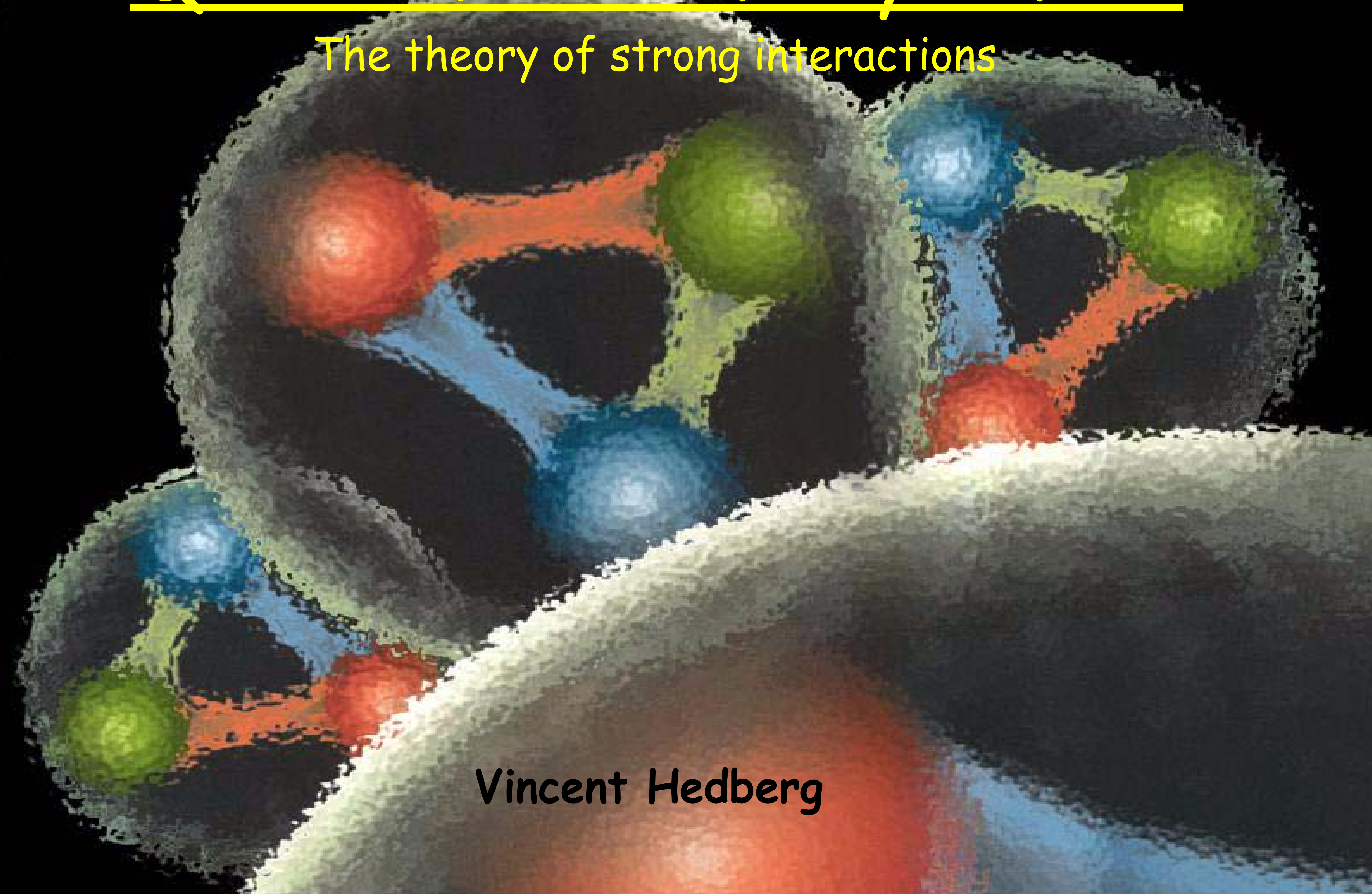
Quantum Chromodynamics

The theory of strong interactions



Vincent Hedberg

Quantum Chromodynamics

➡ Interactions by massless spin-1 particles: Gauge bosons

- Quantum electrodynamics (QED): Photons
  Quantum chromodynamics (QCD): Gluons

- QED: Photons couple to electric charges (Q)
  QCD: Gluons couple to colour charges (Y^c and I_3^c).

- Y^c : colour hypercharge.
  I_3^c : colour isospin charge.

- The strong interaction is flavour-independent:
  It acts the same on u,d,s,c,b and t

Quantum Chromodynamics

- The colour hypercharge (Y^c) + the colour isospin charge (I_3^c)

  ⇨ three colour and three anti-colour quark states

QCD

	Y^c	I_3^c		Y^c	I_3^c
r	1/3	1/2	$\bar{r}$	-1/3	-1/2
g	1/3	-1/2	$\bar{g}$	-1/3	1/2
b	-2/3	0	$\bar{b}$	2/3	0

- **Colour confinement:** Mesons and baryons total colour charge = 0.
- **Colour wave-functions:**

QCD

$$q\bar{q} = \frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})$$

$$q_1q_2q_3 = \frac{1}{\sqrt{6}}(r_1g_2b_3-g_1r_2b_3+b_1r_2g_3-b_1g_2r_3+g_1b_2r_3-r_1b_2g_3)$$

Quantum Chromodynamics

- QCD: Colour hypercharge (Y^C) and colour isospin charge (I_3^C)

  Quark model: Flavour hypercharge (Y) and flavour isospin (I_3)

Quark Model

	Q	Y	I_3		Q	Y	I_3
d	-1/3	1/3	-1/2	$\bar{d}$	1/3	-1/3	1/2
u	2/3	1/3	1/2	$\bar{u}$	-2/3	-1/3	-1/2
s	-1/3	-2/3	0	$\bar{s}$	1/3	2/3	0
c	2/3	4/3	0	$\bar{c}$	-2/3	-4/3	0
b	-1/3	-2/3	0	$\bar{b}$	1/3	2/3	0
t	2/3	4/3	0	$\bar{t}$	-2/3	-4/3	0

- The total wavefunction of hadrons:

$$\Psi_{total} = \Psi_{space} \times \Psi_{spin} \times \Psi_{flavour} \times \Psi_{colour}$$

Quantum Chromodynamics

➡ Photons electric charge = 0 !
Gluons have colour charge !

- Gluons exist in 8 different colour states:

QCD

$\chi^C_{g1} = r\bar{g}$	$I^C_3 = 1$	$Y^C = 0$
$\chi^C_{g2} = \bar{r}g$	$I^C_3 = -1$	$Y^C = 0$
$\chi^C_{g3} = r\bar{b}$	$I^C_3 = 1/2$	$Y^C = 1$
$\chi^C_{g4} = \bar{r}b$	$I^C_3 = -1/2$	$Y^C = -1$
$\chi^C_{g5} = g\bar{b}$	$I^C_3 = -1/2$	$Y^C = 1$
$\chi^C_{g6} = \bar{g}b$	$I^C_3 = 1/2$	$Y^C = -1$
$\chi^C_{g7} = 1/\sqrt{2}\,(g\bar{g} - \bar{r}r)$	$I^C_3 = 0$	$Y^C = 0$
$\chi^C_{g8} = 1/\sqrt{6}\,(g\bar{g} - r\bar{r} - 2\,b\bar{b})$	$I^C_3 = 0$	$Y^C = 0$

- Color confinement ⇨ Gluons do not exist a free particles.

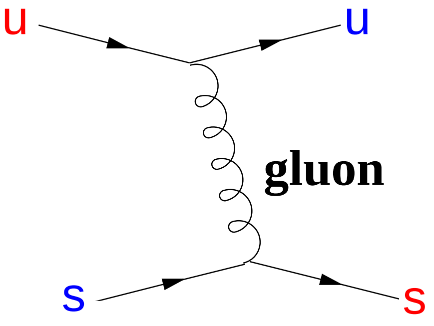
Quantum Chromodynamics

- Colour hypercharge + colour isospin charge are **additive quantum numbers** (like the electric charge).

Example:

$I_3^C = 1/2$ $Y^C = 1/3$

$I_3^C = 0$ $Y^C = -2/3$



$I_3^C = 0$ $Y^C = -2/3$

$I_3^C = 1/2$ $Y^C = 1/3$

Gluon:

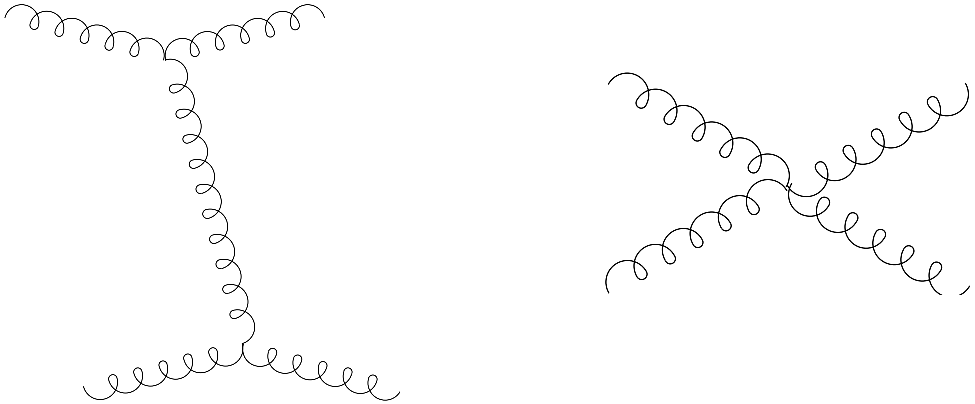
$$I_3^C = I_3^C(r) - I_3^C(b) = \frac{1}{2}$$

$$Y^C = Y^C(r) - Y^C(b) = 1$$

$$\chi_{g3}^c = r\,\bar{b}$$

Quantum Chromodynamics

- Gluons can couple to other gluons
  (since they carry colour charge).



- **QCD: Gluons can form colourless states.**
- **Glueballs**
- **Experiments: Difficult to prove existance of glueballs.**

- Leptons have no colour charge.
  They do not interact strongly.

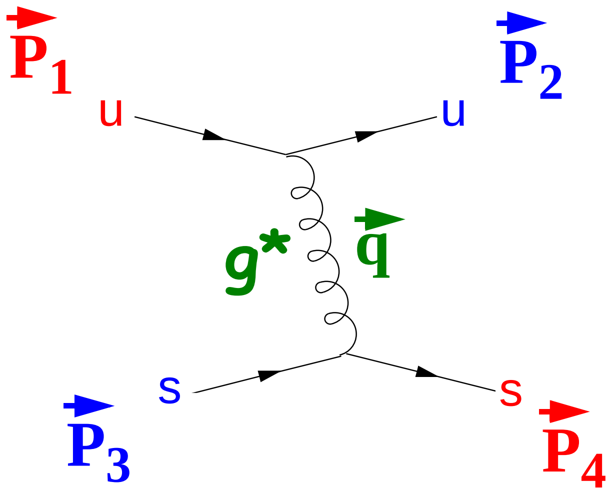
Quantum Chromodynamics

➡ The strong coupling constant

- QED: The electromagnetic coupling contant: α_{em}
  QCD: The **strong couplings constant:** α_s

- α_s: "running constant" ⇨ Decreases with increasing Q^2

- What is Q^2 ?



The 4-vectors of the interacting quarks:

$$\vec{P} = (E,\bar{p}) = (E,p_x,p_y,p_z)$$

The 4-vectors of the quarks ⇨ 4-vector of gluon

$$\vec{q} = (E_q,\bar{q}) = \vec{P}_1 - \vec{P}_2 = (E_1 - E_2 , \bar{P}_1 - \bar{P}_2)$$

The squared 4-vector energy-momentum tranfer:

$$Q^2 = -\vec{q} \bullet \vec{q}$$ (i.e. Q = the "mass" of the gluon)

Quantum Chromodynamics

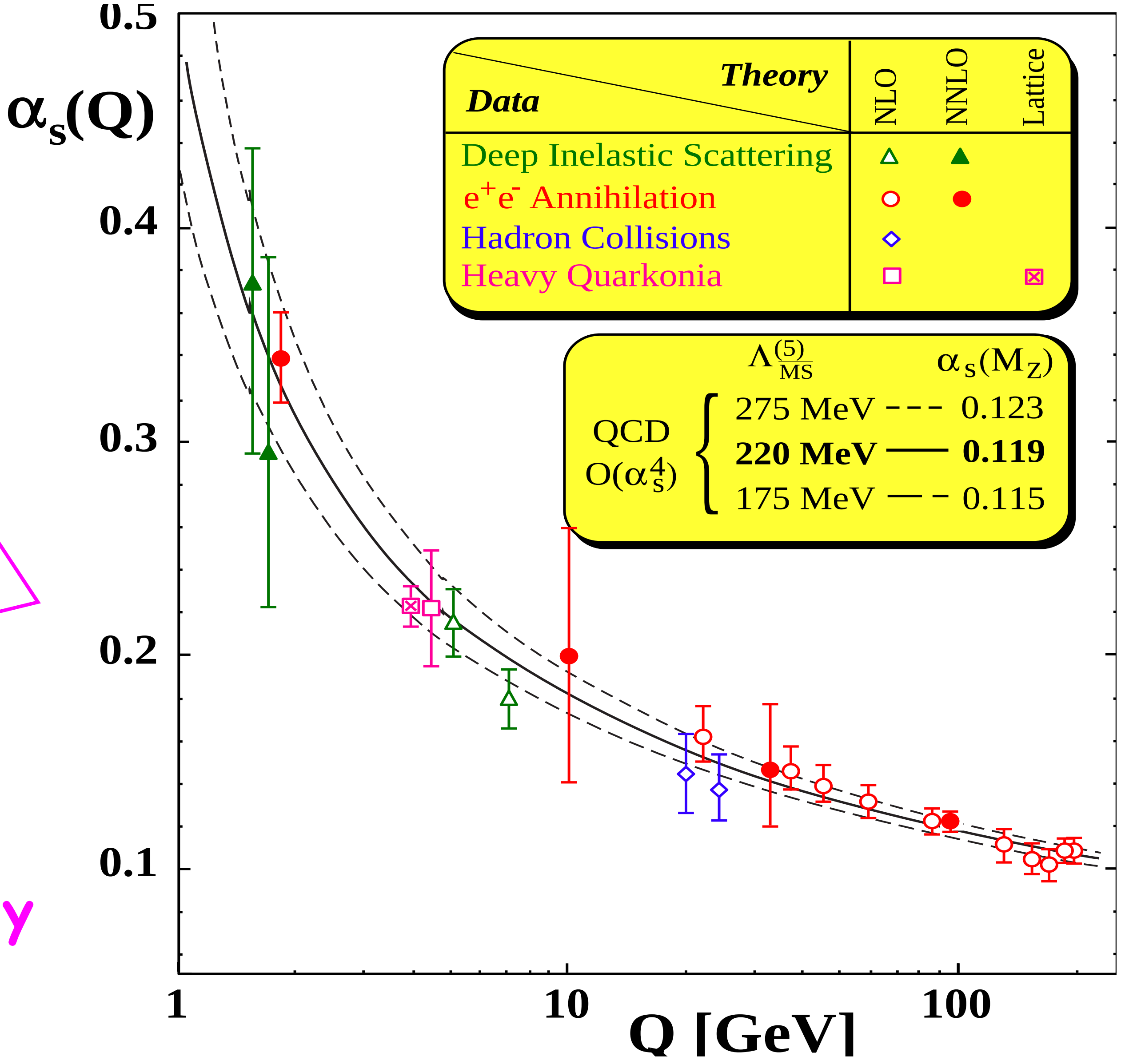
➡ The strong coupling constant

QCD prediction:

$$\alpha_s = \frac{12\pi}{(33-2N_f)\ln(Q^2/\Lambda^2)}$$

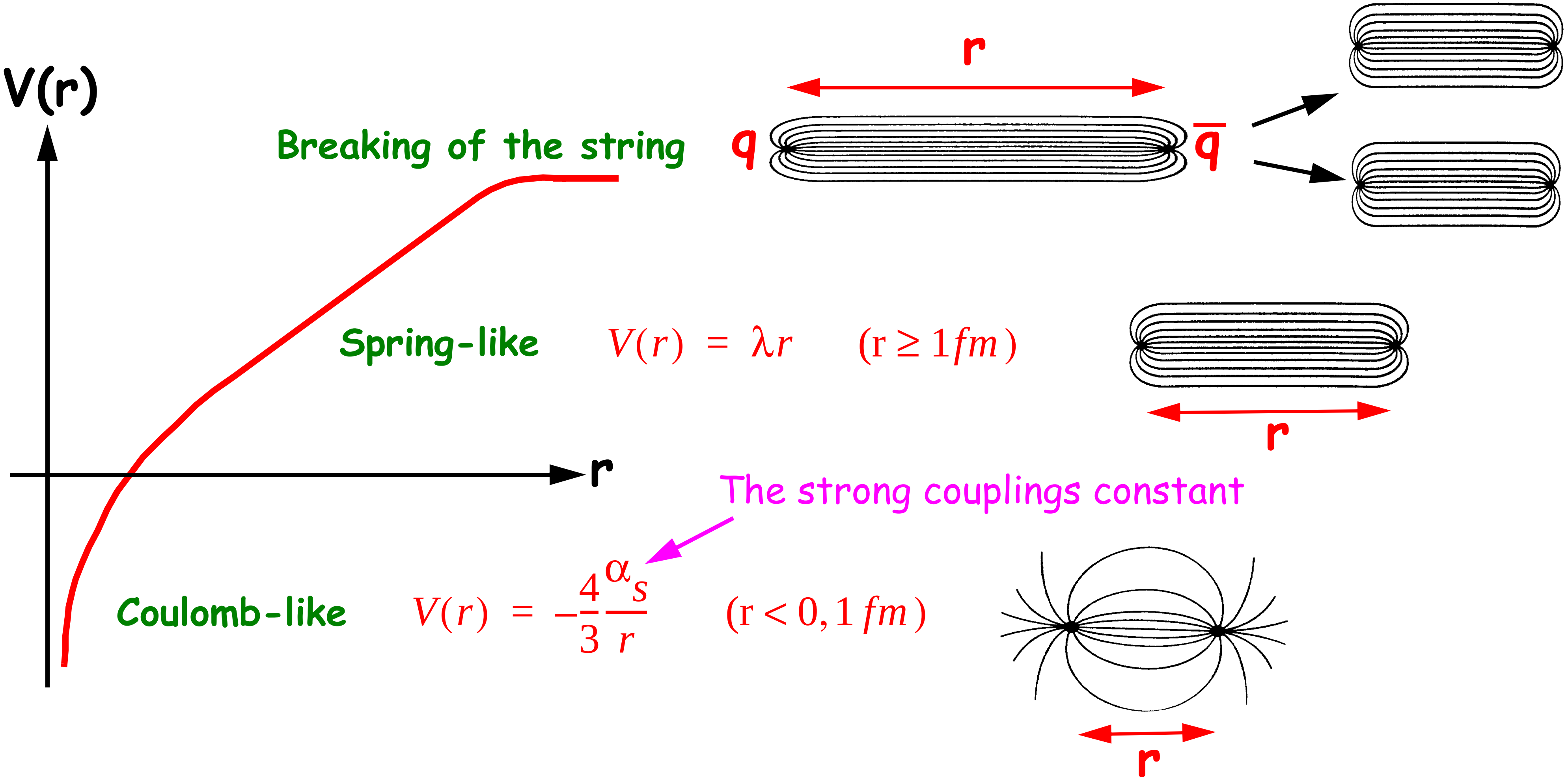
N_f: Number of allowed quark flavours

Λ: QCD scale parameter Determined experimentally ($\Lambda \approx 0.2$ GeV)



Quantum Chromodynamics

➡ The quark-antiquark potential (mesons)



Breaking of the string

Spring-like $V(r) = \lambda r \quad (r \geq 1 fm)$

The strong couplings constant

Coulomb-like $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} \quad (r < 0,1\, fm)$

Quantum Chromodynamics

➡ The principle of asymptotic freedom.

- QCD - Short distances - interaction is weaker
  QCD - Long distances - interaction is stronger

Small distance ⇒ Coulomb-like potential
Small distance ⇒ large Q^2 ⇒ small α_s

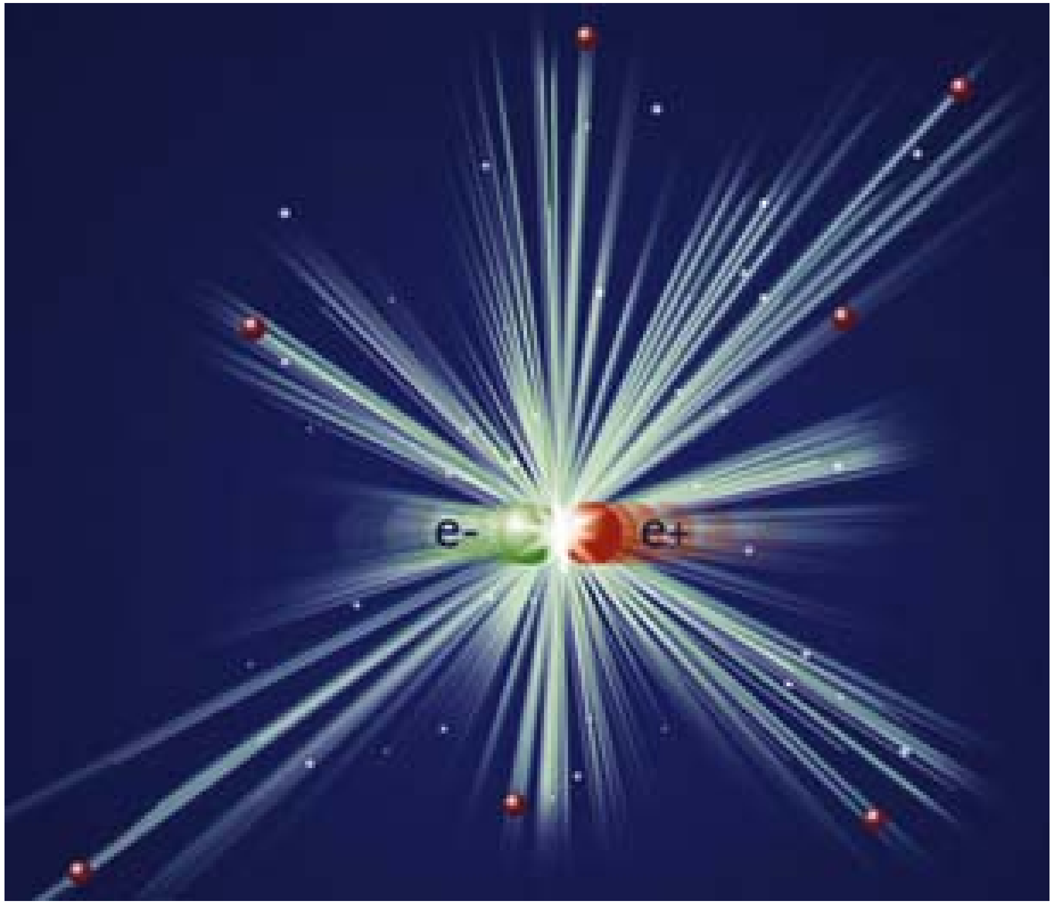
→ quarks and gluons free particles

- Small distances ⇒ first order diagrams.
  Large distances ⇒ higher order diagrams.

- Higher-order diagrams ⇒
  Confinement cannot be calculated analytically

Electron-positron collisions


e^+ e^-



Electron-positron annihilation

➡ The R-value from e^+e^- experiments

- **Measurement:**

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

rate of collisions that give hadrons

rate of collisions that give muons

- **Prediction from theory:**

$$R = N_c \sum e_q^2$$

N_c is the number of colours (=3)

e_q the charge of the quarks.

Electron-positron annihilation

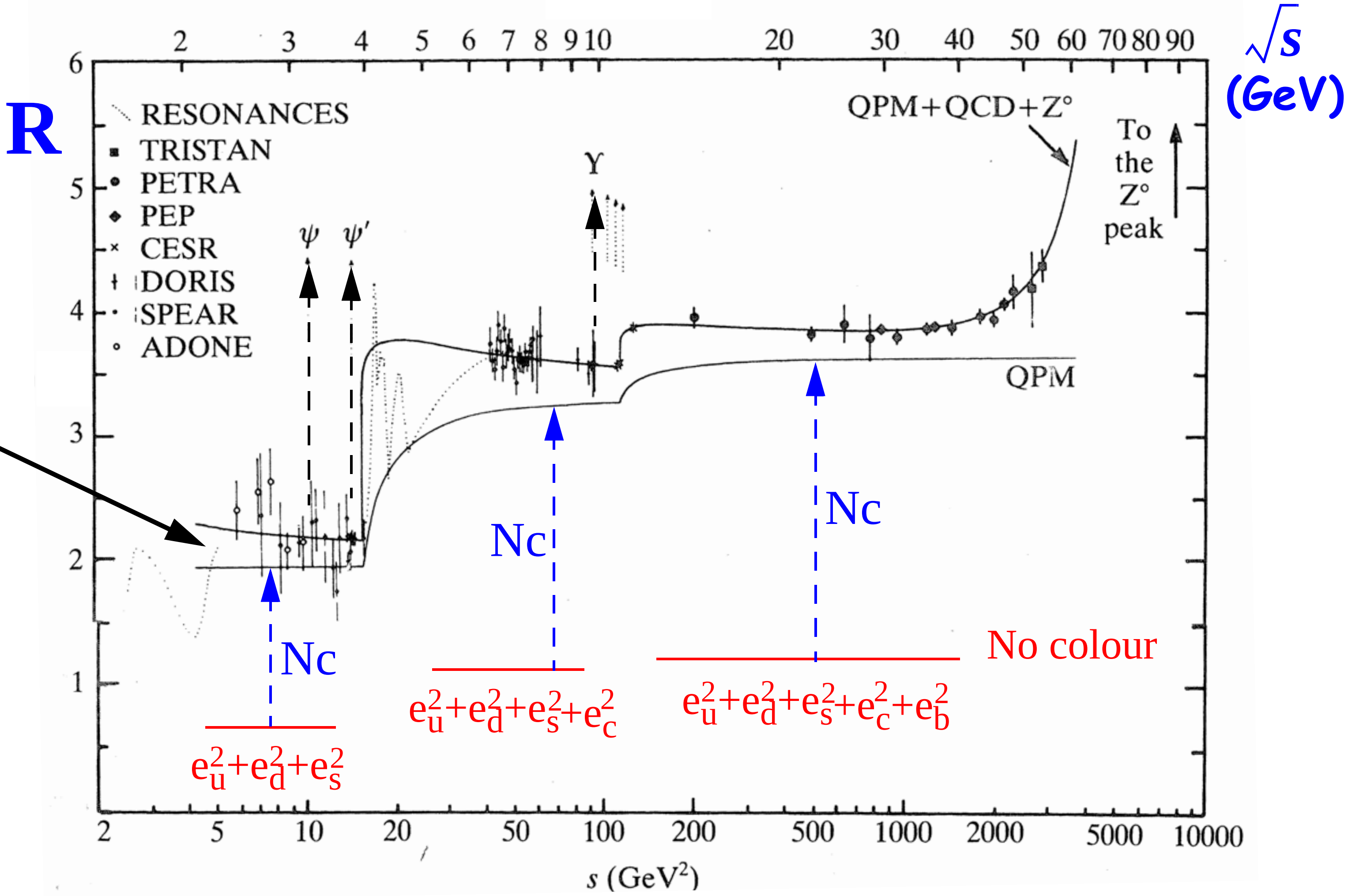
$$R = N_c(e_u^2+e_d^2+e_s^2) = 3\,((\text{-}1/3)^2+(\text{-}1/3)^2+(2/3)^2) = 2 \quad \text{if } \sqrt{s} < m_\psi$$

$$R = N_c(e_u^2+e_d^2+e_s^2+e_c^2)=10/3 \quad \text{if } \sqrt{s} < m_\Upsilon$$

$$R = N_c(e_u^2+e_d^2+e_s^2+e_c^2+e_b^2)=11/3 \quad \text{if } \sqrt{s} > m_\Upsilon$$

$$R = N_c \sum e_q^2 (1 + \alpha_s(Q^2)/\pi)$$

Correction for gluon radiation

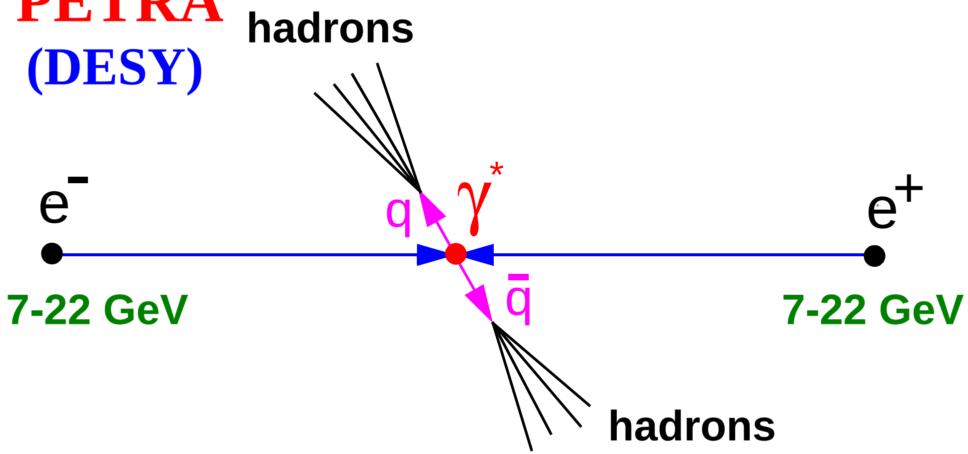


Electron-positron annihilation

→ Jets of particles

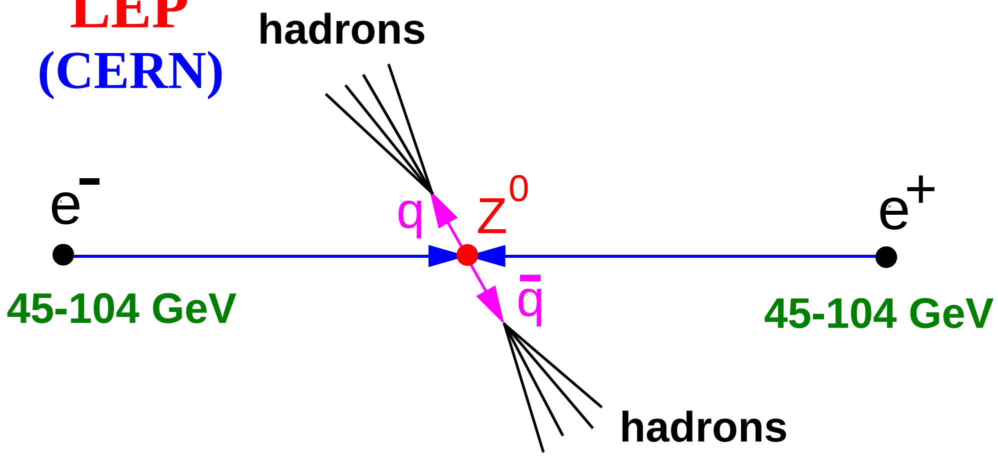
- e^+e^- annihilation process:
  A **photon** or a Z^0 is produced ⇨ **quark-antiquark pair**.
- The quark and the antiquark **fragment** into observable **hadrons**.

PETRA
(DESY)



Length: 2.3 km
Experiments: Tasso, Jade, Pluto, Mark J, Cello

LEP
(CERN)



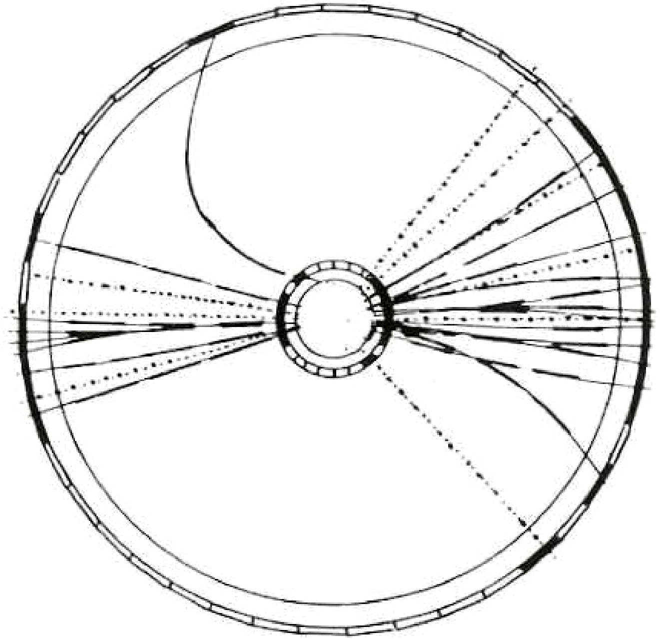
Length: 27 km
Experiments: DELPHI, OPAL, ALEPH, L3

Electron-positron annihilation

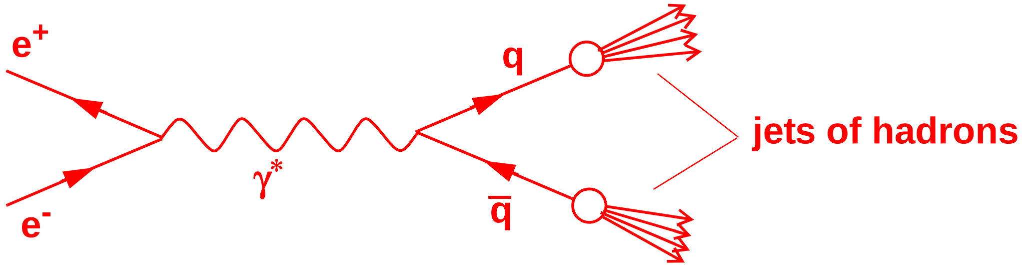
➡ Two jets of particles

- The direction of the jet ⇨ The direction of the quark

Two-jet events in the Jade experiment



Two-jet events in theory:



$$e^+ + e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$$

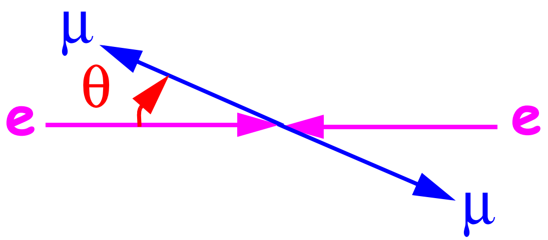
- **Energy and momentum conservation** ⇨
The quark and anti-quark have equal energy and opposite direction
Jets have the same energy and opposite direction

Electron-positron annihilation

➡ The angular distribution of 2 jets and the spin of the quarks.

- $e^+ + e^- \rightarrow \gamma^* \rightarrow \mu^+ + \mu^-$

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$



- $e^+ + e^- \rightarrow \gamma^* \rightarrow q + \bar{q}$

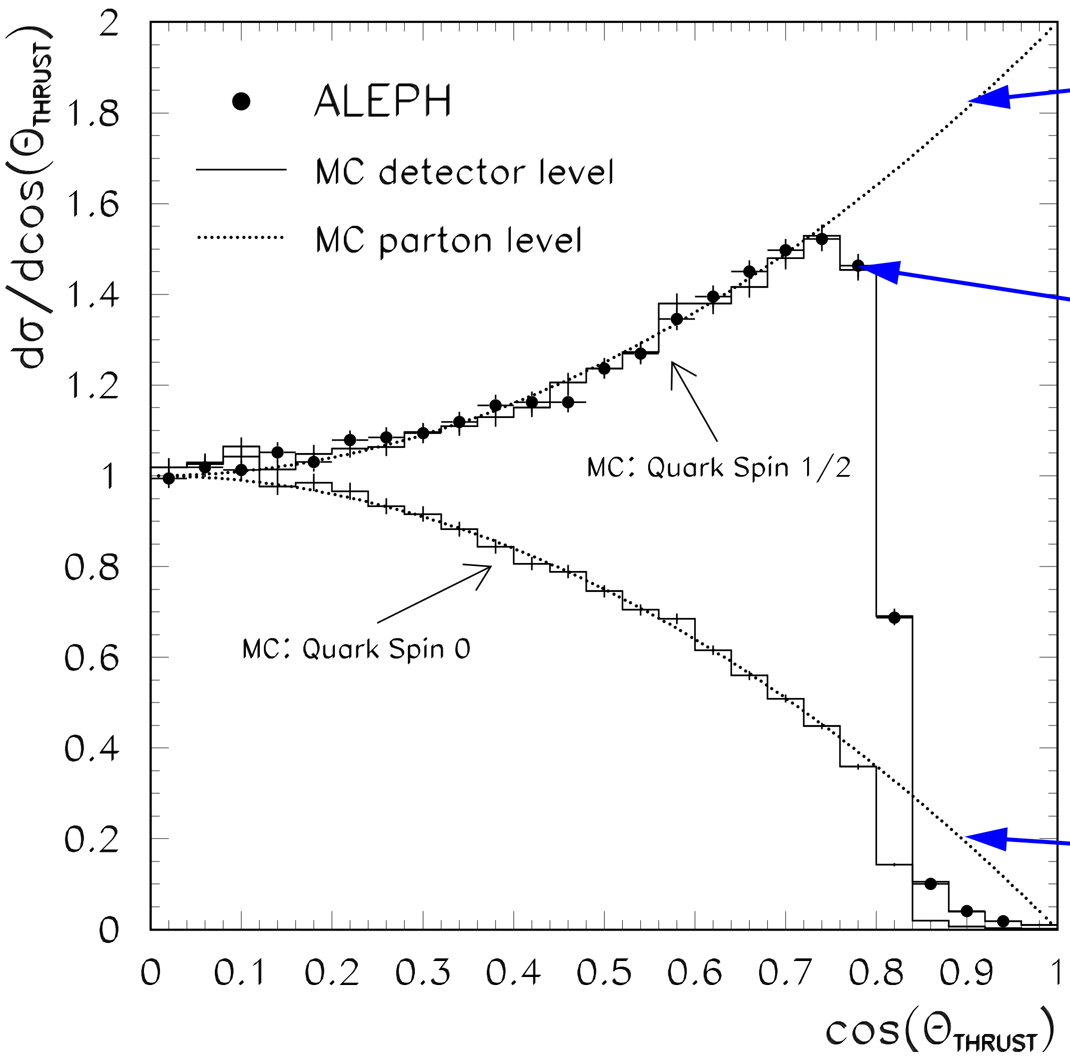
$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$ ⇨ **quark spin = 1/2**

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 - \cos^2\theta)$$ ⇨ **quark spin = 0**

e_q is the quark charge

N_c is the number of colours (=3)

Electron-positron annihilation



$1+\cos^2\theta$ (spin 1/2)

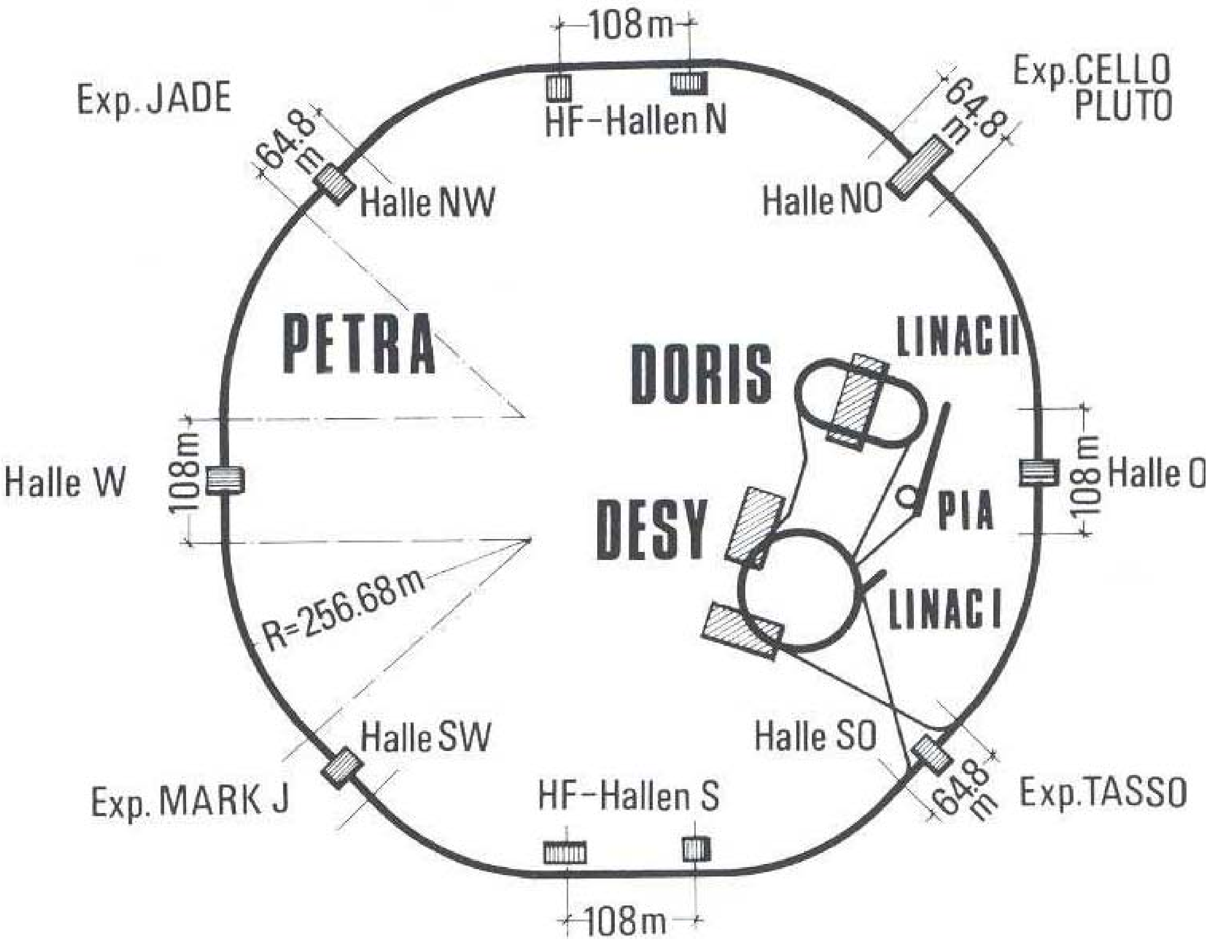
The experimental data

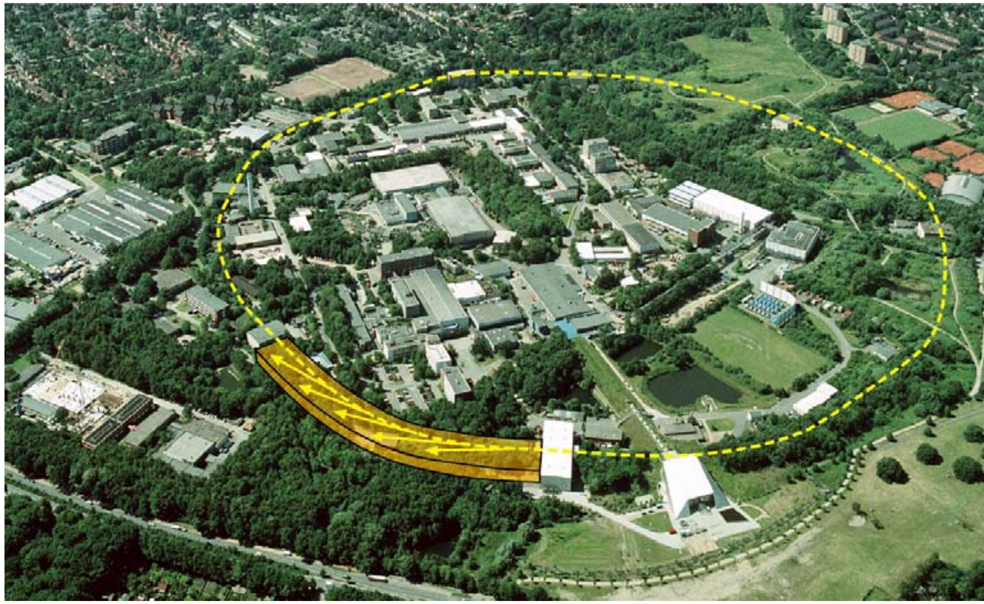
$1-\cos^2\theta$ (spin 0)

Conclusion: Quarks have spin = 1/2 !

The discovery of the gluon

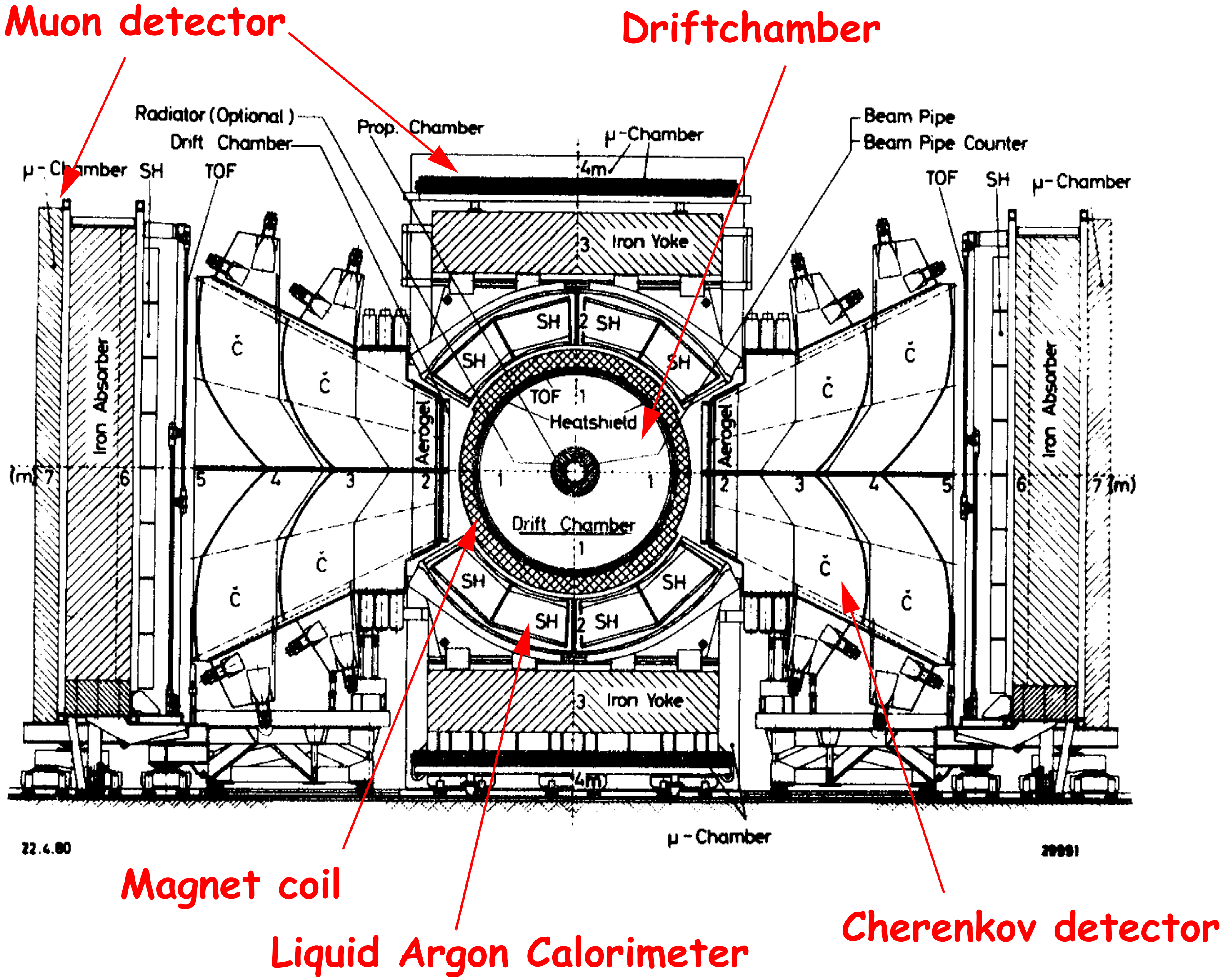
- The accelerator: PETRA at the German laboratory DESY.





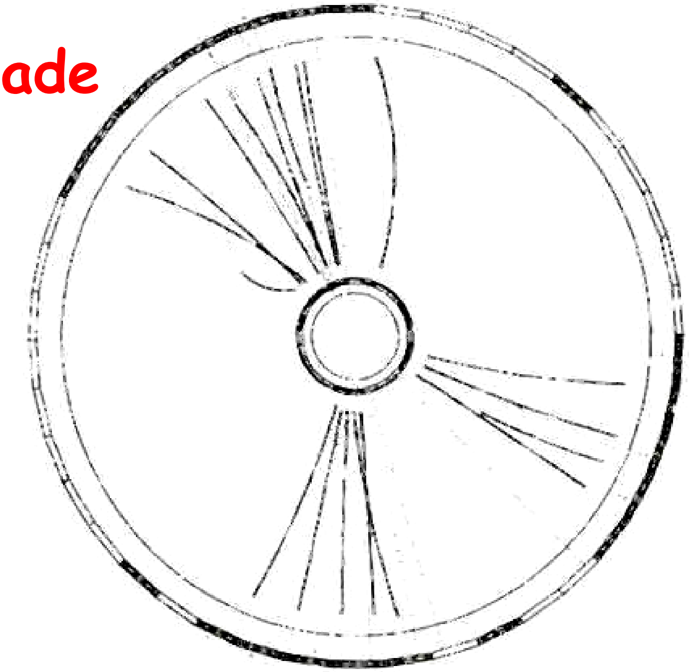
The discovery of the gluon

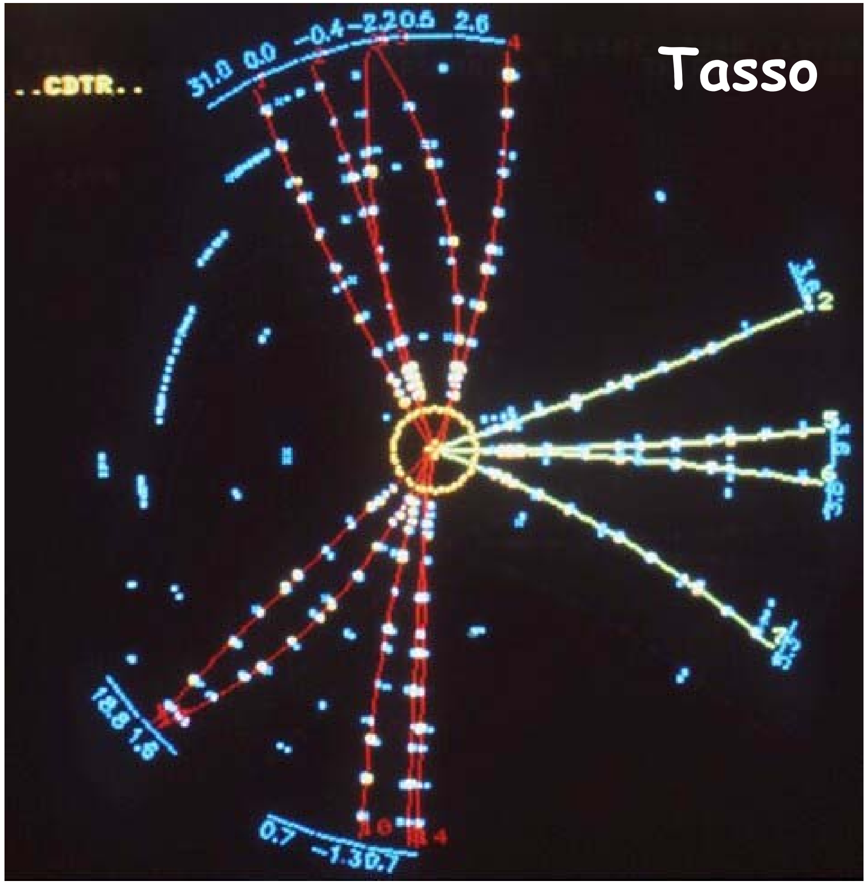
➡ The experiment: TASSO



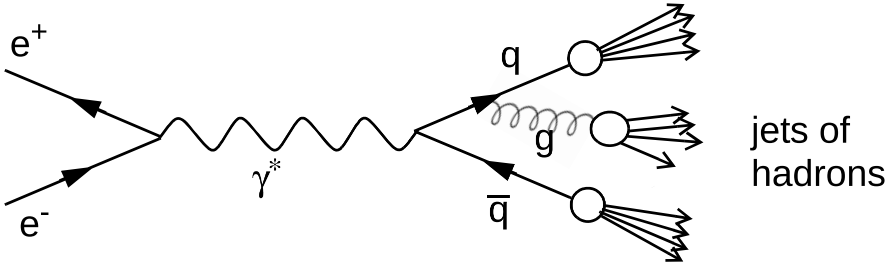
The discovery of the gluon

Jade





Three-jet events in theory:



Three-jet events ⇨ **The third jet from a gluon !**

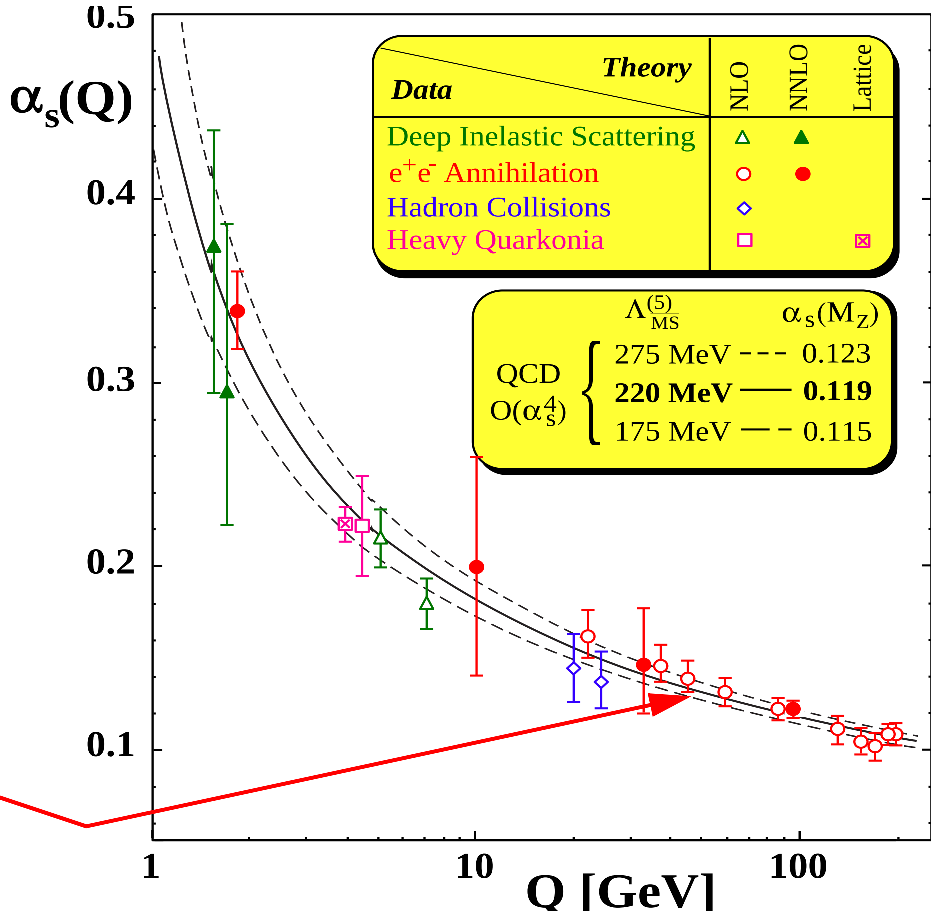
The discovery of the gluon

- The **probability** for **gluon** emission is proportional to α_s.

$$\alpha_s = \frac{\text{Number of three-jet events}}{\text{Number of two-jet events}}$$

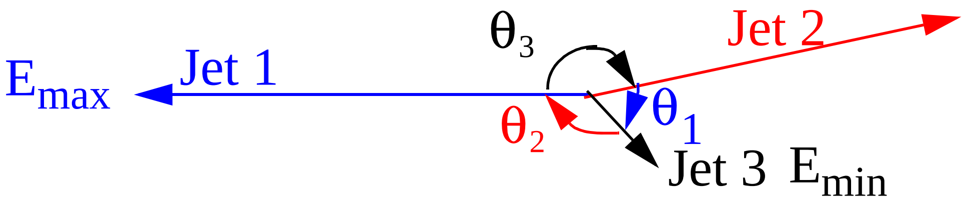
- **PETRA:**

$\alpha_s = 0.15 \pm 0.03$ for $\sqrt{s}$ = 30-40 GeV

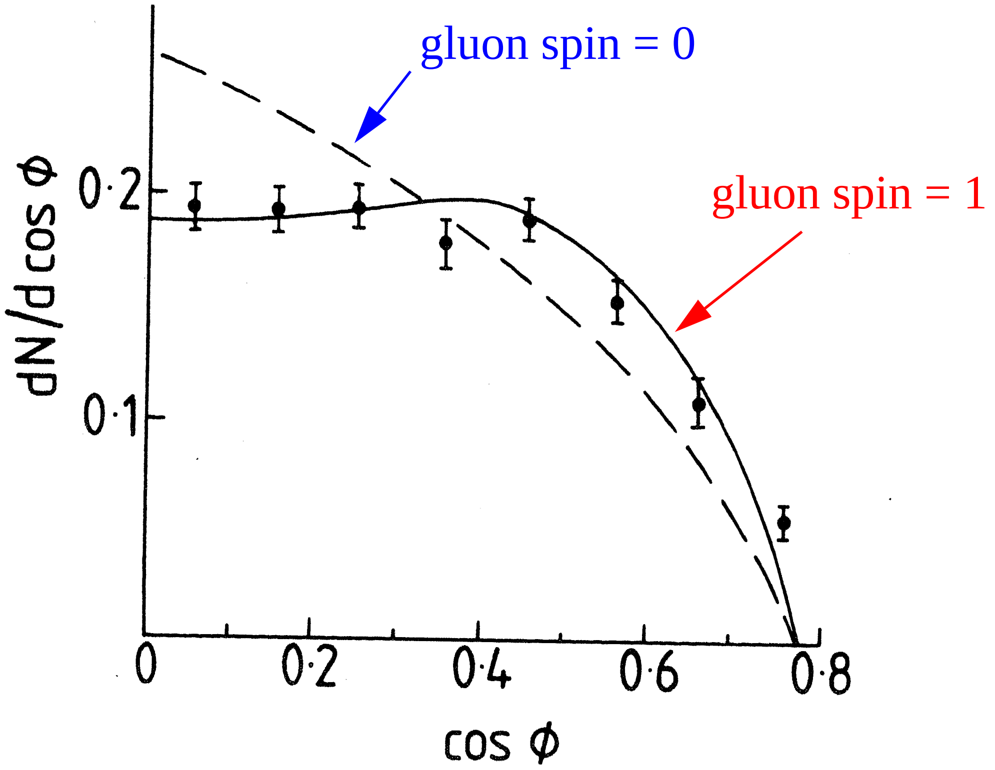


Electron-positron annihilation

➡ Angular distribution of 3 jets and the spin of the gluon.

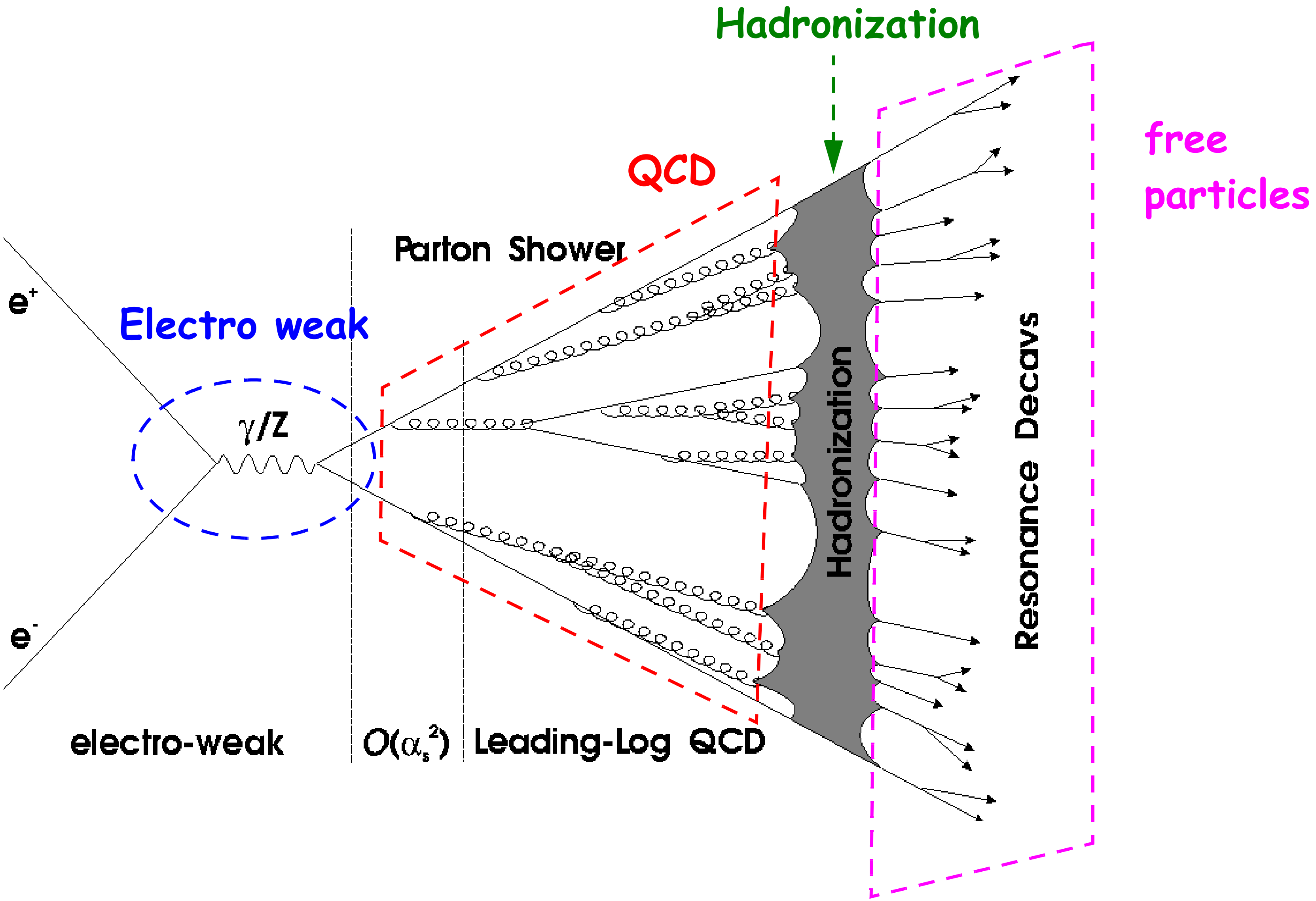


$$\cos\phi = \frac{\sin\theta_2 - \sin\theta_3}{\sin\theta_1}$$

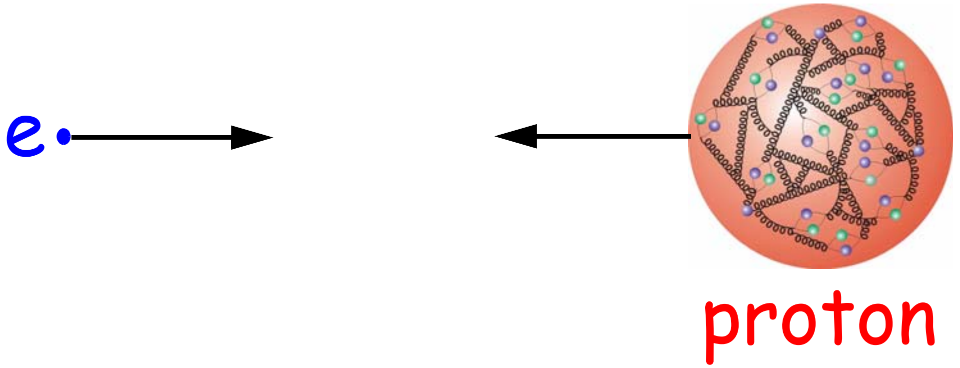


- **Angular distribution** ⇨ **Gluons have spin = 1**

Electron-positron annihilation



Electron-proton collisions



Electron-proton scattering

$\lambda >> r_p$ **Very low electron energies**

Scattering from a "point-like" spin-less object.

$\lambda = r_p$ **Low electron energies**

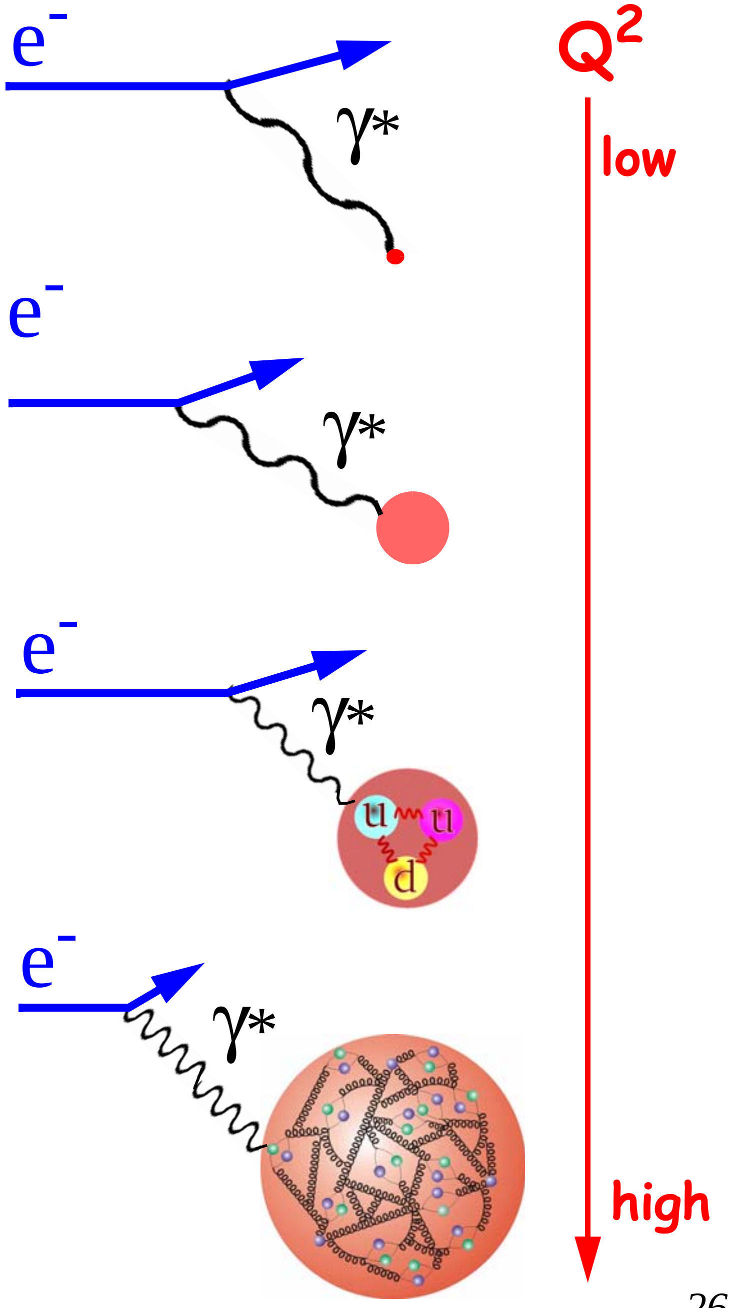
Scattering from an extended charged object.

$\lambda < r_p$ **High electron energies**

Interactions with the valence quarks in the proton.

$\lambda << r_p$ **Very high electron energies**

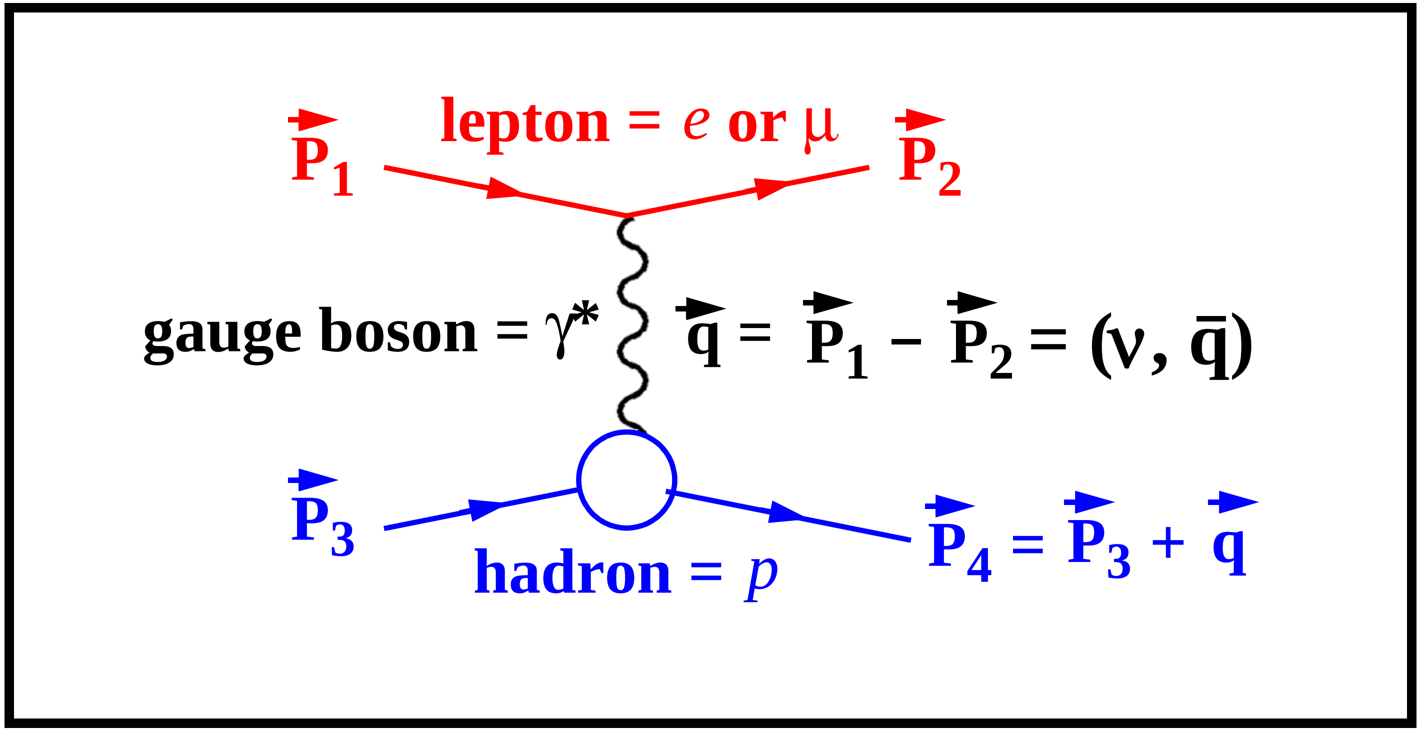
Interactions with the sea of quarks and gluons.



Electron-proton scattering

➡ Elastic scattering

- **Elastic scattering**: The same type of particles before and after.



$\vec{P}_1$ lepton = e or μ $\vec{P}_2$

gauge boson = γ^* $\vec{q} = \vec{P}_1 - \vec{P}_2 = (\nu, \bar{q})$

$\vec{P}_3$ hadron = p $\vec{P}_4 = \vec{P}_3 + \vec{q}$

- Elastic electron-proton scattering ⇨ Size of the proton

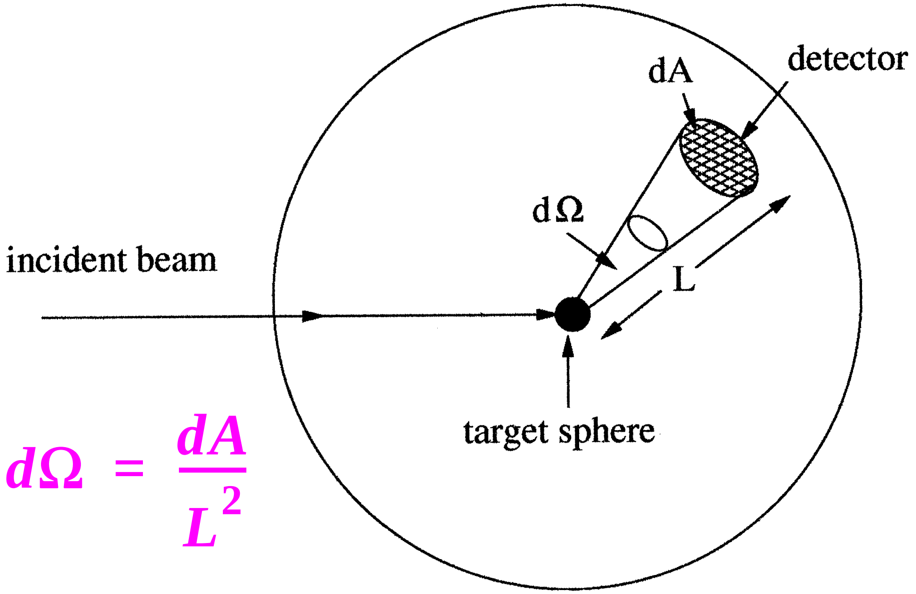
Electron-proton scattering

Differential cross section

- The **differential cross section**

$$\frac{d\sigma(\theta, \varphi)}{d\Omega} \longleftarrow \sin\theta d\theta d\varphi$$

gives the angular distribution.



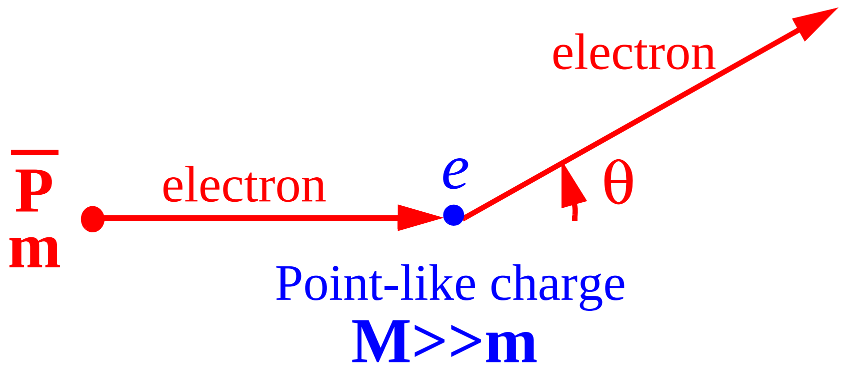
$$d\Omega = \frac{dA}{L^2}$$

- The **total cross section** by integration:

$$\sigma = \int \frac{d\sigma(\theta, \varphi)}{d\Omega} d\Omega = \int_0^{\pi} \int_0^{2\pi} \frac{d\sigma(\theta, \varphi)}{d\Omega} \sin\theta d\theta d\varphi$$

Electron-proton scattering

➡ Elastic scattering on a static point-like charge.



The Rutherford scattering formula

Non-relativistic electron

Point-like electric charge e.

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2\alpha^2}{4p^4\sin^4\left(\frac{\theta}{2}\right)} \longleftarrow \alpha = \frac{e^2}{4\pi}$$

The Mott scattering formula

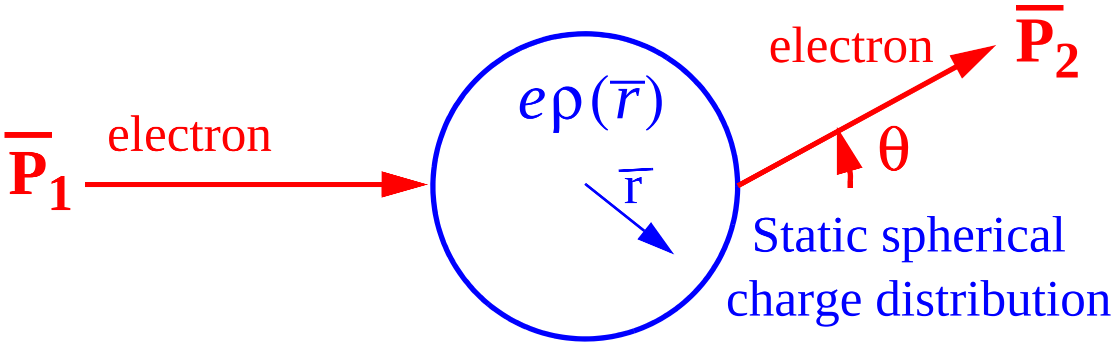
Relativistic electron

Point-like electric charge e.

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2}{4p^4\sin^4\left(\frac{\theta}{2}\right)}\left(m^2 + p^2\cos^2\frac{\theta}{2}\right)$$

Electron-proton scattering

➡ Elastic scattering on an extended charged object.



Momentum transfer

$\bar{q} = \bar{P}_1 - \bar{P}_2$

$q^2 = -\bar{q} \bullet \bar{q}$

- $\rho(r)$: a spherically symmetric density function with $\int \rho(r) d^3\bar{x} = 1$

  $\rho(r)$: describes how the charge e is spread out.

- Not point-like interaction ⇨

  Rutherford formula modified by electric form factor $G_E(q^2)$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2)}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{m^2\alpha^2}{4p^4\sin^4\left(\frac{\theta}{2}\right)}$$

$\bar{q} = \bar{P}_1 - \bar{P}_2$

Electron-proton scattering

- The electric form factor is the Fourier transform of the charge distribution with respect to the momentum transfer $\underline{q}$:

$$G_E(q^2) = \int \rho(r) e^{i\bar{q}\cdot\bar{x}} d^3\bar{x}$$

- The electric form factor has values between 0 and 1:

Low momentum transfer: $G_E(0) = 1 \text{ for } q = 0$

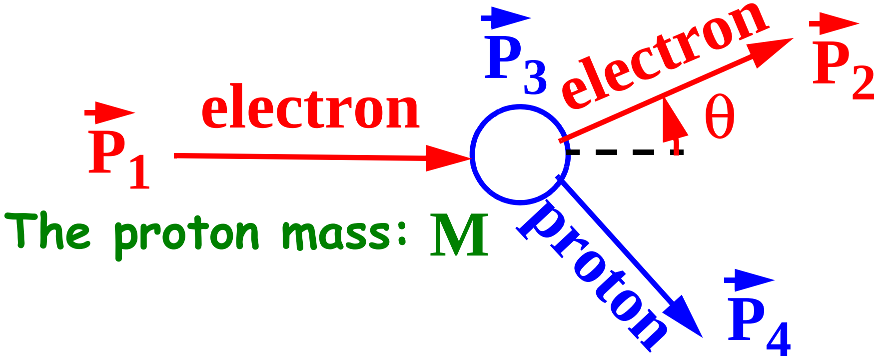
High momentum transfer: $G_E(q^2) \rightarrow 0 \text{ for } q^2 \rightarrow \infty$

- Measurements of the cross-section ⇨ The form factor ⇨ The charge distribution.

- The mean quadratic charge radius $r_E^2 = \int r^2 \rho(r) d^3\bar{x} = -6 \left.\frac{dG_E(q^2)}{dq^2}\right|_{q^2 = 0}$

Electron-proton scattering

➡ Elastic electron-proton scattering



The proton mass: M

4-momentum transfer

$$\vec{q} = \vec{P}_1 - \vec{P}_2$$

$$Q^2 = -\vec{q} \cdot \vec{q}$$

- Protons have charge + magnetic moment

electric formfactor (G_E) + magnetic formfactor (G_M)

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{M} \times \left(G_1(Q^2)\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2} G_2(Q^2)\sin^2\frac{\theta}{2} \right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2} G_M^2}{1 + \frac{Q^2}{4M^2}}$$

$$G_2(Q^2) = G_M^2$$

Electron-proton scattering

- **Measurement of the formfactors:**

i) low Q^2 ($Q<<M$):

G_E dominates the cross section

r_E can be measured $r_E = 0.85 \pm 0.02$ fm

ii) Intermediate Q^2 ($0.02 < Q^2 < 3$ GeV^2):

G_E and G_M give sizable contributions

Parameterization can be used $G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_p} \approx \left(\frac{\beta^2}{\beta^2 + Q^2}\right)^2$

iii) High Q^2 ($Q^2 > 3$ GeV^2):

G_M dominates the cross section

Electron-proton scattering

If protons are point-particles ⇨

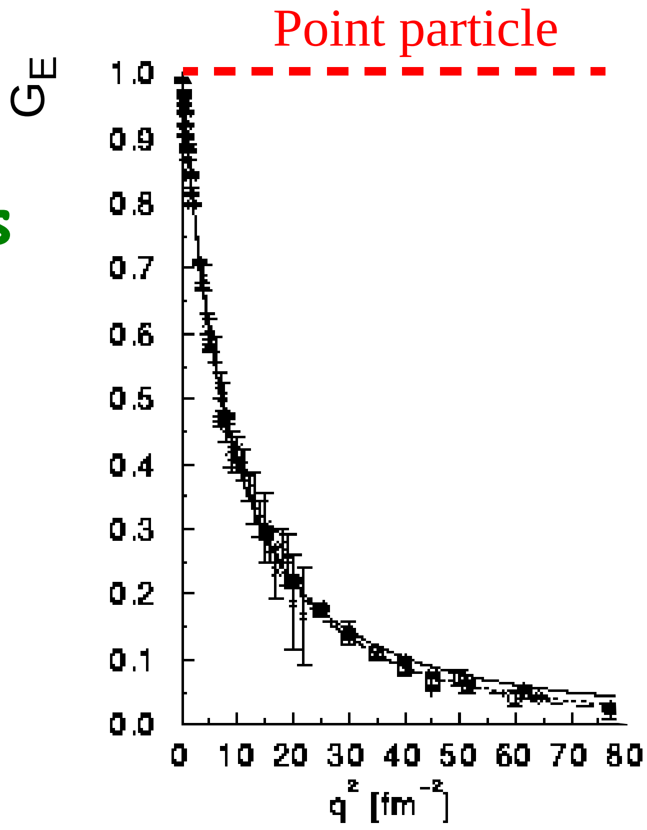
$$G_E(Q^2) = 1$$

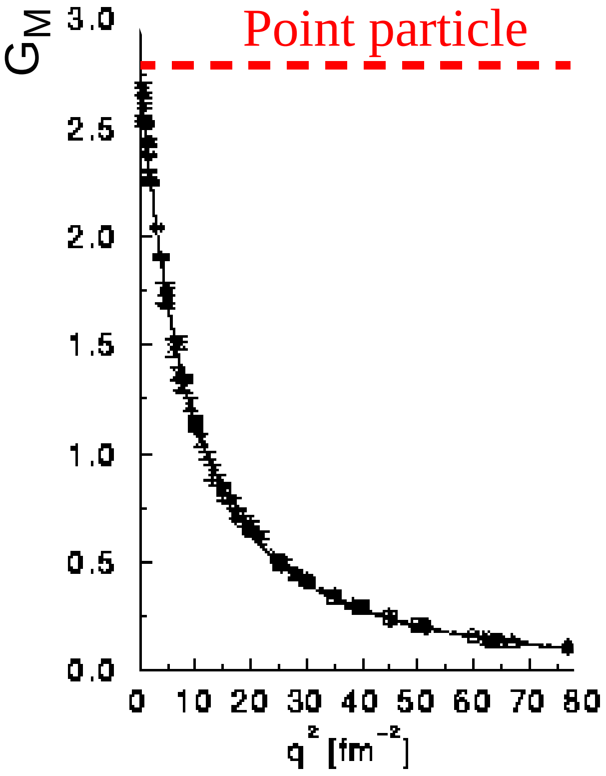
$$G_M(Q^2) = 2.79$$

Electric charge of a proton

Magnetic moment of a proton

Measurements of G_E and G_M of the proton





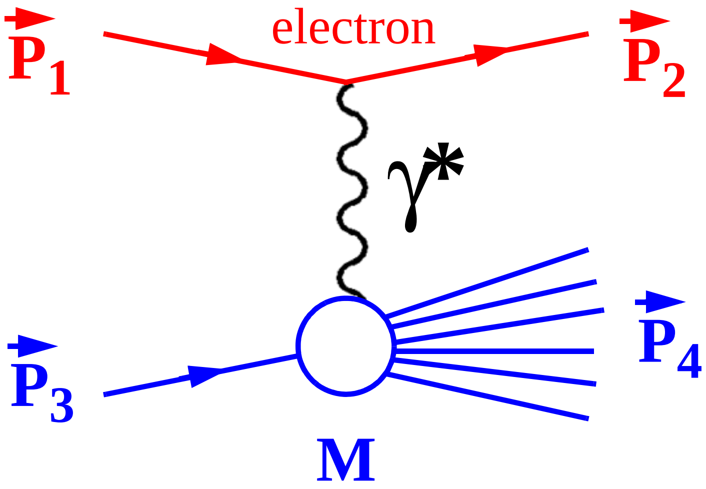
Conclusion:

The proton has an extended charge distribution !

Electron-proton scattering

➡ Inelastic electron-proton scattering

- **Inelastic electron-proton scattering** ⇨ **The proton is broken up**



$\vec{P}_4 = \vec{P}_3 + \vec{q}$

4-momentum transfer

$\vec{q} = \vec{P}_1 - \vec{P}_2 = (\nu, \bar{q})$

$Q^2 = -\vec{q} \cdot \vec{q}$

- Bjorken scaling variable $x = \frac{Q^2}{2M\nu}$ $0 < x < 1$

M is the mass of the proton.

Electron-proton scattering

- The differential cross section for inelastic ep scattering:

$$\frac{d\sigma}{dE_2 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \bullet \left[\frac{1}{\nu} F_2(x, Q^2) \cos^2\left(\frac{\theta}{2}\right) + \frac{2}{M} F_1(x, Q^2) \sin^2\left(\frac{\theta}{2}\right)\right]$$

Dimensionless structure functions

- $F_{1,2}(x,Q^2)$ parameterize the photon-proton interaction in the same way a $G_1(Q^2)$ and $G_2(Q^2)$ do it in elastic scattering.

- Bjorken scaling or scale invariance:

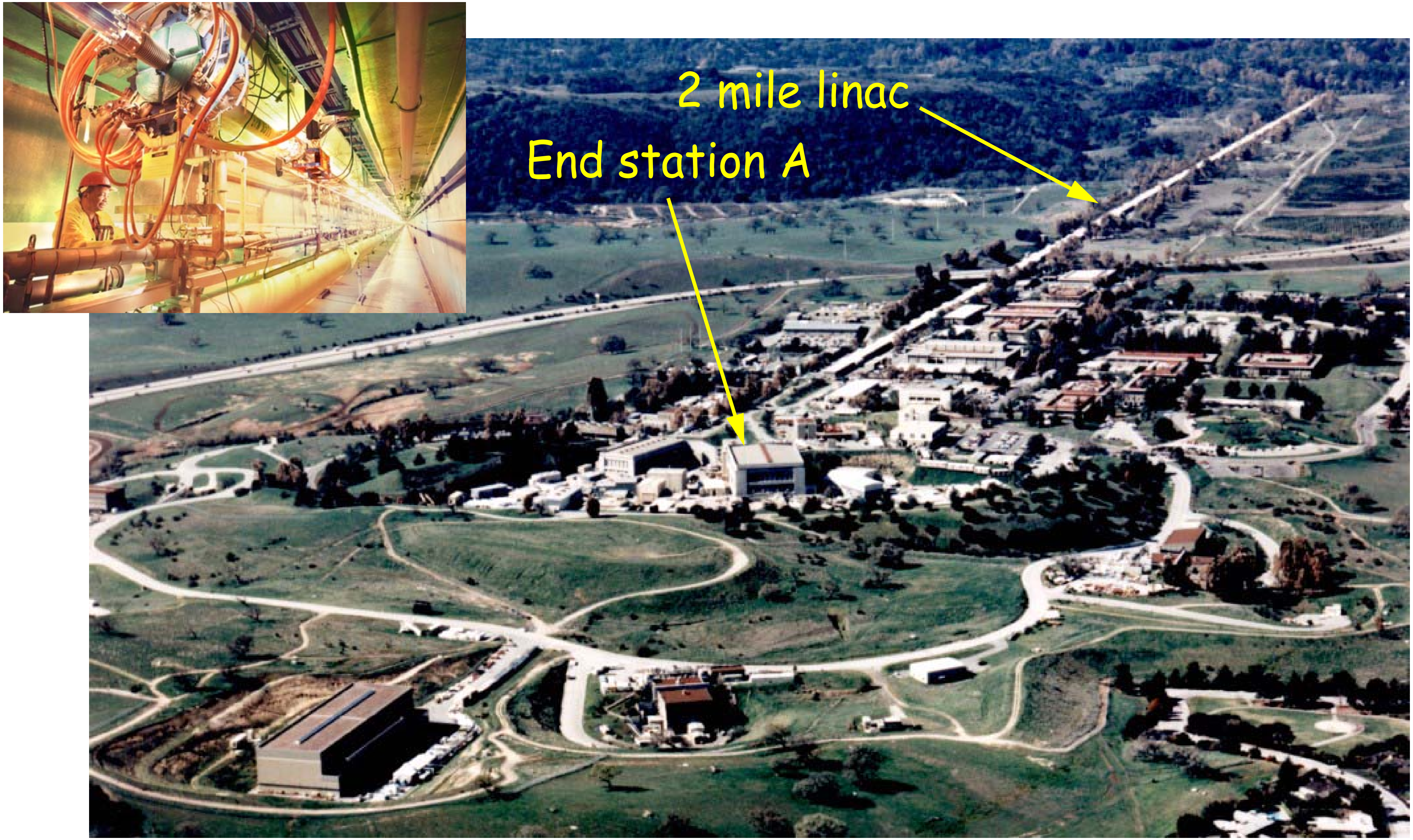
$F_{1,2}(x, Q^2) = F_{1,2}(x)$ when $Q^2 \to \infty$ and x is fixed and finite.

i.e. the structure functions are independent on Q^2 for Q>>M .

- Scaling: $F_{1,2}(x,Q^2)$ do not change if masses, energies and momenta are multiplied by a scale factor.

Electron-proton scattering

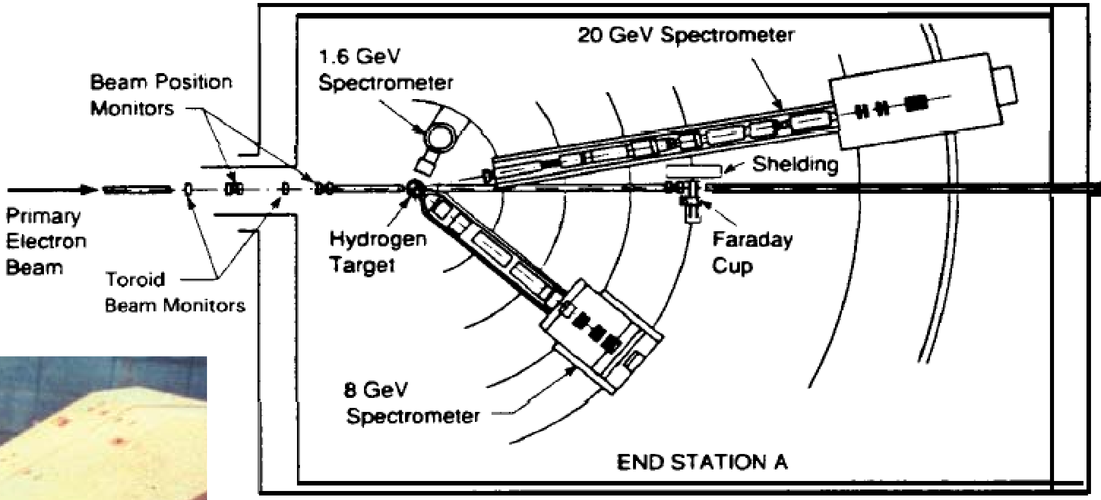
- The discovery of quarks at the SLAC 2 mile LINAC

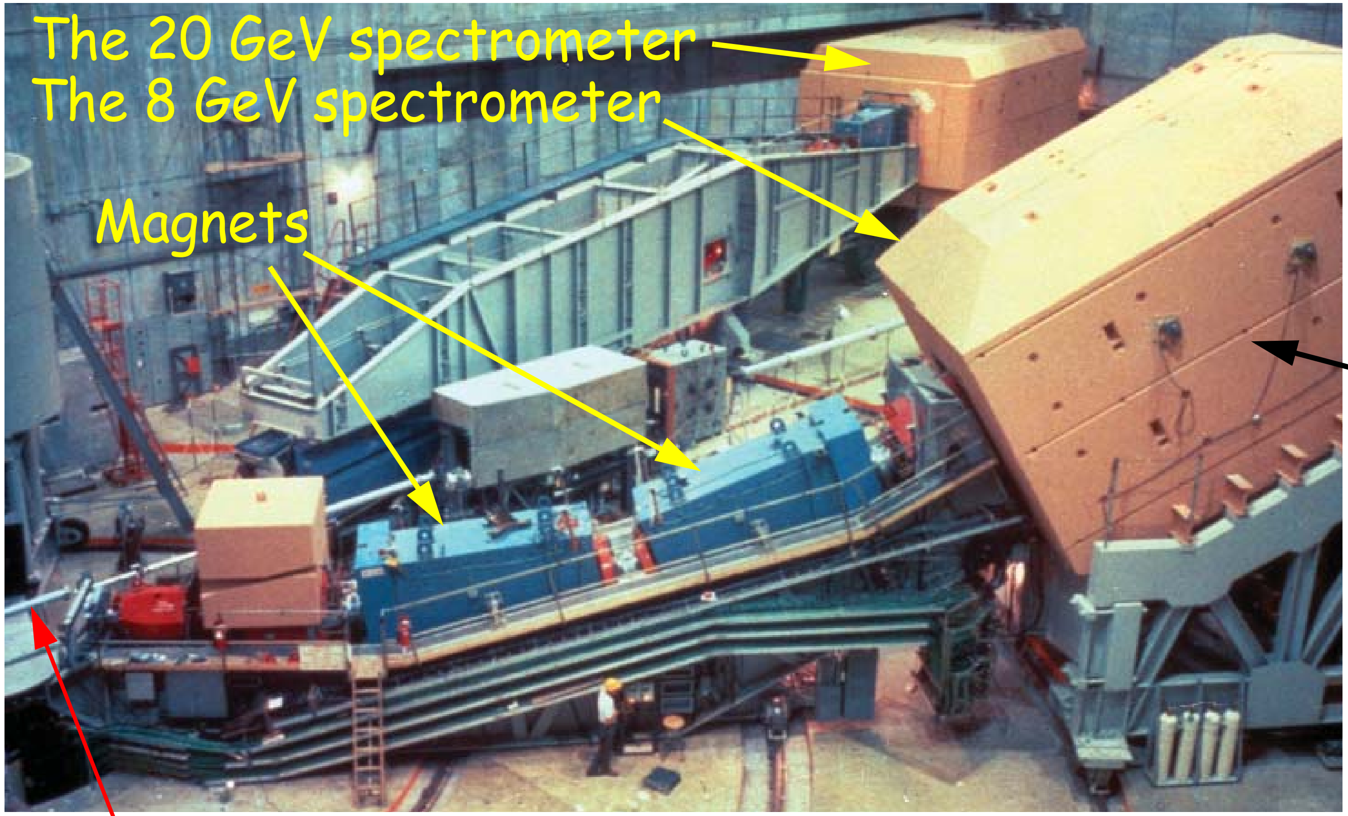


Electron-proton scattering

➡ The discovery of quarks

The MIT-SLAC experiment



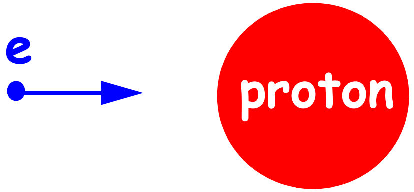


Cerenkov detectors

Scintillators

Detectors for e/π separation

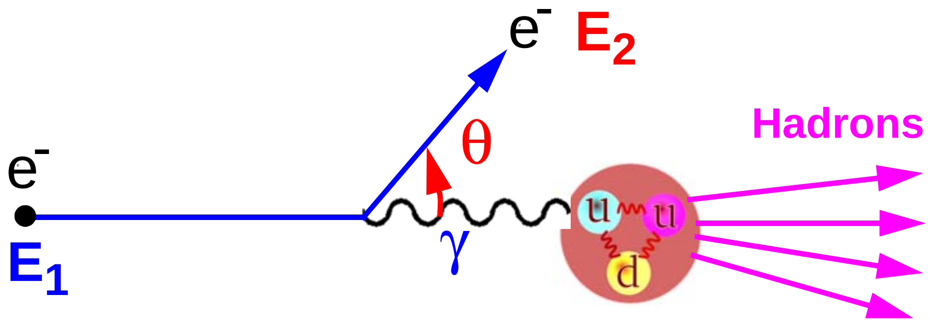
8 GeV electrons hits a hydrogen target



Electron-proton scattering

➡ The discovery of quarks

- Calculate x and Q^2 from the energy and angle of the electron.



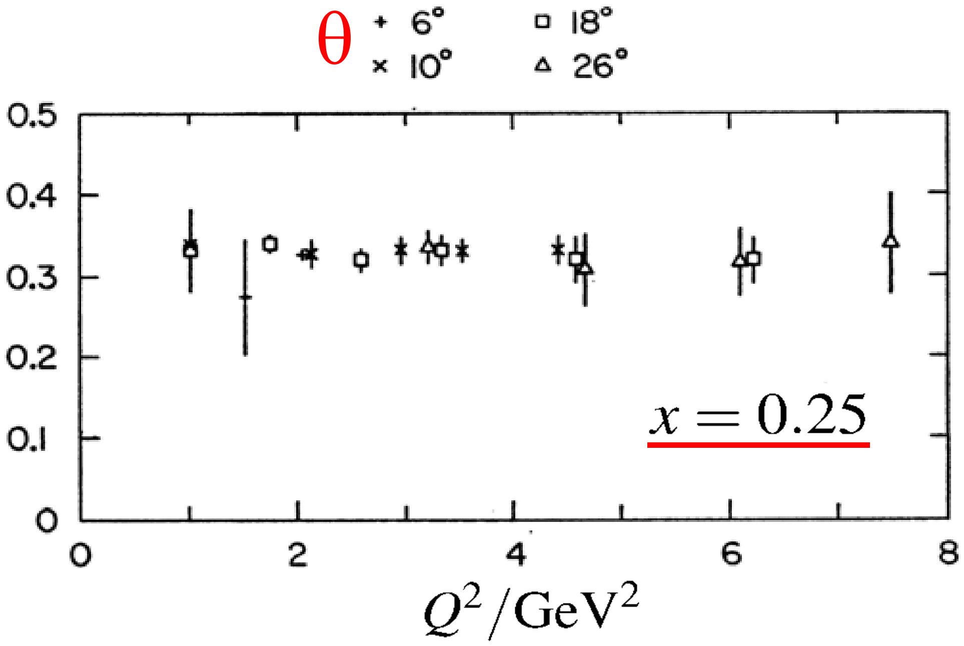
$$Q^2 = 4 \cdot E_1 \cdot E_2 \sin^2\left(\frac{\theta}{2}\right)$$

$$x = \frac{Q^2}{2 \cdot M \cdot (E_1 - E_2)}$$

- Cross section measurement ⇨ F_2

- F_2 does not depend on Q^2

- Protons have a sub-structure (partons)

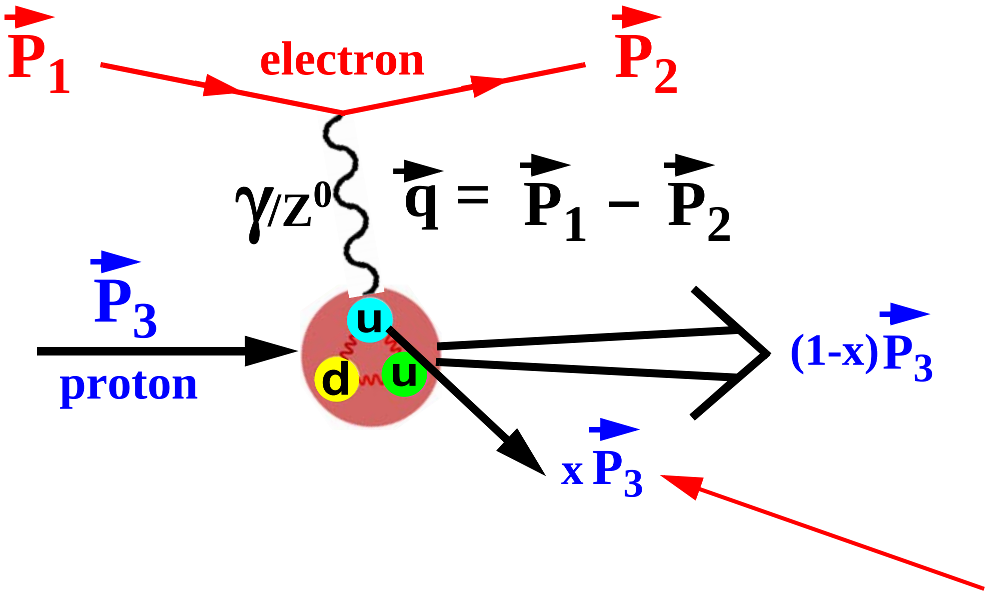


Electron-proton scattering

Deep inelastic electron-proton scattering

- **The parton model:** Scale invariance ⇨ Scattering on point-like constituents (partons) in the proton.

- **The quark model:** Partons = Quarks



$$Q^2 = -\vec{q}\cdot\vec{q} = 4\cdot E_1\cdot E_2\sin^2\left(\frac{\theta}{2}\right)$$

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2\cdot M\cdot(E_1-E_2)}$$

- Parton/quark model ⇨ **Fraction of the proton momentum** carried by the **struck quark** is given by **Bjorken x.**

Electron-proton scattering

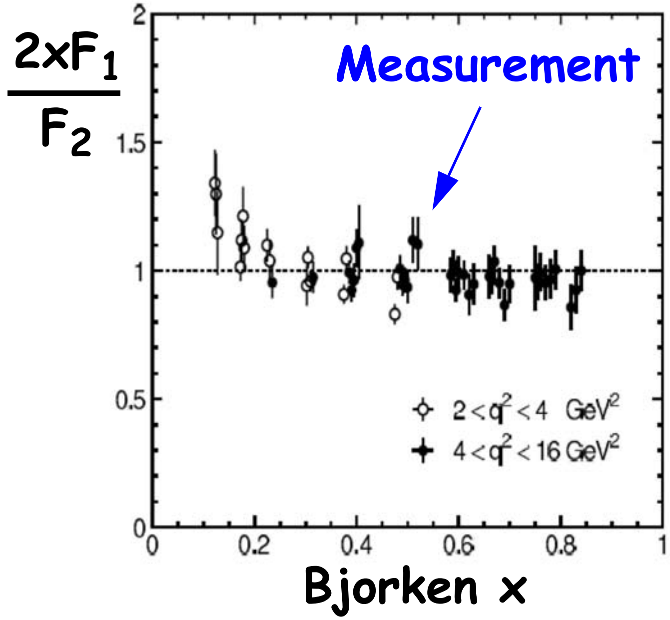
Deep inelastic electron-proton scattering

- Parton model ⇨ F_1 depends on the spin of the partons (quarks)

Prediction: $F_1(x, Q^2) = 0$ quark spin =0

The Callan-Gross relation: $2xF_1(x, Q^2) = F_2(x, Q^2)$ quark spin=1/2

Measurement:



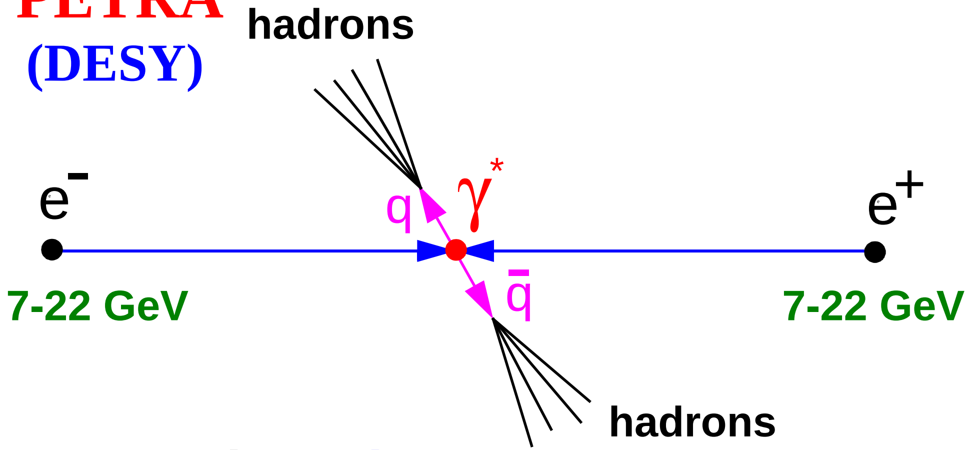
Theory prediction:

Spin = 1/2

Spin = 0

Electron-proton scattering

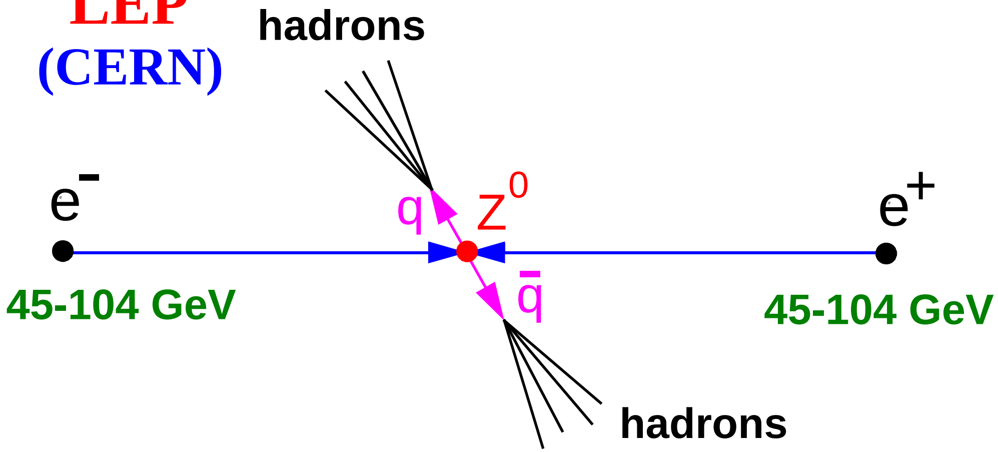
PETRA
(DESY)



Length: 2.3 km

Experiments: Tasso, Jade, Pluto, Mark J, Cello

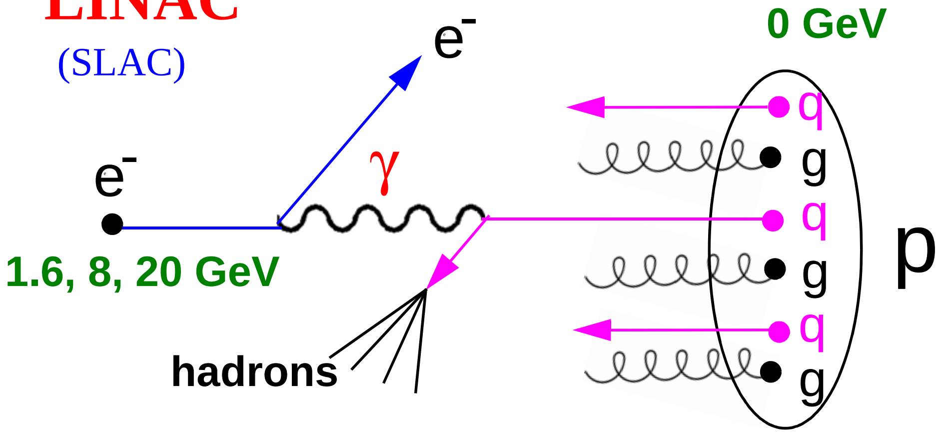
LEP
(CERN)



Length: 27 km (4184 magnets)

Experiments: DELPHI, OPAL, ALEPH, L3

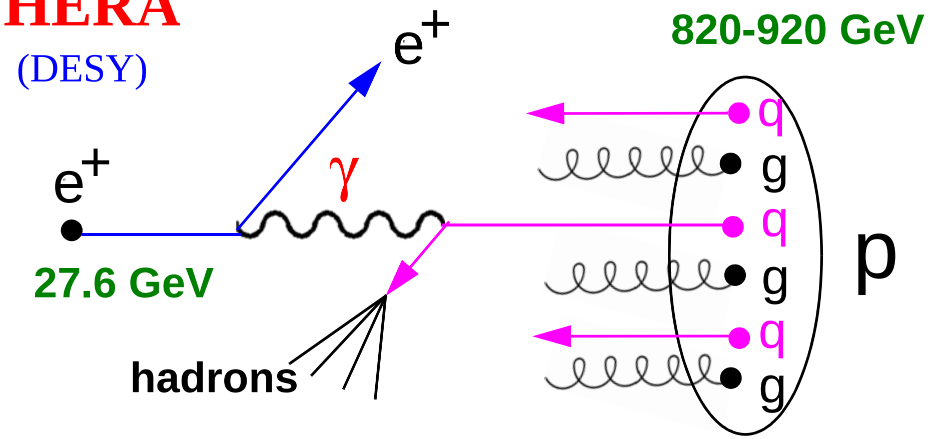
LINAC
(SLAC)



Length: 3 km

Experiments: SLAC-MIT

HERA
(DESY)



Length: 6 km (1650 magnets)

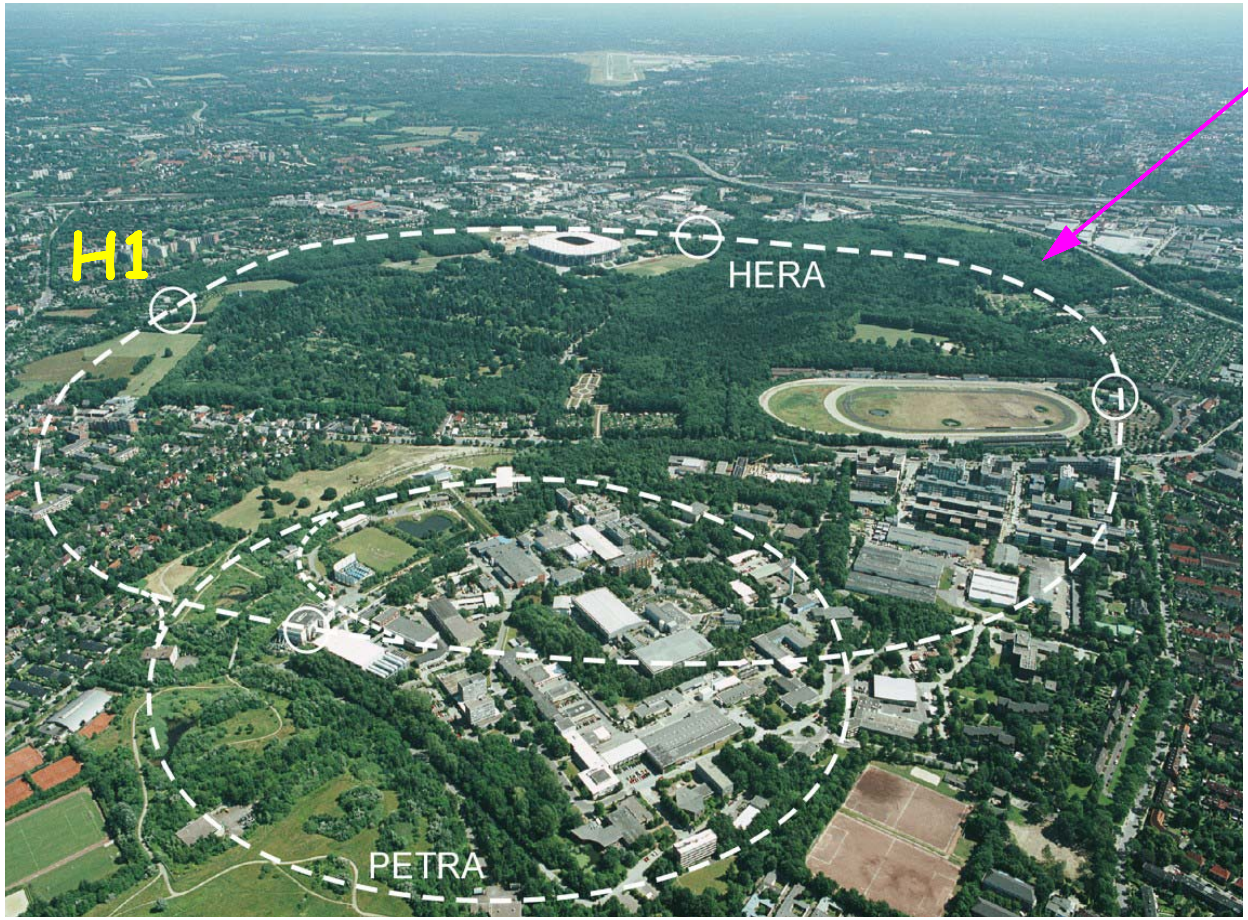
Experiments: H1, ZEUS

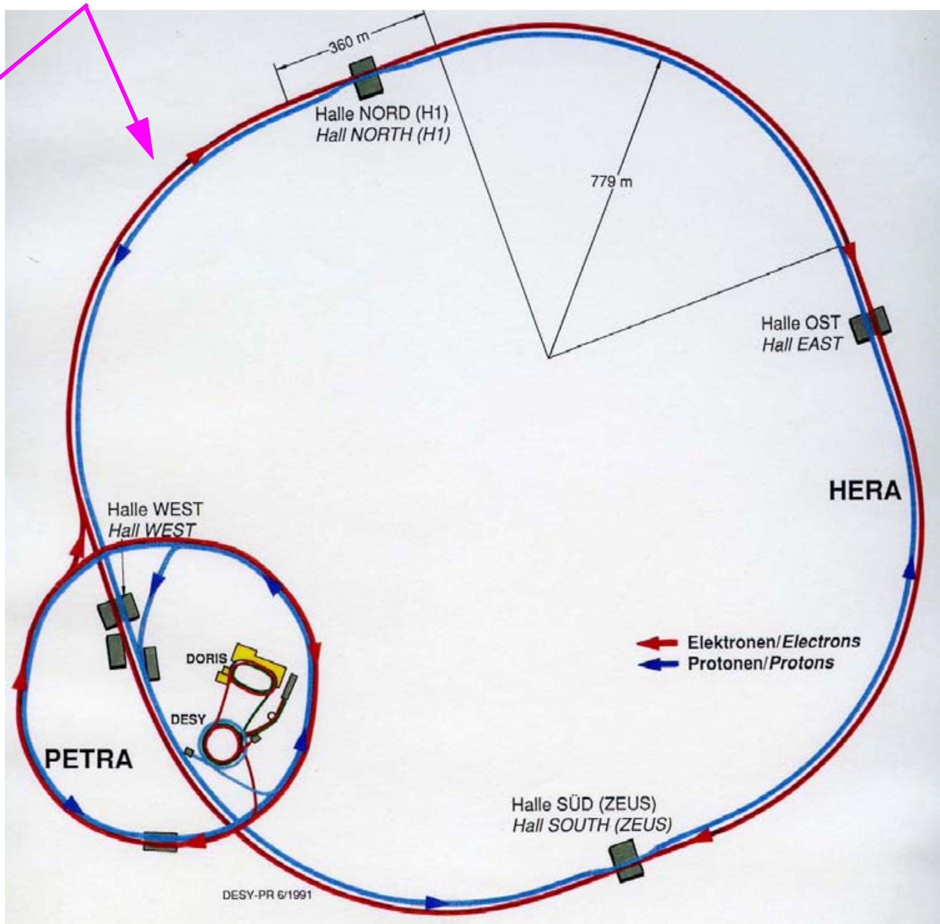
Electron-proton scattering

➡ The HERA accelerator at DESY

- **HERA accelerator:** only large electron-proton collider ever built.
- Petra was pre-accelerator.

6 km long





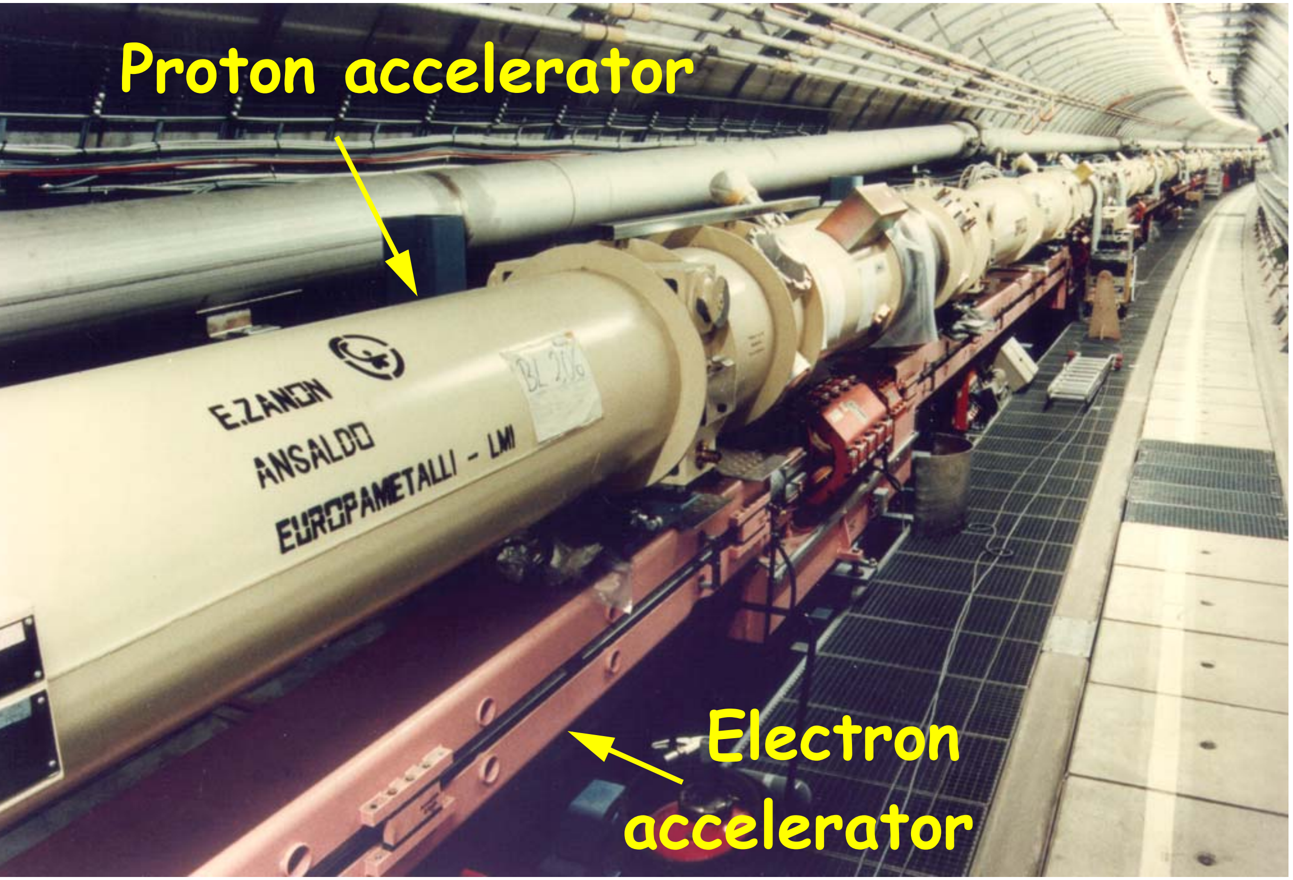
Electron-proton scattering

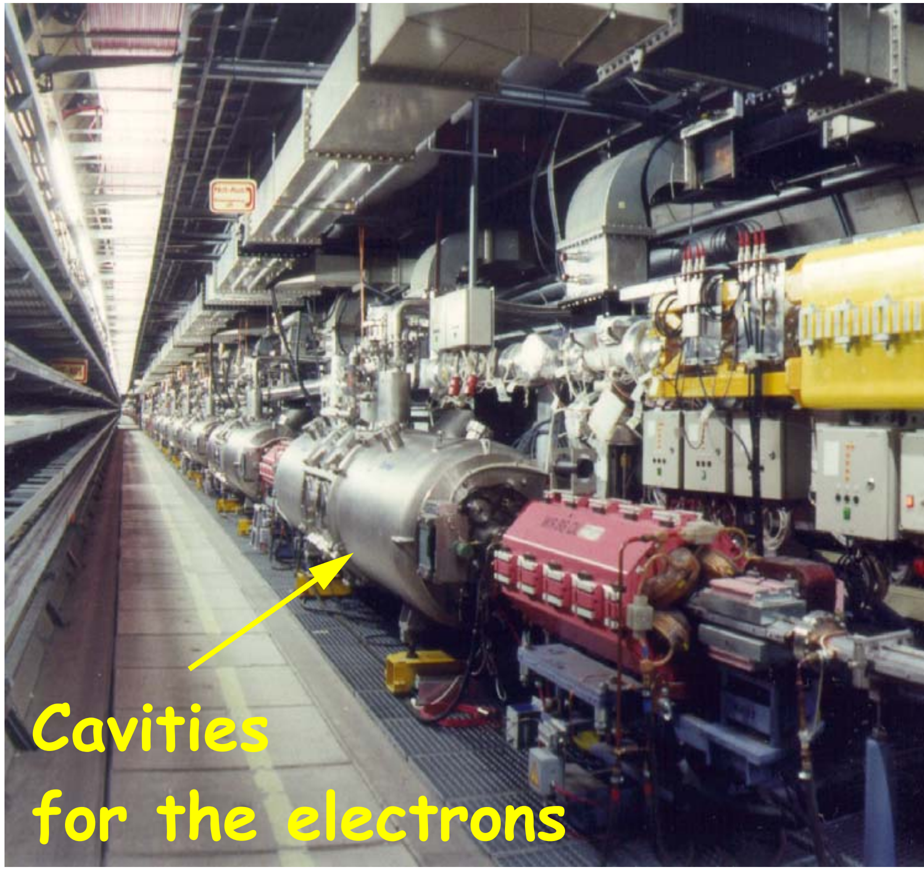
HERA

Proton accelerator: Super conducting magnets. Energy = 920 GeV

Electron accelerator: Normal warm magnets. Energy = 28 GeV

Collision energy = 320 GeV (54000 GeV fixed target)

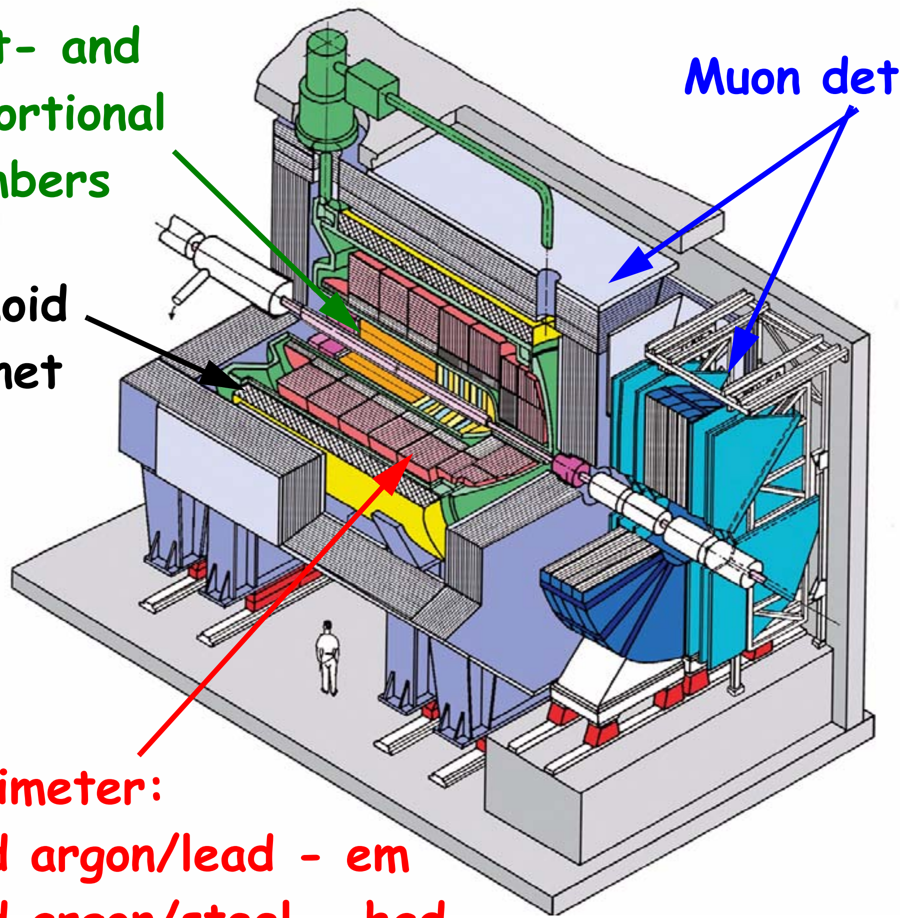




Electron-proton scattering

➡ The H1 Experiment

- **Events at HERA were boosted in the proton direction due to the large difference in electron and proton beam energies.**

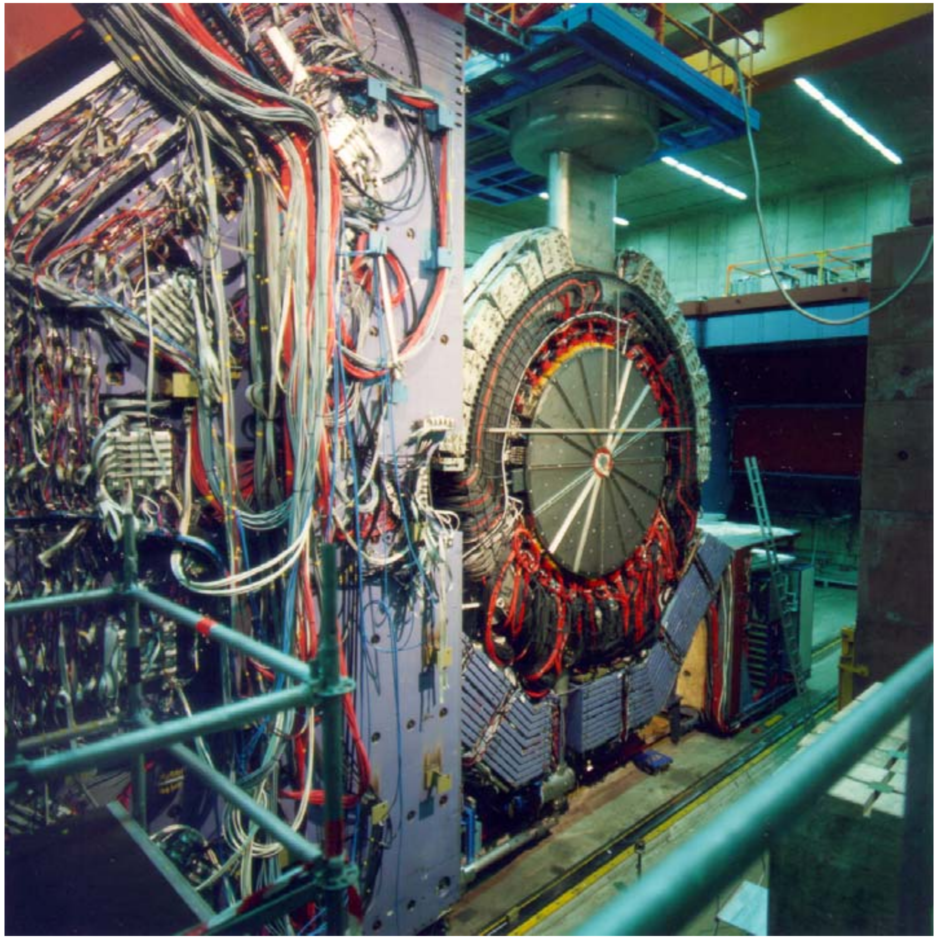


Tracker:
Drift- and
Proportional
chambers

Muon detectors

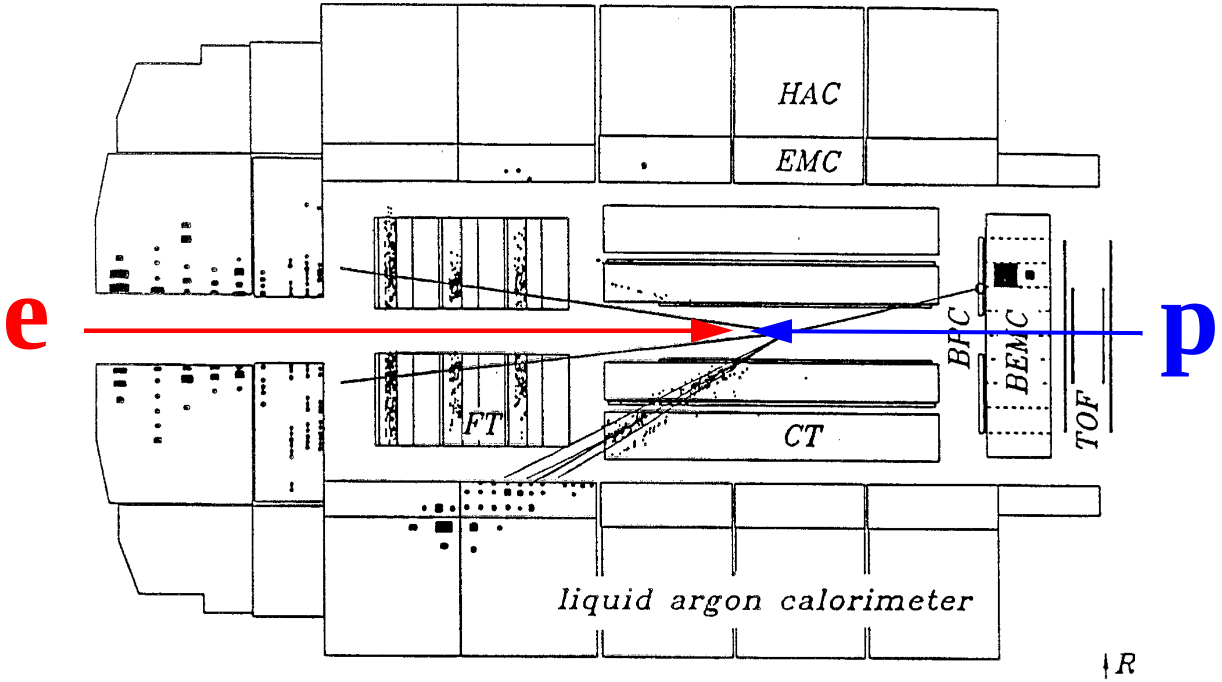
Solenoid
magnet

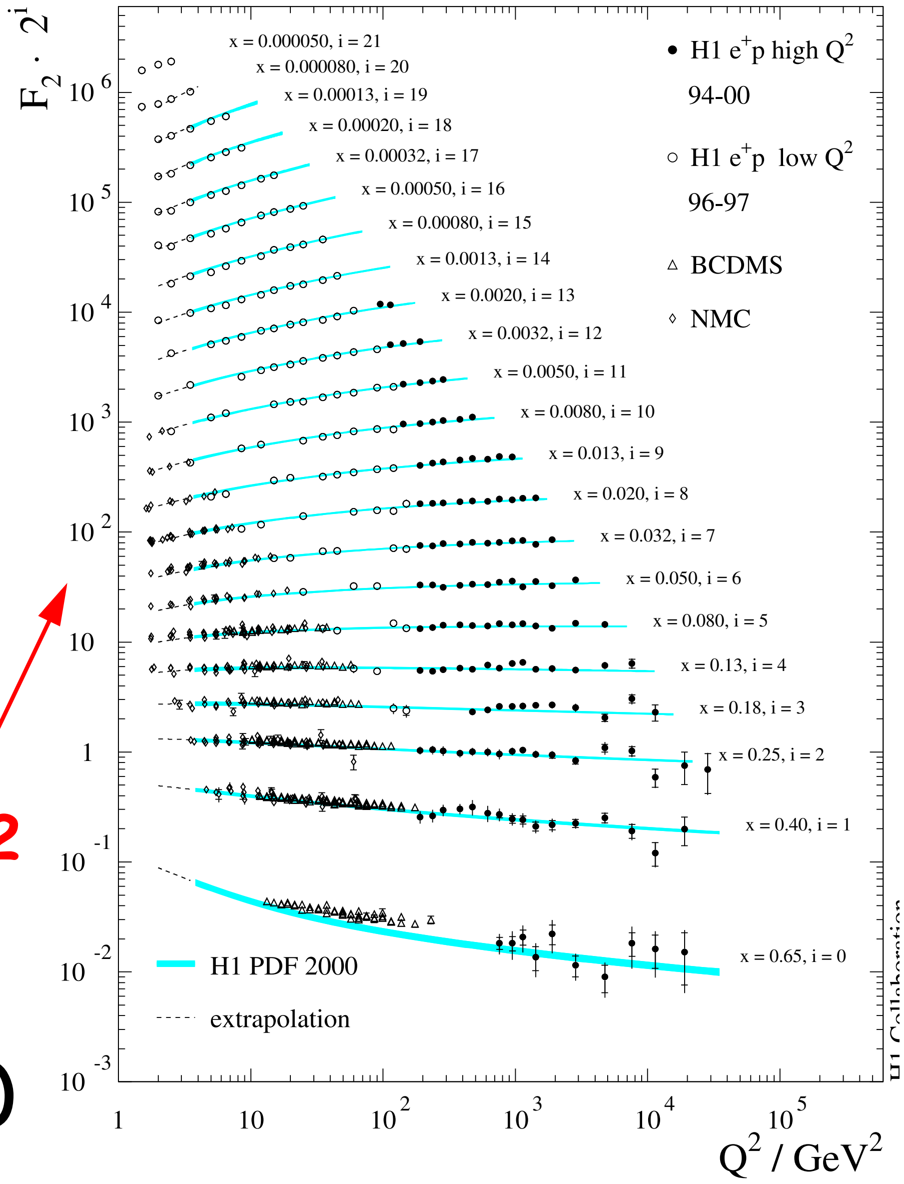
Calorimeter:
Liquid argon/lead - em
Liquid argon/steel - had



Electron-proton scattering

➡ Measurement of structure functions



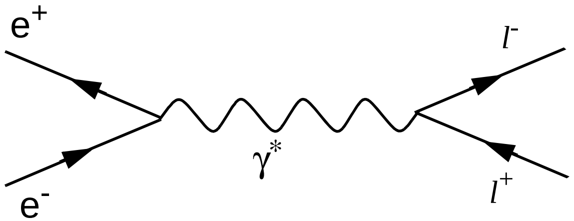


- The cross section + the energy and scattering angle of the electron ⇨ F_2

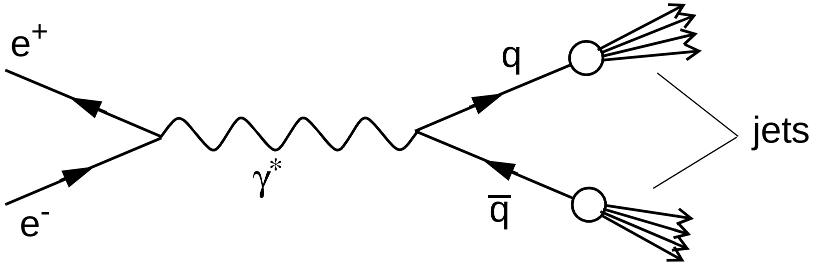
- No quark sub-structure was observed down to 10^{-18}m (1/1000th of a proton)

Summary of scattering formulas

SUMMARY: Electron-Positron interactions

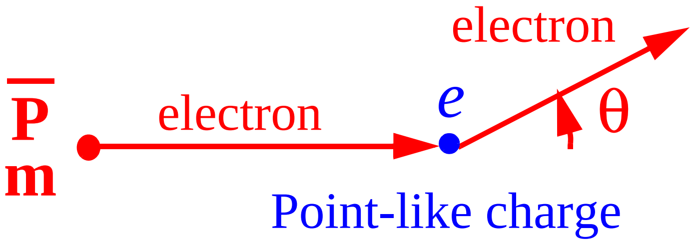


$$\frac{d\sigma}{d\cos\theta}\left(e^+e^- \to l^+l^-\right) = \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$



$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = N_c e_q^2 \frac{\pi\alpha^2}{2Q^2}(1 + \cos^2\theta)$$

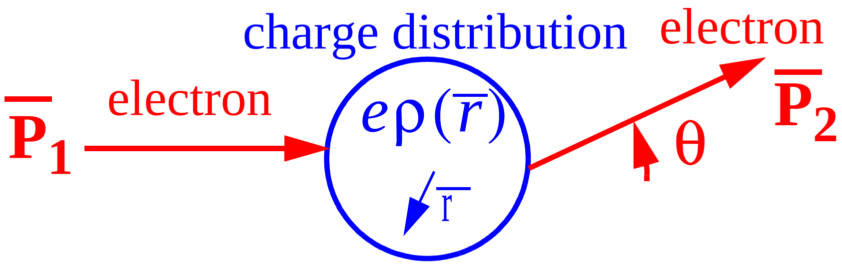
SUMMARY: Elastic electron-proton scattering



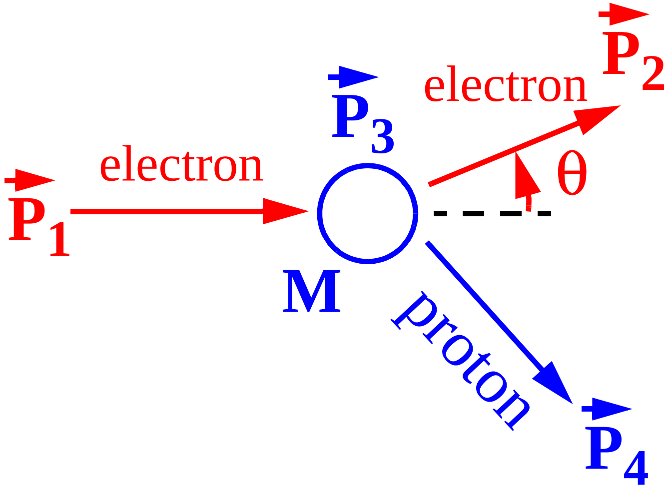
Point-like charge
M>>m

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \underbrace{\frac{\alpha^2}{4p^4\sin^4(\frac{\theta}{2})}}_{\big\downarrow}\left(m^2+p^2\cos^2\frac{\theta}{2}\right)$$

Static spherical
charge distribution



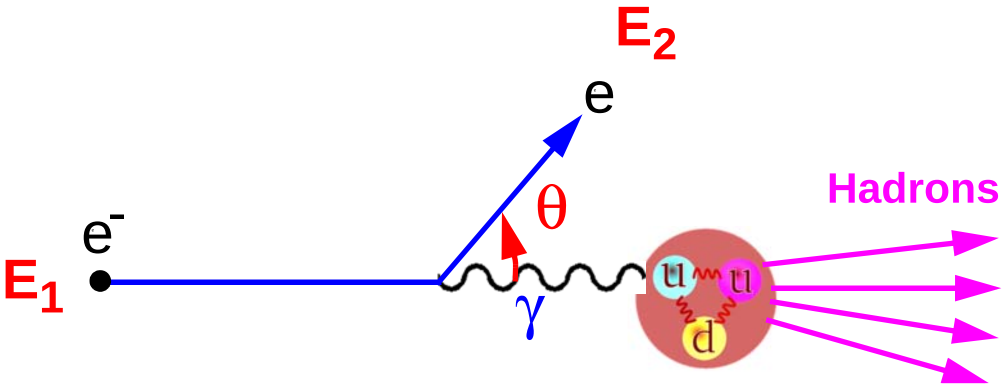
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R G_E^2(q^2)$$

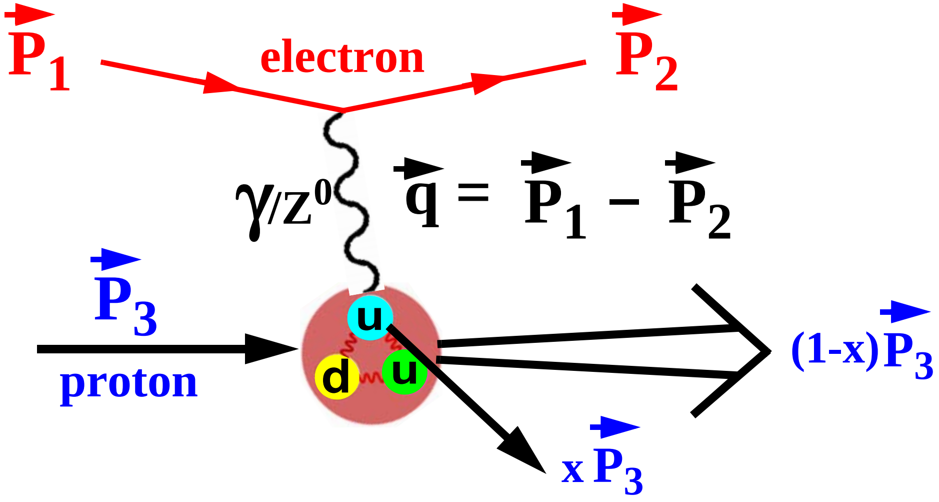


$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \mathrm{x} \left(G_1(Q^2)\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}G_2(Q^2)\sin^2\frac{\theta}{2}\right)$$

$$G_1(Q^2) = \frac{G_E^2 + \frac{Q^2}{4M^2}G_M^2}{1+\frac{Q^2}{4M^2}} \qquad G_2(Q^2) = G_M^2$$

SUMMARY: Inelastic electron-proton scattering





$$Q^2 = -\vec{q} \cdot \vec{q}$$

$$x = \frac{Q^2}{2M\nu}$$ **Bjorken - x**

$$\frac{d\sigma}{dE_2\,d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\left(\frac{\theta}{2}\right)} \cdot \frac{1}{\nu} \cdot \left[F_2(x, Q^2)\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{xM^2}F_1(x, Q^2)\sin^2\left(\frac{\theta}{2}\right)\right]$$