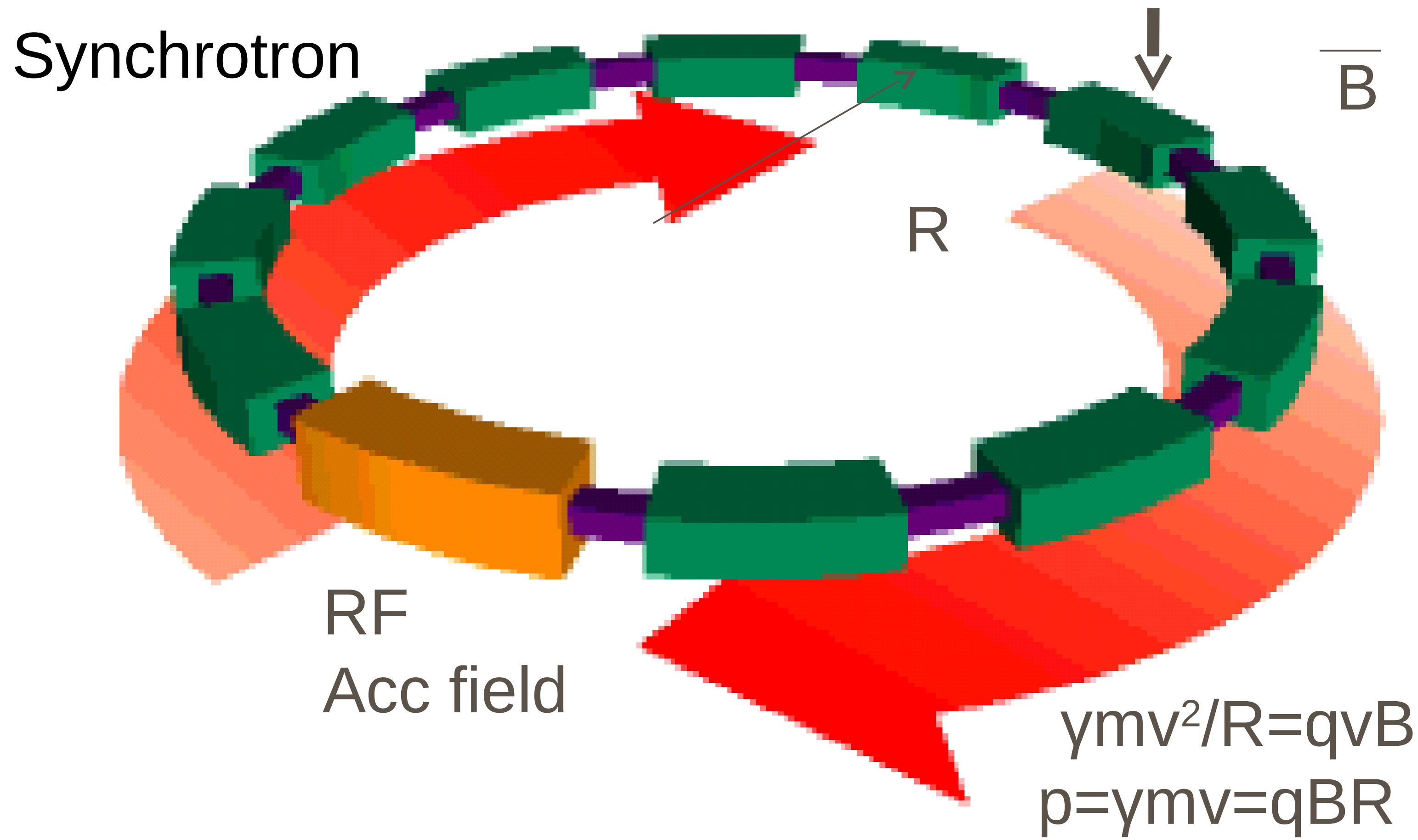
Lectures on accelerator physics

- Lecture 5 and 6: Advanced topics
  - Transverse motion, strong focusing, and LHC
- Material borrowed from
  - Lecture by Anders Oskarsson
  - Lecture by Eric Torrence (University of Oregon)
  - **LHC lectures by Danillo Vranic (GSI)**
- Weak focusing follows “Principles of Charged Particle Acceleration” by Stanley Humpries Jr. Chapter 7.

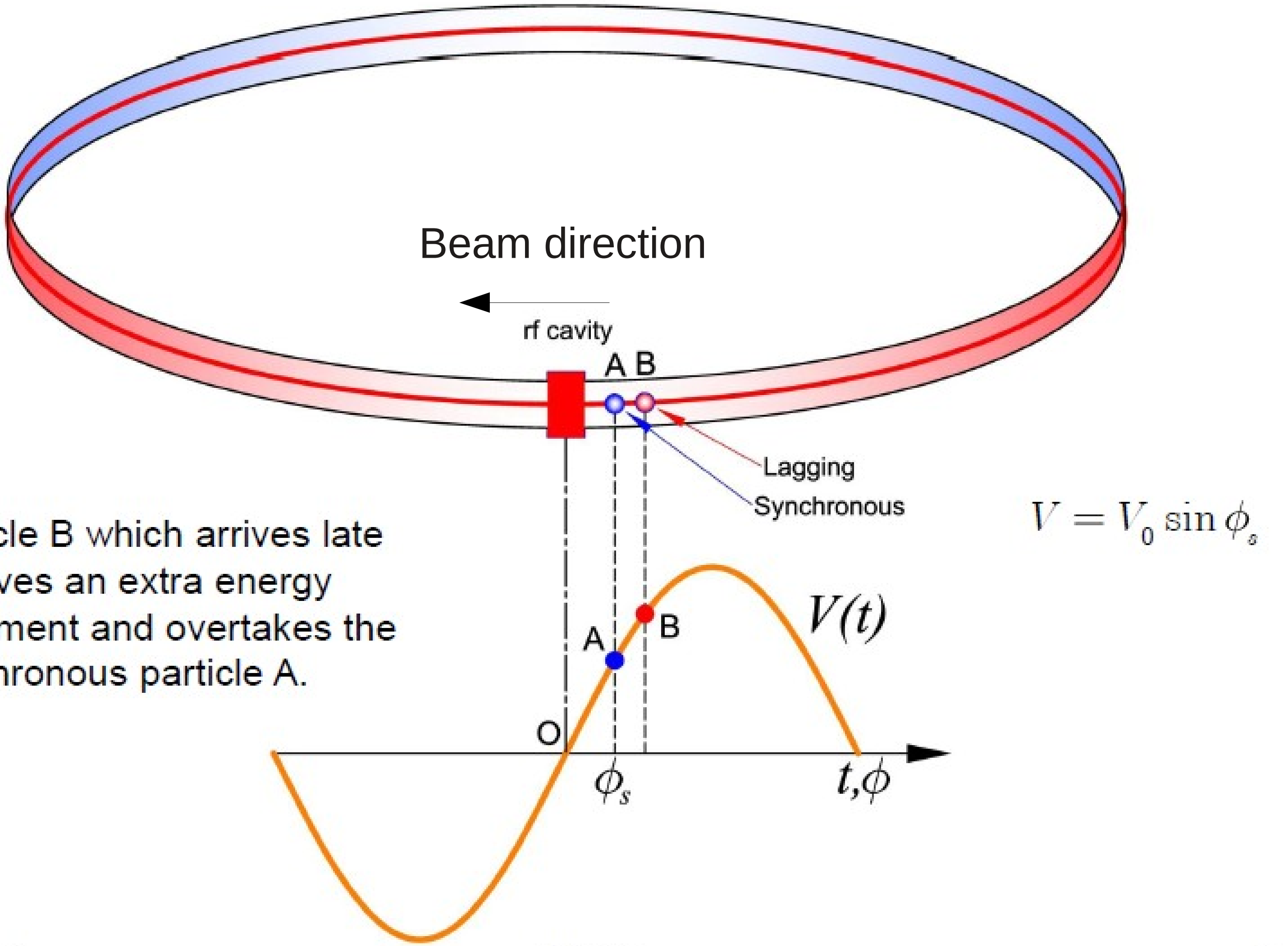
Towards the synchrotron



 Dipole magnet give circular motion

 Acceleration in E-field

LONGITUDINAL DYNAMICS



Particle B which arrives late receives an extra energy increment and overtakes the synchronous particle A.

8/25/2010 D. Vranic 2

HARMONIC NUMBER

A particle circulates around the machine with period: $\tau = \dfrac{L}{\beta c}$

Then the number of turns (circulation frequency) is: $f_r = \dfrac{1}{\tau}$

L is the circumference and βc is the velocity.

For LHC $L = 26658.8832m$ and $\tau = 88.92\mu s \rightarrow f_r \approx 11245.5Hz$

The **synchronous particle** is defined as that particle which always arrives at the desired synchronous phase lag ϕ_s behind the zero-crossing of the rf wave. For this to occur, the rf frequency f_a must be an integer multiple of f_r

$$f_a = h \cdot f_r$$

where integer h is known as the **harmonic number.**

$$h = \frac{\text{RF frequency}}{\text{Circulation frequency}}$$

For LHC h is chosen to be **35640**. Then we have $f_a \approx 400.8MHz$

BUNCHES AND BUCKETS

Harmonic number determines number of places on the circumference where a particle could be located and arrive synchronously in the accelerating cavity. The segments of the circumference centred on these points are called **buckets**. The groups of particles in these buckets are called **bunches**.
Not all buckets need to be filled with bunches. In LHC only 2808 out of 35640 buckets will be filled with minimal distance of 10 buckets.

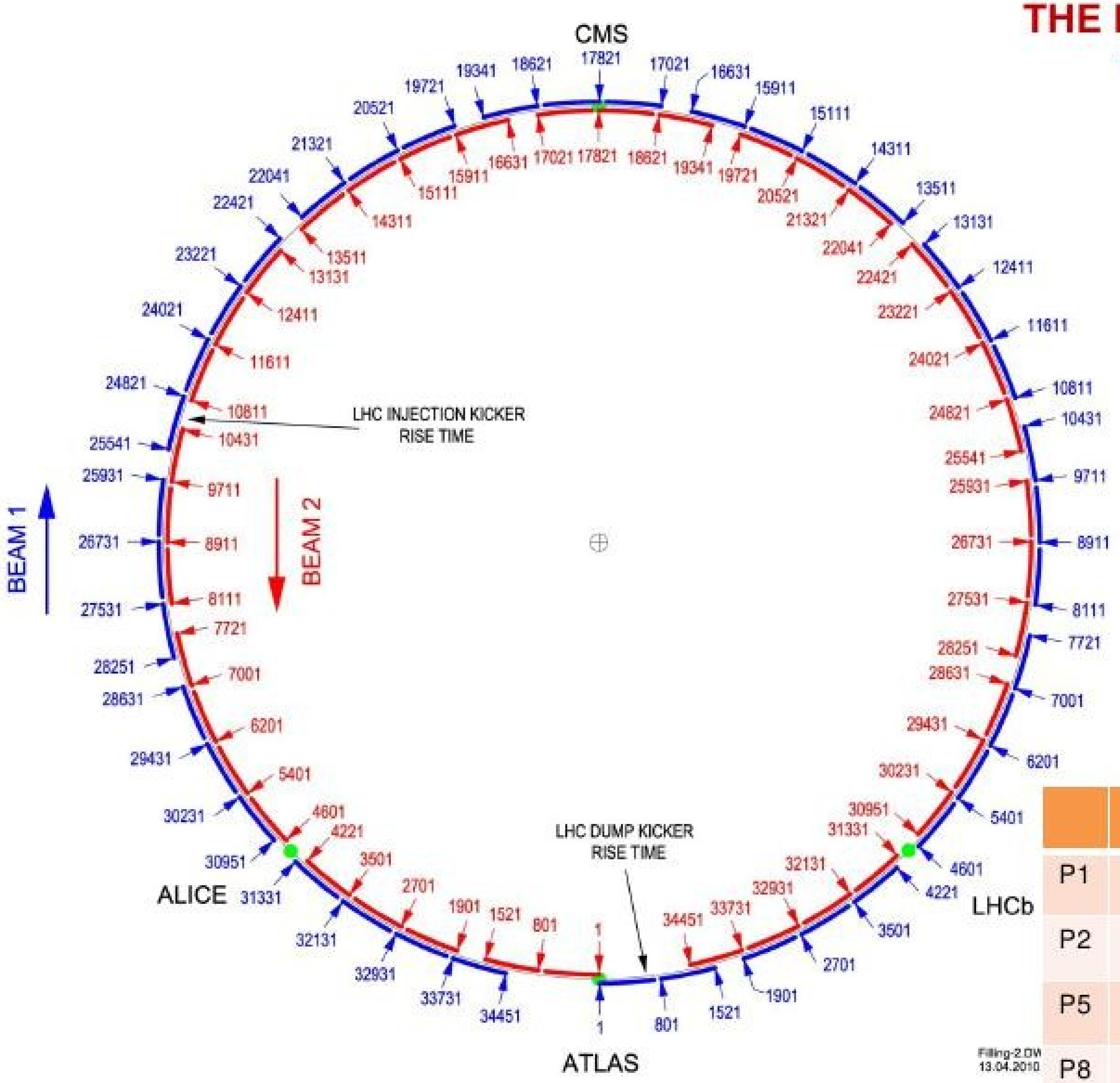
Number of buckets = h	35640
Bucket spacing (time)	2.5 ns
Bucket spacing (space)	74.8cm
Max umber of bunches	2808
Min bunch spacing (time)	25 ns
Min bunch spacing (space)	7.48 m
RMS bunch length	7.5 cm

Bucket spacing (space) is constant:

LHC circumference / Number of buckets =26658.8832/35640=0.748m

8/25/2010 D. Vranic 4

THE INITIAL POSITIONS OF THE BUNCHES



EACH BEAM 39x72 =2808 BUNCHES

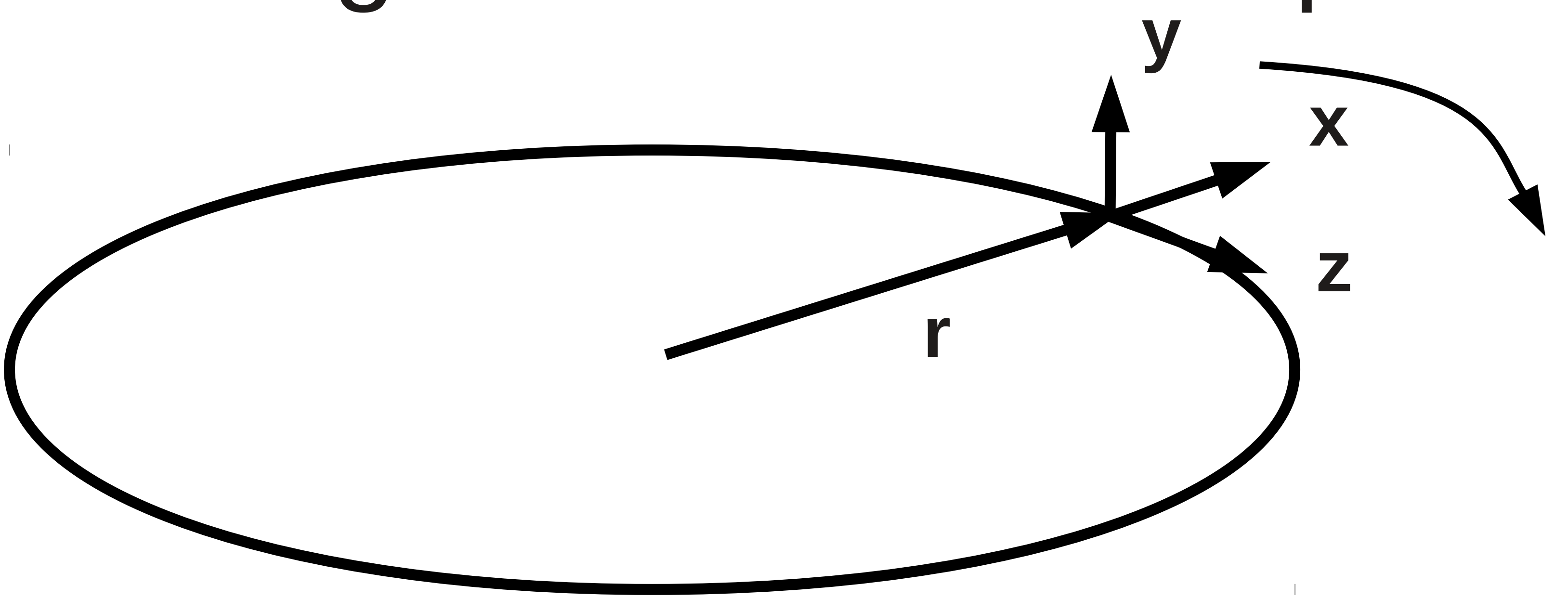
PS=1/11 SPS

SPS=7/27 LHC

IP POSITIONS

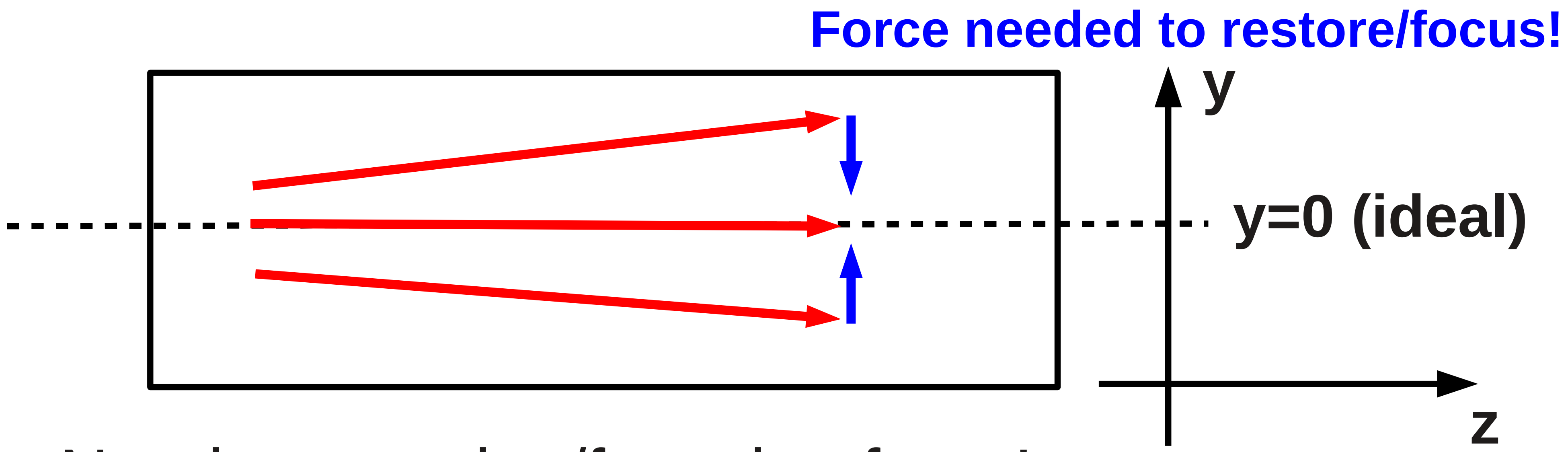
	BEAM 1	BEAM 2
P1	1	1
P2	31186(EMPTY)	4456(EMPTY)
P5	17821	17821
P8	4471(EMPTY)	31171(EMPTY)

Focusing in the transverse plane



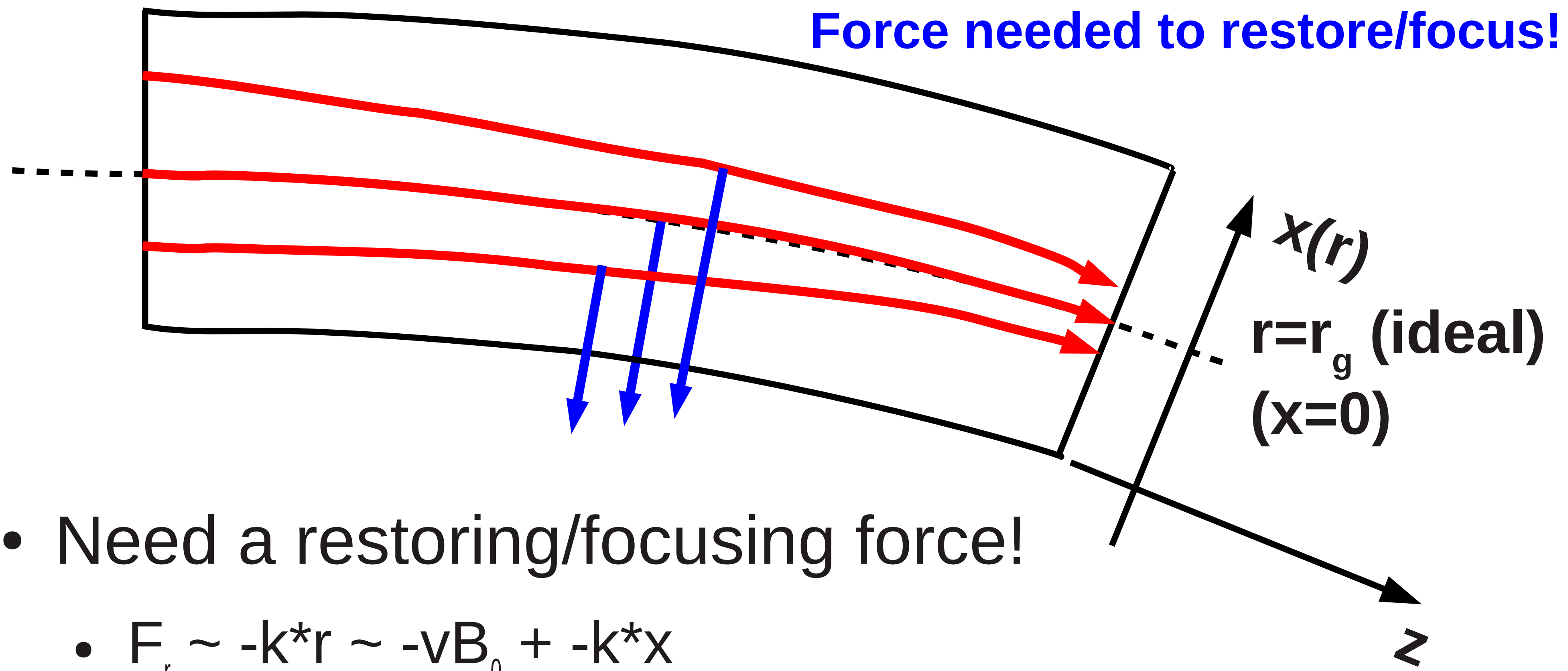
- Assume (v_x,v_y,v_z)~(0,0,v) and v~c = constant!
  - Very good assumption!
- z=vt → t=z/v (~z/c)
  - d/dt ~ v d/dz (~c d/dz)

Transverse focusing in y direction



- Need a restoring/focusing force!
  - $F_y \sim -k*y$
- Harmonic oscillator (like string)

Transverse focusing in x direction



- Need a restoring/focusing force!
  - $F_r \sim -k^*r \sim -vB_0 + -k^*x$
  - = central force (r_g) + harmonic oscillator in x

Let us first solve harmonic equation (ignoring magnet realities!)

$$\gamma m \frac{d^2 y}{dt^2} = \gamma m v^2 \frac{d^2 y}{dz^2} = -ky$$

$$y(z) = y_0 \cos\left(\frac{2\pi}{\lambda} z + \varphi\right),$$

where

$$\lambda = 2\pi \sqrt{\frac{\gamma m v^2}{k}}.$$

- Note that the wavelength does not depend on the amplitude y_0. There is only one wavelength for all amplitudes!

Tune interlude

- One defines
  - Q (ν[nu]) = C/λ, where $C=2\pi r_g$ is the circumference of the synchrotron ring
- Q is the number of transverse (betatron) oscillations per turn
- It is different for x and y directions
- Very important for beam stability!

TUNE

The tune is the **number of betatron oscillations per turn.**

It is very important that tune is not integer or a simple fraction

$$Q \neq \frac{p}{n}$$ (where n and p are integers)

otherwise, over one or more revolutions, particle will repeat its path in the accelerator and 'see' the same field imperfections. These will then build up a resonant growth and beam will be lost.

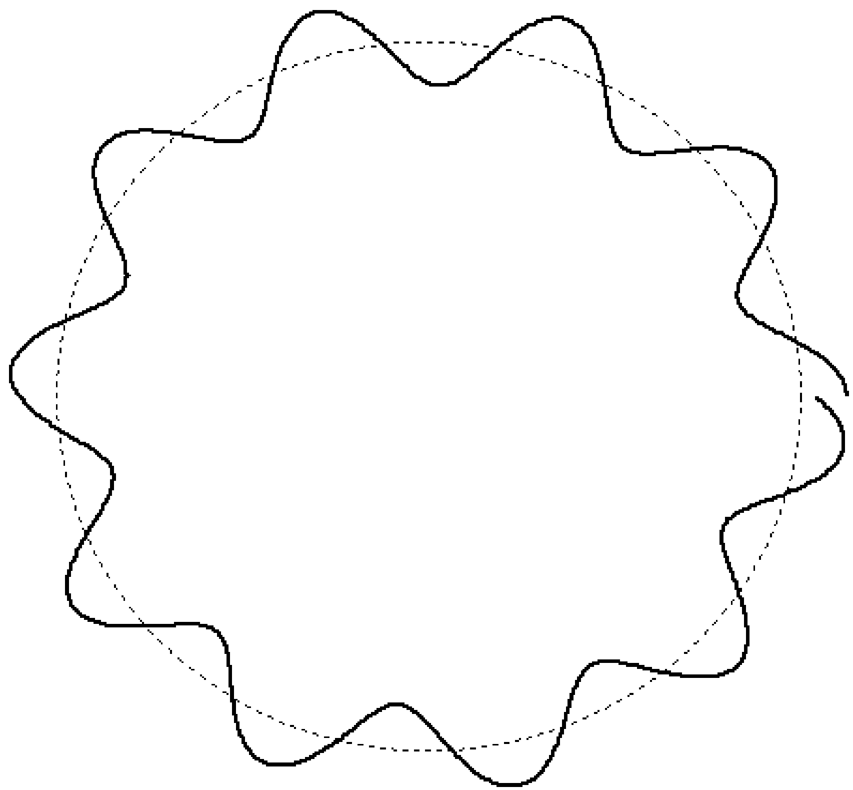
NO HARMONY!

REMARK:
As opposite to old 'theories' about solar system, there is no 'harmony' and that is the reason why it lasts so long. Planet between Mars and Jupiter was 'in the harmony'.

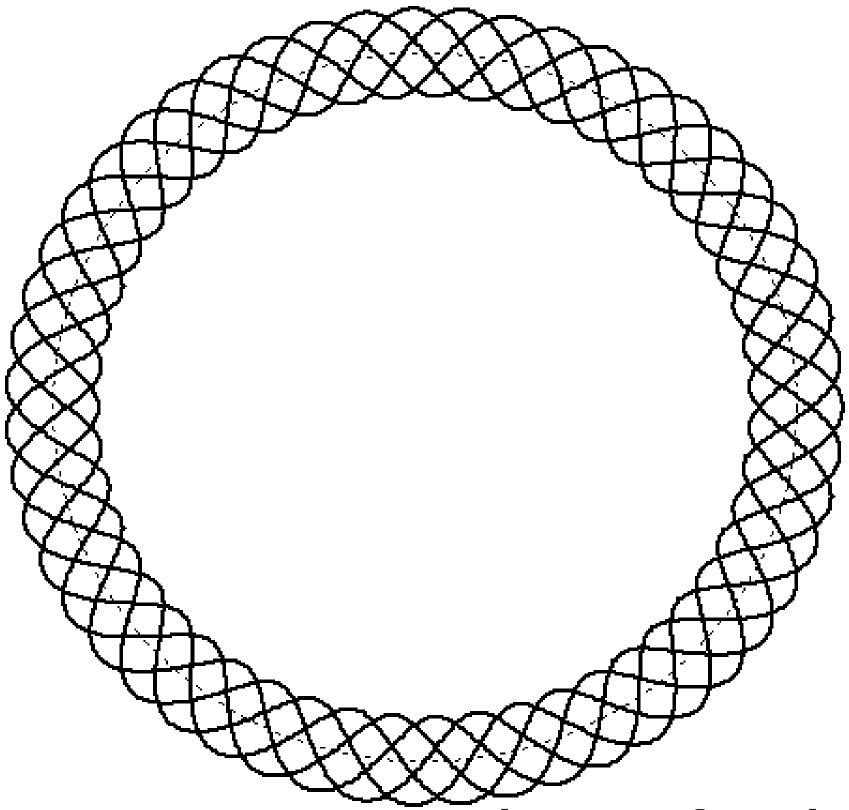
8/25/2010 D. Vranic 9

Bad harmonic tune (Q=10.2)

1 turn



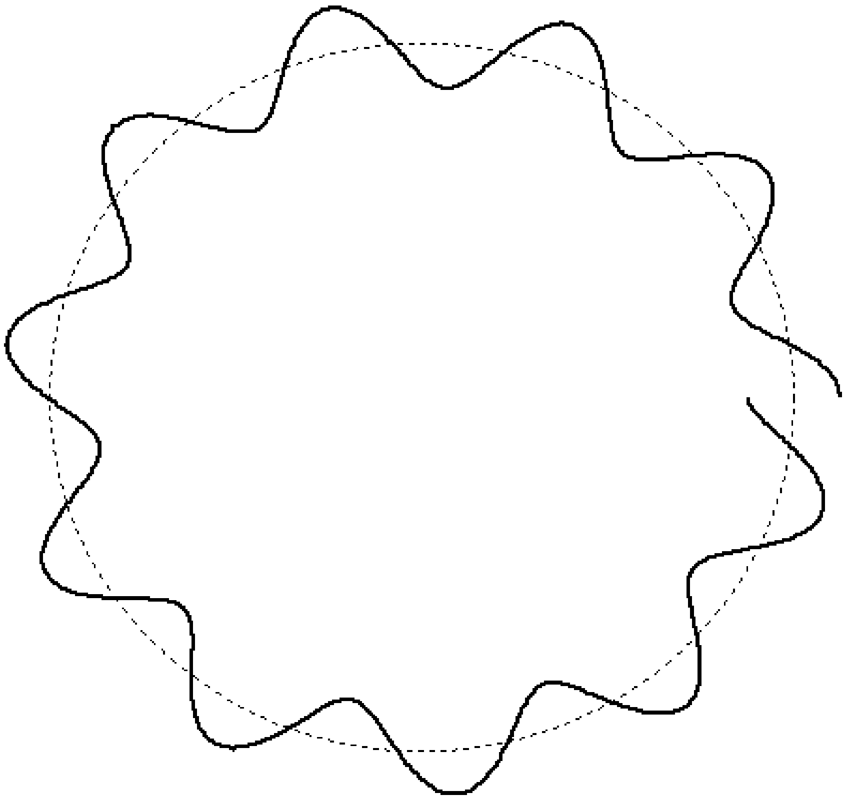
100 turns



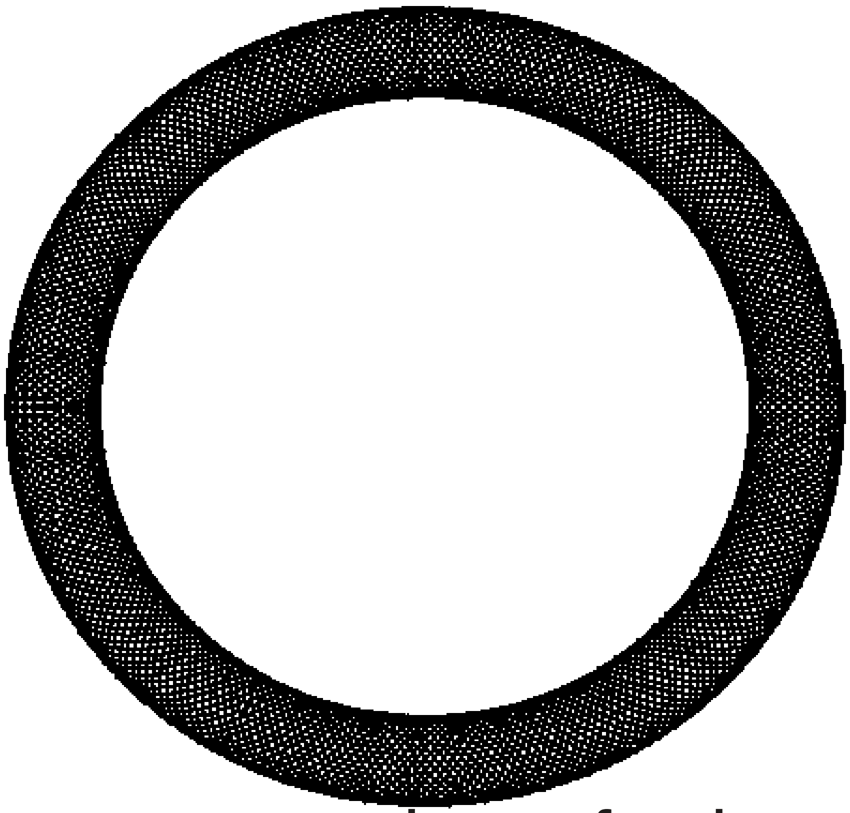
Problem: tune does not integrate out magnet imperfections

Better (less harmonic) tune (Q=10.48)

1 turn

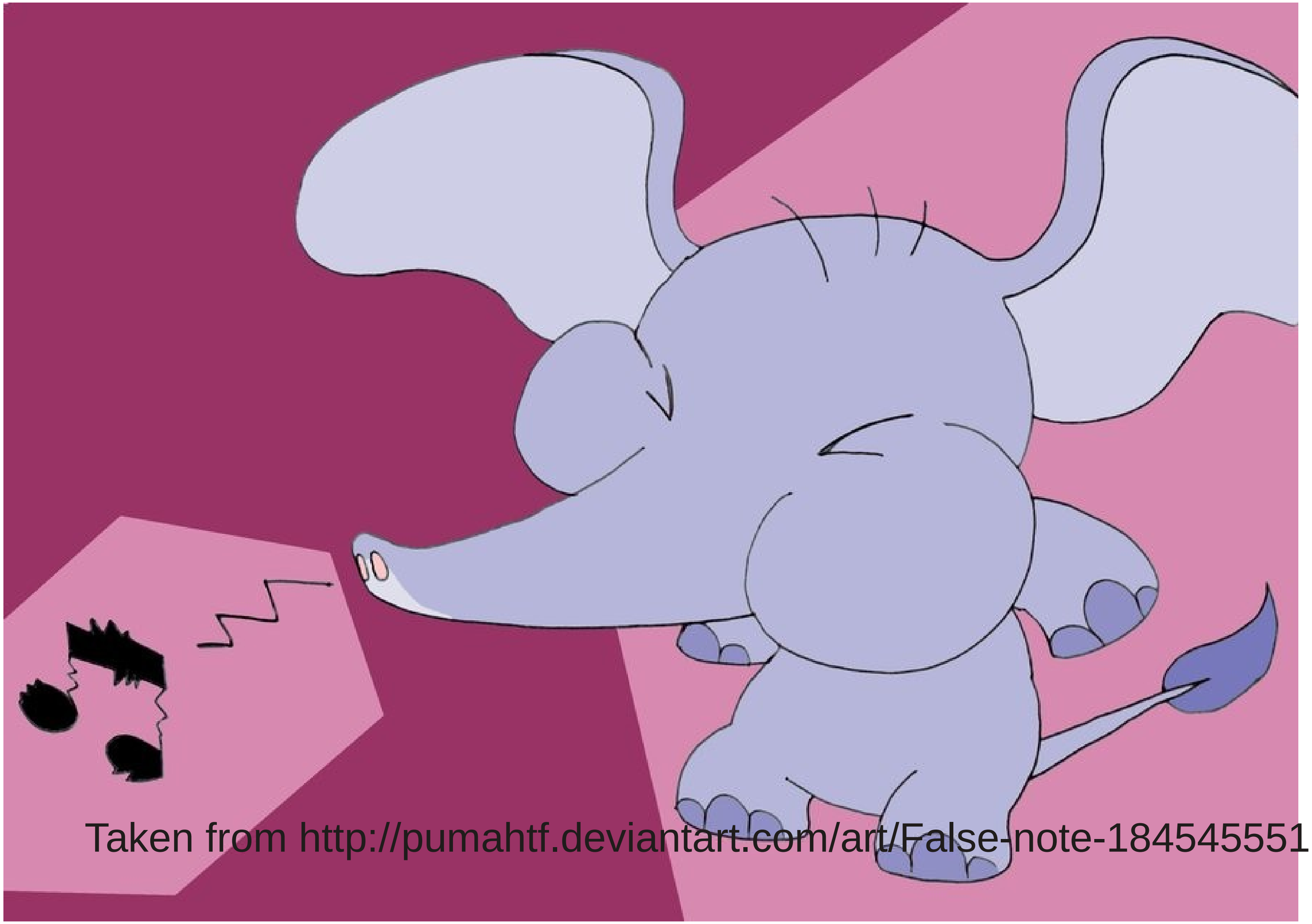


100 turns



Tune is better at integrating out magnet imperfections

Lesson: bad musicians makes great accelerator physicists



Taken from http://pumahtf.deviantart.com/art/False-note-184545551

LHC TUNES

AT 7TeV

HORIZONTAL TUNE: $Q_x = 64.31$

VERTICAL TUNE: $Q_y = 59.32$

$$\Delta Q \leq 3 \cdot 10^{-3}$$

AT 450GeV

HORIZONTAL TUNE: $Q_x = 64.28$

VERTICAL TUNE: $Q_y = 59.31$

Betatron tunes should avoid linear coupling resonances at

$$nQ_x + mQ_y = p$$

8/25/2010 D. Vranic 10

Back to transverse motion and magnet realities!

- Taylor expanding the dipole magnetic field AND fulfilling Maxwell equations gives
  - $(B_x, B_y, B_z) \sim (-(n_0 B_0/r_g)y, B_0 - (n_0 B_0/r_g)*x, 0)$
    - NB! note that – sign is not good!
- Ideally we want n_0 as large as possible to confine the beam!
  - (And make the magnet as small as possible)
- Let us look at solution for x (r)!

The equation of motion for x

$$\gamma m \frac{d^2 r}{dt^2} = \gamma m v^2 \frac{d^2 r}{dz^2} = \gamma m \frac{v^2}{r} - qvB_y$$

$$\frac{d^2 r}{dz^2} = \frac{1}{r} - \frac{q}{\gamma m v} B_y$$

Substituting $x = r - r_g$ + expanding $\frac{1}{r}$:

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{1}{r_g^2} x - \frac{q}{\gamma m v} B_y$$

Inserting the Taylor expansion of B_y:

$$\frac{d^2 x}{dz^2} = \frac{1}{r_g} - \frac{qB_0}{\gamma m v} - \frac{1}{r_g^2} x + \frac{q n_0 B_0}{\gamma m v r_g} x$$

The first two terms gives the solution for the ideal trajectory $\rightarrow$: $\frac{1}{r_g} = \frac{qB_0}{\gamma m v}$ so that:

$$\frac{d^2 x}{dz^2} = -\frac{1}{r_g^2}(1 - n_0)x.$$

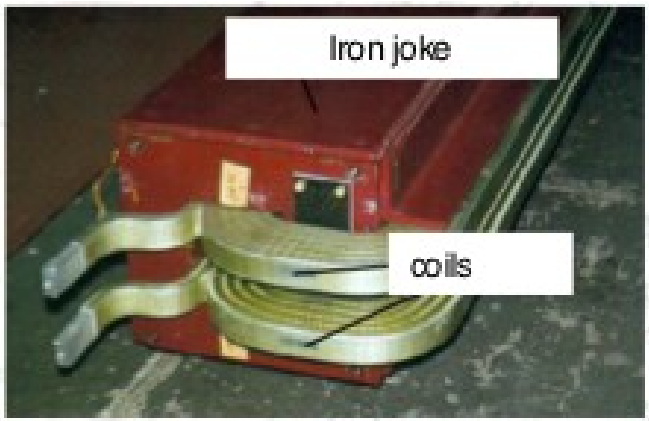
Weak focusing: $0 < n_0 < 1$

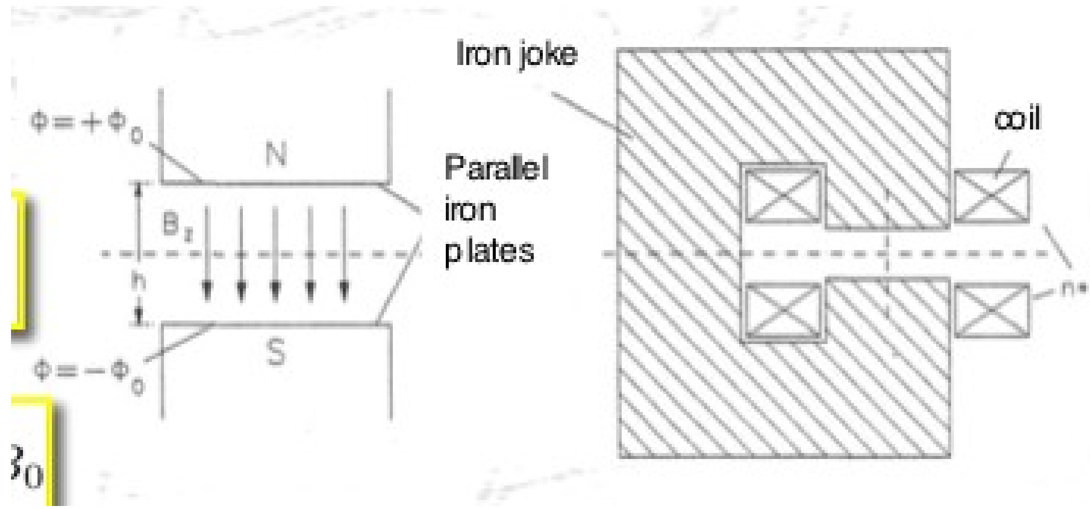
$$\frac{d^2x}{dz^2} = -\frac{1}{r_g^2}(1 - n_0)x.$$

- Only harmonic oscillation solution when $(1-n_0)>0$ (and y equation requires $n_0>0$)
  - Otherwise exponential growth!
- This means that the focusing is limited!
  - That is why this solution is called weak focusing

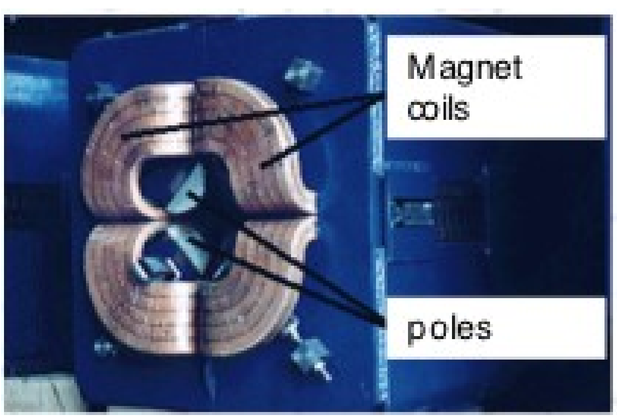
Can we find better focusing? Beamline Elements

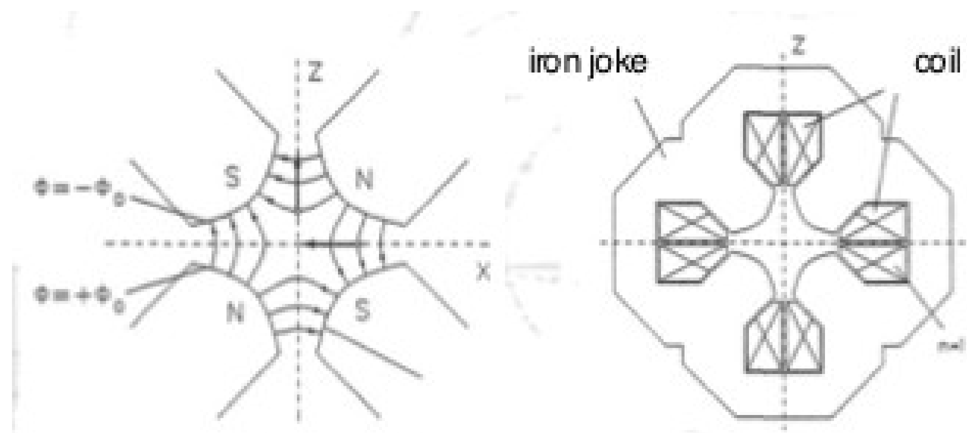
Dipole (bend) magnets





Quadrupole (focusing) magnets



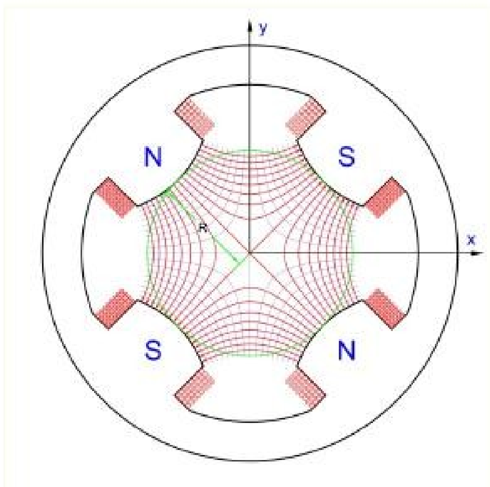


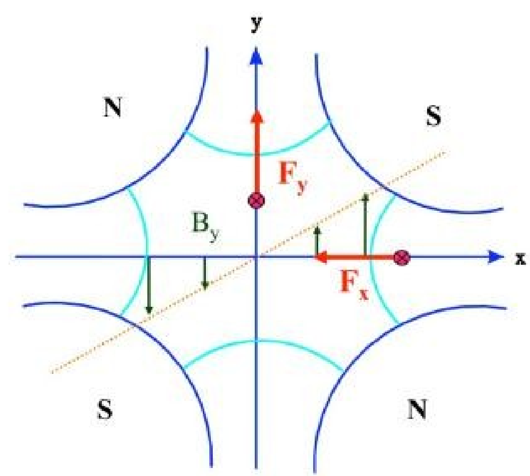
Also Sextupoles and beyond

Quadropoles has similar problem!

FOCUSING OF THE PROTON BEAM

Quadrupole looks good – field increases linarly with distance from the center.





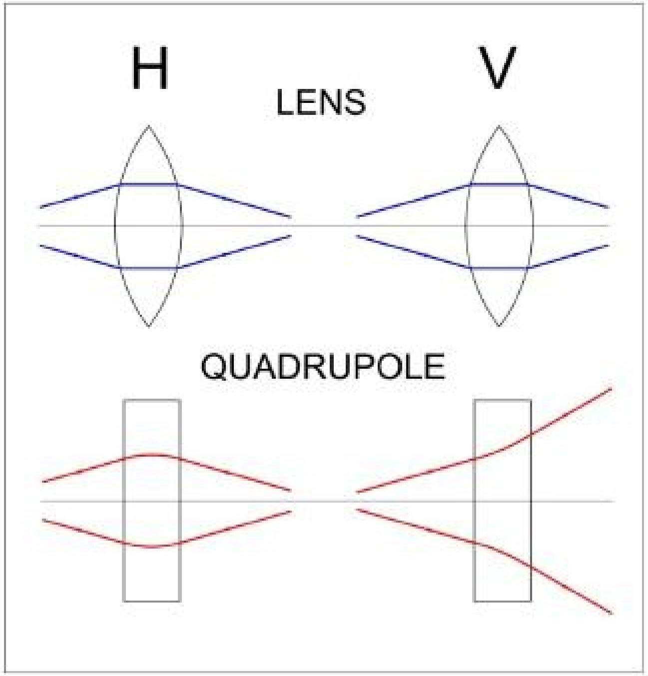
F_y has wrong direction! It doesn't work!

No solution: Maxwell tells us $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ $\oint_{\partial S} \mathbf{B} \cdot d\boldsymbol{l} = \mu_0 I$

8/25/2010 D. Vranic 6

PROBLEM

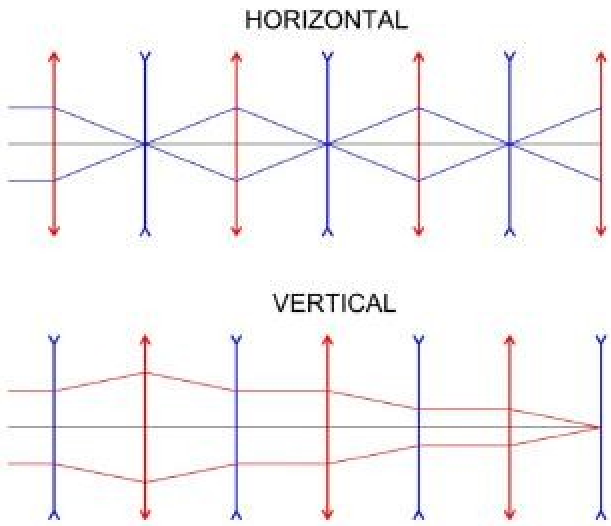
Quarupole is convergent lens in horizontal, but divergent in vertical direction!



There was no solution until 1952, and it is beautiful and simple:

8/25/2010 D. Vranic 7

SOLUTION: AG OR STRONG FOCUSING

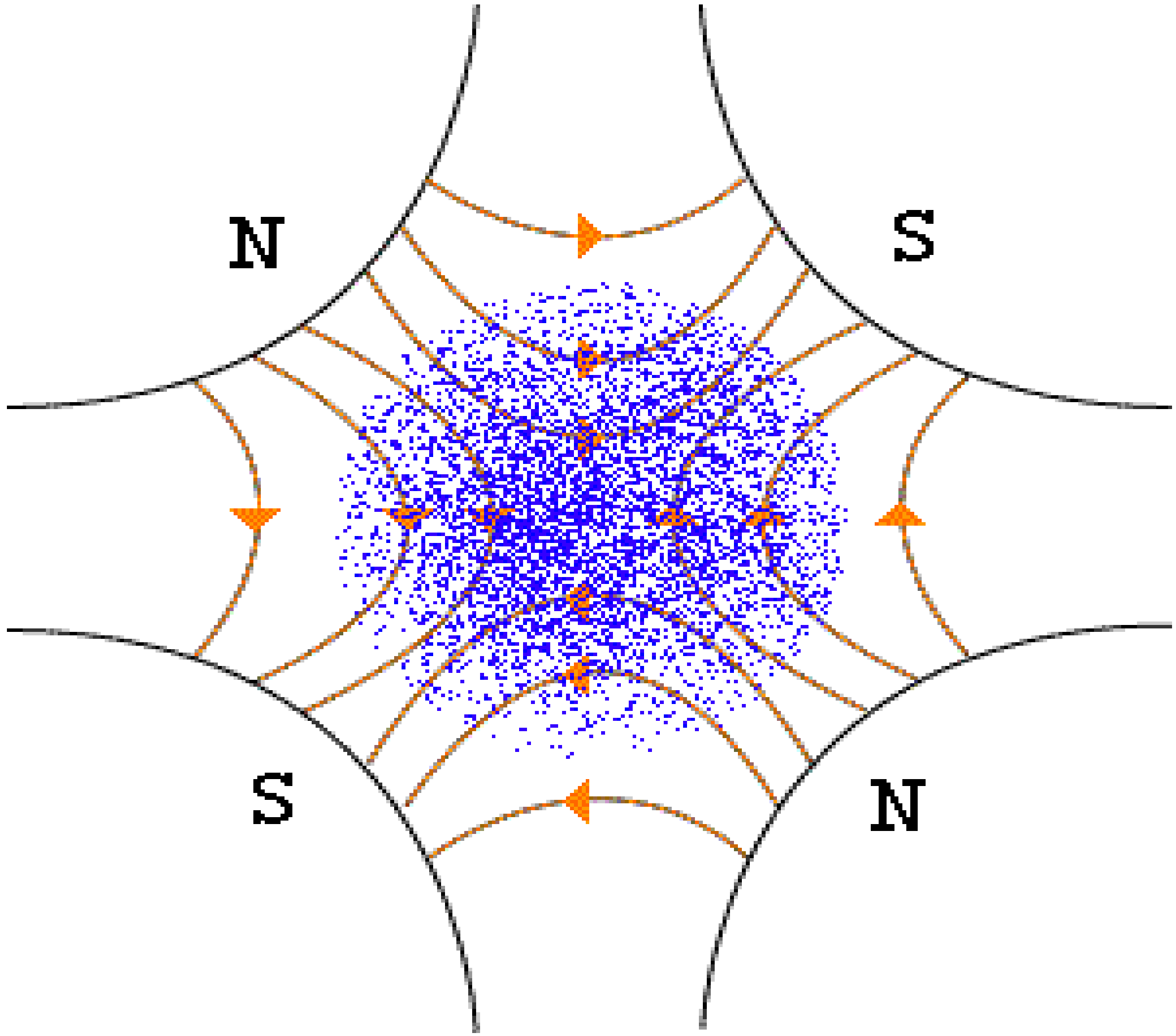


FODO LATTICE

F - focusing
D - defocusing
O - drift space or dipoles

If we have alternating convergent and divergent lenses with right spacing overall effect is focusing!

8/25/2010 D. Vranic 8

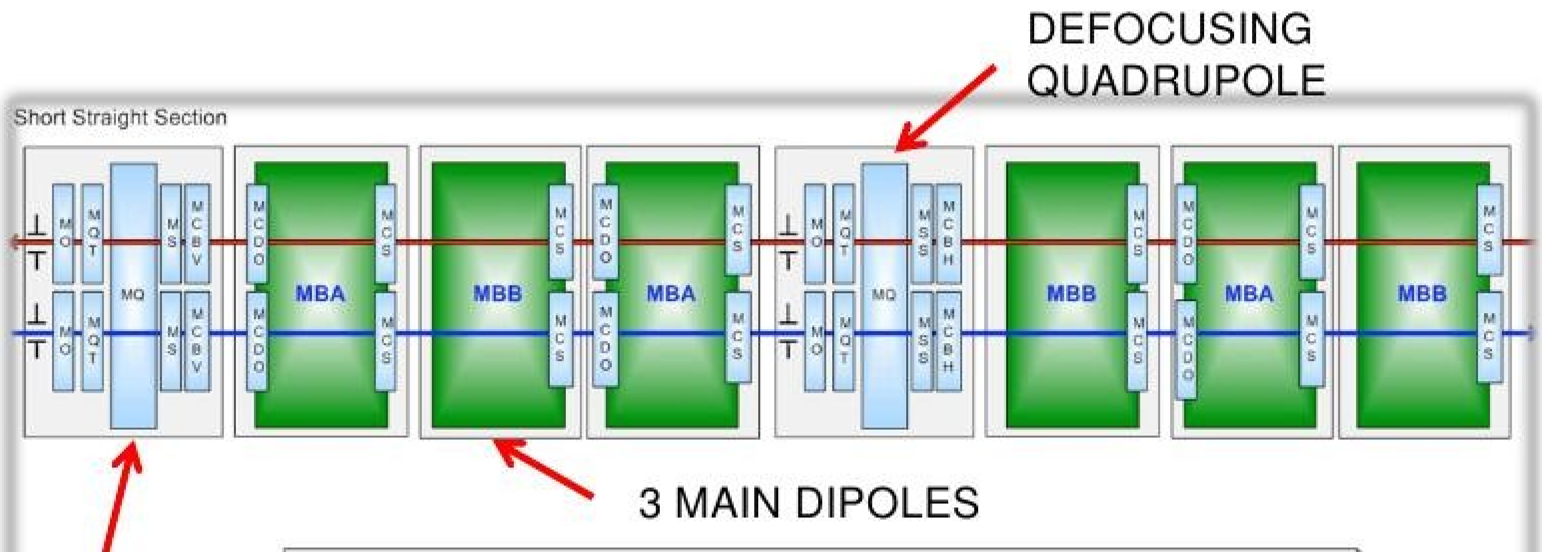


Animation taken from The Physics of Accelerators

Slides by C.R. Prior Rutherford Appleton Laboratory and Trinity College, Oxford

LHC FODO LATTICE CELL (106.9 m)

The pattern of bending and focusing magnets is called lattice.



FOCUSING
QUADRUPOLE

MCS: Sextupole corrector (b3)

MCDO: Assembly of spool correctors consists of an octupole insert **MCO** (b4) and a decapole magnet **MCD** (b5)

MQT: Trim quarupole corrector

MS: arc sextupole corrector

MQS: skew quad lattice corrector

MCBH: Horizontal dipole corrector

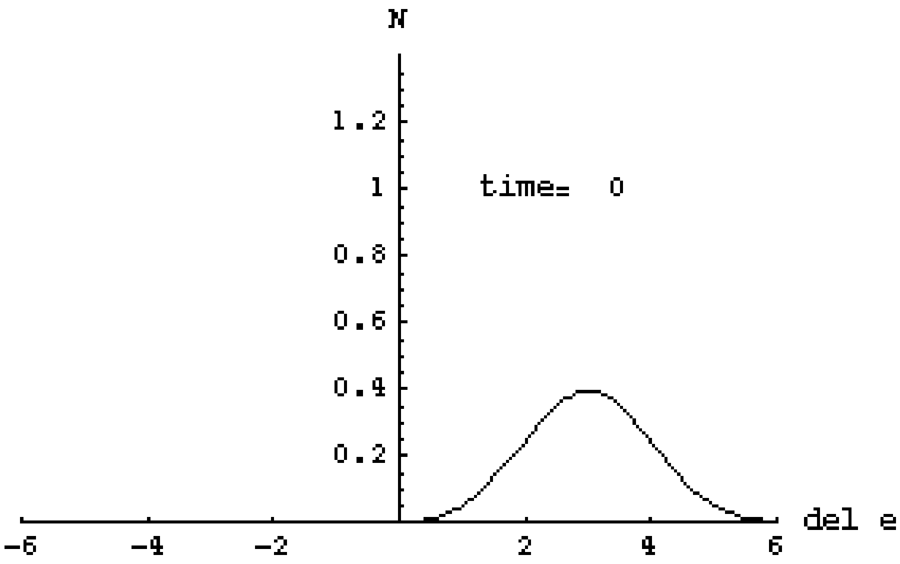
MCBV: Vertical dipole corrector

MO: Lattice octupole

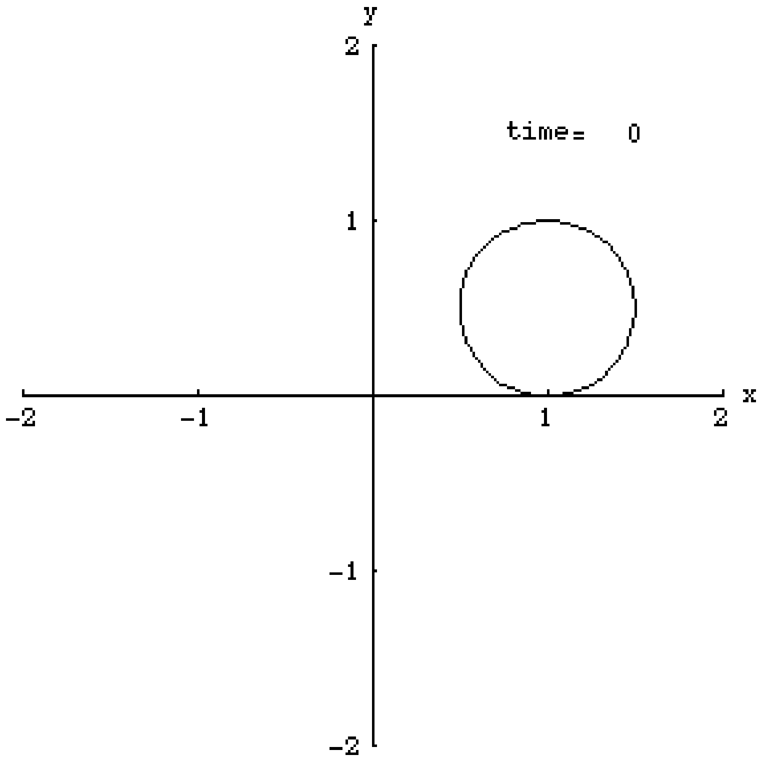
8/25/2010 D. Vranic 4

Energy adjusting by AC (longitudinal) & transverse strong focusing

Longitudinal

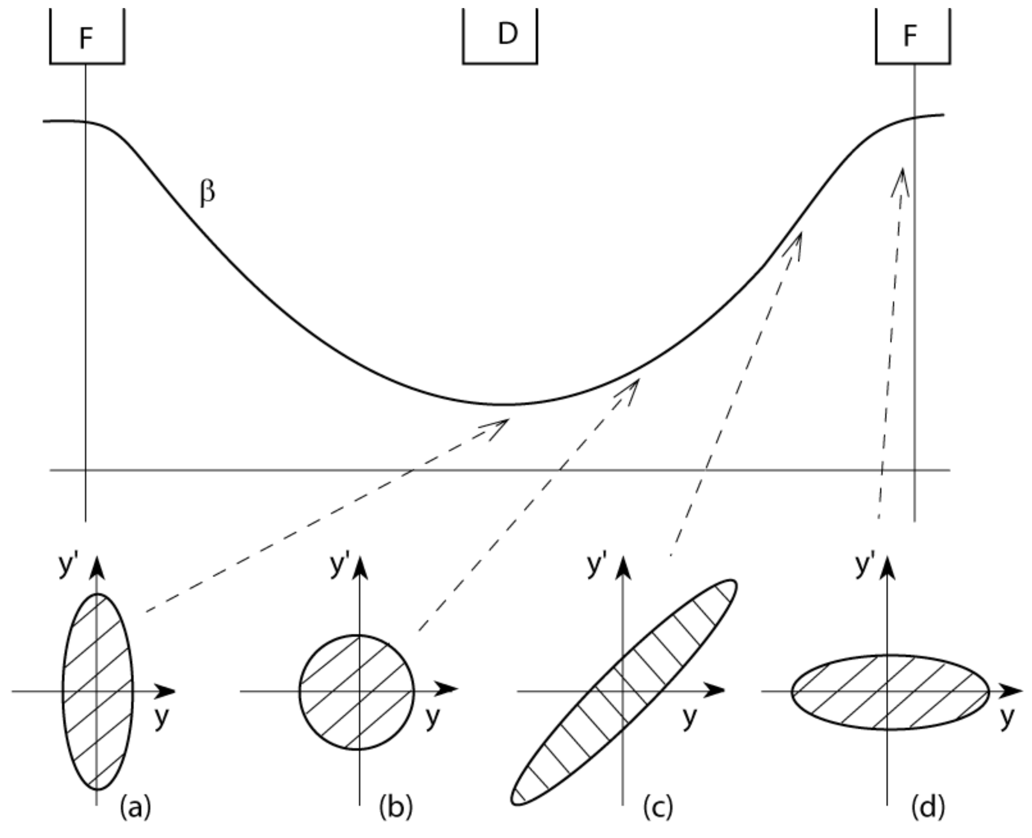


Transverse



- "Catching the beam" animations taken from
  - http://www.lns.cornell.edu/~dugan/USPAS/

Phase space limit (1/2)



- Liouville's theorem states that for most beams the phase space area cannot change

Phase space limit (2/2)

- In reality the limits are on p_y*y:
  - Emittance ε ~ p_y*y is constant
    ~ γmβ_y*y where β_y= dy/dt
    ~ γmβ*y'*y where y' = dy/dz
  - So the phasespace limit implies that the area of the phase space ellipse for y'*y (the emittance) decreases as 1/p
  - This is called adiabatic damping. The physical size of the beam decreases as it is accelerated. The width decreases as 1/sqrt(p) [the other 1/sqrt(p) is the decreasing divergence].

Recall lecture 1 and 2

Collision rate is defined to be the number of 'events' per second, i.e. the number of collisions happening in the center of one of the experiments (depends on the cross section)

The collision rate can be increased if:

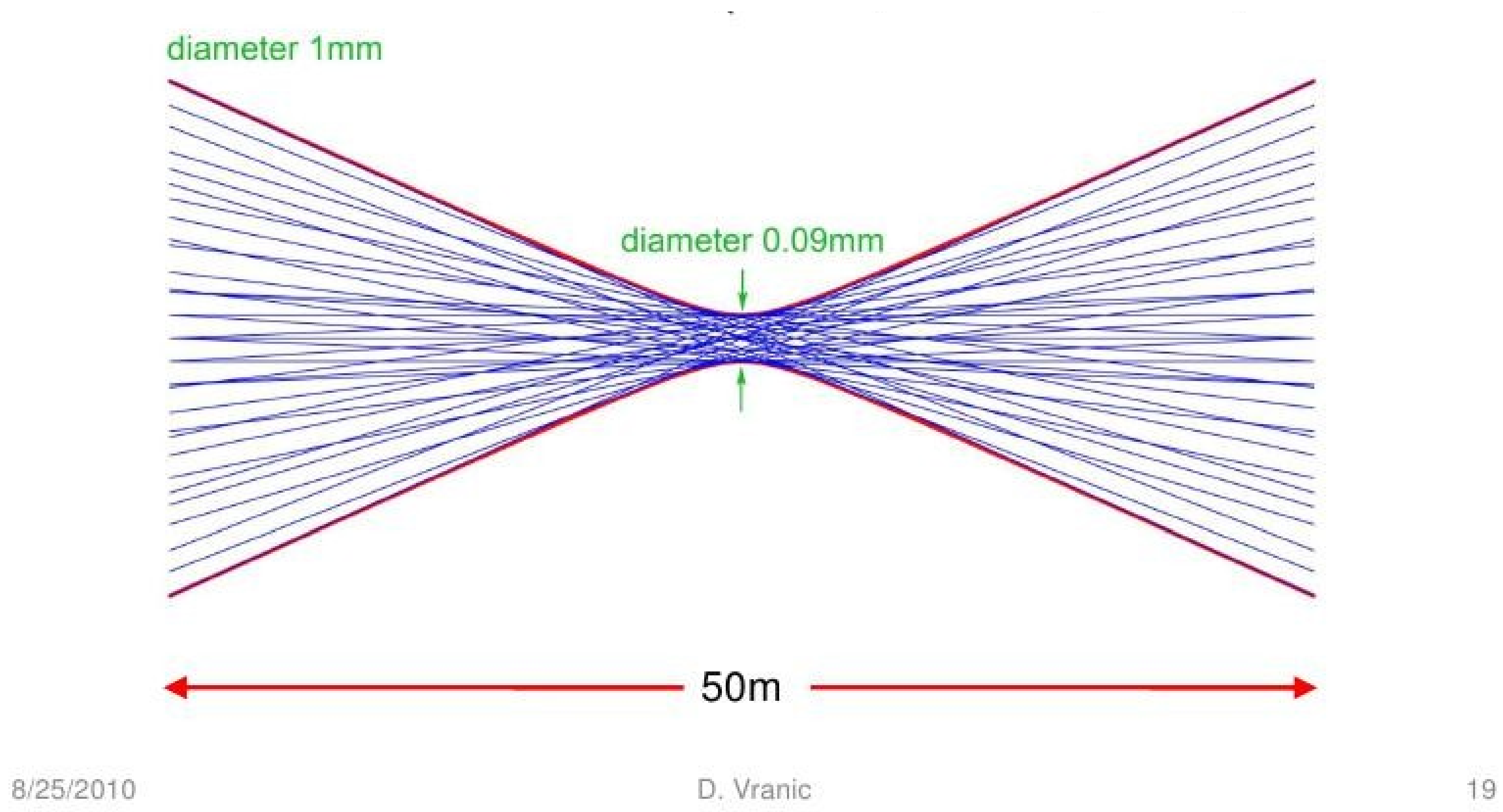
- There is more beam/bunch in the two rings (N_B,N_Y)
- There are more bunches colliding (k_b)
- **The beam profiles, the size of the beam, at the interaction point, is small (σ_x,σ_y) -> β***

$$L = \frac{N_B N_Y}{4\pi\, \sigma_x \sigma_y} k_b\, f_{rev} \quad (\mathrm{cm}^{-2}\mathrm{s}^{-1})$$

$$R = L \cdot \sigma$$

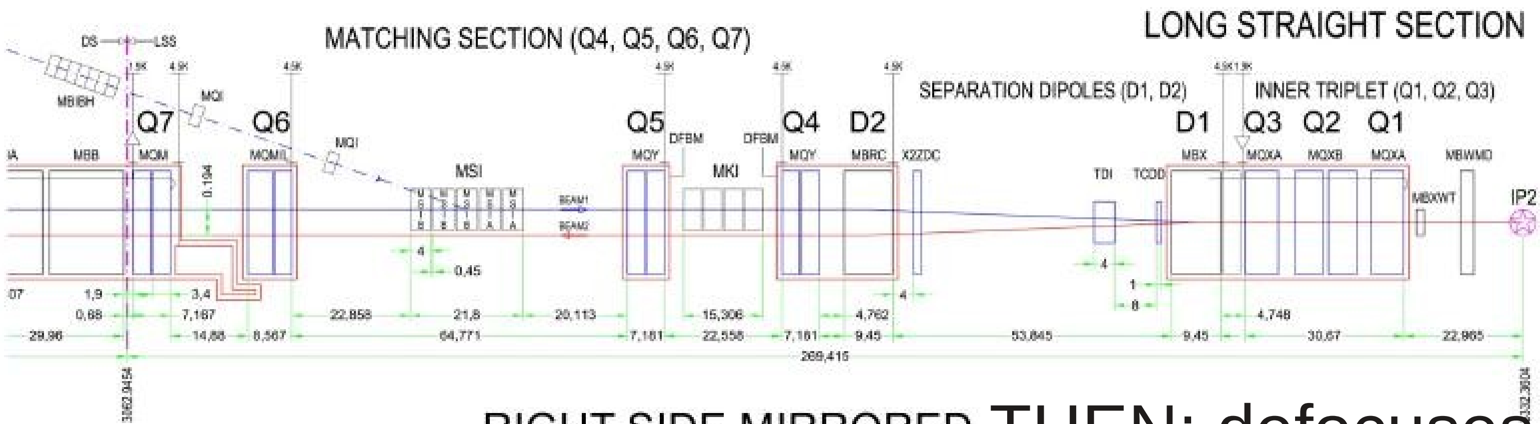
σ is the cross-section
R is the number of events per Second (corresponding to σ)

Example of focusing for collisions at P2 (ALICE)

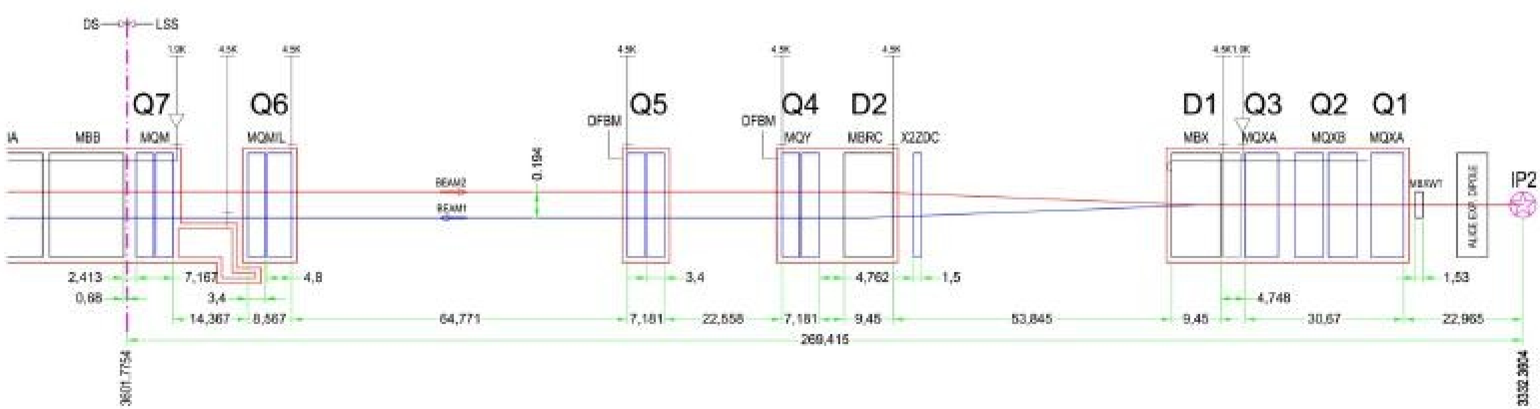


SYMMETRY!

LEFT SIDE EXAMPLE: Focuses beam!



RIGHT SIDE MIRRORED THEN: defocuses beam!



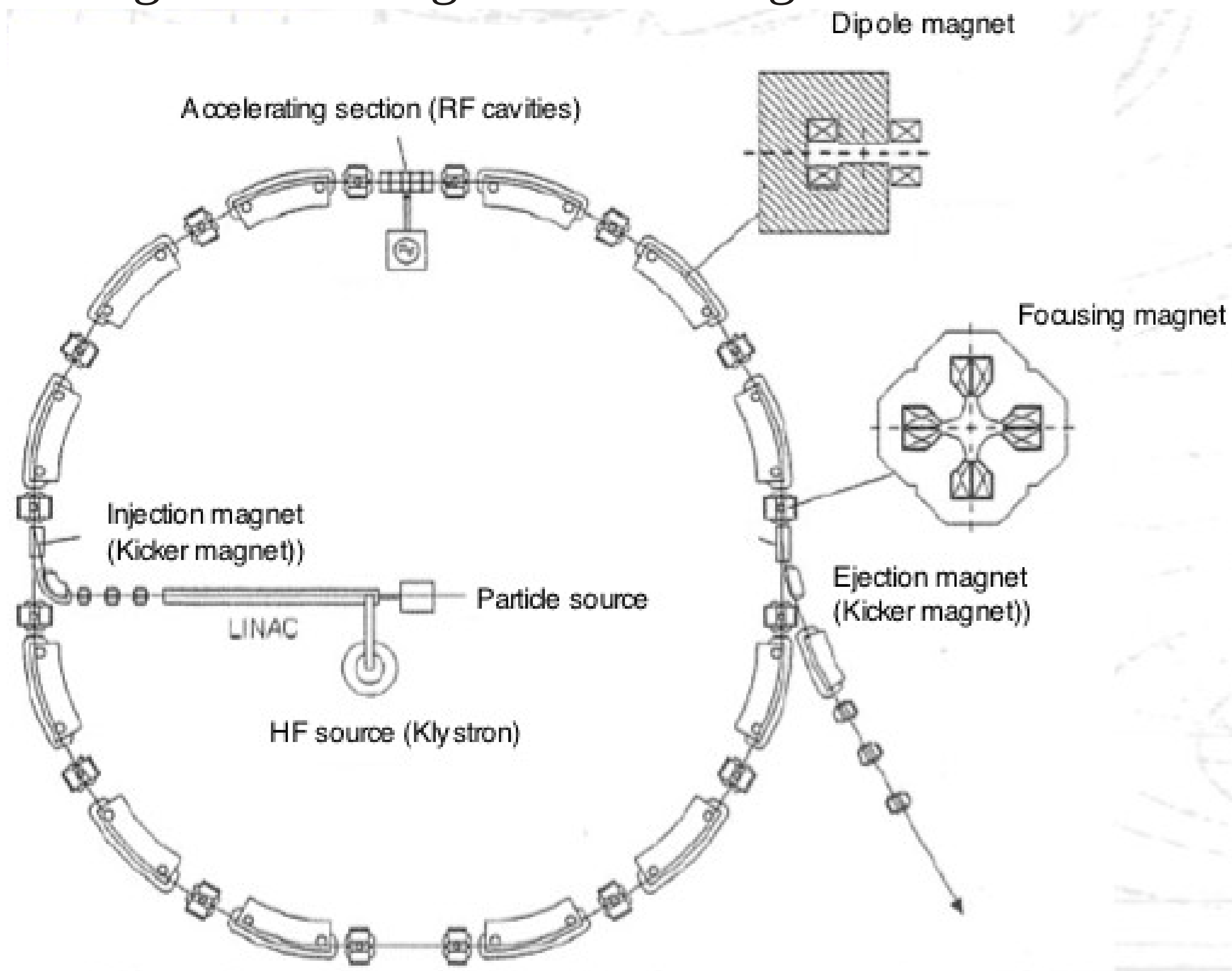
8/25/2010 D. Vranic 15

Synchrotrons

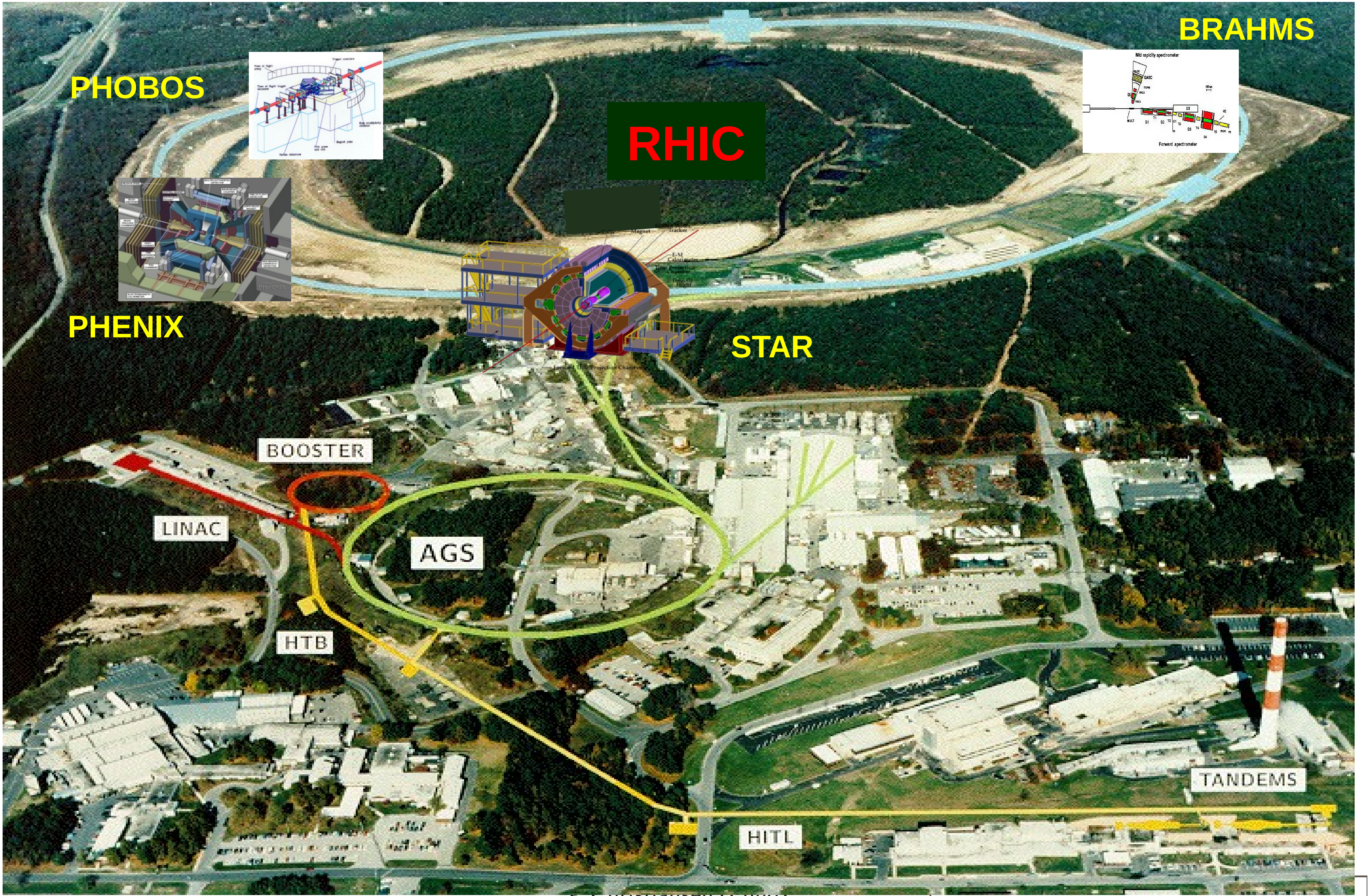
Use smaller magnets in a ring + accelerating station

3 GeV protons
BNL 1950s

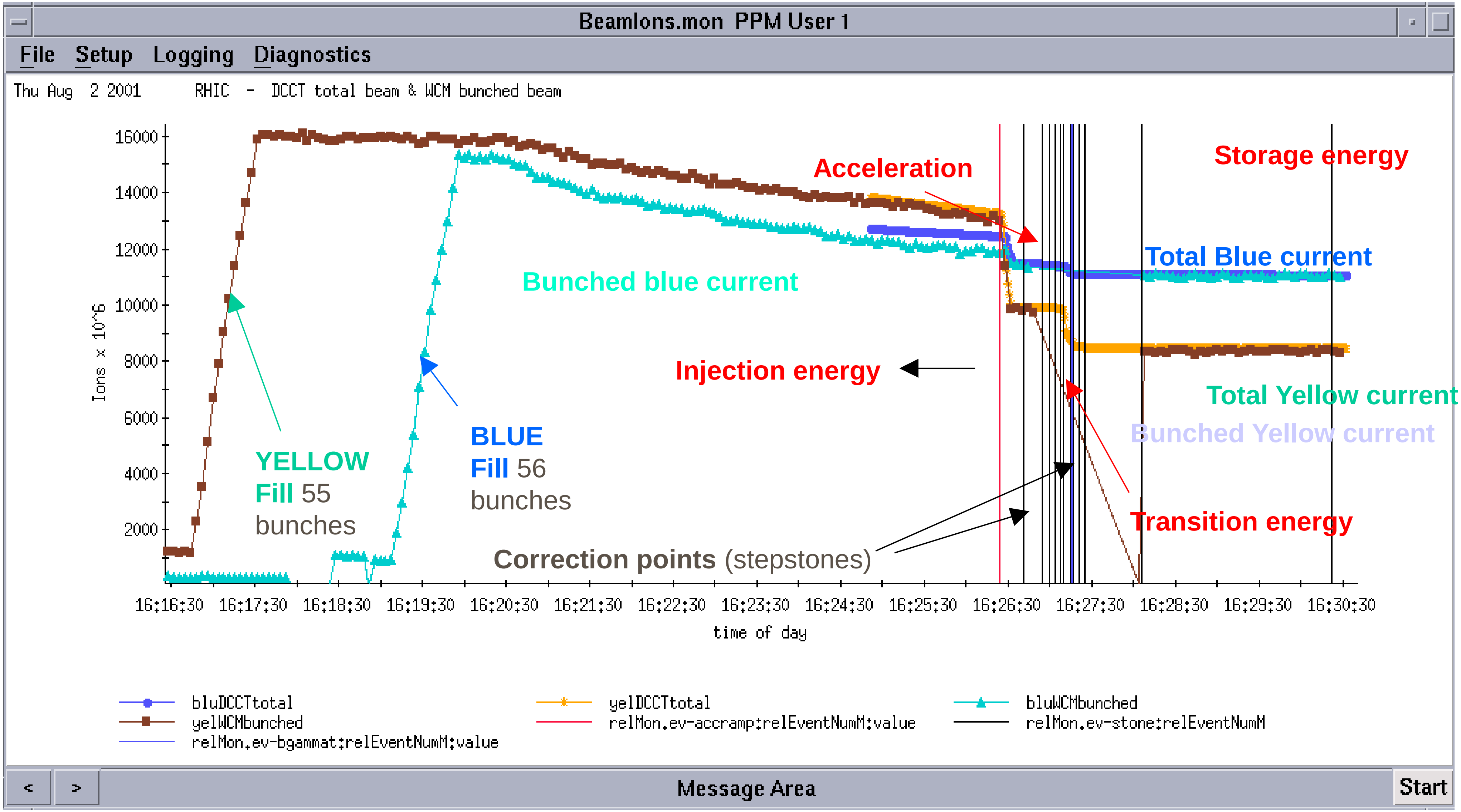
Basis of all circular machines built since



at Brookhaven National

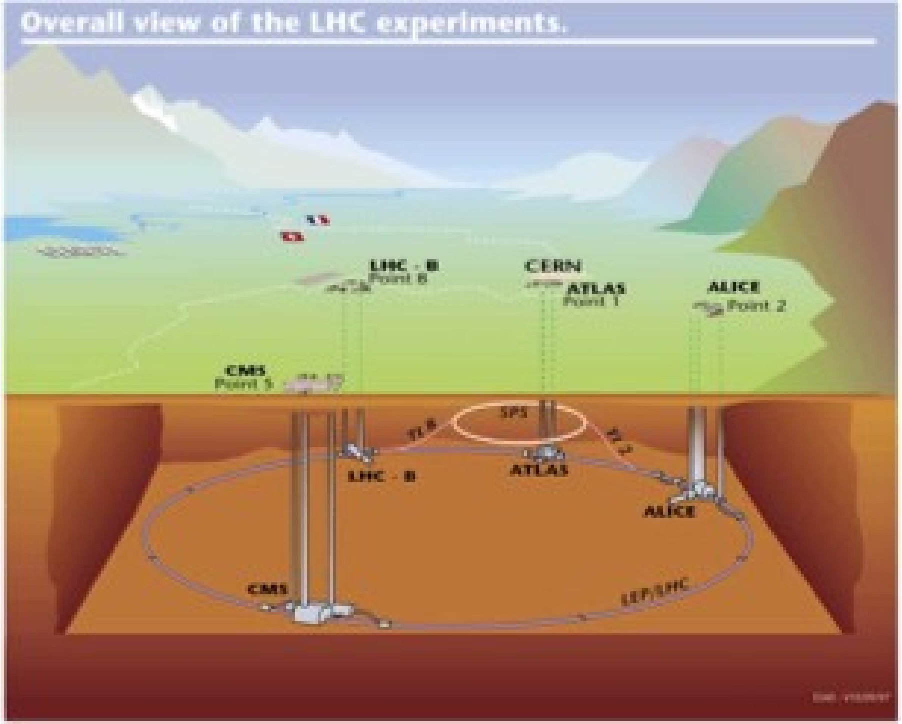


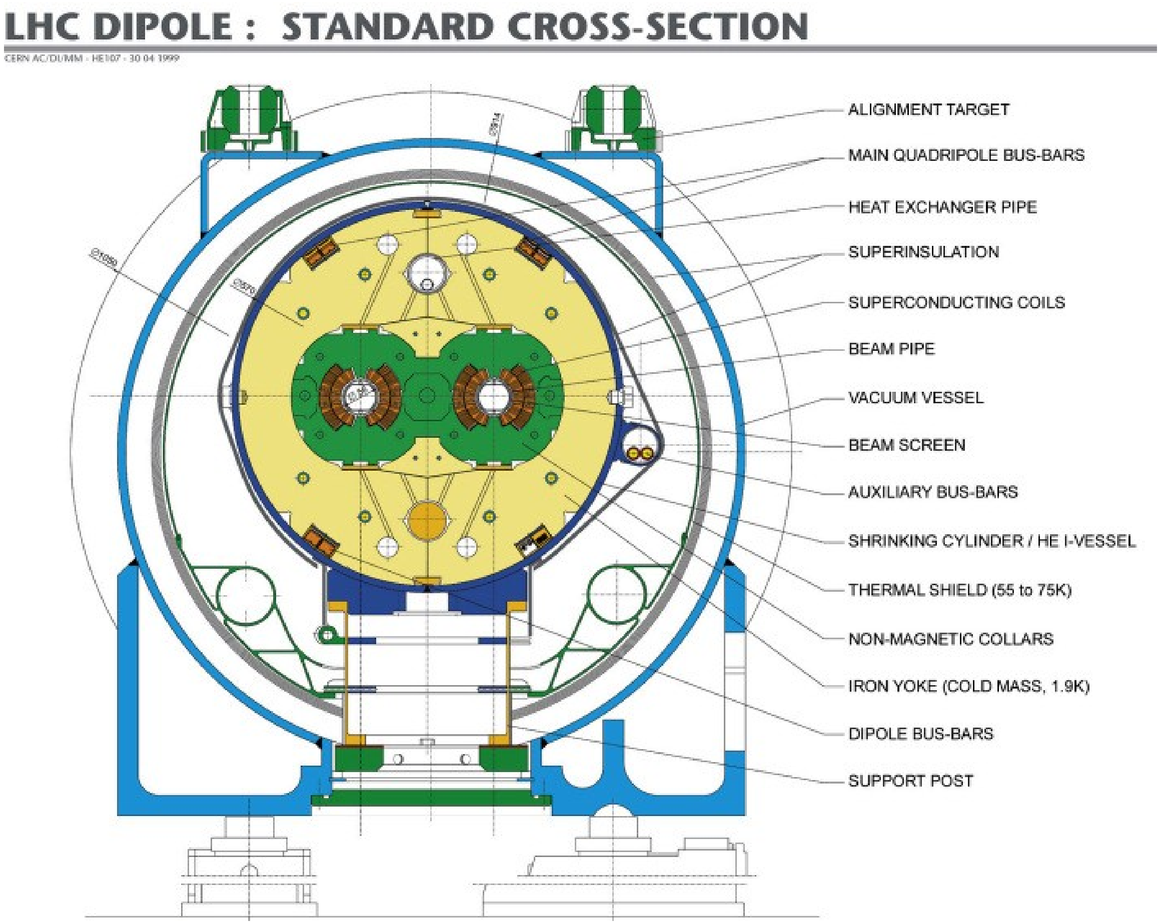
RHIC ramp with 56 bunches

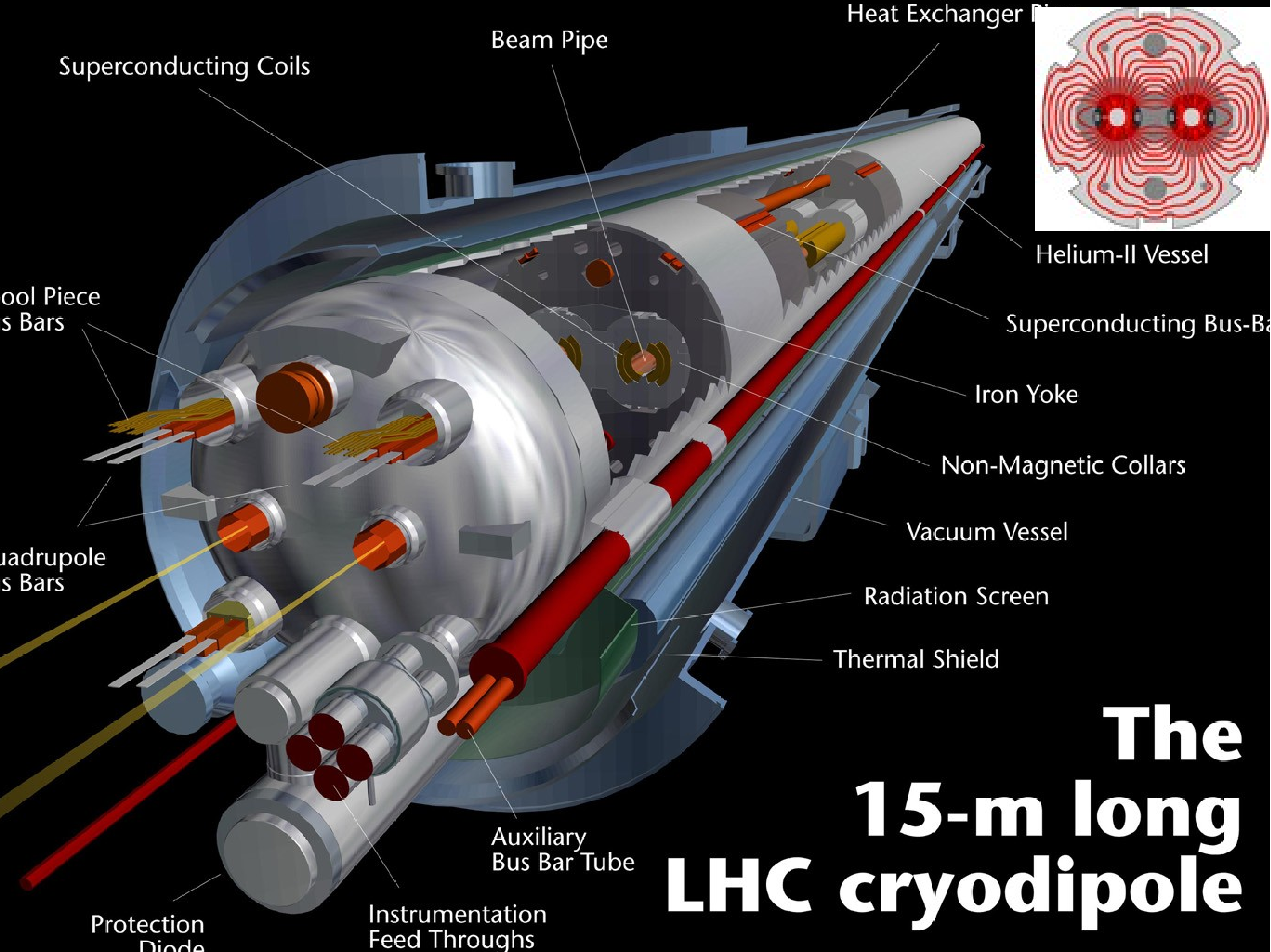


The beam is accelerated from Injection Energy (10 GeV) to Storage Energy (100 GeV). The acceleration process is called “ramp”.

CERN Large Hadron Collider

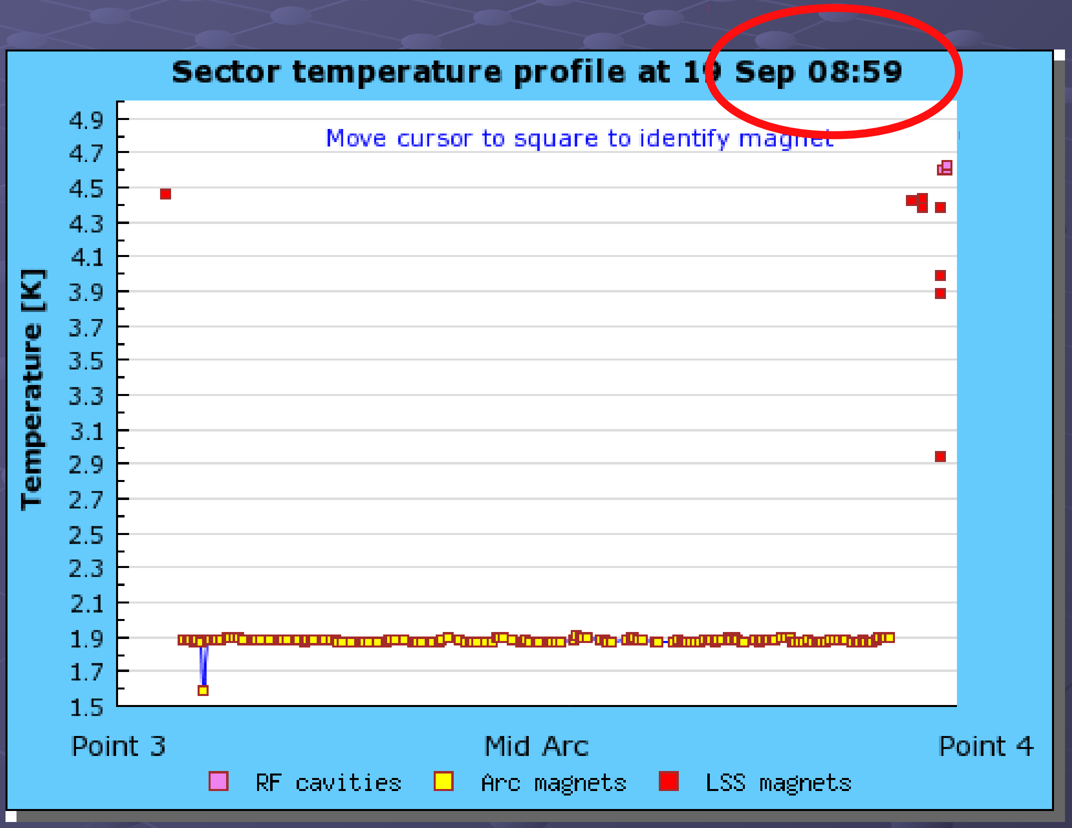






Heat Exchanger
Beam Pipe
Superconducting Coils
Helium-II Vessel
ool Piece
s Bars
Superconducting Bus-Ba
Iron Yoke
Non-Magnetic Collars
Vacuum Vessel
uadrupole
s Bars
Radiation Screen
Thermal Shield
Auxiliary
Bus Bar Tube
Protection
Instrumentation
Feed Throughs
The
15-m long
LHC cryodipole

The 19 September 2008 accident

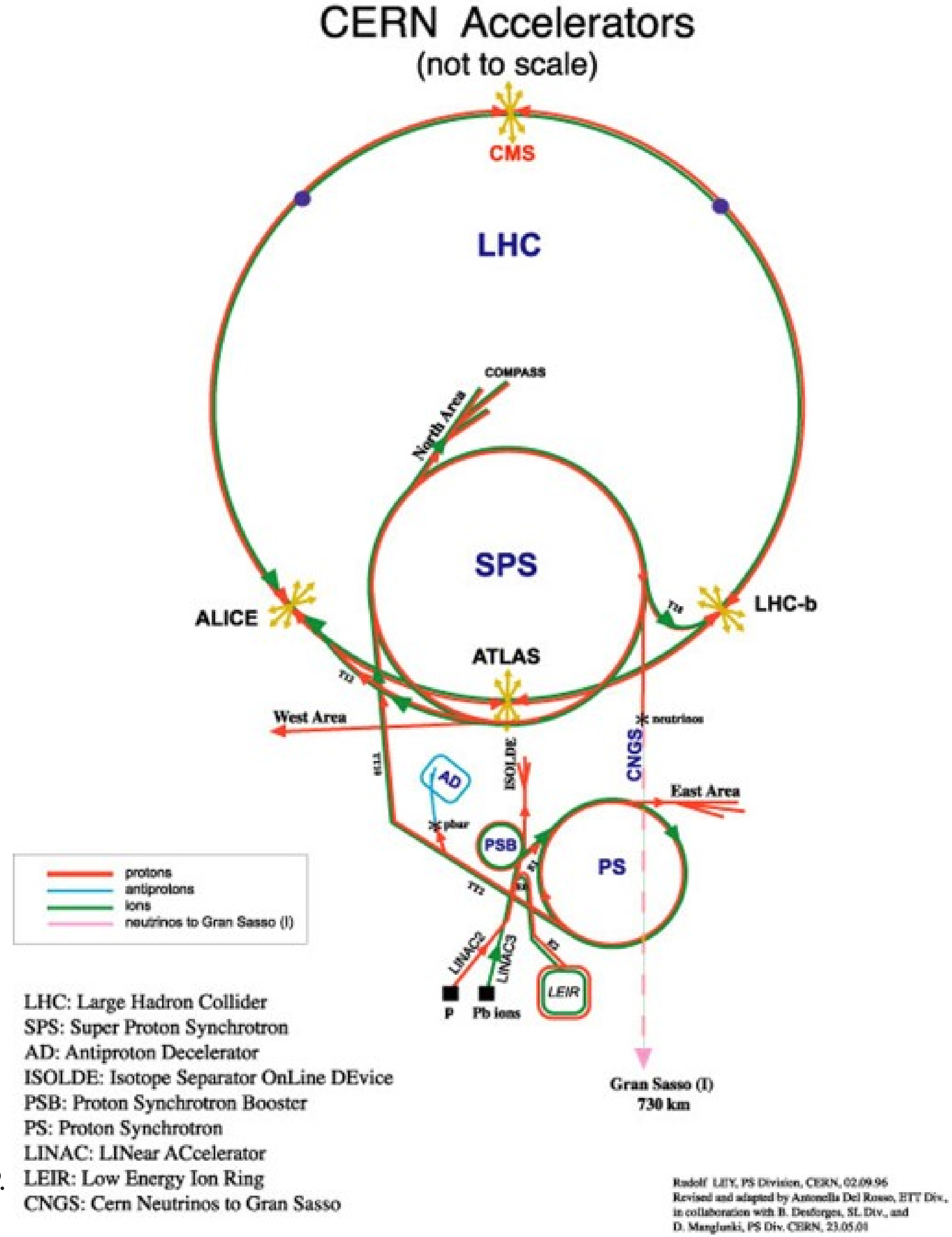


THE 19 SEPTEMBER 2008 INCIDENT

CERN Complex

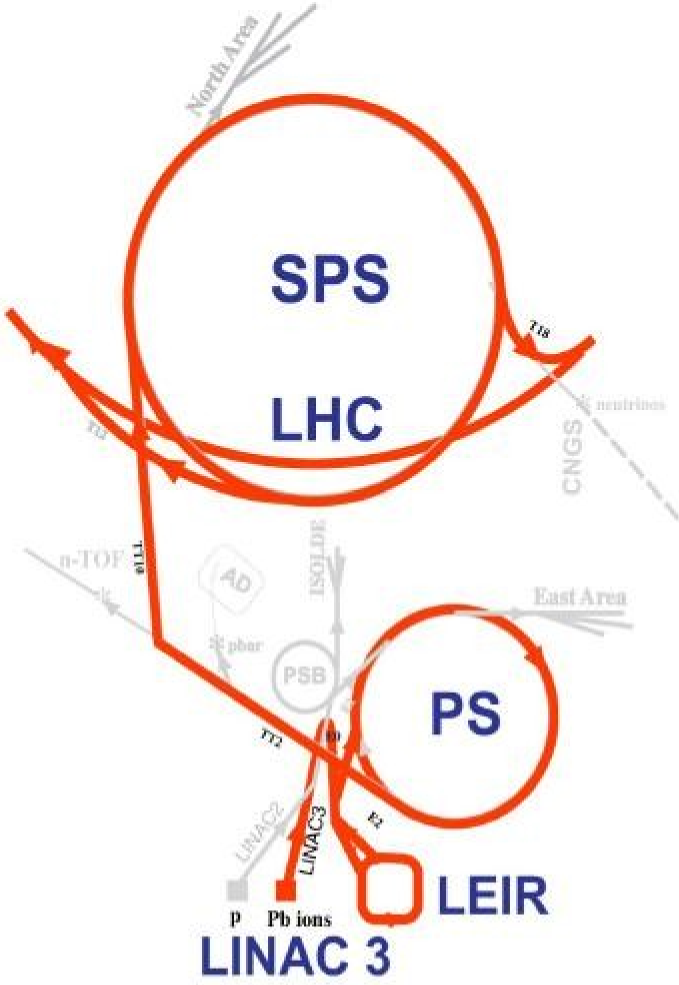
Old rings still in use

Many different programs



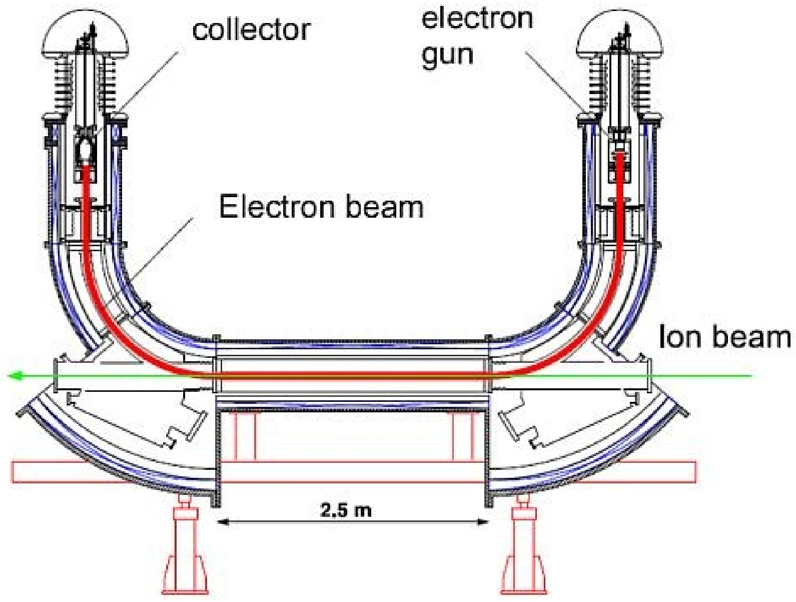
ION BEAM IN THE LHC

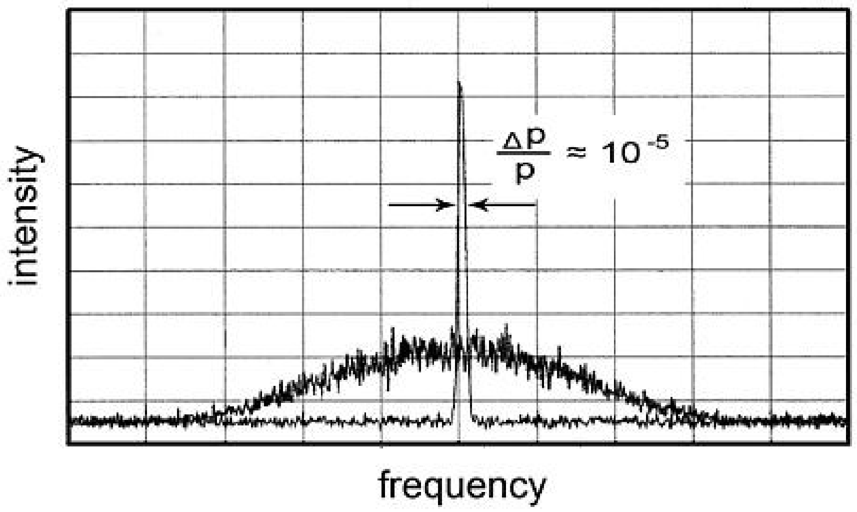
- ECR ion source
– Provide highest possible intensity of Pb29+
- RFQ + Linac 3
– Adapt to LEIR injection energy
– strip to Pb54+
- LEIR
– Accumulate and cool Linac 3 beam
– Prepare bunch structure for PS
- PS
– Define LHC bunch structure
– Strip to Pb82+
- SPS
– Define filling scheme



11/23/2010 D. Vranic 2

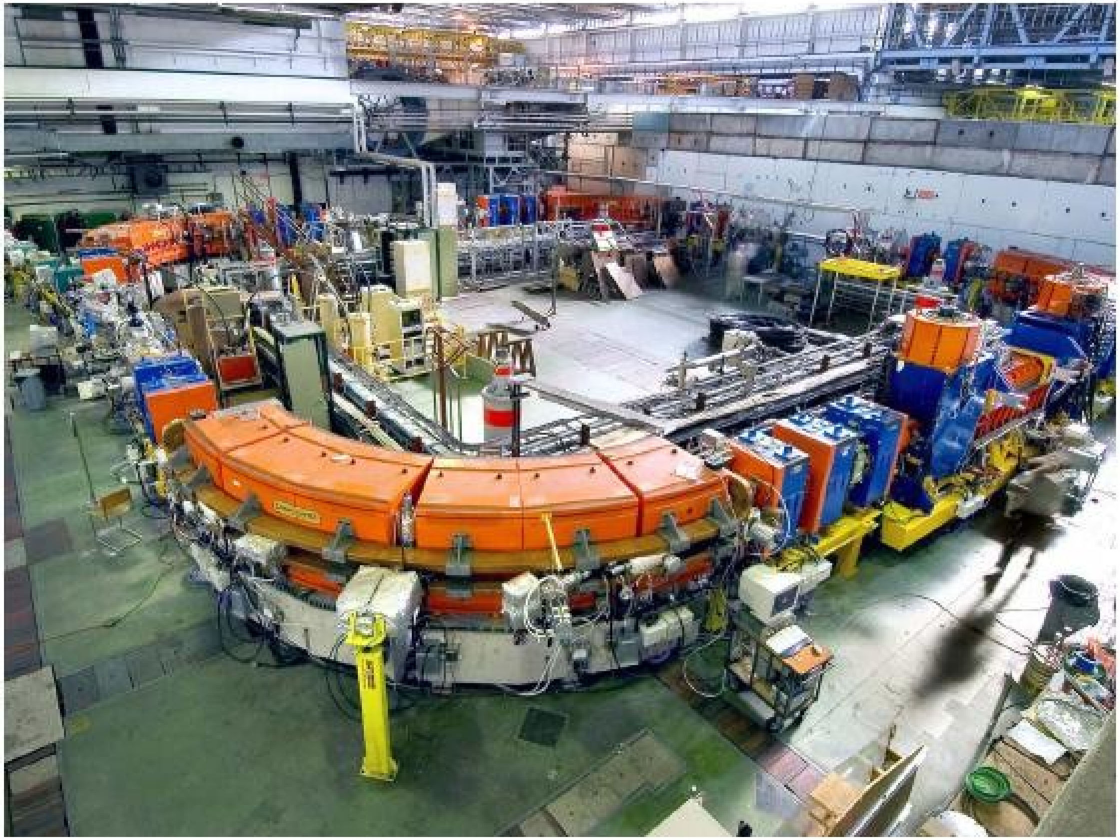
LEIR: Electron cooling example





- From: http://web-docs.gsi.de/
- Elastic collision e+ion will decrease the relative momentum spread in the beam

LEIR

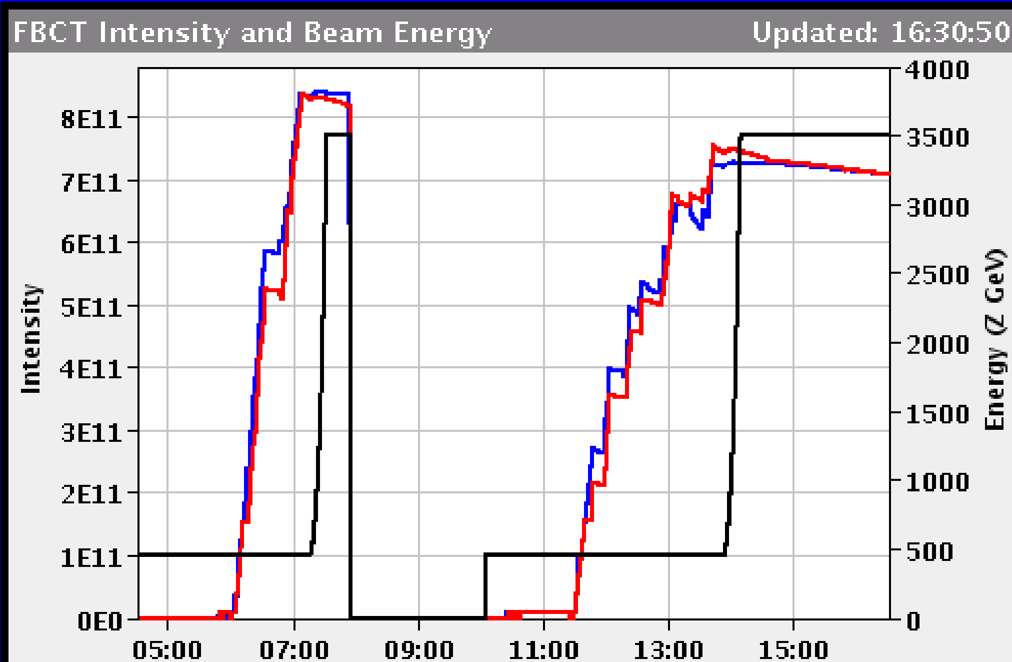


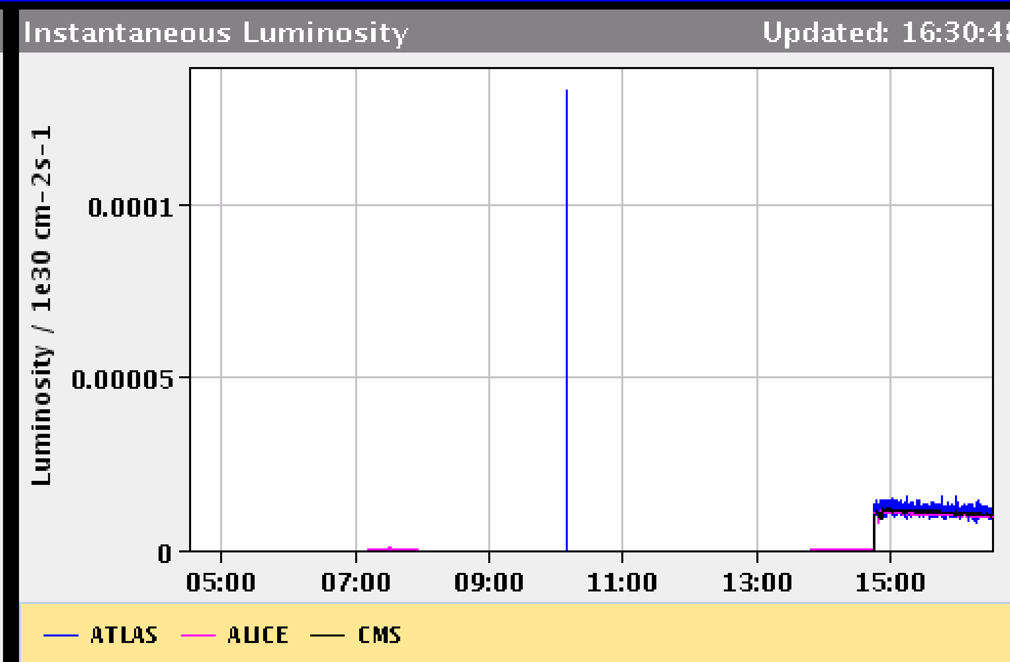
11/23/2010 D. Vranic 9

LHC Page1 Fill: 1530 E: 3500 Z GeV 29-11-2010 16:30:51

ION PHYSICS: STABLE BEAMS

Energy:	3500 Z GeV	I(B1):	8.32e+11	I(B2):	7.57e+11





Comments 29-11-2010 14:54:46 :

*** STABLE BEAMS ***

All points optimized

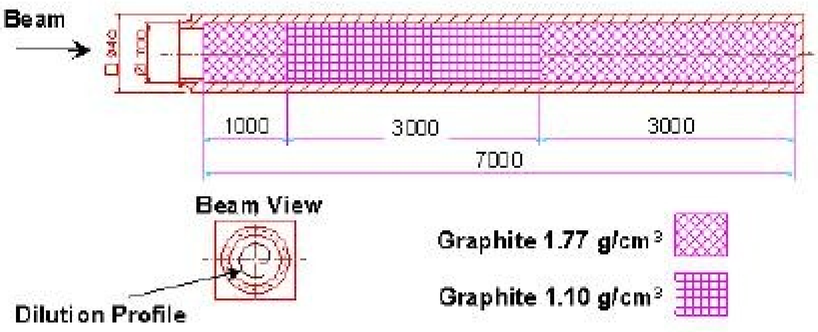
BIS status and SMP flags	B1	B2
Link Status of Beam Permits	true	true
Global Beam Permit	true	true
Setup Beam	false	false
Beam Presence	true	true
Moveable Devices Allowed In	true	true
Stable Beams	true	true

AFS: 500ns_121b_113_114_0_4bpi31inj_IONS

PM Status B1 ENABLED PM Status B2 ENABLED

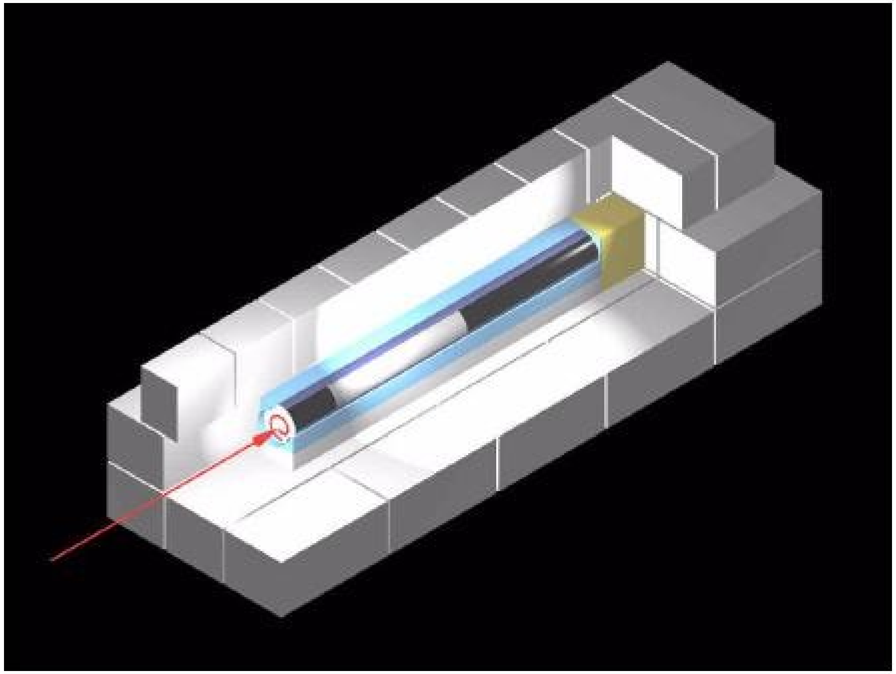
DUMP CORE TDE

7m long C / C-C TDE in steel shrink-cylinder, followed by 1m Al, 2m Fe
~1000 T of concrete shielding



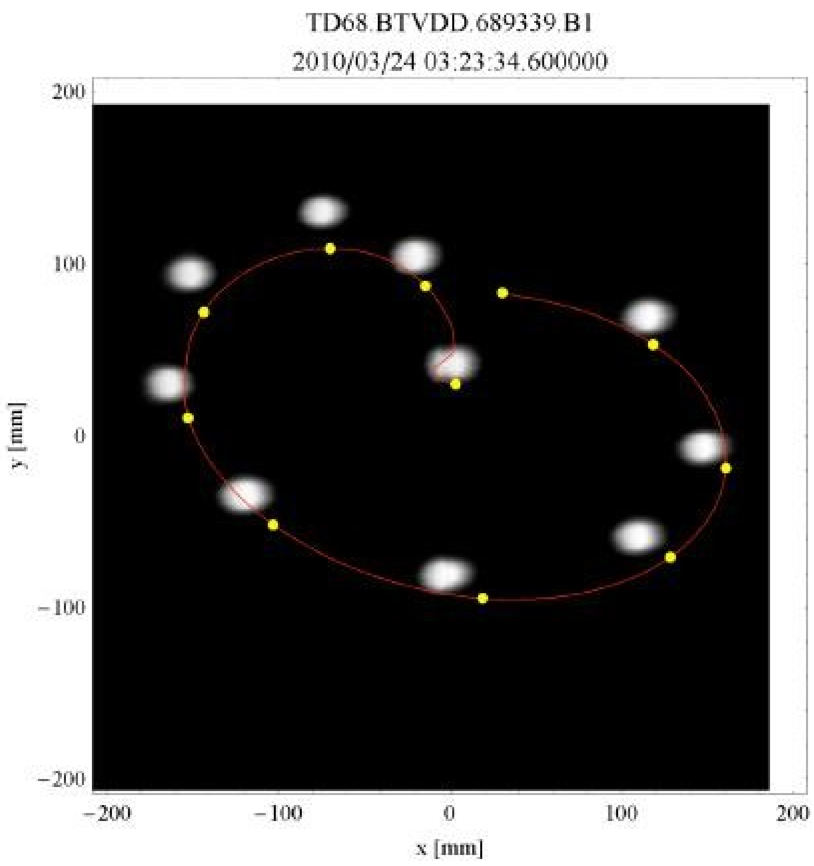
This is the **ONLY element in the LHC that can** withstand the impact of the full 7 TeV beam !
Nevertheless, the dumped beam must be painted to keep the peak energy densities at a tolerable level !

Why graphite? If the material were heavy, all the beam's energy would concentrate in the first half meter of the block.



8/25/2010 D. Vranic 11

The beam size has increased to an extent where the sigma is 1.6mm in both planes.



8/25/2010 D. Vranic 12