IX. Weak Interactions: W and Z bosons

- Like in QED and QCD, the weak force carriers are *spin-1 bosons*; they couple to *quarks and leptons*
  - Weak interactions are carried out by three *intermediate vector bosons*: W^+ and W^- (mass 80.4 GeV), and Z^0 (91.2 GeV)
  - Since these bosons are **very massive** particles, weak interactions have very short range (order of 2×10^{-3} fm)
  - Historically, all observed weak processes were *charged current* reactions mediated by W^+ or W^- bosons (like β-decay) . Electroweak theory predicted existence of *neutral current* reactions caused by the Z^0 boson

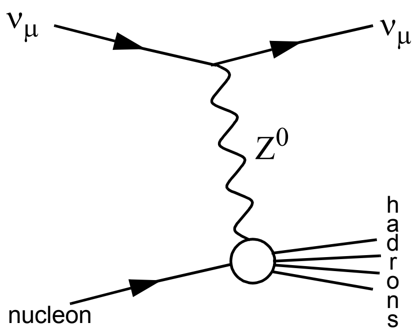


Figure 127: Predicted neutral current reaction: no muon in the final state

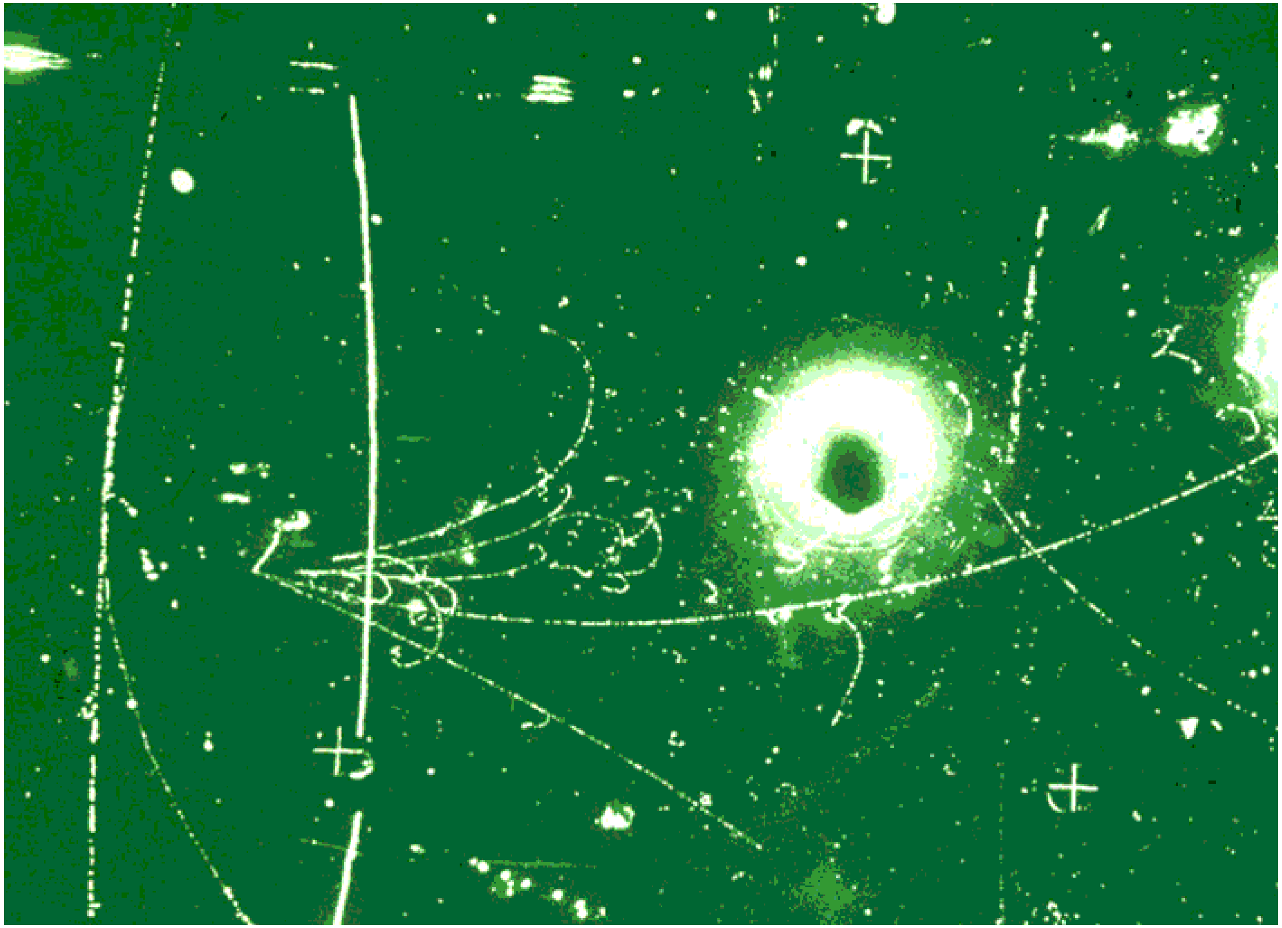


Figure 128: One of the first neutral current reactions as seen by the Gargamelle bubble chamber in 1973

Brief history of the W and Z bosons discovery

- First dedicated study of vector bosons: detectors UA1 and UA2 at the proton-antiproton collider SPS (Super Proton Synchrotron) at CERN (started in 1981)
  - Search for leptonic decays of the W and Z bosons produced in $p\bar{p}$ collisions:

$$\begin{aligned}\bar{p} + p &\rightarrow W^{+} + X\\ &\qquad \llcorner\!\!\longrightarrow l^{+} + \nu_{l}\end{aligned} \tag{155}$$

$$\begin{aligned}\bar{p} + p &\rightarrow W^{-} + X\\ &\qquad \llcorner\!\!\longrightarrow l^{-} + \bar{\nu}_{l}\end{aligned} \tag{156}$$

$$\begin{aligned}\bar{p} + p &\rightarrow Z^{0} + X\\ &\qquad \llcorner\!\!\longrightarrow l^{+} + l^{-}\end{aligned} \tag{157}$$

  - W and Z can decay into quarks as well, but in hadron collisions this can not be identified

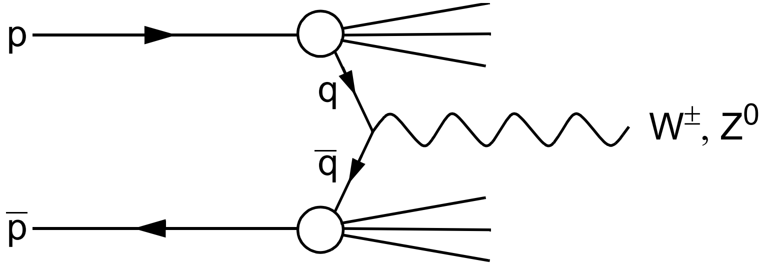


Figure 129: The mechanism of $W^{\pm}$ and Z production in $p\bar{p}$ annihilation

From the quark point of view, processes (155)-(157) are quark-antiquark annihilations:

$$u + \bar{d} \rightarrow W^{+} \quad , \quad d + \bar{u} \rightarrow W^{-} \tag{158}$$

$$u + \bar{u} \rightarrow Z^{0} \quad , \quad d + \bar{d} \rightarrow Z^{0} \tag{159}$$

To obtain sufficient centre-of-mass energies for quark-antiquark collisions (~90 GeV), proton and antiproton beams at SPS had an energy of 270 GeV each.

W bosons

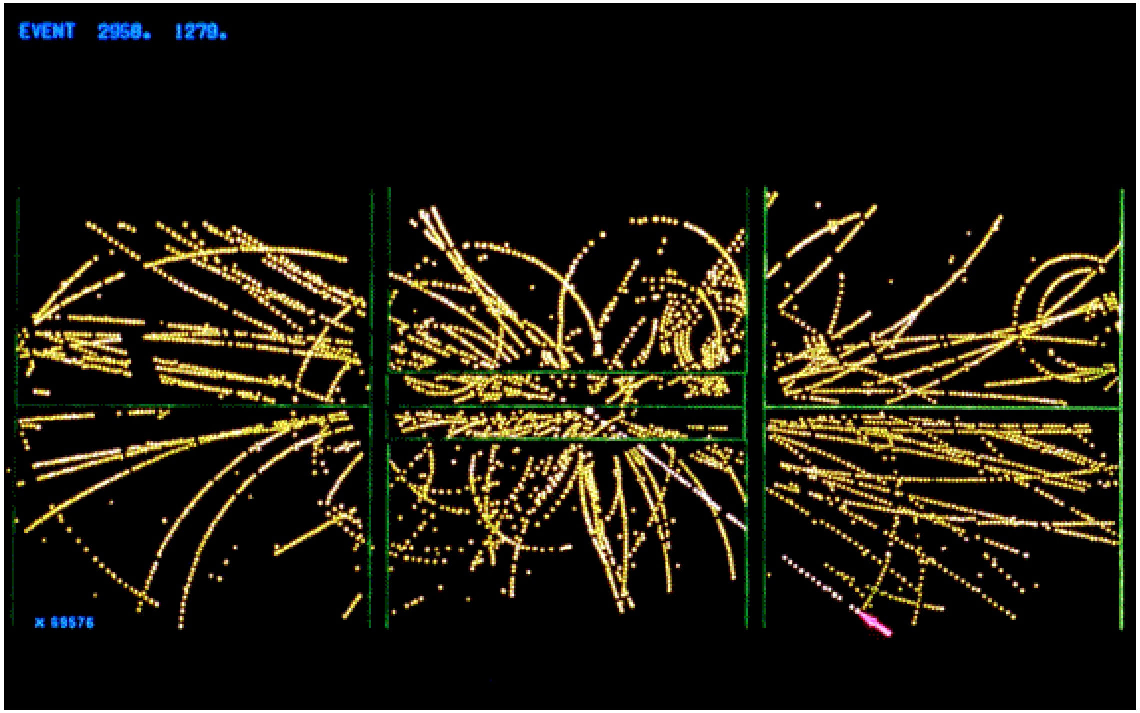


Figure 130: A W boson observed by UA1 detector in 1982; a high transverse momentum electron is indicated by the arrow

- Signature of a W boson:
  – a lepton with large momentum ($p(l)>10\ GeV/c$) is emitted at a wide angle to the beam ($\theta>5^\circ$)
  – large “*missing transverse momentum*” ($p_T=p\sin\theta$) carried away by neutrino

- Neutrinos can not be detected, but we know that in a symmetric collision, sum of all the momenta must be 0. Events with large missing momentum (>15 GeV in UA1) indicate presence of energetic neutrinos

If $p_T(W)=0 \Rightarrow \not{p}_T = p_T(l)$: the missing transverse momentum is equal to the transverse momentum of the detected lepton

From 43 events observed by UA1, the mass of W^+ and W^- was defined as

$$M_W = 80.33 \pm 0.15\ GeV/c^2 \qquad (160)$$

and the decay width as

$$\Gamma_W = 2.07 \pm 0.06\ GeV \qquad (161)$$

which corresponds to the lifetime of 3.2×10^{-25} s

- Branching ratios of leptonic decay modes of $W^\pm$ are about 11% for each lepton generation

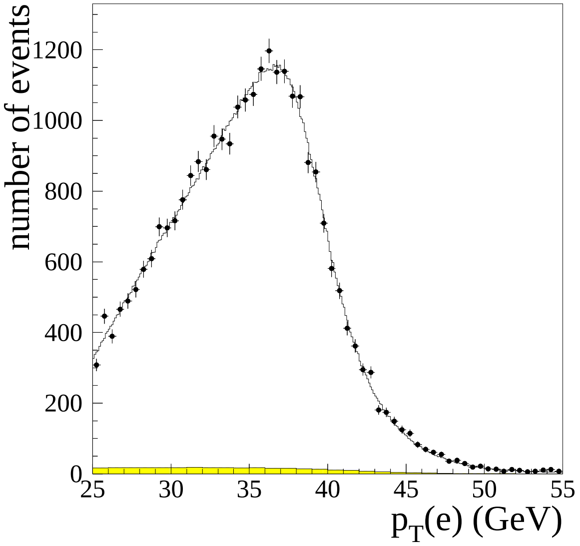


Figure 131: A later result from the D0 experiment at the Tevatron (also proton-antiproton collisions); fit gives M_W=80.48± 0.09 GeV

W bosons can be pair-produced in e^+e^- annihilation, and the up-to-date world average for the W mass is

$$M_W = 80.385 \pm 0.015\ GeV/c^2 \tag{162}$$

Z^0 boson

- Signature of a Z^0 boson in $p\bar{p}$ collision: pair of leptons (e^+e^-) with very large momenta.
- Mass of the Z^0 then equals to the invariant mass of leptons

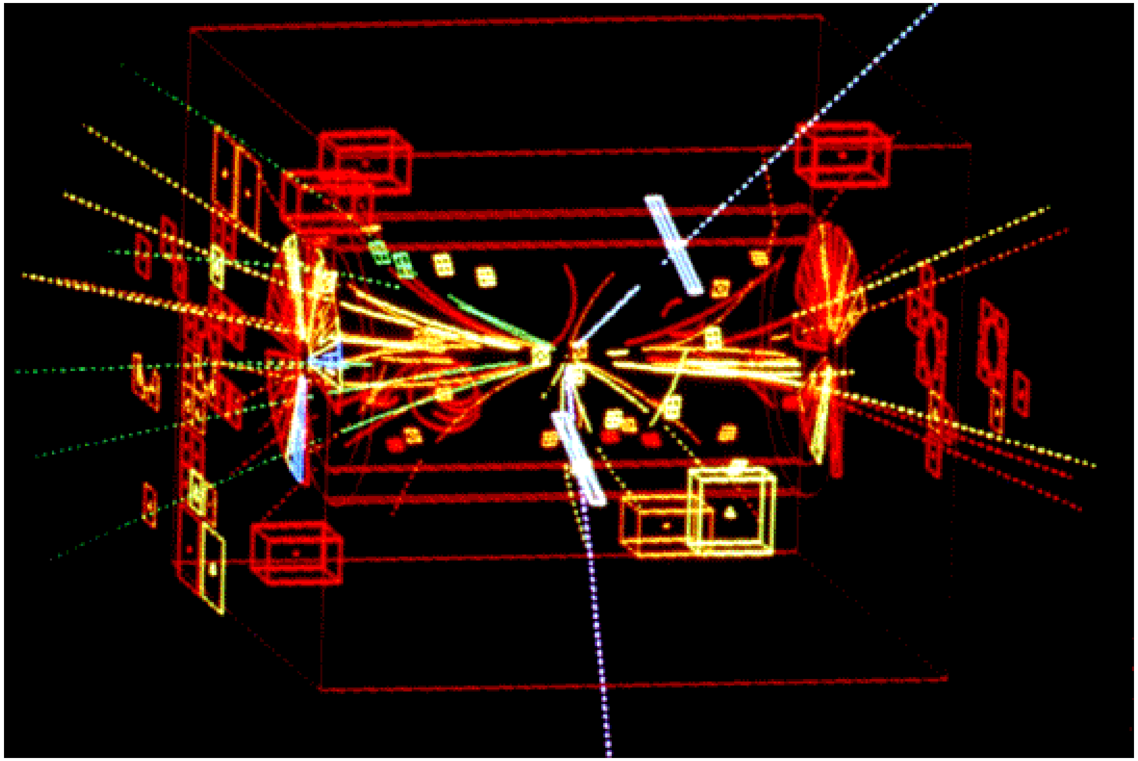


Figure 132: A Z^0 production event in the UA1 detector.

Knowing M_W, the mass of Z^0 was predicted to be $M_Z \approx 90$ GeV/c^2

- From the first 18 electron and 10 muon events measured by UA1:

$$M_Z = 93.0 \pm 1.4\ GeV/c^2 \qquad (163)$$

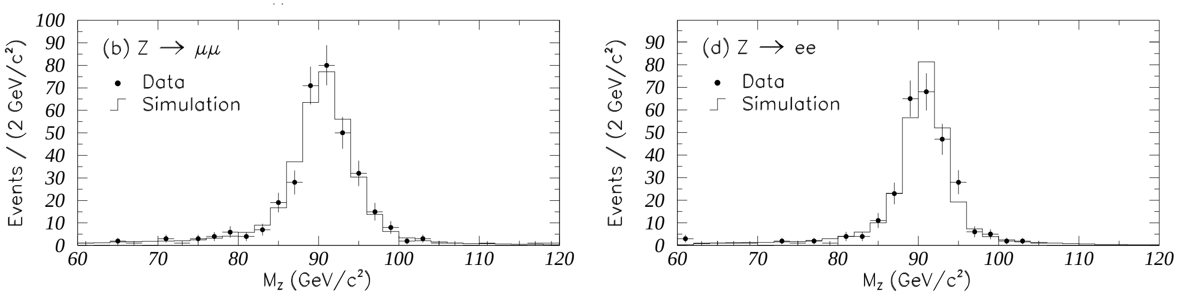


Figure 133: Dilepton mass spectra near the Z^0 peak at Tevatron

More precise methods and new data from e^+e^- collisions at LEP give

$$M_Z = 91.1876 \pm 0.0021\ GeV/c^2 \qquad \Gamma_Z = 2.4952 \pm 0.0023\ GeV/c^2 \qquad (164)$$

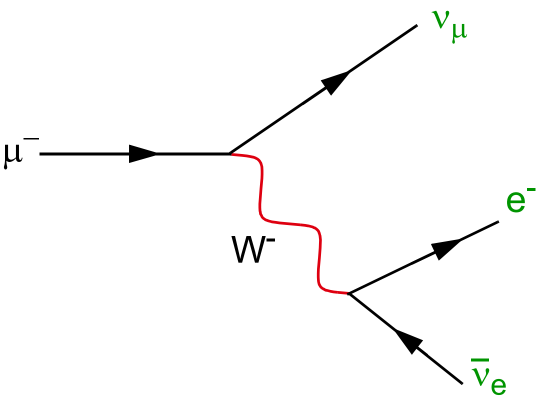
which corresponds to the lifetime of 2.6×10^{-25} s.

- Branching ratios of leptonic decay modes of Z^0 are around 3.4% for each lepton generation

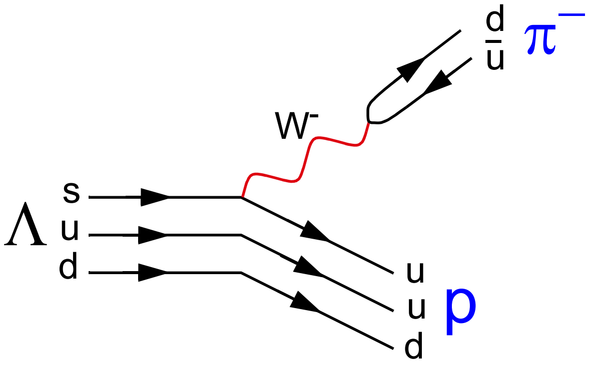
Charged current reactions

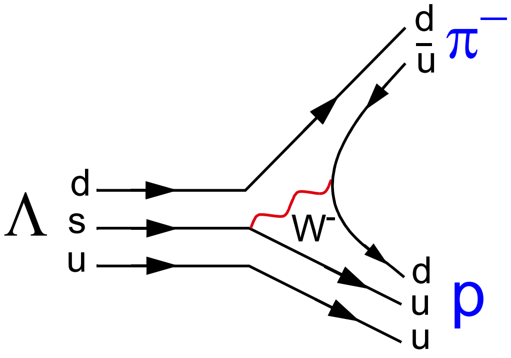
- *Charged current* reactions are weak interactions mediated by the charged W bosons:

1) purely *leptonic* processes: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

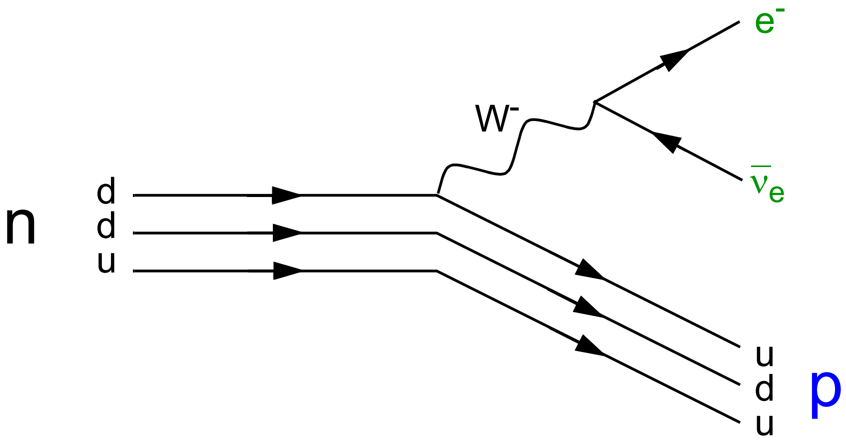


2) purely hadronic processes: $\Lambda \to \pi^- + p$





3) *semileptonic* reactions: $n \rightarrow p + e^- + \bar{\nu}_e$



- **Reminder**: all the electromagnetic interactions can be built from eight basic interactions:

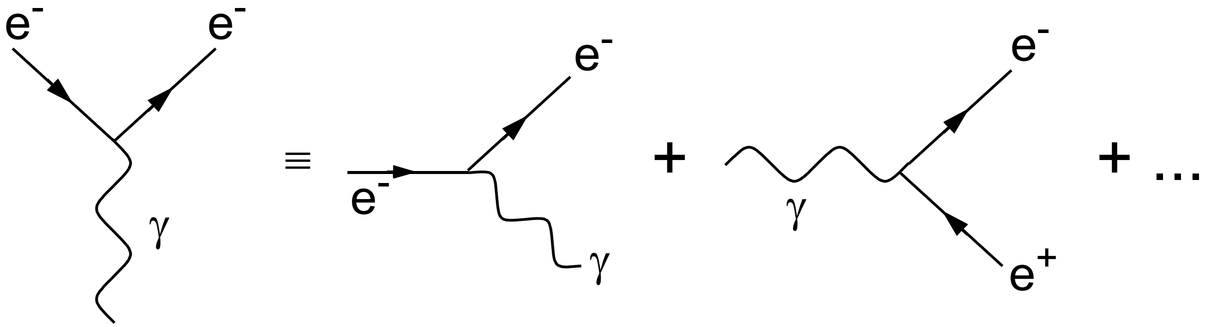


Figure 134: The basic vertex for electron-photon interactions

- In a similar way, leptonic weak interaction processes can be built from a certain number of reactions corresponding to basic vertices:

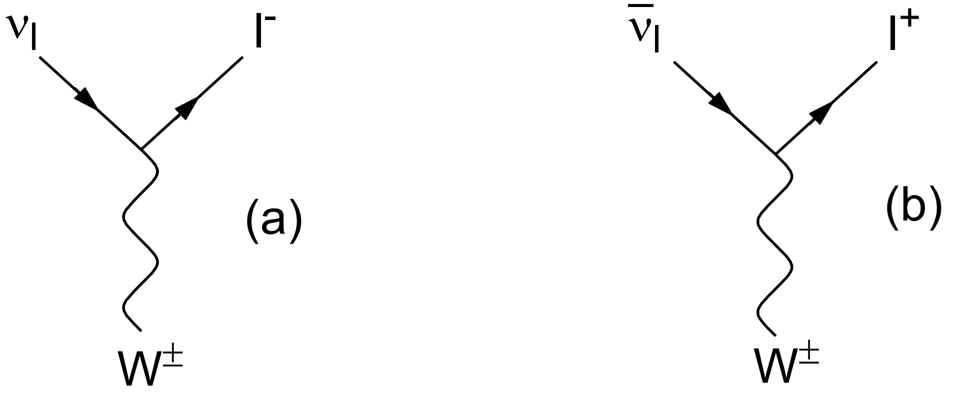


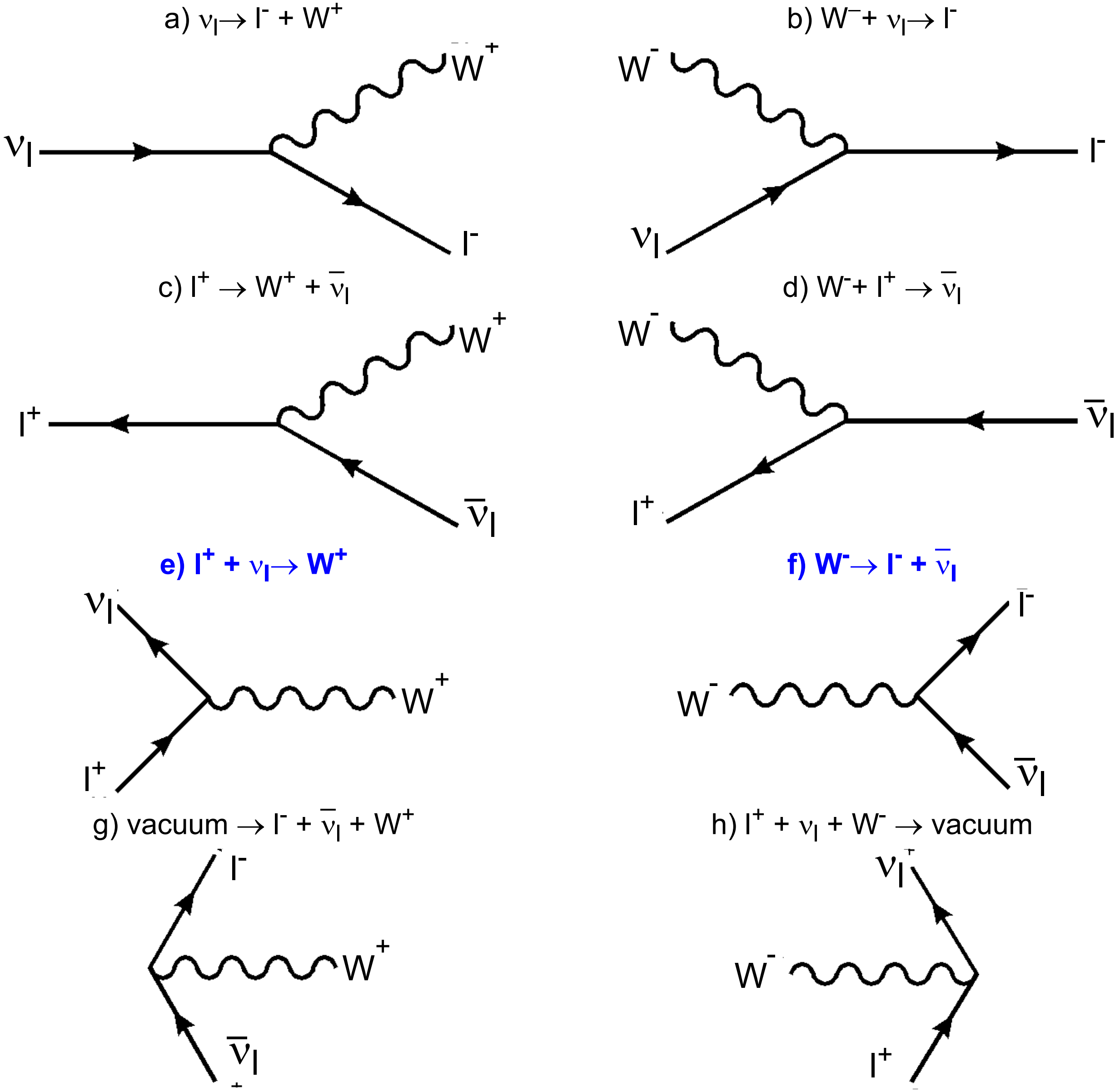
Figure 135: The two basic vertices for $W^\pm$-lepton interactions

- Weak interactions always conserve lepton quantum numbers

Diagram-wise this conservation is guaranteed by:

– at each vertex, there is one arrow pointing in and one pointing out

– lepton indices “l” are the same on both lines



a) $\nu_l \rightarrow l^- + W^+$

b) $W^- + \nu_l \rightarrow l^-$

c) $l^+ \rightarrow W^+ + \bar{\nu}_l$

d) $W^- + l^+ \rightarrow \bar{\nu}_l$

e) $l^+ + \nu_l \rightarrow W^+$

f) $W^- \rightarrow l^- + \bar{\nu}_l$

g) vacuum $\rightarrow l^- + \bar{\nu}_l + W^+$

h) $l^+ + \nu_l + W^- \rightarrow$ vacuum

Figure 136: Eight basic weak current reactions

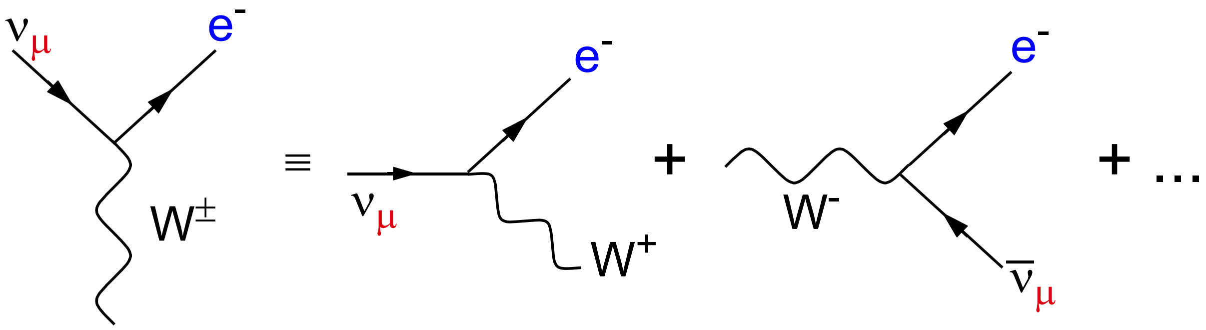


Figure 137: Vertices **violating** lepton number conservation (forbidden)

- Processes of Figure 136 are virtual, so that two or more have to be combined to conserve energy
- However, processes like in Fig.136(e) and 136(f) do not violate energy conservation if

$$M_W > M_l + M_{\nu l} \qquad (l = e, \mu, \tau)$$

- In particular, reactions 155 and 156, used to detect the W bosons, are dominated by mechanisms shown in Fig.136(e) and 136(f).

- Leptonic vertices are characterized by the corresponding strength parameter α_W independently on lepton type involved

Knowing the decay rate of $W \to e\nu$, one can estimate α_W to the first order:

- Since the process involves only one vertex and lepton masses are negligible ⇒

$$\Gamma(W \to e\nu) \approx \alpha_W M_W \approx 80\alpha_W \; GeV \tag{165}$$

- Measured decay rate:

$$\Gamma(W \to e\nu) \approx 0.2 \; GeV \tag{166}$$

which gives

$$\alpha_W \approx 1/400 \; = \; O(\alpha_{em}) \tag{167}$$

hence the “strength” of the weak interaction is comparable with the electromagnetic one

- Weak interaction is still much weaker at low energies $E \ll M_W$

Analogues of electron-electron scattering by photon exchange:

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e \qquad (168)$$

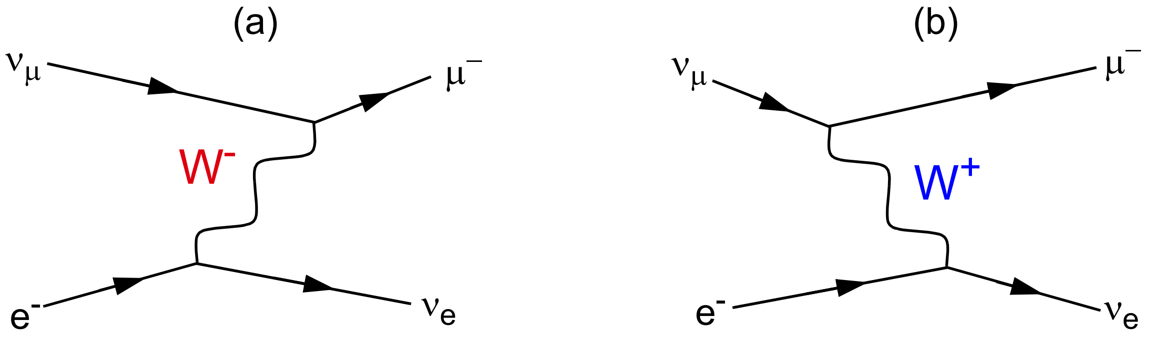


Figure 138: Time-ordered diagrams for inverse muon decay (168)

- Time ordering implies changing the sign of the current!
  - A conventional muon decay is depicted involving W^- :

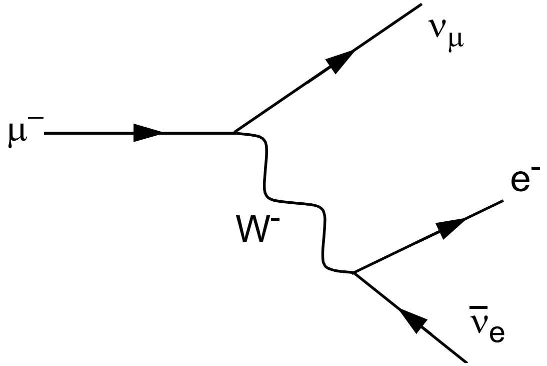


Figure 139: Dominant diagram for muon decay

Including higher order diagrams, inverse muon decay (168) can look like:

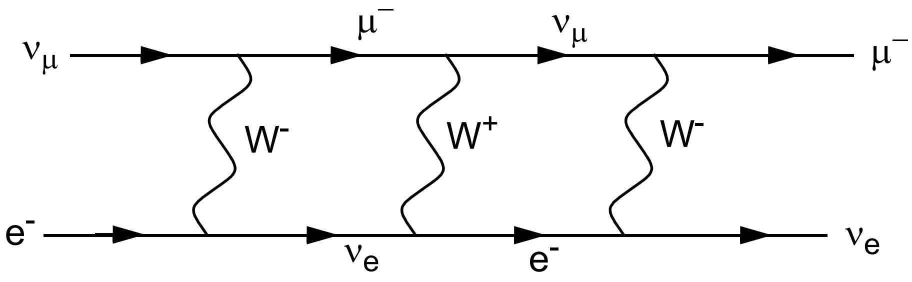


Figure 140: Some higher order contributions to inverse muon decay

- A diagram like Fig.140 gives a negligible contribution of order α_W^6 to the total cross section, analogously to the case of electromagnetic photon exchange

Since W bosons are very heavy, at $E \ll M_W$ interactions like (168) can be approximated by a zero-range interaction:

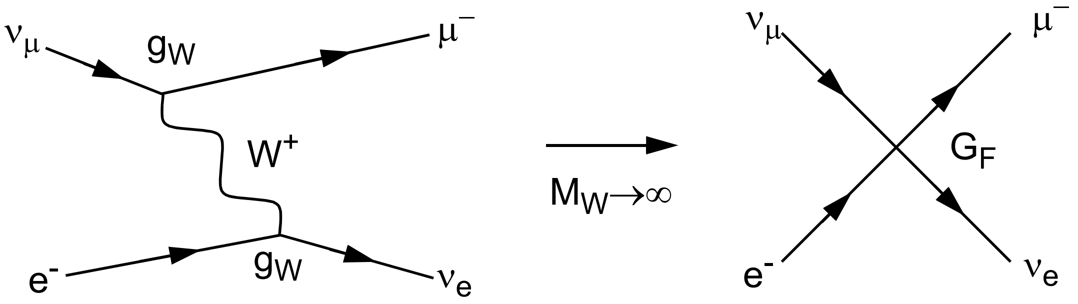


Figure 141: Low-energy zero-range interaction in muon decay

- Taking into account spin effects, the relation between α_W and G_F in zero-range approximation is:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} = \frac{4\pi\alpha_W}{M_W^2} \qquad (169)$$

where g_W is the coupling constant in W-vertices, $\alpha_W \equiv g_W^2/4\pi$ by definition.

This gives the estimate of $\alpha_W=4.2\times10^{-3}=0.58\alpha_{em}$, which is perfectly compatible with estimate (167)

  - α_W is indeed slightly smaller than α_{em} in the low energy approximation

- Weak interaction rates are only small at low energies, because the very large M_W enters (169) as the inverse square

- Weak interactions of hadrons: constituent quarks emit or absorb W bosons

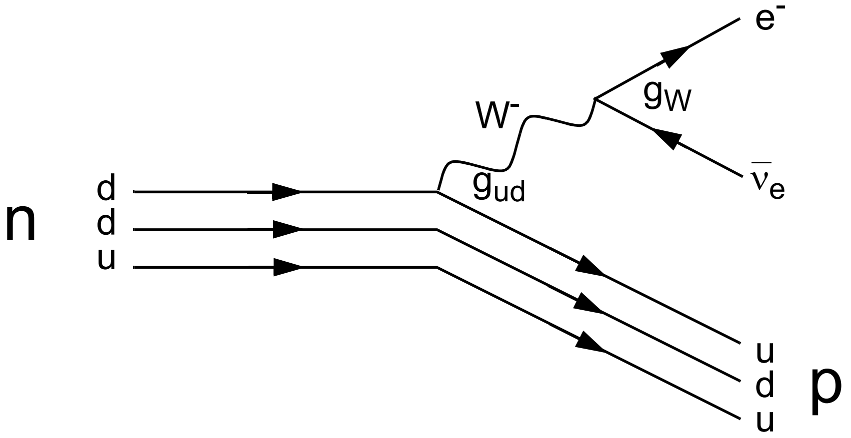


Figure 142: Neutron β-decay

- *Lepton-quark symmetry*: corresponding generations of quarks and leptons have identical weak interactions:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \leftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \leftrightarrow \begin{pmatrix} c \\ s \end{pmatrix}, \; etc.$$

The coupling constants do not change upon exchange of quarks/leptons:

$$g_{ud} = g_{cs} = g_W \tag{170}$$

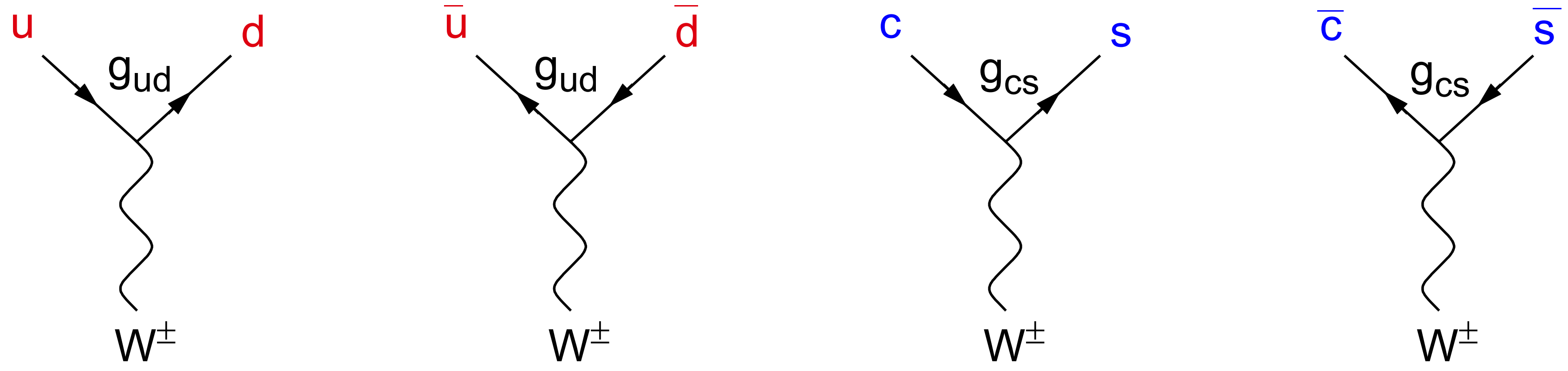


Figure 143: W-quark vertices assumed by lepton-quark symmetry

An example of an allowed reaction:

$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad (d\bar{u} \to \mu^- + \bar{\nu}_\mu) \tag{171}$$

- However, some observed reactions are not consistent with the lepton-quark symmetry:

$$K^- \to \mu^- + \bar{\nu}_\mu \qquad (s\bar{u} \to \mu^- + \bar{\nu}_\mu) \tag{172}$$

(branching ratio of this process is 0.63 - quite a common decay)

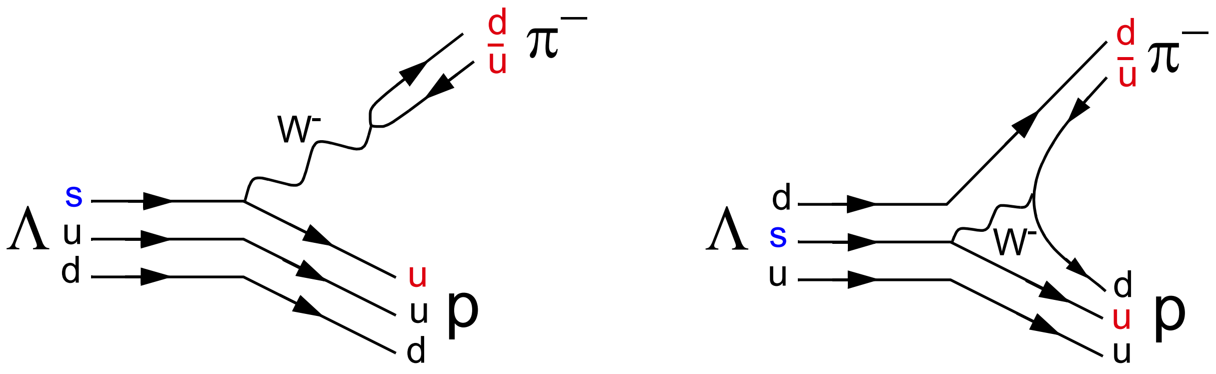


Figure 144: Dominant quark diagrams for Λ decay

To solve the contradiction, the “*quark mixing*” hypothesis was introduced by Cabibbo:

- d- and s-quarks participate the weak interactions via the linear combinations:

$$\begin{aligned} d' &= d\cos\theta_C + s\sin\theta_C \\ s' &= -d\sin\theta_C + s\cos\theta_C \end{aligned} \tag{173}$$

Parameter θ_C is called *Cabibbo angle*

- Quark-lepton symmetry applies to doublets like

$$\begin{pmatrix} u \\ d' \end{pmatrix} \textit{ and } \begin{pmatrix} c \\ s' \end{pmatrix}$$

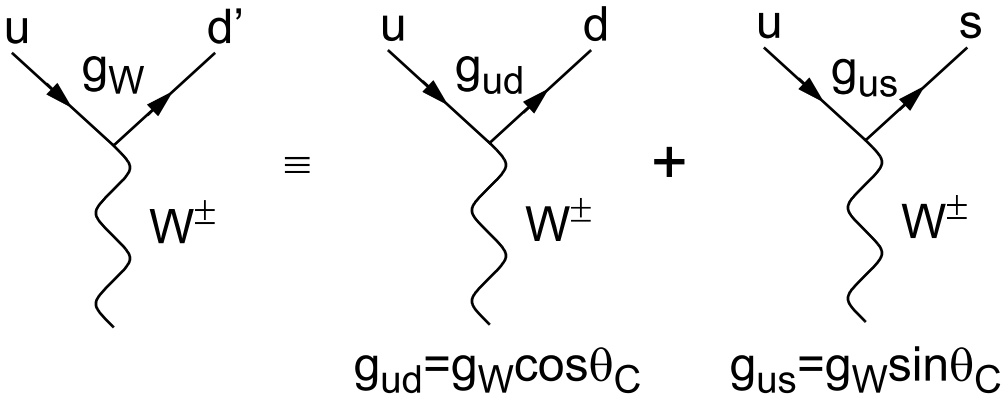


Figure 145: Interpretation of quark mixing

Quark mixing hypothesis allows some more W-quark vertices:

$$g_{ud} = g_{cs} = g_W cos\theta_C \tag{174}$$

$$g_{us} = -g_{cd} = g_W sin\theta_C \tag{175}$$

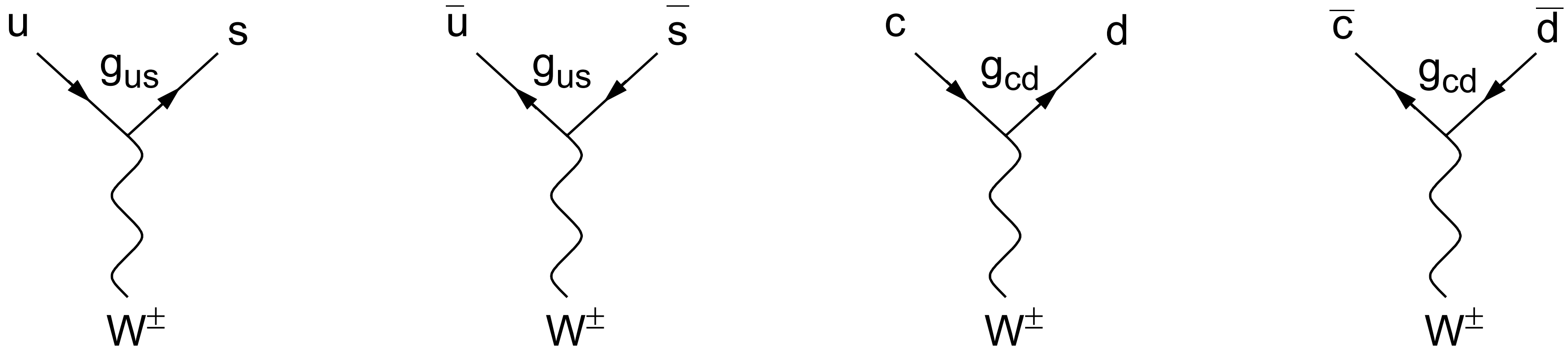


Figure 146: Additional W-quark vertices assumed by lepton-quark symmetry with quark mixing

Cabibbo angle is not given by the theory and has to be measured experimentally, for example, comparing decay rates:

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2\theta_C$$

which corresponds to

$$\theta_C = 12.7° \pm 0.1° \tag{176}$$

Charmed quark couplings g_{cd} and g_{cs} are measured in neutrino scattering experiments and give

$$\theta_C = 12° \pm 1°$$

It can be seen that decays involving couplings (175) are *Cabibbo-suppressed*: they rates are reduced by an order

$$\frac{g_{us}^2}{g_{ud}^2} = \frac{g_{cd}^2}{g_{cs}^2} = \tan^2\theta_C = \frac{1}{20}$$

On the other hand, decays like $c \to s l^+ \nu_l$ and $c \to s u \bar{d}$ are *Cabibbo-allowed*, hence:

- charmed particles almost always decay into strange ones.

Adding the third generation

- Existence of c-quark was first predicted from the lepton-quark symmetry
- After discovery of τ, ν_τ, and b, the sixth quark has been predicted to complete the symmetry: the top-quark was confirmed with the mass of 173 GeV/c^2
  - Announced in March 1995 by the two dedicated independent experiments at the Tevatron: CDF and D0

For two generations, form (173) is conveniently written in a matrix form as:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \qquad (177)$$

Adding the third generation, mixing between all of them must be allowed:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{178}$$

- The 3x3 matrix of (178) is the so-called *CKM matrix $V_{\alpha\beta}$ (Cabibbo-Kobayashi-Maskawa)*

Coupling constants are then:

$$g_{\alpha\beta} = g_W V_{\alpha\beta} \qquad (\alpha = u, c, t; \beta = d, s, b) \tag{179}$$

The two-generation mixing model agrees well with the experimental data, hence V_{ub}, V_{cb}, V_{td} and V_{ts} ought to be very small.

In the limit that mixing between the b quark and (d,s) ones can be neglected, the CKM matrix is

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (180)$$

and hence b’=b

Matrix (180) suggests that b-quarks can’t decay; they however do:

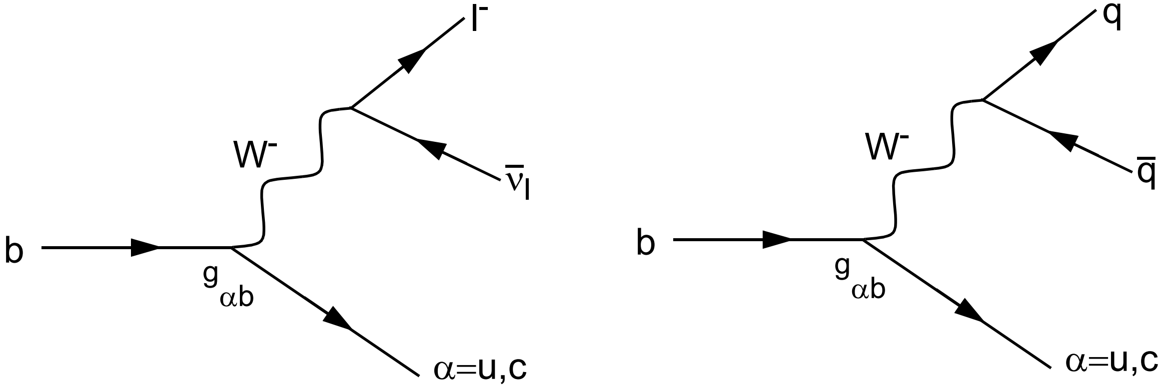


Figure 147: Dominant decays of b-quark

Decay modes of Fig.147 have rates proportional to squared couplings:

$$|g_{ub}|^2 = |V_{ub}|^2 g_W^2 \quad or \quad |g_{cb}|^2 = |V_{cb}|^2 g_W^2 \qquad (181)$$

If V_{ub} and V_{cb} are indeed 0, b-quark should be stable. In reality, it decays, with the rather long lifetime of

$$\tau_b \approx 10^{-12} s \qquad (182)$$

If otherwise $g_{ub}=g_{cb}=g_W$, lifetime has to be shorter, like in the case of τ decays (Fig.148).

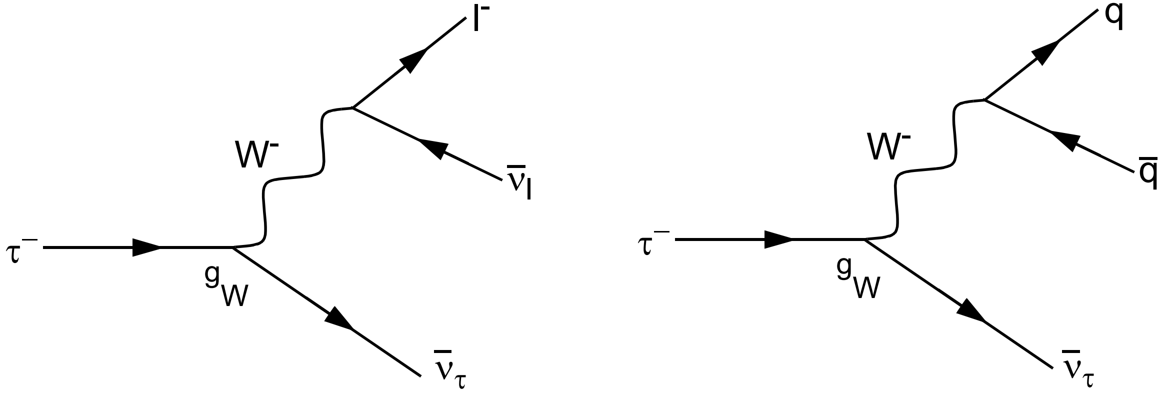


Figure 148: Dominant decays of τ lepton

Knowing the lifetime of τ lepton $\tau_\tau \approx 3 \times 10^{-13}$ s, and assuming there is no suppression of b decay ($V_{ub}=V_{cb}=1$), the lifetime of b-quark should be:

$$\tau_b \approx \frac{1}{N}\left(\frac{m_\tau}{m_b}\right)^5 \approx 10^{-15} s$$

where N is number of possible b-quark decays per analogous τ-decays (3 for the leptonic mode and 4 - for semileptonic)

This contradicts experimental results (b quark lives much longer); more precise recent measurements yield

$$\left|V_{ub}\right| = (4.15 \pm 0.49) \times 10^{-3} \quad and \quad \left|V_{cb}\right| = (40.9 \pm 1.1) \times 10^{-3} \qquad (183)$$

which is still small enough for practical purposes.

- The top-quark is much heavier than even W bosons and can produce them by a decay like:

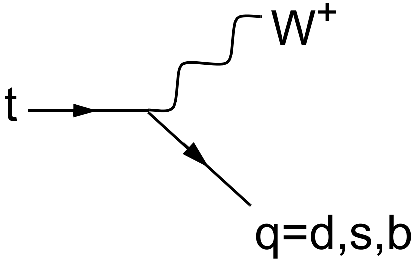


Figure 149: Decay t→ W^+ + q

- As can be seen from CKM matrix (V_{td} and V_{ts} are ~0), the <u>only</u> significant decay mode of t-quark is

$$t \to W^+ + b \qquad (184)$$

with a rate proportional to

$$\alpha_W = g_W^2 / 4\pi \approx 4.2\times10^{-3}$$

Estimate of decay width $\Gamma \sim \alpha_W m_t \sim 1\ GeV$ suggests very short lifetime:

$$\tau_t \approx 4\times10^{-25} s$$

- Top-quarks do not form hadrons because of the too short lifetime

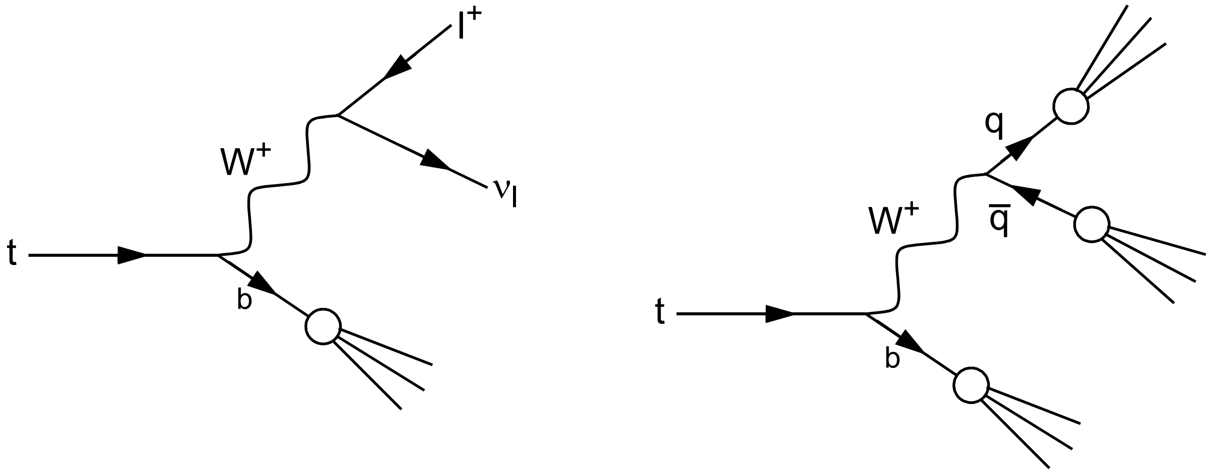


Figure 150: Decays of top-quark

Boson factories in pictures

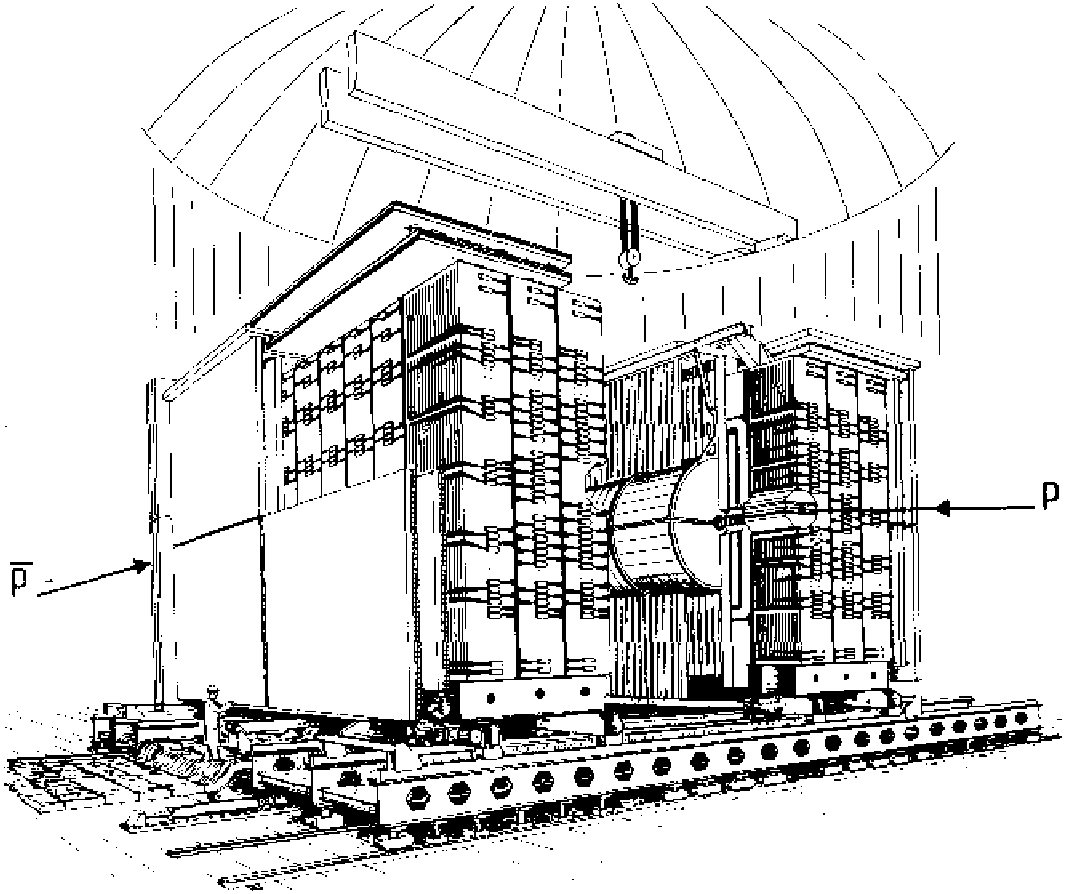


Figure 151: UA1 detector layout (proton-antiproton collisions); solid angle is fully covered down to 0.2°

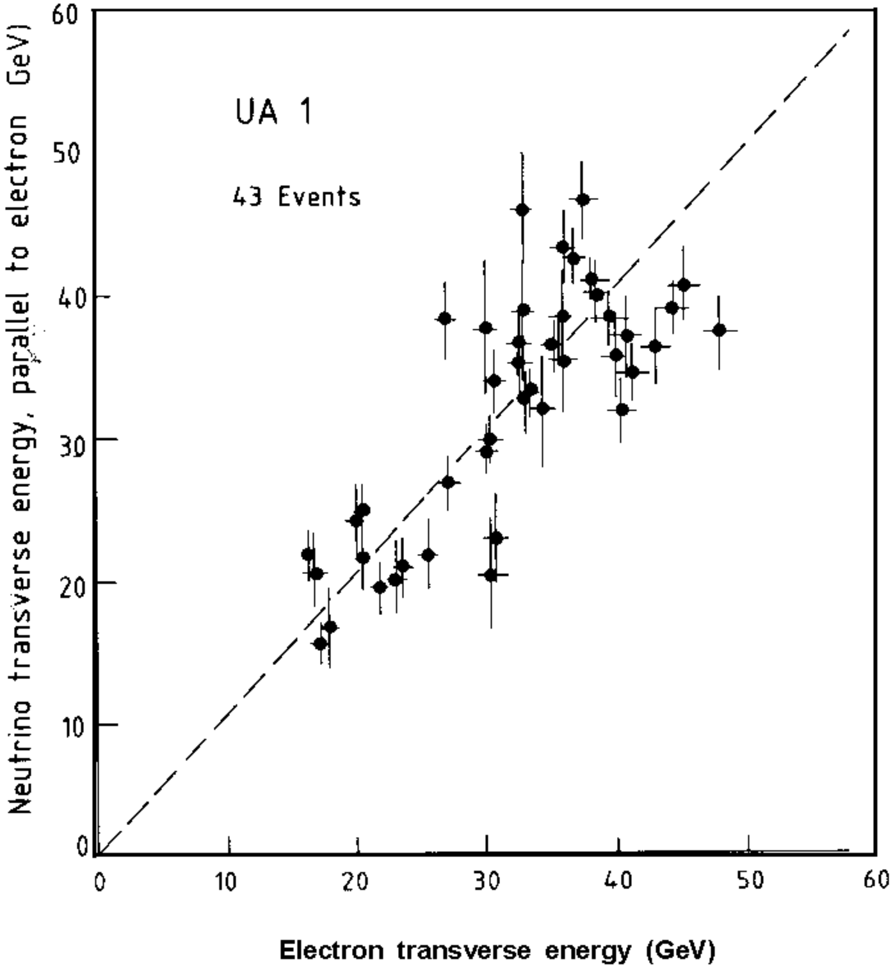


Figure 152: Correlation between the electron and neutrino transverse energies in W measurements by UA1.

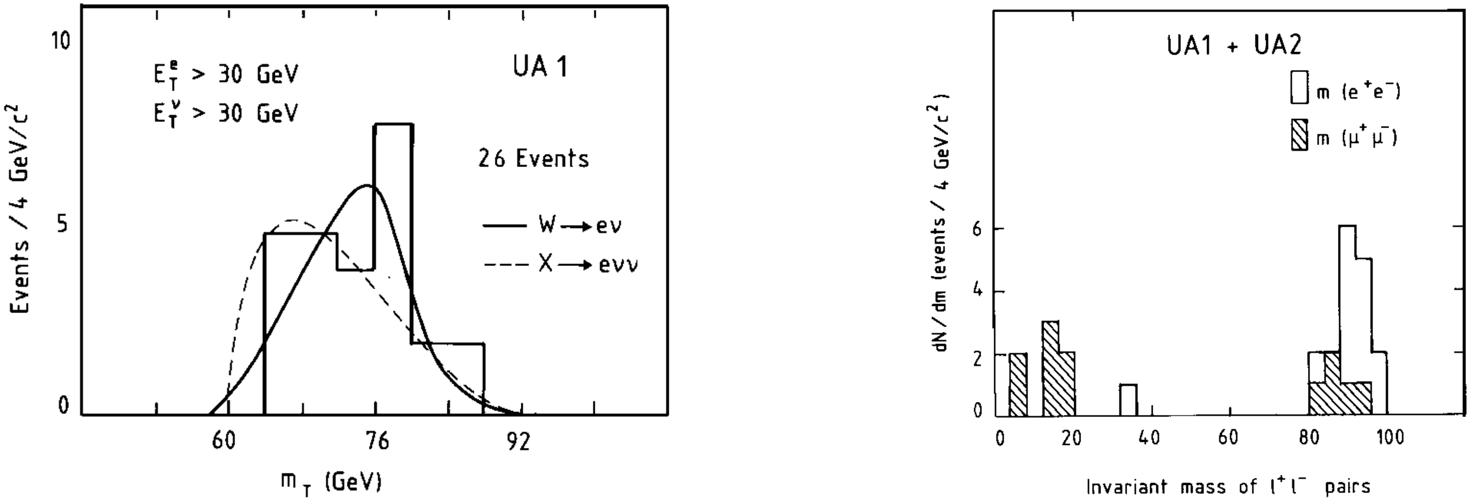
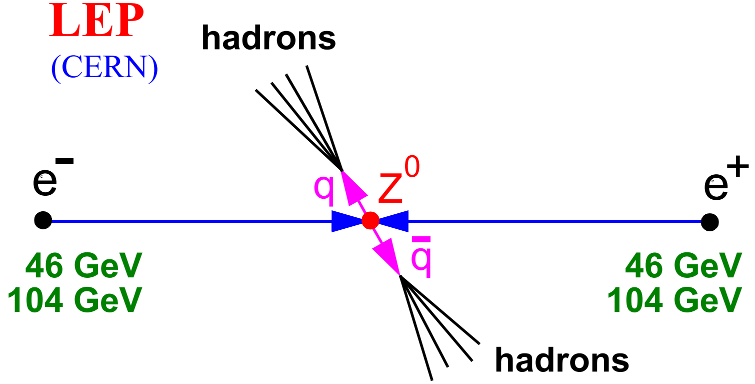


Figure 153: W and Z masses as measured by UA1 (W) and UA1 and UA2 (Z) experiments

Precision studies of W and Z bosons

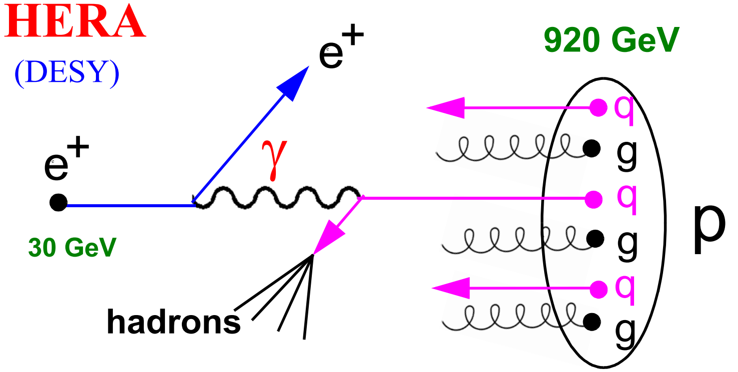
All modern colliders produce copious amounts of weak bosons

LEP
(CERN)



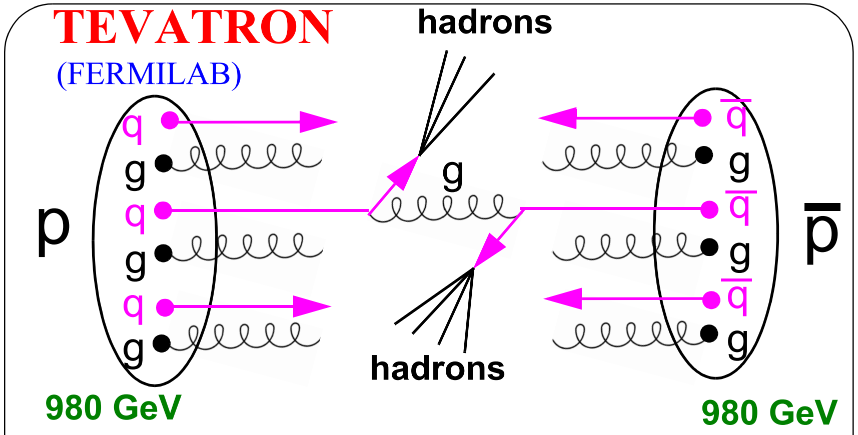
Length: 27 km (4184 magnets)
Experiments: DELPHI, OPAL, ALEPH, L3

HERA
(DESY)



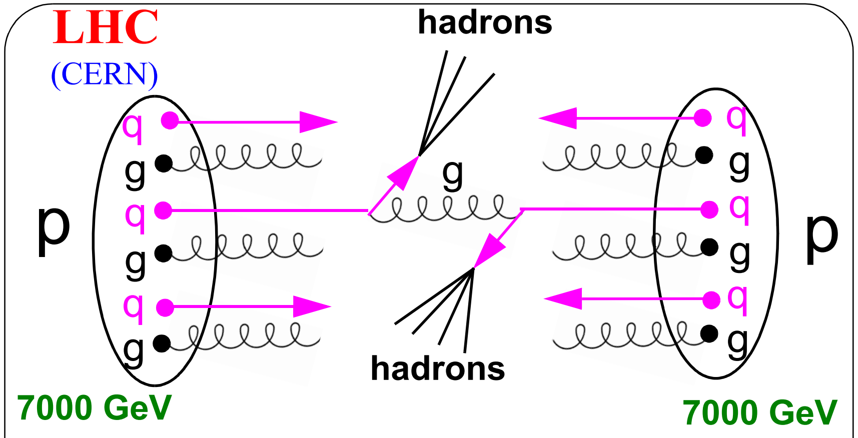
Length: 6 km (1650 magnets)
Experiments: H1, ZEUS

TEVATRON
(FERMILAB)



Length: 6 km (990 magnets)
Experiments: CDF, D0

LHC
(CERN)



Length: 27 km (1232 + 386 magnets)
Experiments: ATLAS, CMS, LHCb, ALICE

- Between 1989 and 1995 LEP operated at 45.6 GeV/beam (for Z studies)

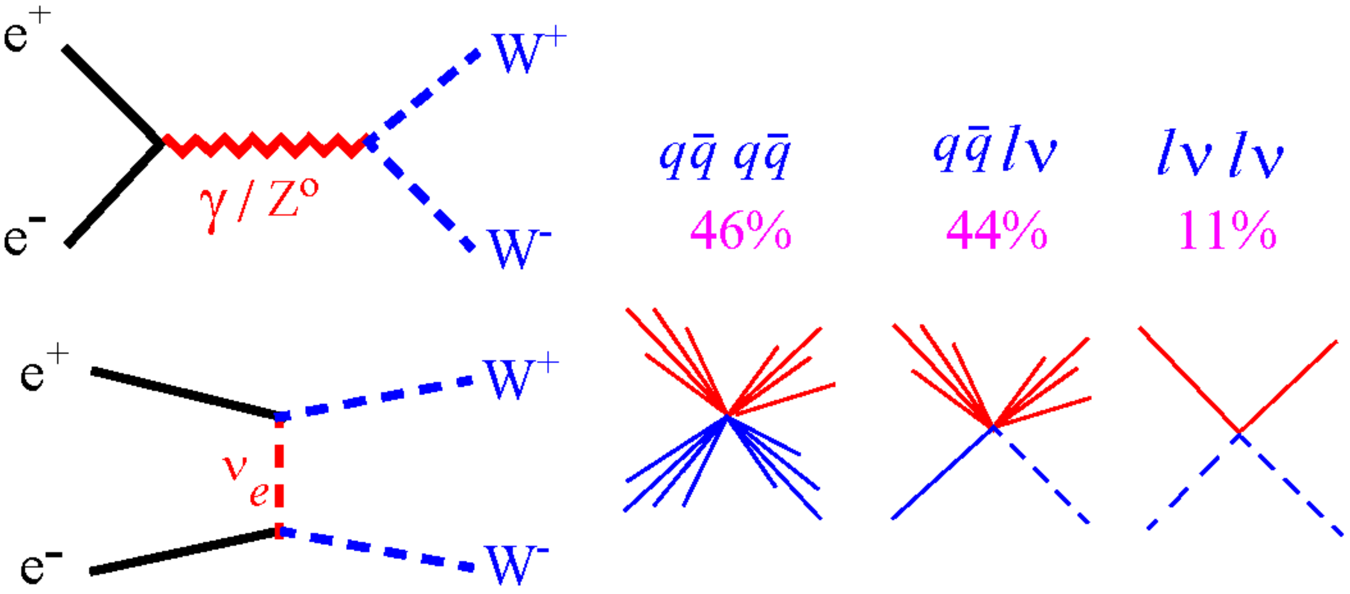


Figure 154: W pair production modes in electron-positron annihilation

- From 1996 to 2000, energy increased gradually to 104 GeV/beam, allowing precision studies of W bosons produced in pairs, and even Higgs near-discovery

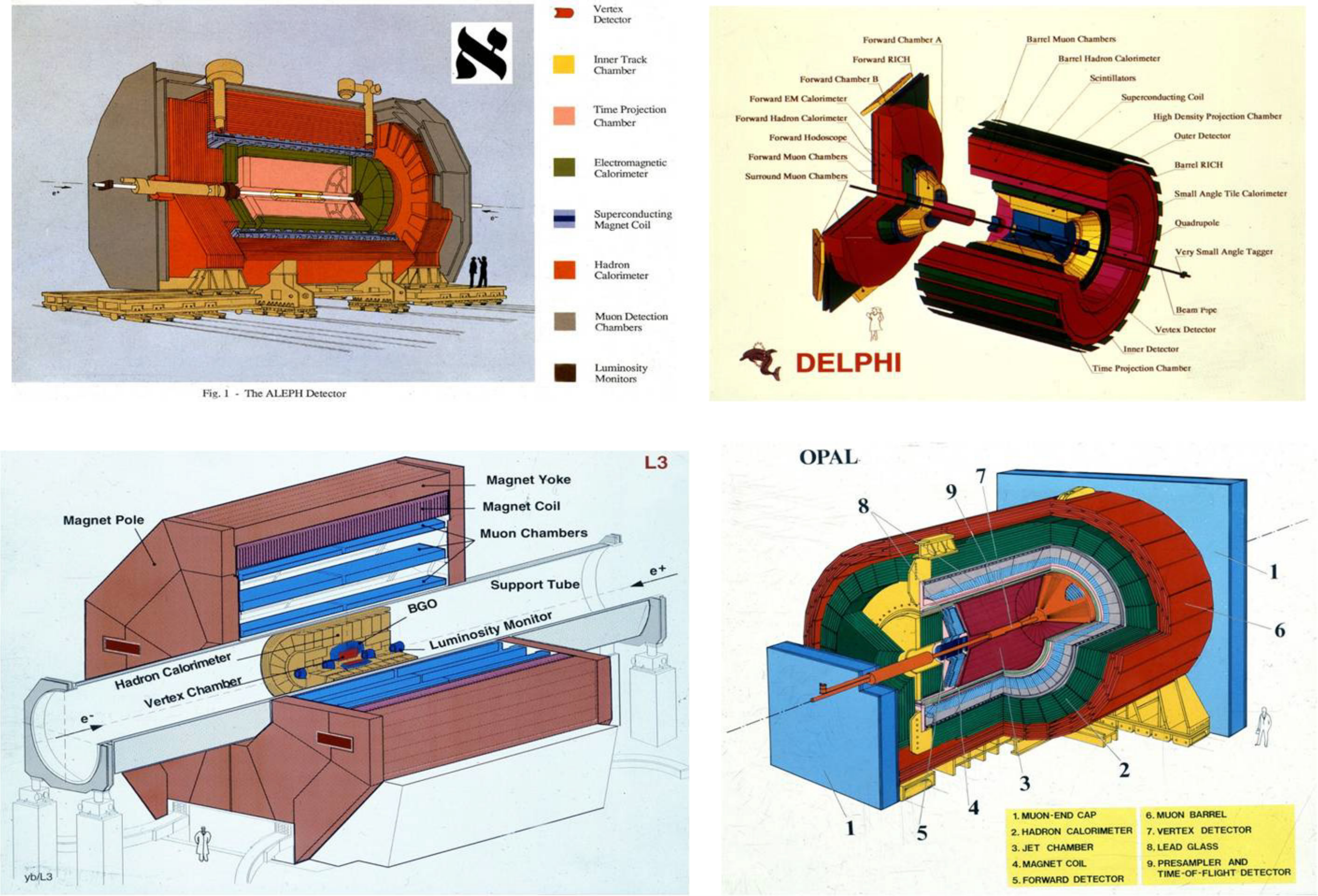


Figure 155: The four LEP experiments (data taking 1989-2000):ALEPH, DELPHI, L3 and OPAL

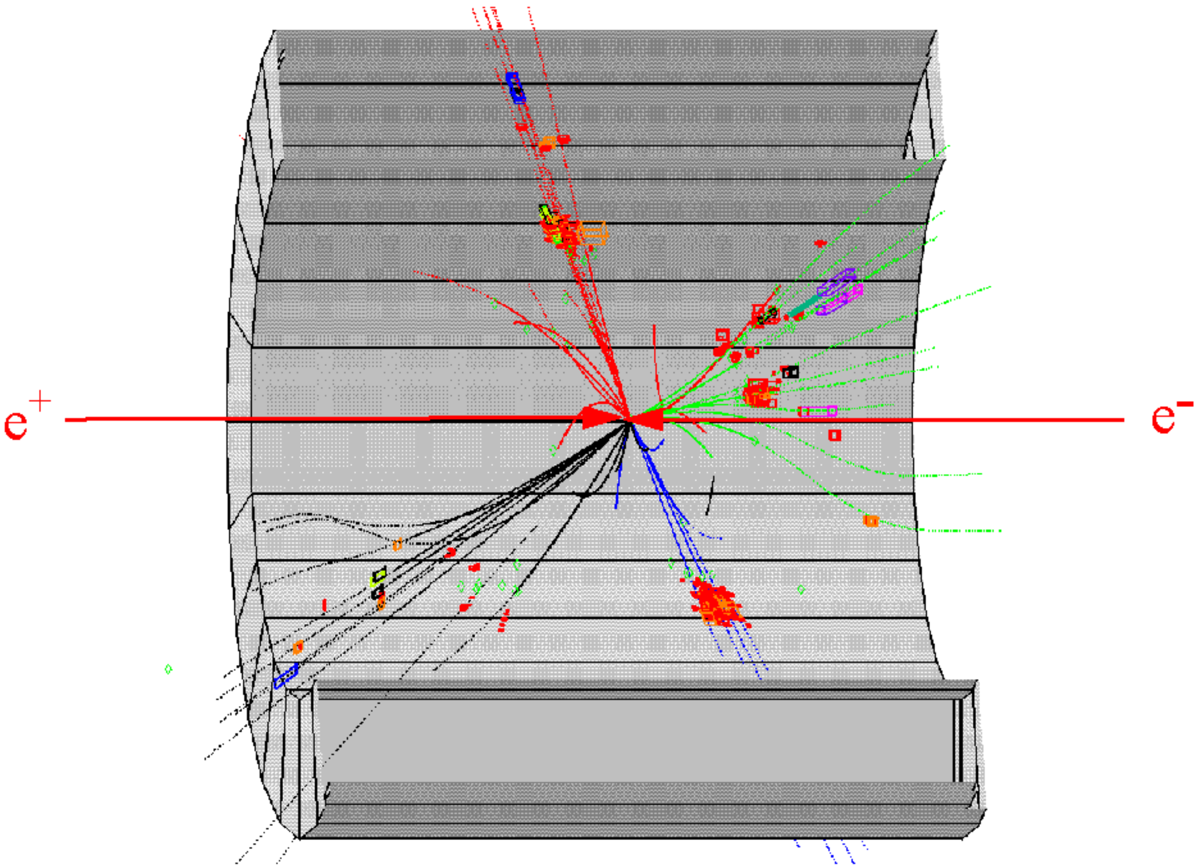


Figure 156: A 4-jet WW event as registered by the DELPHI detector at LEP

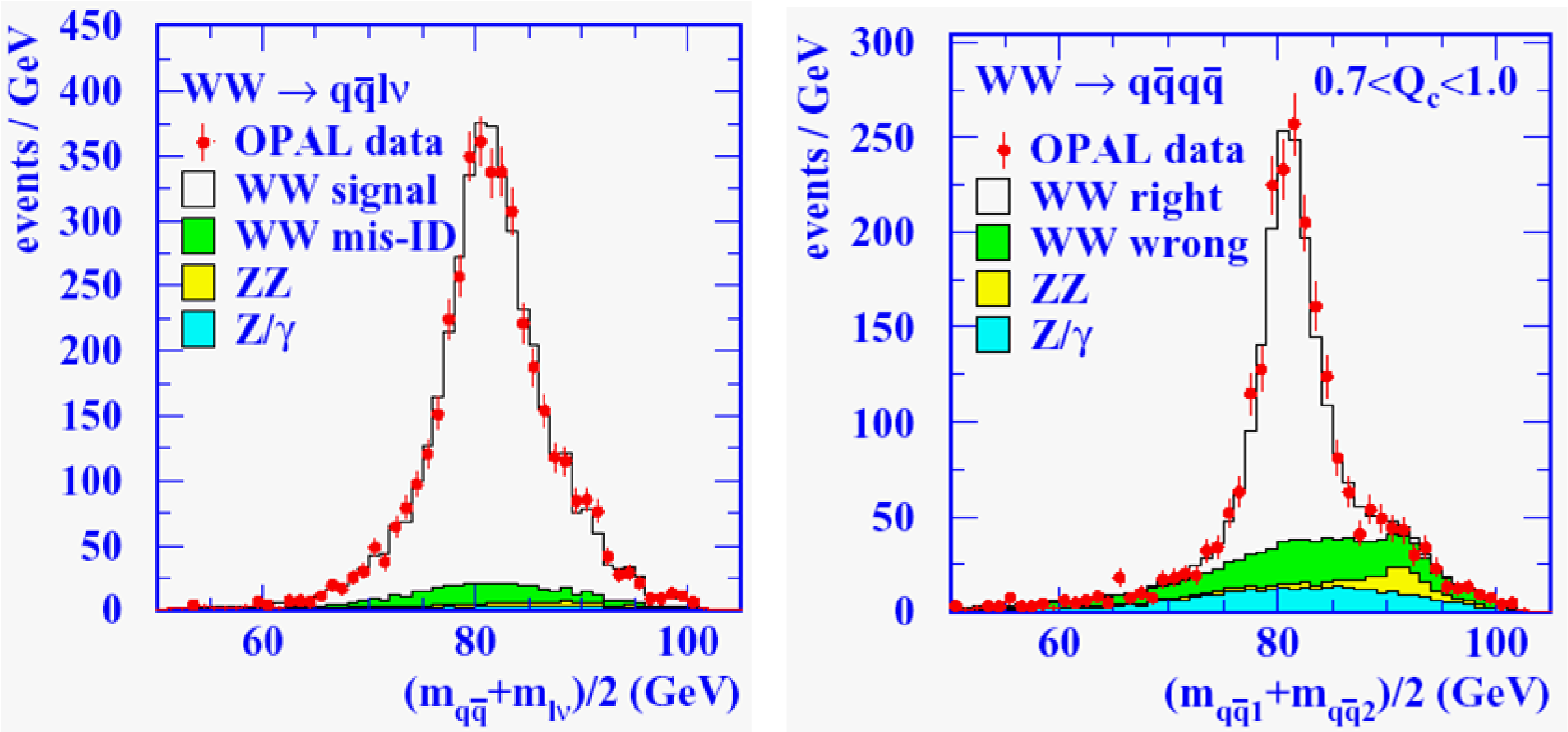
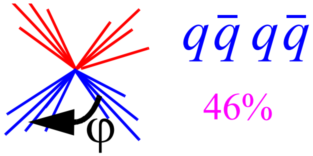


Figure 157: W mass reconstruction by OPAL experiment at LEP. The qqlν channel is the *golden* channel: best measurement



$$M_W^2 = (\bar{P}_q + \bar{P}_{\bar{q}})^2 \qquad \text{(4-vectors)}$$

$$M_W^2 = 2E_qE_{\bar{q}}(1-\cos\varphi) \quad (\text{if } m_q = 0)$$

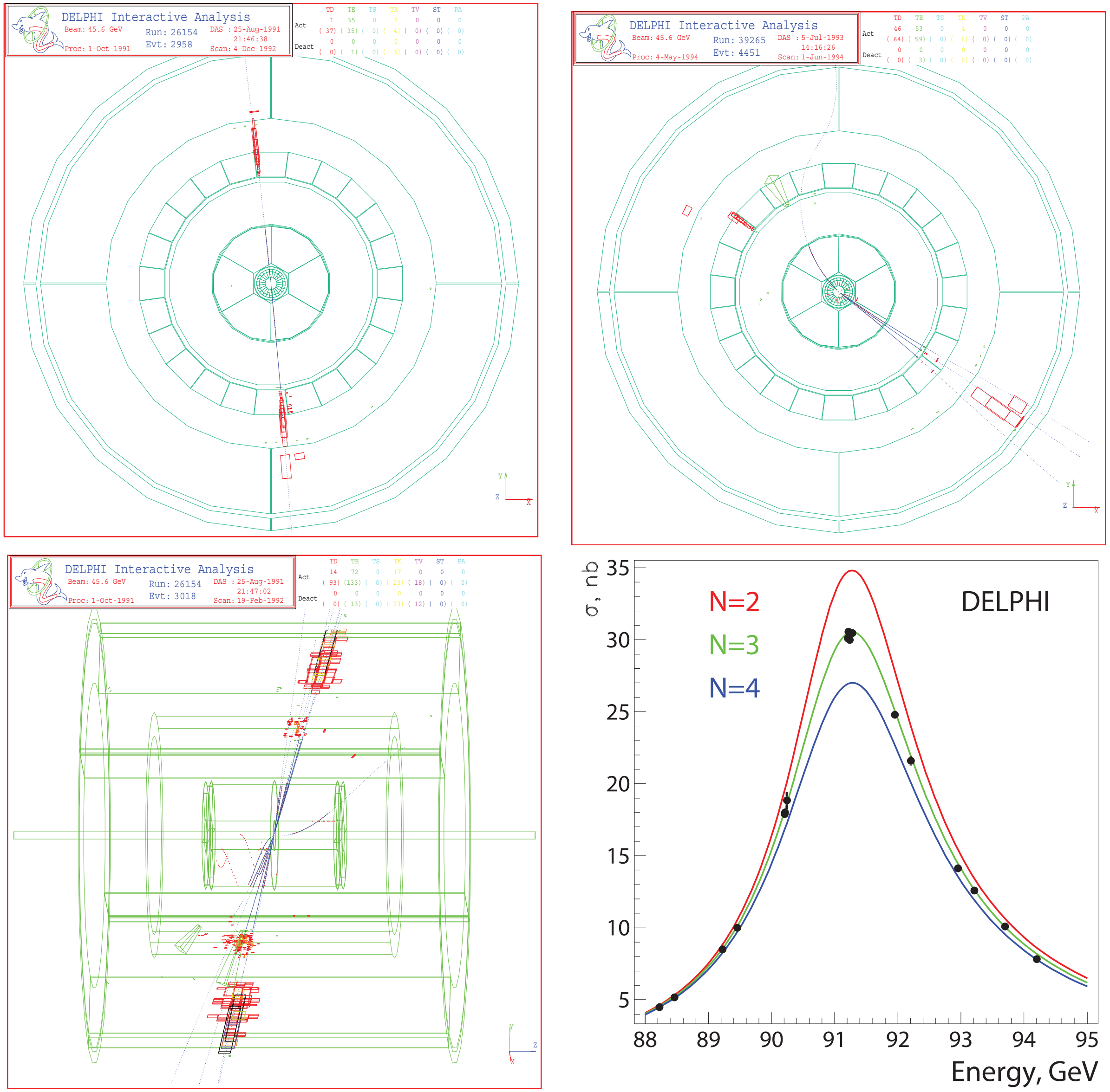


Figure 158: Decays of the Z into e^+e^-, $\tau^+\tau^-$ and $q\bar{q}$ and precision scan for the Z mass, by DELPHI experiment at LEP