X. Electroweak unification

- Neutral weak bosons were predicted by the electroweak theory

Modern quantum field theories are *gauge invariant* theories, i.e. they are theories were the main equations do not change when a gauge transformation is performed

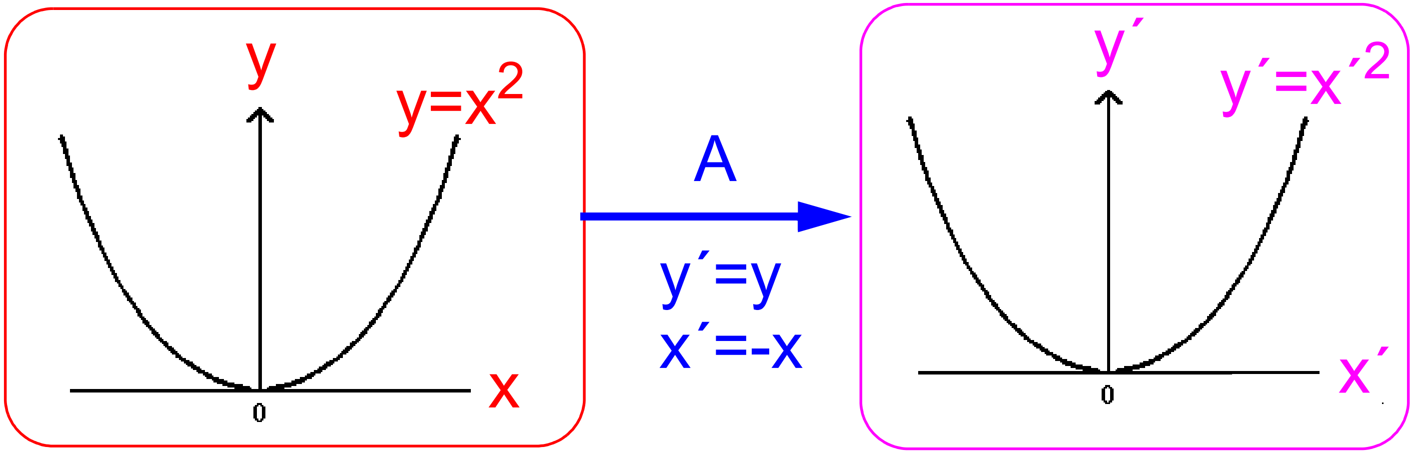


Figure 159: The equation $y=x^2$ is symmetric (invariant) under transformation A, i.e. it looks the same before and after the transformation

- Gauge transformation: certain alteration of a quantum field variables that leaves basic properties of the field unchanged; a symmetry transformation

By requiring that theories are gauge invariant one can in fact *deduce* various *interactions*

- There are several forms of gauge invariance corresponding to different interactions

In QED, Schrödinger equation must be invariant under phase transformation of the wavefunction (a U(1) transformation):

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{iq\alpha(\vec{x},t)}\psi(\vec{x},t) \qquad (185)$$

Here $\alpha(\vec{x},t)$ is an arbitrary continuous function, q is electric charge.

If a particle is free, then

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi \qquad (186)$$

- Transformed wavefunction $\psi'(\vec{x},t)$ can not be a solution of the Schrödinger equation (186) since it leads to extra q-dependent terms:

$$i\frac{\partial\psi'}{\partial t} = \left[\frac{1}{2m}(\nabla + iq\alpha)^2 - 2mq\frac{\partial\alpha}{\partial t}\right]\psi' \qquad (187)$$

- *Gauge principle*: to keep the invariance condition satisfied, a minimal field should be added to the Schrödinger equation, i.e., an interaction should be introduced

Recall that electric field is:

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t}$$

Gauge transformation corresponds to an interaction that changes the potentials in such a way that ensures invariance:

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla\alpha \quad \text{and} \quad \phi \rightarrow \phi' = \phi - \frac{\partial\alpha}{\partial t}$$

The gauge-invariant Schrödinger equation is then:

$$i\frac{\partial\psi}{\partial t} = \left[\frac{1}{2m}(\nabla - qA)^2 + q\phi\right]\psi \qquad (188)$$

The unified *electroweak theory* was introduced in 1960-ies by Glashow, Weinberg and Salam

- It is a quantum field theory, details of which are outside this course's scope - we will focus on its predictions
- The theory introduces weak isospin (I_3^W) and weak hypercharge (Y^W) which are related to the electric charge Q as: $Q=I_3^W+Y^W/2$
- It also introduces massless gauge particles (W^+, W^-, W^0 and B^0) that interact with massless fermions in order to make the theory gauge-invariant
  - In QED, transition from one electron state to another with different phase, $e^- \to e^-$, demands emission (or absorption) of a photon: $e^- \to e^-\gamma$
  - Electroweak theory generalizes it to transformations like:

$$e^- \to \nu_e \qquad \nu_e \to e^- \qquad e^- \to e^- \qquad \nu_e \to \nu_e$$

which leads via the gauge principle to interactions:

$$e^- \to \nu_e W^- \quad \nu_e \to e^- W^+ \quad e^- \to e^- W^0 \quad \nu_e \to \nu_e W^0$$

- W^+, W^-, and W^0 are corresponding spin-1 gauge bosons

While W^+ and W^- are the well-known (though massive) charged currents, W^0 as such has not been observed experimentally

- Theory of weak interactions only by means of $W^\pm$ bosons leads to *divergence*: cross-sections of processes involving two W bosons grow *infinitely* with increasing energy

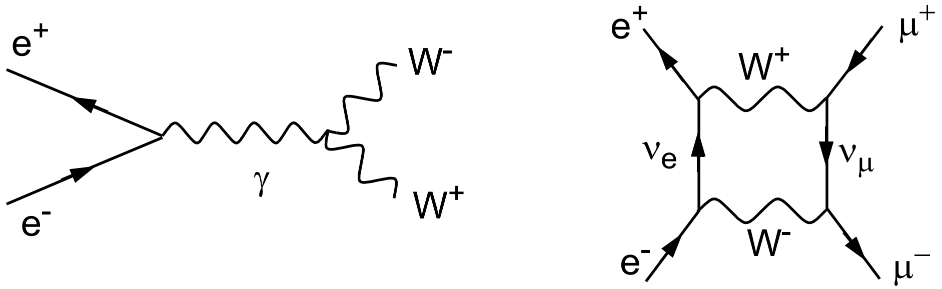


Figure 160: Examples of divergent processes

- A “good” theory (such as QED) must be *renormalizable*: all expressions can be made finite by re-expressing them in a finite number of physical parameters (like e, m_e and $\hbar$ in QED)
  - Electroweak theory is actually renormalizable, though demonstration of it is highly non-trivial
- Introduction of Z^0 boson fixes the divergence problem: Z^0 can couple to two W bosons and thus cancel the divergence

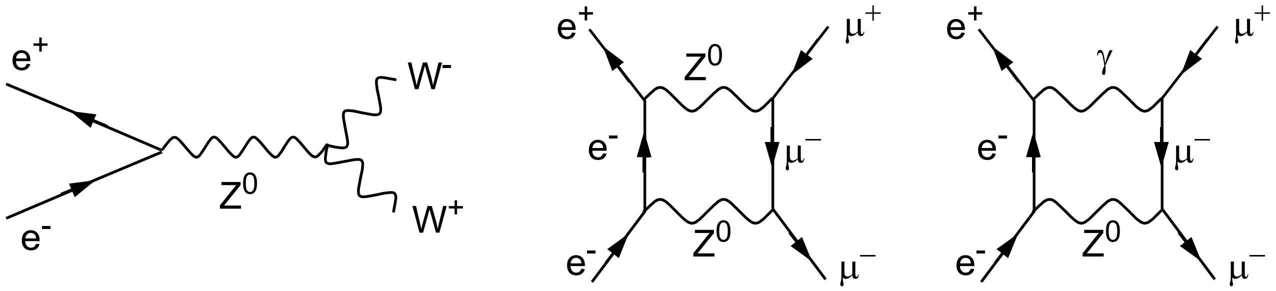


Figure 161: Additional processes to cancel divergence

  - The divergence can also be cancelled by introducing a “heavy electron”, but experimental evidence unambiguously favors Z^0
  - Introduction of neutral bosons makes electroweak theory gauge-invariant

Rules for Z^0 boson vertices:

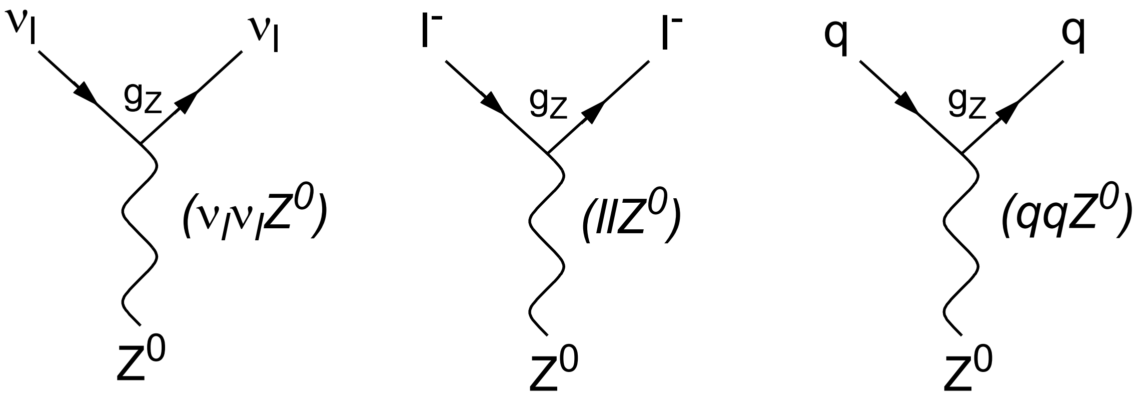


Figure 162: Basic vertices for Z^0-lepton and Z^0-quark couplings

- Conserved lepton numbers
- Conserved quark flavour (*remember,* in W vertices, quark flavour is not conserved)
  - By applying quark-lepton symmetry and assuming there is quark mixing:

$$d'd'Z^0 + s's'Z^0 = (d\cos\theta_C + s\sin\theta_C)(d\cos\theta_C + s\sin\theta_C)Z^0 +$$

$$+(-d\sin\theta_C + s\cos\theta_C)(-d\sin\theta_C + s\cos\theta_C)Z^0 = ddZ^0 + ssZ^0$$

  - Therefore, it is actually not necessary to apply quark mixing in Z^0 vertices

Experimental test of flavour conservation at Z^0 vertex:

$$K^+ \to \pi^0 + \mu^+ + \nu_\mu \qquad \text{(a)} \qquad\qquad K^+ \to \pi^+ + \nu_l + \bar{\nu}_l \qquad \text{(b)}$$

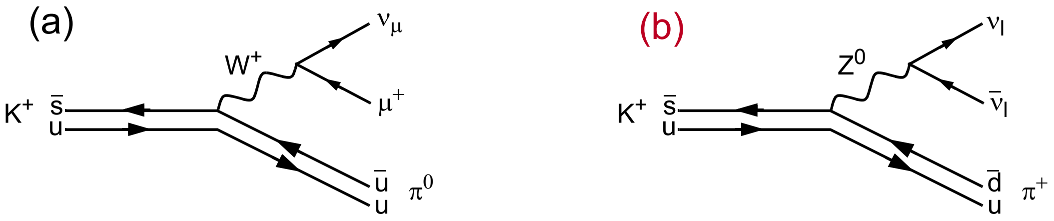


Figure 163: Decay (a) is allowed; decay (b) – forbidden

- Experiments E787 and E949 at the Brookhaven National Laboratory (BNL): a dedicated rare kaon decay experiment
  - K^+ beam (created by 24 GeV protons from the AGS accelerator) is deposited onto a fixed target

Measured upper limit on the ratio of the decay rates is:

$$\frac{\sum_l \Gamma(K^+ \to \pi^+ + \nu_l + \bar{\nu}_l)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)} = 7,8 \times 10^{-11}$$

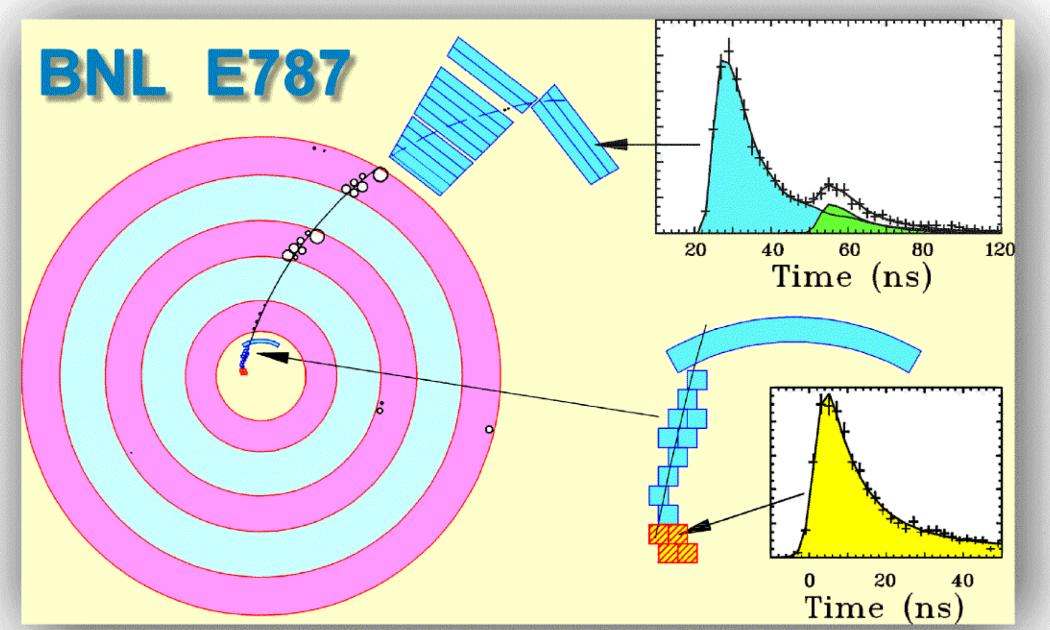


Figure 164: Picture of a rare event in E787 (a single pion track). Only 7 such events have been observed by E787/949.

- Other experiments looking for the same decay: J-PARC KOTO (K_L, running), CERN NA62 (under construction, start in 2014), FNAL ORKA (R&D)
- With this rate, the observed decays can not be due to the flavor-violating Z^0 decays

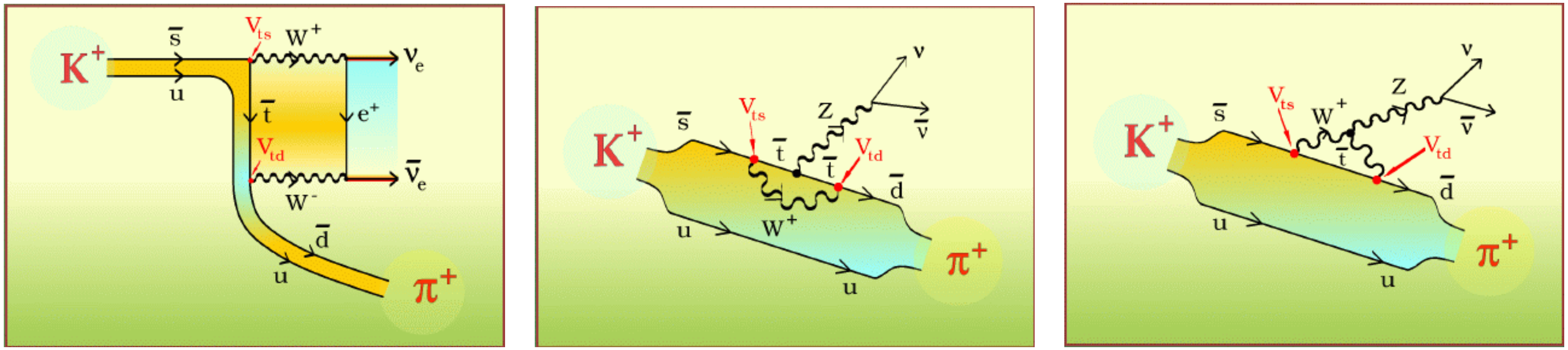


Figure 165: Second-order charged interactions that can explain the observed rare kaon decays.

- Thanks to the t-d vertex in the third diagram above, one can estimate the V_{td} element of the CKM matrix:

$$0.007 < |V_{td}| < 0.030$$

Unification condition and boson masses

- Comparing vertices involving γ, $W^{\pm}$ and Z^{0}, one can conclude that they are not independent and can be expressed via the same coupling constant

For a consistent electroweak theory, two conditions are introduced:

- The *unification condition* relates coupling constants α_{em}, g_w and g_z:

$$\sqrt{\frac{\pi \cdot \alpha_{em}}{2}} = g_W \sin\theta_W = g_Z \cos\theta_W \tag{189}$$

here θ_W is the *weak mixing angle*, or *Weinberg angle*:

$$\cos\theta_W = \frac{M_W}{M_Z} \tag{190}$$

- The *anomaly condition* relates electric charges of leptons and quarks:

$$\sum_l Q_l + 3\sum_q Q_q = 0 \tag{191}$$

In the zero-range approximation for heavy bosons (see also Eq.(169)):

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} \Rightarrow M_W^2 = \frac{g_W^2\sqrt{2}}{G_F} = \frac{\pi\alpha_{em}}{\sqrt{2}G_F \sin^2\theta_W} \tag{192}$$

If we introduce also the neutral current coupling (in low energy zero-range approximation, as usual):

$$\frac{G_Z}{\sqrt{2}} = \frac{g_Z^2}{M_Z^2} \tag{193}$$

the weak mixing angle can then be expressed through:

$$\frac{G_Z}{G_W} = \frac{g_Z^2}{g_W^2}\frac{M_W^2}{M_Z^2} = \sin^2\theta_W \tag{194}$$

From measurements of rates of charged and neutral currents reactions,

$$\sin^2\theta_W = 0.227 \pm 0.014$$

which allowed to predict masses of W (using Eq.(192)) and hence Z^0, as:

$$M_W = 78.3 \pm 2.4\ GeV/c^2;\ M_Z = 89.0 \pm 2.0\ GeV/c^2$$

The most precise result (at Z^0 *pole*):

$$\sin^2\theta_W = 0.23116 \pm 0.00012 \tag{195}$$

However, the most precise value for mass ratio is somewhat different:

$$1 - \frac{M_W^2}{M_Z^2} = 0.22318 \pm 0.0052$$

The difference comes from higher-order diagrams, e.g. *loops*:

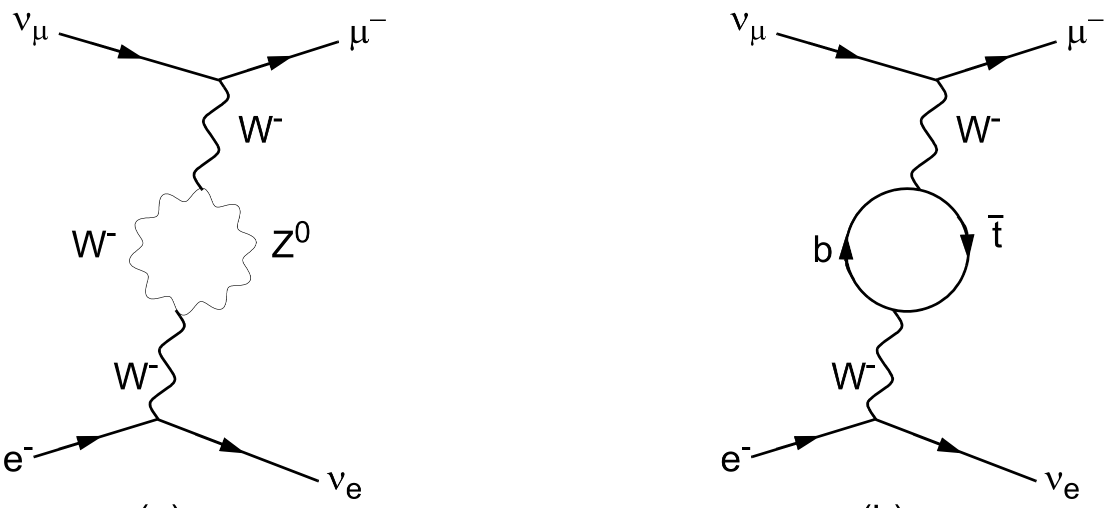


Figure 166: Examples of higher order contributions to inverse muon decay

From higher order corrections, the prediction for the top-quark mass was:

$$m_t = 170 \pm 30\ GeV/c^2 \tag{196}$$

Direct observation gives the value of $m_t = 173,07 \pm 0,88\ GeV/c^2$

- Predictions for W, Z and top masses were the most impressive successes of the electroweak theory

❖ In any process in which a photon is exchanged, a Z^0 boson can be exchanged as well; in addition, Z^0 couples to <u>neutrinos</u>:

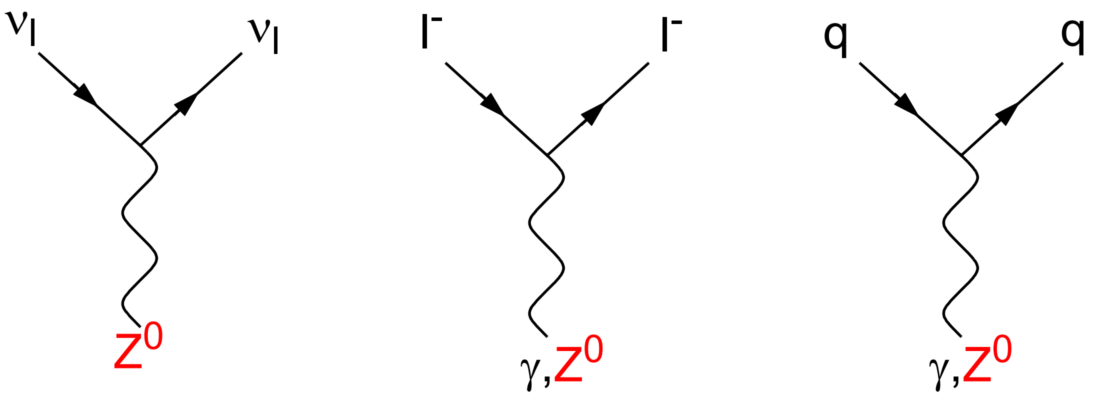


Figure 167: Z^0 and γ couplings to leptons and quarks

Example: reaction $e^+e^- \rightarrow \mu^+\mu^-$ has two dominant contributions:

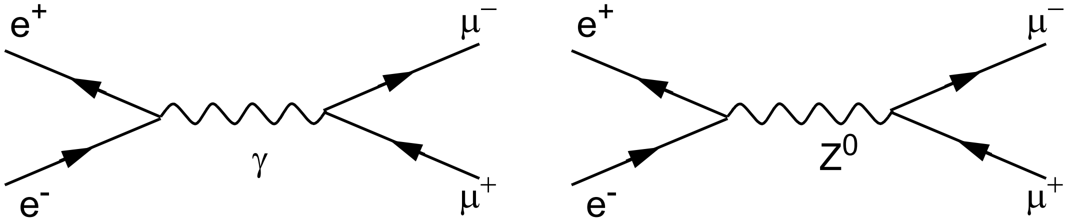


Figure 168: Dominant contributions to the e^+e^- annihilation into muons

With simple dimensional arguments one can estimate the cross section for the photon- and Z-exchange process *at low energy*:

$$\sigma_\gamma \approx \frac{\alpha^2}{E^2} \qquad \sigma_Z \approx G_Z^2 E^2 \tag{197}$$

From Eq.(197), ratio of σ_Z and σ_γ is:

$$\frac{\sigma_Z}{\sigma_\gamma} \approx \frac{E^4}{M_Z^4} \tag{198}$$

At low energies, photon exchange dominates. At high energies ($E=M_Z$), this low-energy approximation fails.

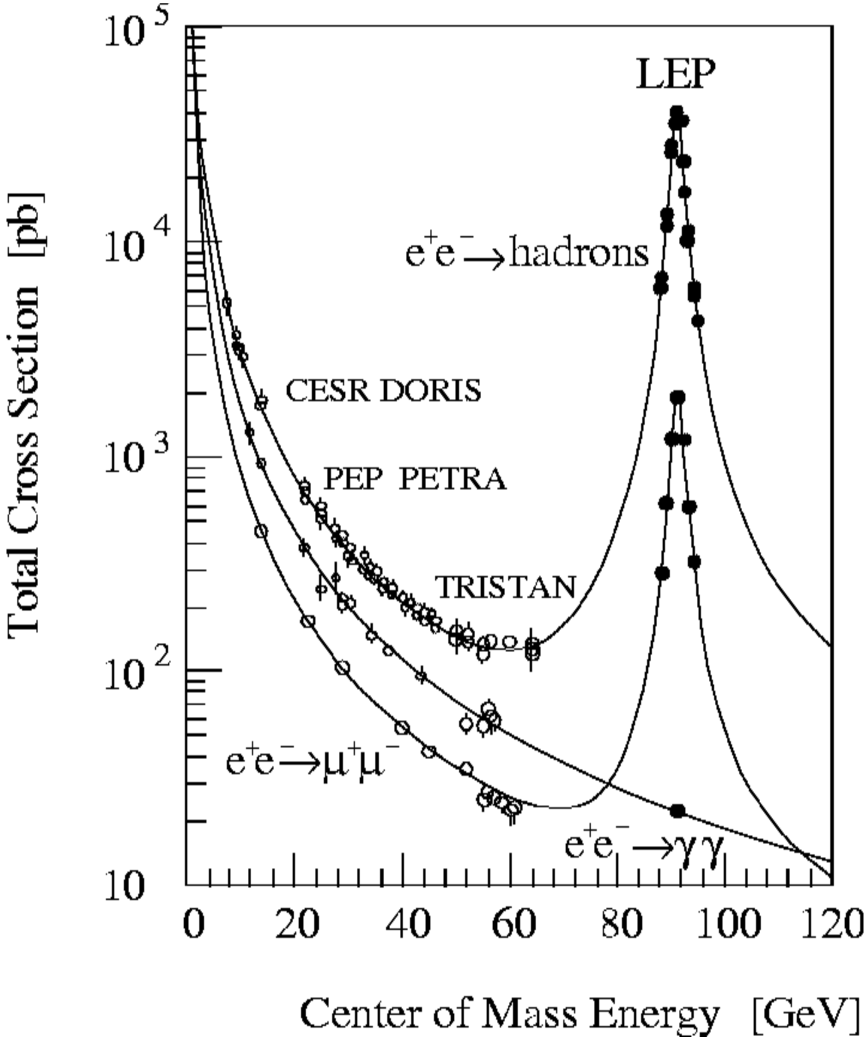


Figure 169: Total cross sections of e^+e^- annihilation

- Z^0 peak is described by the Breit-Wigner formula:

$$\sigma(e^+e^- \to X) = \frac{12\pi M_Z^2}{E_{CM}^2}\left[\frac{\Gamma(Z^0 \to e^+e^-)\Gamma(Z^0 \to X)}{(E_{CM}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right] \qquad (199)$$

Here Γ_Z is the total Z^0 decay rate, and $\Gamma_Z(Z^0 \to X)$ are decay rates to other final states.

Height of the peak (at $E_{CM}=M_Z$) is then proportional to the product of branching ratios:

$$B(Z^0 \to e^+e^-)B(Z^0 \to X) \equiv \frac{\Gamma(Z^0 \to e^+e^-)}{\Gamma_Z}\frac{\Gamma(Z^0 \to X)}{\Gamma_Z} \qquad (200)$$

Fitted SM parameters of the Z^0 peak:

$$\begin{aligned} M_Z &= 91.1874 \pm 0.0021\ GeV/c^2 \\ \Gamma_Z &= 2.4961 \pm 0.0010\ GeV \end{aligned} \qquad (201)$$

- Fitting the peak with Eq.(199), not only M_Z and Γ_Z can be found, but also partial decay rates:

$$\Gamma(Z^0 \to hadrons) = 1.7426 \pm 0.0010\ GeV \qquad (202)$$

$$\Gamma(Z^0 \to l^+ l^-) = 0.084005 \pm 0.000015\ GeV \qquad (203)$$

- Decays $Z^0 \to l^+l^-$ and $Z^0 \to hadrons$ account for only about 80% of all Z^0 decays

- Remaining decays are those containing only <u>neutrinos</u> in the final state

$$\Gamma_Z = \Gamma(Z^0 \to hadrons) + 3\Gamma(Z^0 \to l^+ l^-) + \\ + N_\nu \Gamma(Z^0 \to \nu_l \overline{\nu}_l) \qquad (204)$$

From Eqs.(201)-(203):

$$N_\nu \Gamma(Z^0 \to \nu_l \overline{\nu}_l) = 0.498 \pm 0.009\ GeV$$

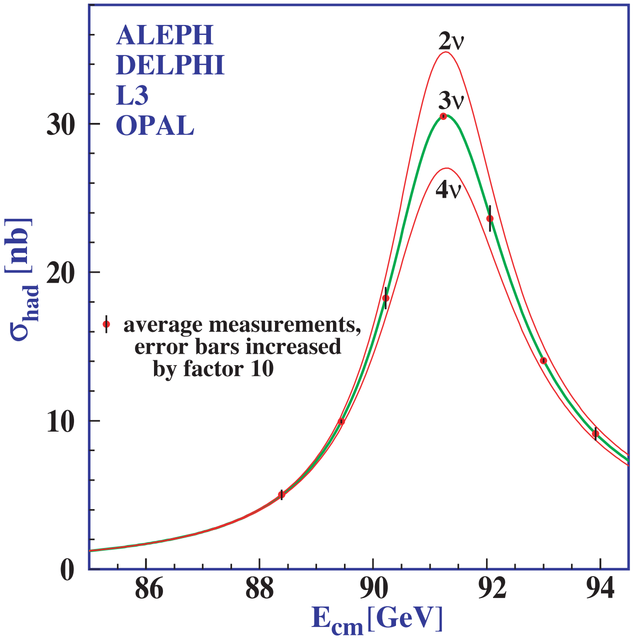
Decay rate to neutrino pairs is calculated from diagrams of Figure 167:

$$\Gamma(Z^0 \rightarrow \nu_l \bar{\nu}_l) = 0.166\ GeV \tag{205}$$

which means that $N_\nu \approx 3$. More precisely, from SM fits to LEP data,

$$N_\nu = 2.984 \pm 0.008 \tag{206}$$

- There are no explicit restrictions on number of generations in the SM
- However, analysis of Z^0 line shape shows that there are 3 and only 3 kinds of massless neutrinos.
  - If neutrinos are assumed to have negligible masses w.r.t. Z^0, there must be only 3 generations of leptons and quarks in the SM



- Electroweak unification regards both Z^0 and γ as mixtures of W^0 and yet another neutral boson B^0:

$$\begin{aligned} \gamma &= B^0 \cos\theta_W + W^0 \sin\theta_W \\ Z^0 &= -B^0 \sin\theta_W + W^0 \cos\theta_W \end{aligned} \qquad (207)$$

The corresponding gauge transformation is:

$$\psi_l(\vec{x}, t) \to \psi_l'(\vec{x}, t) = e^{i g_Z y_l \alpha(\vec{x}, t)} \psi_l(\vec{x}, t) \qquad (208)$$

here l stands for electron or neutrino and y_l are corresponding constants

Introduction of B^0 leads to extra vertices:

$$e^- \to e^- B^0 \qquad \nu_e \to \nu_e B^0$$

with new couplings $g_Z y_{e^-}$ and $g_Z y_{\nu_e}$. If the unification condition (189) is satisfied, first mixture in (207) indeed has the coupling of a photon.

- Generally, experimental data agree very well with gauge invariant electroweak theory predictions
- But gauge invariance implies that spin-1 bosons have <u>zero masses</u> if they are the only bosons in the theory (photon and even gluon nicely comply with this requirement)

$$\Downarrow$$

one more field should exist!

The Higgs Boson

- The scalar *Higgs field* (introduced in 1964) solves the problem by implying a yet another SM particle, the *Higgs boson:*

– Higgs field has a non-zero value ϕ_0 in vacuum

– The filed has a corresponding *Higgs boson* H^0 which is a spin-0 particle

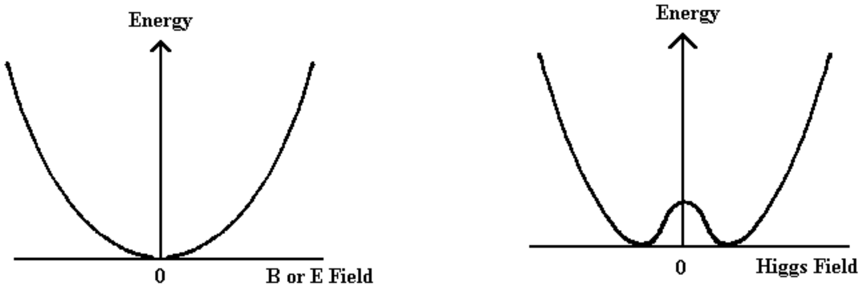


Figure 170: Comparison of the electric and Higgs fields

- The vacuum value ϕ_0 is not gauge invariant ⇒ *hidden gauge invariance*, or *spontaneously broken symmetry*

- Vacuum hence is supposed to be populated with massive Higgs bosons ⇒ when a gauge field interacts with the Higgs field it acquires mass (e.g. W and Z bosons become massive)

- In the same manner, fermions acquire masses by interacting with Higgs bosons:

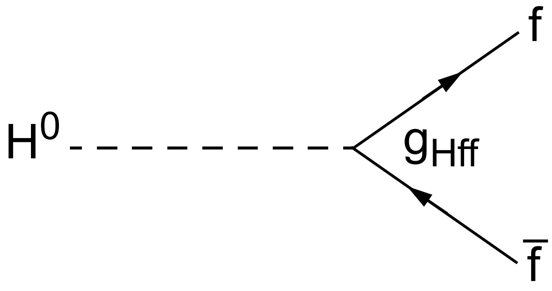


Figure 171: A basic vertex for Higgs-fermion interactions

The coupling constant is related to the fermion mass:

$$g_{Hff}^2 = \sqrt{2}G_F m_f^2 \tag{209}$$

- The mass of the Higgs itself is not predicted by the theory, only its couplings to other particles are predicted (as in Eq.(209))
  - There are other ways to satisfy the gauge invariance requirements, but they are less elegant and produced no experimental proof (yet)

Possible signatures of Higgs

a) If H^0 was much lighter than Z^0 ($M_H \leq 50\ GeV/c^2$), then Z^0 could decay by

$$Z^0 \to H^0 + l^+ + l^- \tag{210}$$

$$Z^0 \to H^0 + \nu_l + \bar{\nu}_l \tag{211}$$

But the branching ratio would be very low:

$$3 \times 10^{-6} \leq \frac{\Gamma(Z^0 \to H^0 l^+ l^-)}{\Gamma_{tot}} \leq 10^{-4}$$

With the large LEP statistics, they still could have been detectable; since the reactions (210) and (211) have not been observed, the *lower limit* set by LEP1 was $M_H > 58\ GeV/c^2$

b) If H^0 is significantly heavier than 60 GeV/c^2, it can be produced in e^+e^- annihilation at higher energies:

$$e^+ + e^- \rightarrow H^0 + Z^0 \qquad (212)$$

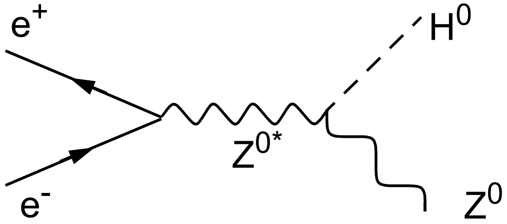


Figure 172: “Higgsstrahlung” in e^+e^- annihilation

- In such a reaction, Higgs with mass of up to 90 GeV/c^2 could have been detected by observing H^0 decaying into a $b\bar{b}$ pair (74%) and Z to a $q\bar{q}$ pair (70%) - 4 jets, of which 2 are *b-tagged*
- In the closing days of LEP, ALEPH experiment reported a couple of such candidate Higgs events. Other experiments saw no events of this kind.

The final lower limit established by LEP is:

$$M_H > 114.4\ GeV/c^2 \qquad (213)$$

SM fit to electroweak parameters measured at LEP was quite good at predicting Higgs mass:

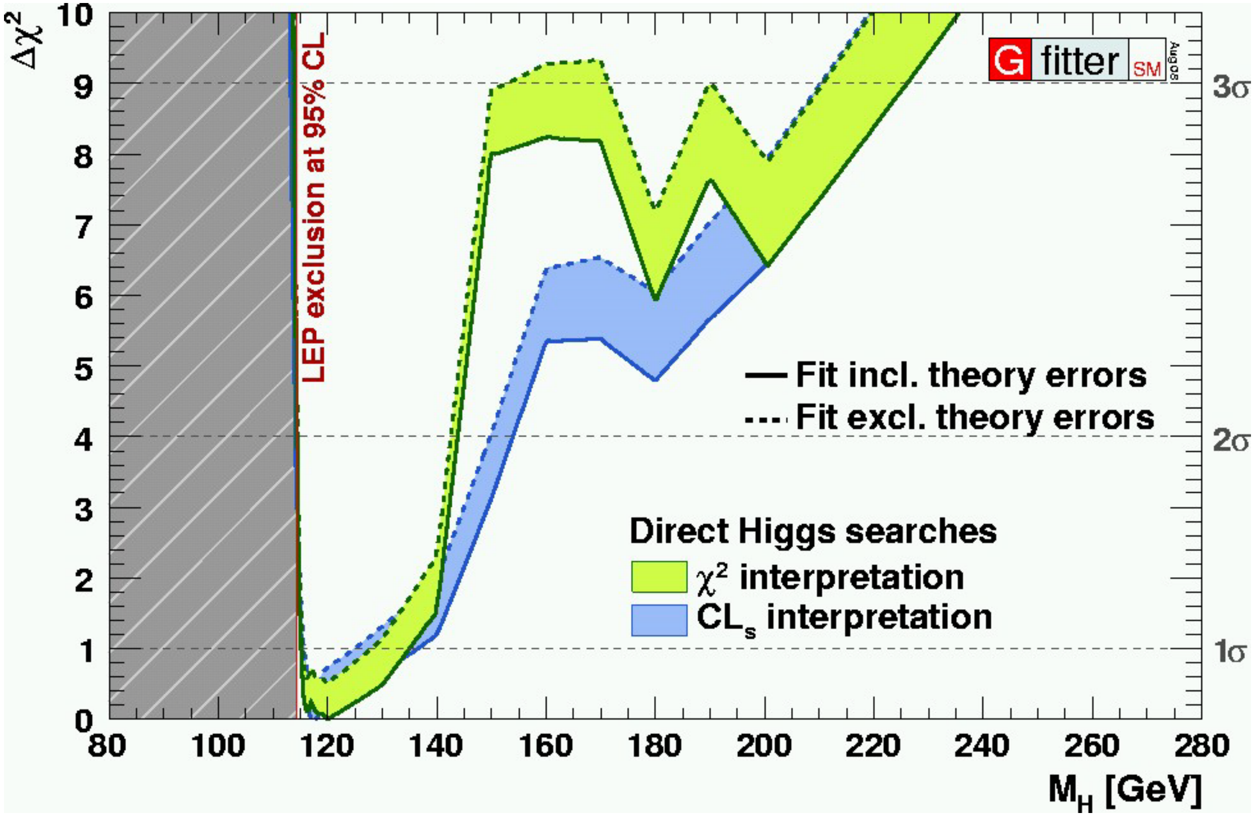


Figure 173: Fit of the Higgs mass made by DELPHI in 2008

c) Higgs with masses up to 1 TeV can be observed at the LHC:

$$p + p \rightarrow H^0 + X \tag{214}$$

where H^0 is produced in electroweak interaction between quarks or gluons, e.g.:

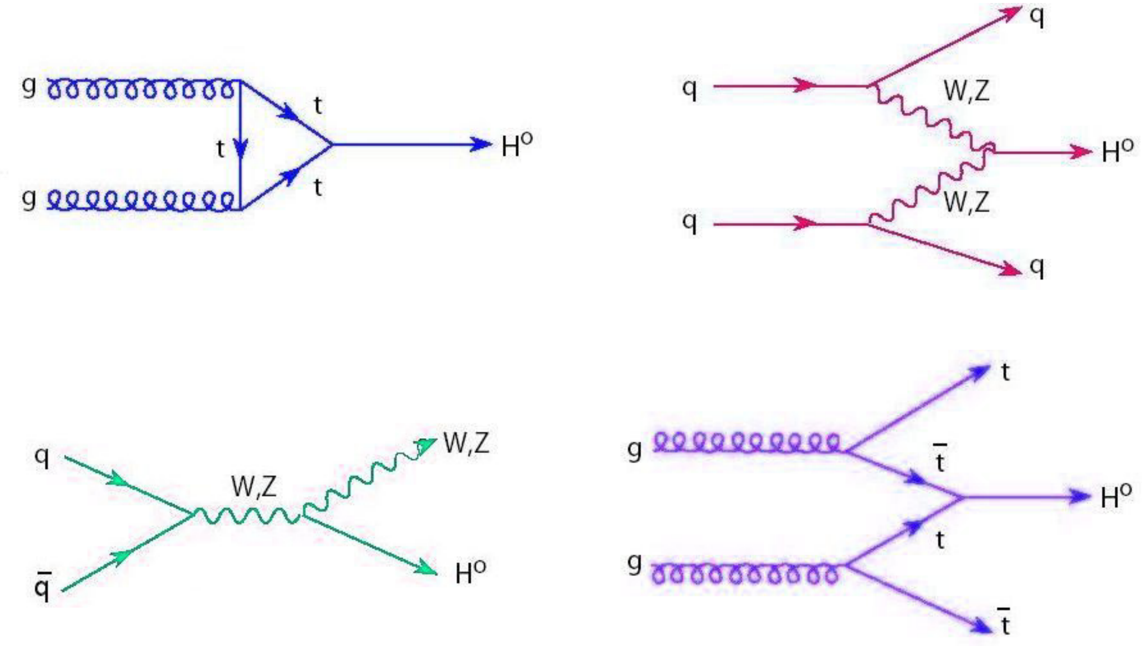


Figure 174: Examples of Higgs production processes at LHC

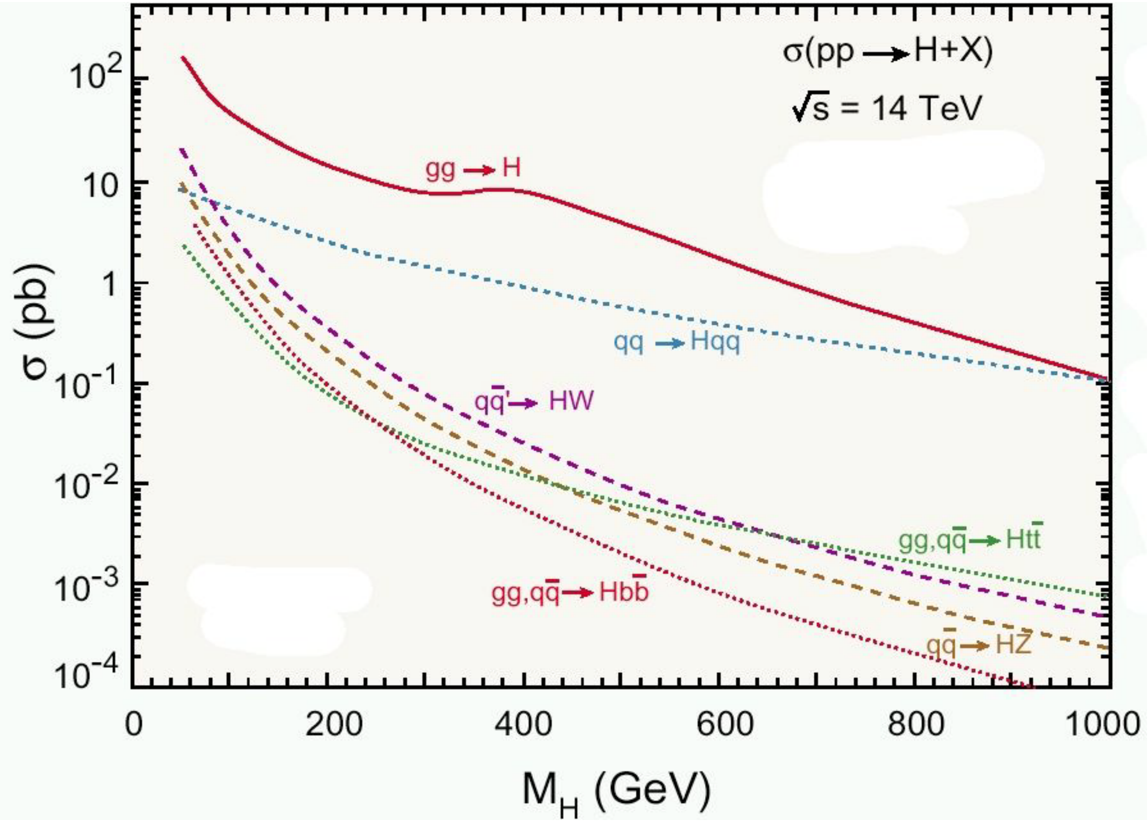


Figure 175: Predicted Higgs production cross sections at LHC

- Gluon fusion is the dominant production process

Due to the heavy background, good signatures have to be considered:

- If $M_H > 2M_Z$, then the dominant decay modes would be:

$$H^0 \to Z^0 + Z^0 \tag{215}$$

$$H^0 \to W^- + W^+ \tag{216}$$

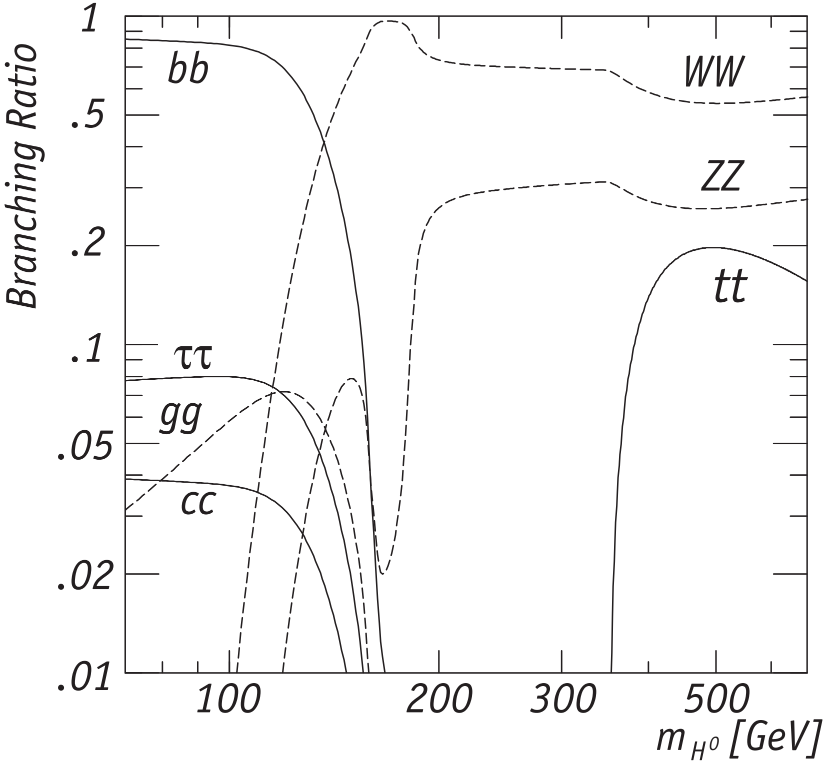


Figure 176: Branching ratios for the main decays of the SM Higgs boson

The most clear signal is when both Z^0 decay into electron or muon pairs:

$$H^0 \to l^+ + l^- + l^+ + l^- \tag{217}$$

This will mean 200 GeV/c$^2 \leq M_H \leq$ 500 GeV/c^2, but only 3% of all decays

- It is also possible to use the 4-lepton channel if $M_H < 2M_Z$, but then one of the Z^0 will be virtual

- If $M_H < 2M_W$, the dominant decay mode (~57%) is

$$H^0 \to \mathrm{b} + \bar{\mathrm{b}} \tag{218}$$

but this gives indistinguishable signal at LHC. Another mode is

$$H^0 \to \gamma + \gamma \tag{219}$$

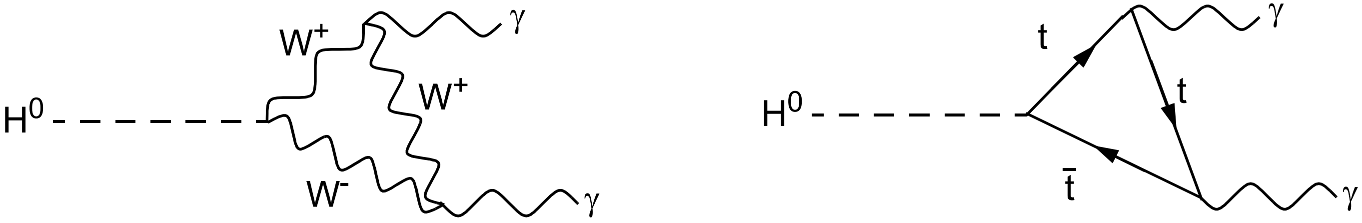


Figure 177: Dominant mechanisms for the decay (219)

- Branching ratio of this kind of processes is about only 0.23%, but easy to observe

- A strong signal at ~125 GeV was reported by ATLAS and CMS experiments at LHC on July 4, 2012

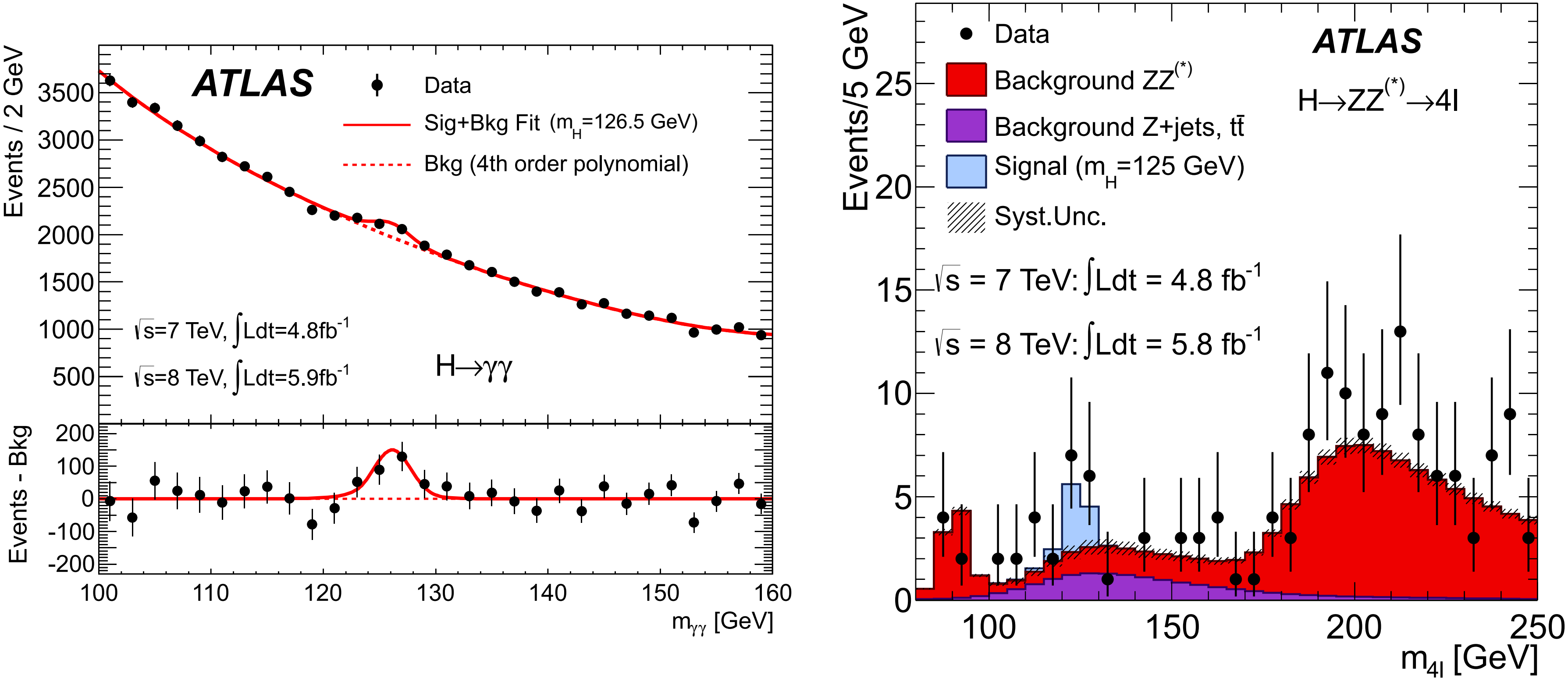


Figure 178: First ATLAS Higgs results in 2-photon and 4-lepton channels

Current Higgs mass values measured at LHC in different channels:

- ATLAS, 4-lepton: $M_H = 124.3^{+0.6}_{-0.5}{}^{+0.5}_{-0.3}\ GeV$
- ATLAS, 2-photon: $M_H = 126.8\ GeV \pm 0.2 \pm 0.7\ GeV$
- CMS, 2-photon: $M_H = 125.4\ GeV \pm 0.8\ GeV$
- Other channels also show signals in the same area, but require more data

- Analysis of angular distributions favours spin 0 and parity +1 ($J^P=0^+$)
- There is also evidence of couplings to fermions, compatible with SM predictions
- The neutral Higgs is a minimal SM requirement; there might exist more complicated variants, including charged higgs-particles.

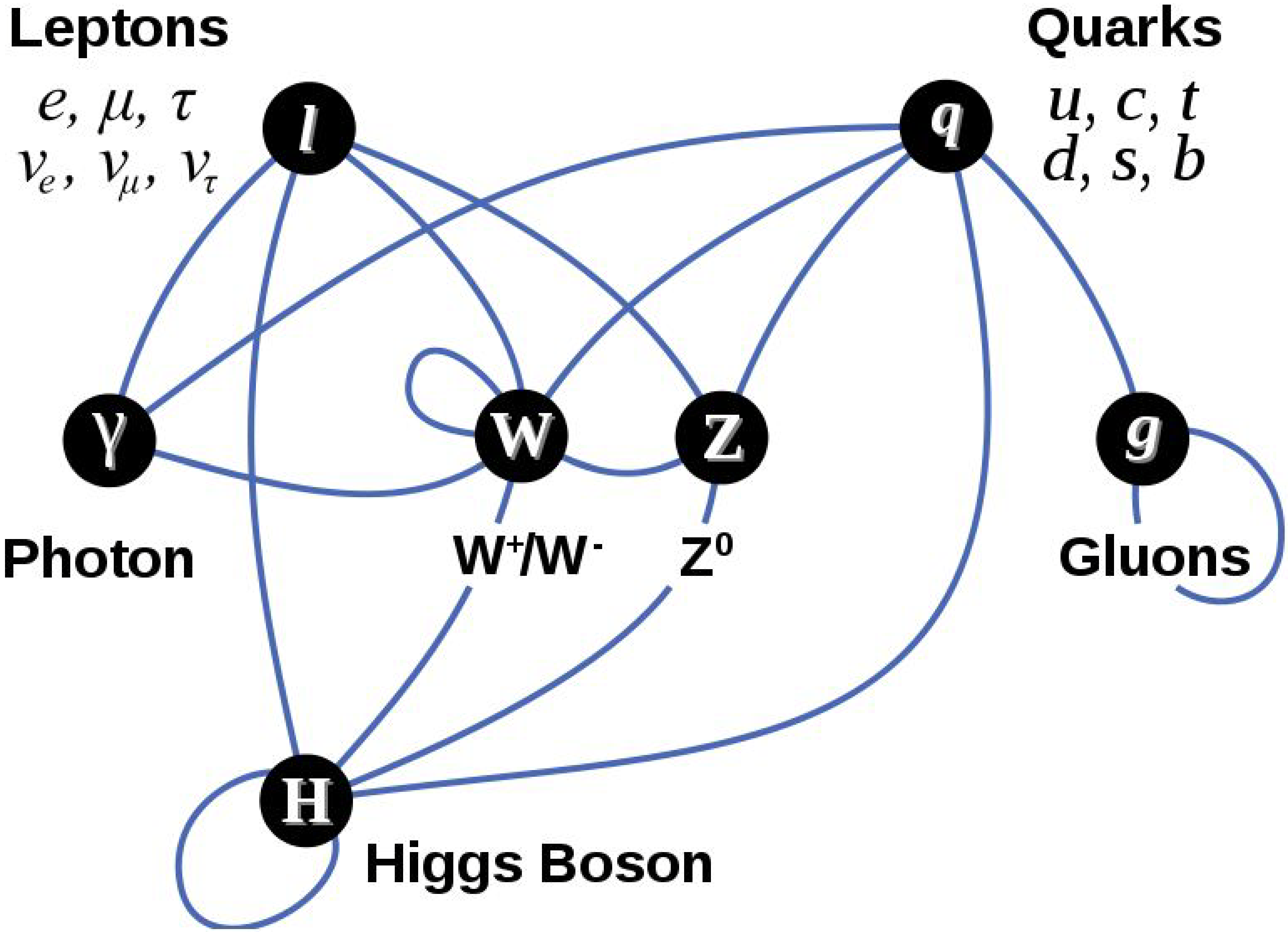


Figure 179: Summary of SM constituents and couplings