0 | -1 | 1 | 0 | 0 |
| Q | -1 | 0; -1 | 1; 0; -1 | 2; 1; 0; -1 | 0 | 0; -1 | 1; 0 | 0 | -1; 1 |
| I | 0 | 1/2 | 0; 1 | 3/2; 1/2 | 0 | 1/2 | 1/2 | 0; 1 | 0; 1 |

In all observed interactions (save pentaquarks) isospin-related criteria are satisfied as well, confirming once again the quark model.

- This allows predictions of possible multiplet members: suppose we observe production of the $\Sigma^+$ baryon in a strong interaction:

$$K^- + p \to \pi^- + \Sigma^+$$

which then decays weakly :

$$\Sigma^+ \to \pi^+ + n$$
$$\Sigma^+ \to \pi^0 + p$$

It follows that $\Sigma^+$ baryon quantum numbers are: B = 1, Q = 1, $S$ = -1 and hence Y = 0 and $I_3 = 1$.

- Since $I_3 > 0 \Rightarrow I \neq 0$ and there are more multiplet members!

- When a baryon has $I_3$=1, the only possibility for isospin is I=1, and we have a triplet:

$$S^+, S^0, S^-$$

Indeed, all such particles have been observed:

$$K^- + p \to \pi^0 + \Sigma^0$$
$$\hookrightarrow \Lambda + \gamma$$
$$K^- + p \to \pi^+ + \Sigma^-$$
$$\hookrightarrow \pi^- + n$$

Masses and quark composition of Σ-baryons are:

$$\Sigma^+(1189) = uus \; ; \; \Sigma^0(1193) = uds \; ; \; \Sigma^-(1197) = dds$$

It indicates that d-quark is heavier than u-quark, under following assumptions:

(a) strong interactions between quarks do not depend on their flavour and give contribution of $M_o$ to the baryon mass

(b) electromagnetic interactions contribute as $\delta \sum e_i e_j$, where $e_i$ are quark charges and $\delta$ is a constant

The simplest attempt to calculate mass difference of up- and down-quarks:

$$M(\Sigma^-) = M_0 + m_s + 2m_d + \delta/3$$

$$M(\Sigma^0) = M_0 + m_s + m_d + m_u - \delta/3$$

$$M(\Sigma^+) = M_0 + m_s + 2m_u$$

$$\Downarrow$$

$$m_d - m_u = [\, M(\Sigma^-) + M(\Sigma^0) - 2M(\Sigma^+) \,] / 3 = 3.7 \text{ MeV/c}^2$$

- NB : this is a very simplified model, as under these assumptions $M(\Sigma^0) = M(\Lambda)$ , while their mass difference $M(\Sigma^0) - M(\Lambda) \approx 77$ Mev/$c^2$ .

Generally, combining other methods:

$$2 \leq m_d - m_u \leq 4 \text{ ( MeV/c}^2 \text{ )}$$

which is negligible comparing to hadron masses (but not if compared to estimated u and d masses themselves)

# Resonances

❖ *Resonances* are highly unstable particles that decay by strong interaction (lifetimes about $10^{-23}$ s)

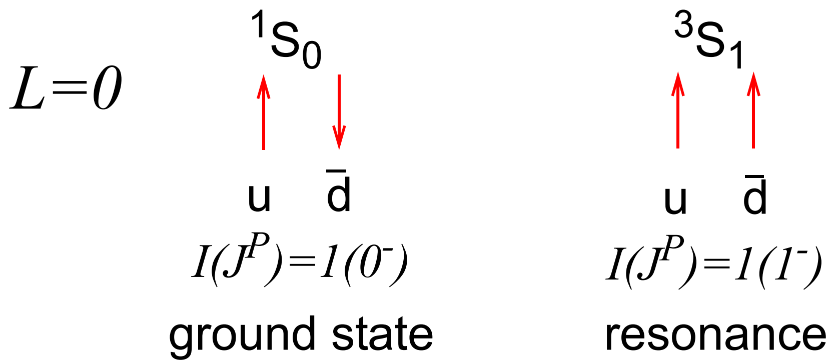


Figure 89: Example of a $q\bar{q}$ system in ground and first excited states

❖ If a ground state is a member of an isospin multiplet, then resonant states will form a corresponding multiplet too

Since resonances have very short lifetimes, they can only be detected by registering their *decay products*:

$$\pi^- + p \rightarrow n + X$$
$$\quad\quad\quad\quad\quad\quad\quad \hookrightarrow A + B$$

- Invariant mass of a particle is measured via energies and masses of its decay products (see 4-vectors in Chapter I.):

$$W^2 \equiv (E_A + E_B)^2 - (\vec{p}_A + \vec{p}_B)^2 = E^2 - \vec{p}^2 = M^2 \qquad (106)$$

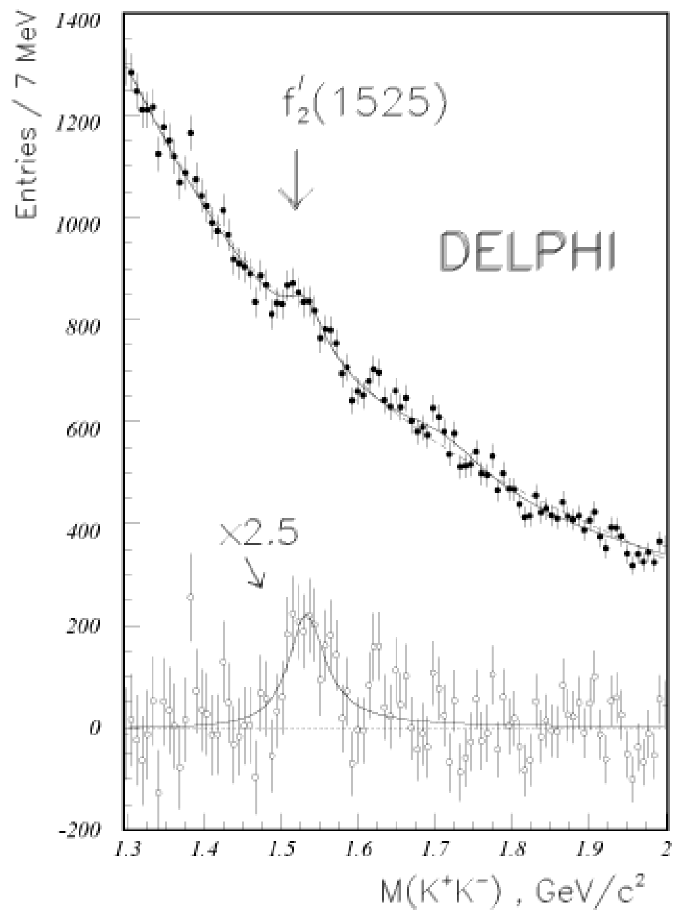


Figure 90: A typical resonance peak in $K^+K^-$ invariant mass distribution

- Resonance peak shapes are approximated by the *Breit-Wigner* formula:

$$N(W) = \frac{K}{(W-W_0)^2 + \Gamma^2/4} \qquad (107)$$

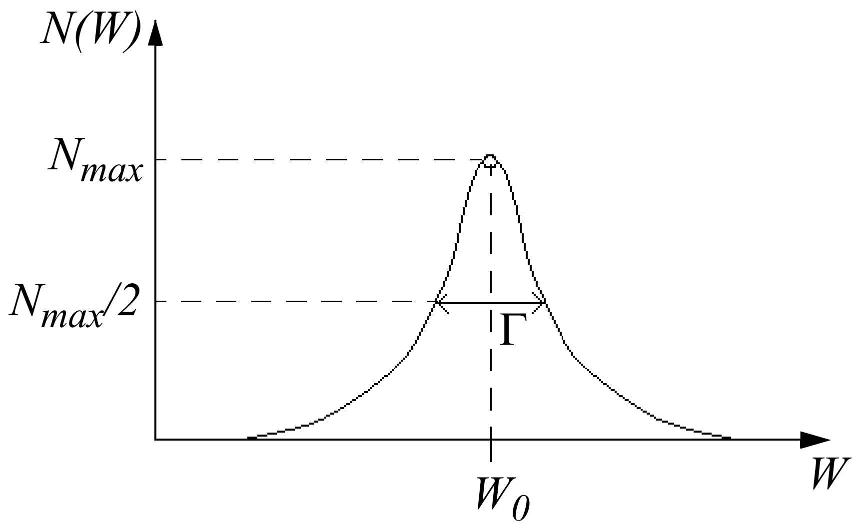


Figure 91: Breit-Wigner shape

- Mean value of the Breit-Wigner shape is the mass of a resonance: $M=W_0$
- $\Gamma$ is the width of a resonance, and it has the meaning of inverse mean lifetime of particle at rest: $\Gamma \equiv 1/\tau$

Internal quantum numbers of resonances are also derived from their decay products:

$$X^0 \rightarrow \pi^+ + \pi^-$$

for such $X^0$: $B = 0; S = C = \tilde{B} = T = 0; Q = 0 \Rightarrow$ Y=0 and $I_3$=0.

- When $I_3$=0, to determine whether I=0 or I=1, searches for isospin multiplet partners have to be done.

Example: $\rho^0(769)$ and $\rho^0(1700)$ both decay to $\pi^+\pi^-$ pair and have isospin partners $\rho^+$ and $\rho^-$ :

$$\pi^\pm + p \rightarrow p + \rho^\pm$$
$$\hookrightarrow \pi^\pm + \pi^0$$

By measuring angular distribution of $\pi^+\pi^-$ pair, the relative orbital angular momentum of the pair $L$ can be determined, and hence spin and parity of the resonance $X^0$ are (S=0):

$$J = L; P = P_\pi^2(-1)^L = (-1)^L; C = (-1)^L$$

- Some excited states of pion:

| resonance | $I(J^{PC})$ |
|---|---|
| $\rho^0(769)$ | $1(1^{--})$ |
| $f_2^0(1275)$ | $0(2^{++})$ |
| $\rho^0(1700)$ | $1(3^{--})$ |

- B=0 : *meson resonances*, B=1 : *baryon resonances*.

## Many baryon resonances can be produced in pion-nucleon scattering:

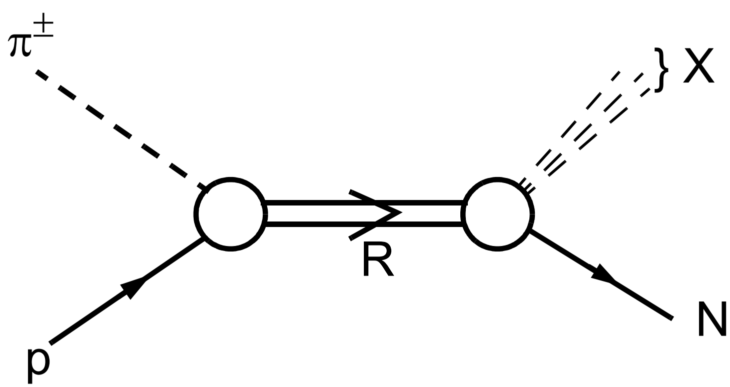


Figure 92: Formation of a resonance R and its decay into a nucleon N

- Peaks in the observed total cross-section of the $\pi^{\pm}$p-reaction correspond to resonance formation

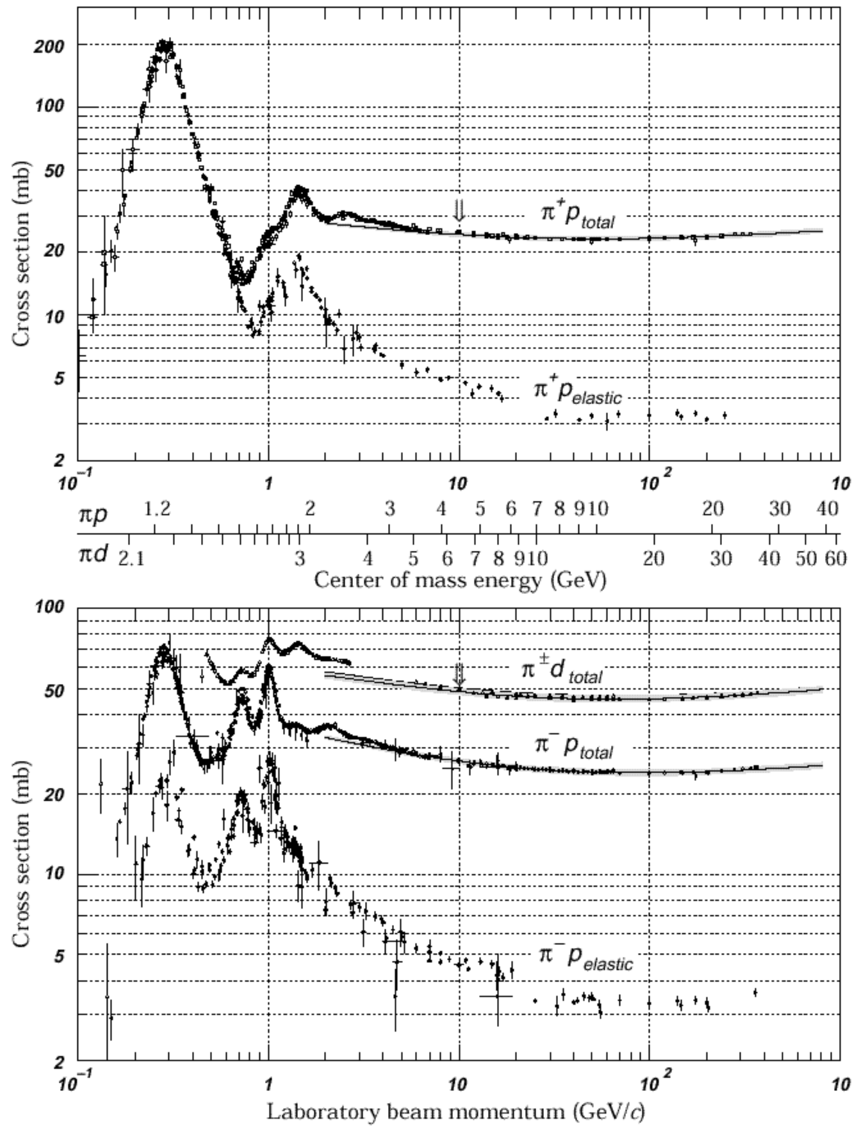


Figure 93: Scattering of $\pi^+$ and $\pi^-$ on proton

All the resonances produced in pion-nucleon scattering have the same internal quantum numbers as the initial state:

$$B = 1; S = C = \tilde{B} = T = 0$$

and thus Y=1 and $Q=I_3+1/2$

Possible isospins are $I=1/2$ or $I=3/2$, since for pion $I=1$ and for nucleon $I=1/2$

- $I=1/2 \Rightarrow$ N-resonances ($N^0$, $N^+$)
- $I=3/2 \Rightarrow$ Δ-resonances ($\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$)

Figure 93: peaks at $\approx$1.2 GeV/$c^2$ correspond to $\Delta^{++}$ and $\Delta^0$ resonances:

$$\pi^+ + p \to \Delta^{++} \to \pi^+ + p$$

$$\pi^- + p \to \Delta^0 \to \pi^- + p$$

$$\hookrightarrow \pi^0 + n$$

- Fits by the Breit-Wigner formula show that both $\Delta^{++}$ and $\Delta^0$ have approximately same mass of $\approx$1232 MeV/$c^2$ and width $\approx$120 MeV/$c^2$.
    - Studies of angular distributions of decay products show that $I(J^P) = \frac{3}{2}\left(\frac{3}{2}\right)^+$
    - Remaining members of the multiplet are also observed: $\Delta^+$ and $\Delta^-$
- There is no lighter state with these quantum numbers $\Rightarrow$ $\Delta$ is a *ground state*, although observed as a resonance.

## *Quark diagrams*

- Quark diagrams are convenient way of illustrating strong interaction processes

Consider an example:

$$\Delta^{++} \to p + \pi^+$$

- The only 3-quark state consistent with $\Delta^{++}$ quantum numbers (Q=2) is (uuu), while p=(uud) and $\pi^+$=($u\bar{d}$)

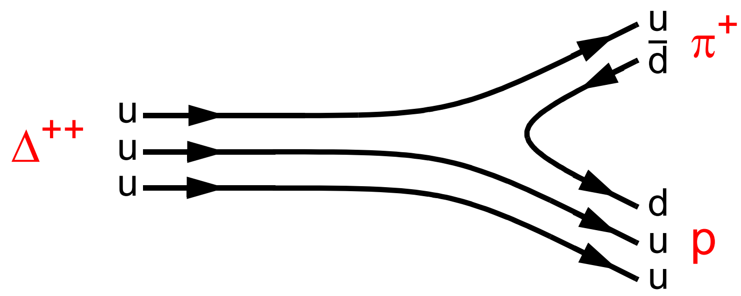


Figure 94: Quark diagram of the reaction $\Delta^{++} \rightarrow p + \pi^+$

## Analogously to Feinman diagrams:

- arrow pointing rightwards denotes a particle, and leftwards – antiparticle
- time flows from left to right

## Allowed resonance formation process:

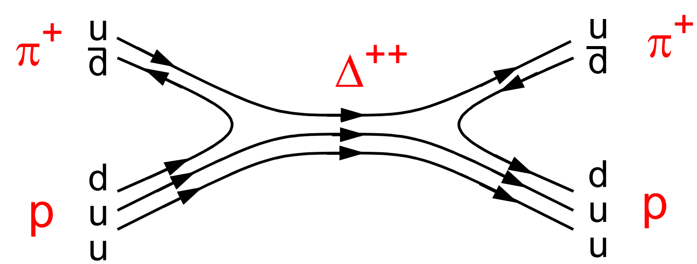


Figure 95: Formation and decay of $\Delta^{++}$ resonance in $\pi^+ p$ elastic scattering

# Hypothetical exotic resonance:

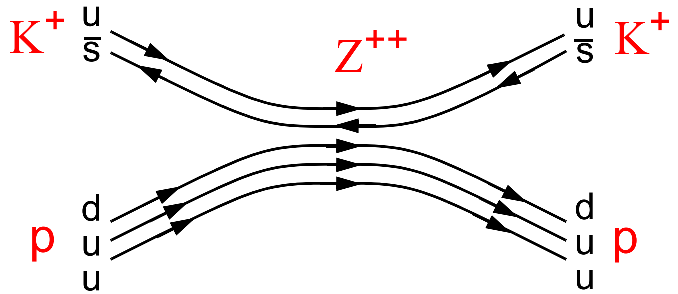


Figure 96: Formation and decay of an exotic resonance $Z^{++}$ in $K^+p$ elastic scattering

- Quantum numbers of such a particle $Z^{++}$ are exotic. There are no resonance peaks in the corresponding cross-section, but data are scarce:

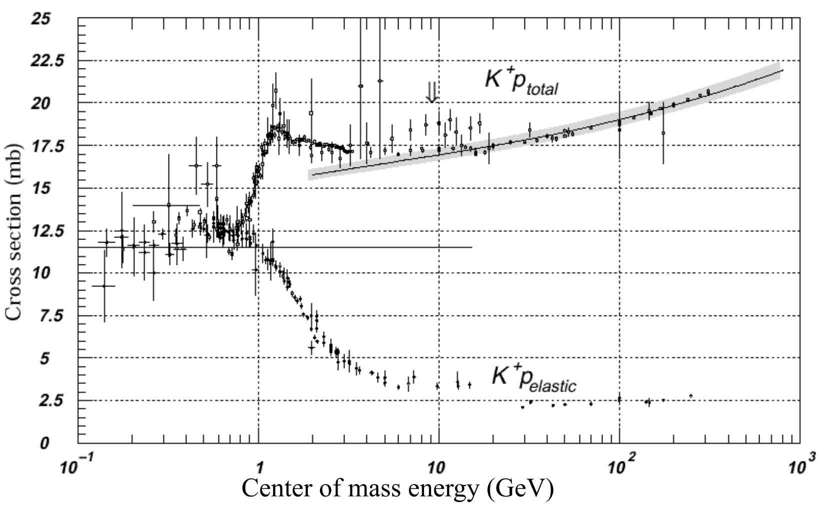


Figure 97: Cross-section for $K^+p$ scattering

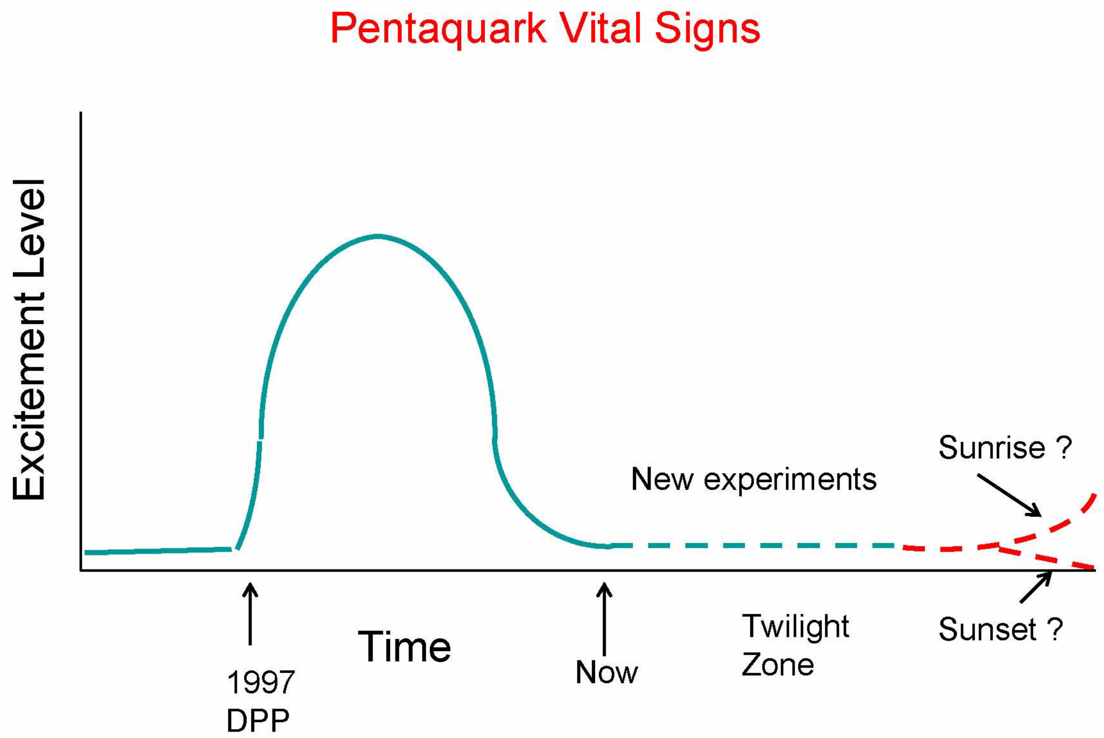


Figure 98: Pentaquark searches status as of October 2005, by Paul Stoler