Accelerator
Division

Introduction to RF for Particle Accelerators
Part 2: RF Cavities

Dave McGinnis

RF Cavity Topics

- Modes
  - Symmetry
  - Boundaries
  - Degeneracy
- RLC model
- Coupling
  - Inductive
  - Capacitive
  - Measuring
- Q
  - Unloaded Q
  - Loaded Q
  - Q Measurements
- Impedance Measurements
- Power Amplifiers
  - Class of operation
  - Tetrodes
  - Klystrons
- Beam Loading
  - De-tuning
  - Fundamental
  - Transient

RF and Circular Accelerators

For circular accelerators, the beam can only be accelerated by a time-varying (RF) electromagnetic field.

Faraday's Law

$$\oint_C \vec{E} \bullet d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \bullet d\vec{S}$$

The integral

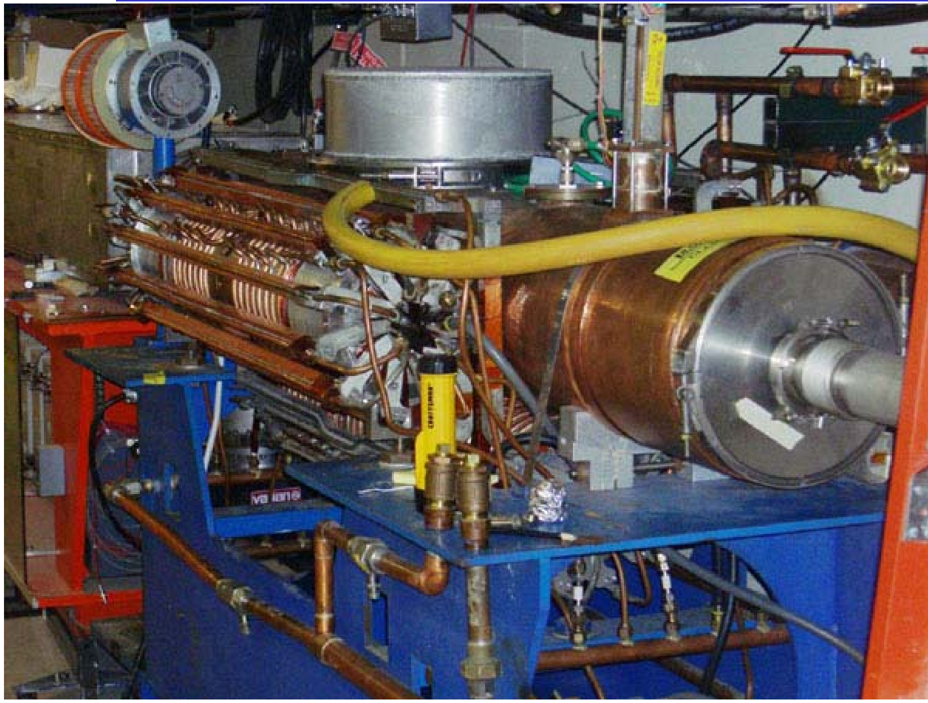
$$q \oint_C \vec{E} \bullet d\vec{l}$$

is the energy gained by a particle with charge q during one trip around the accelerator.

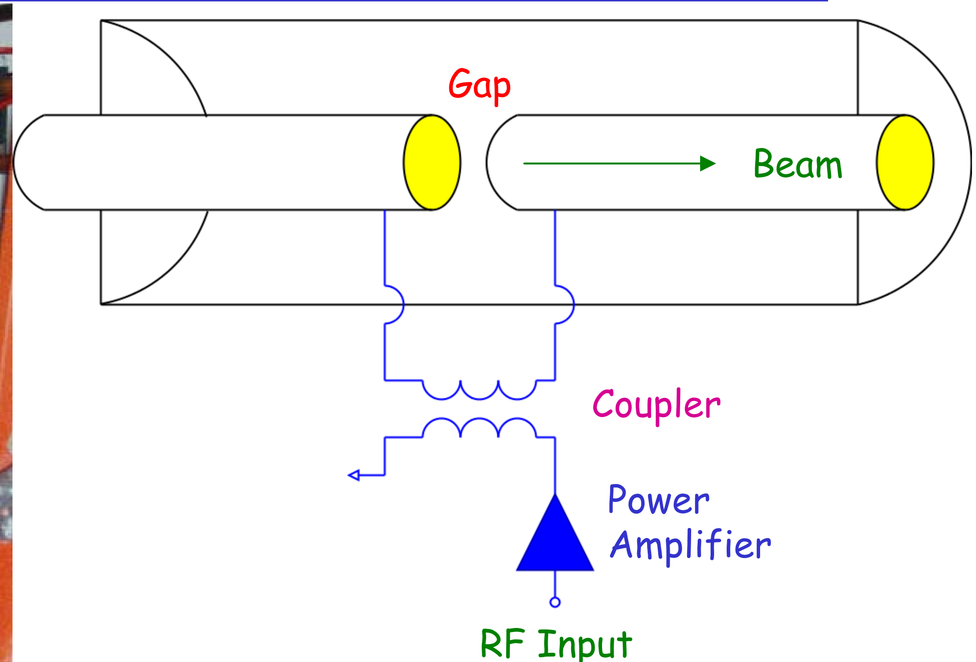
For a machine with a fixed closed path such as a synchrotron, if

$$\frac{\partial \vec{B}}{\partial t} = 0 \qquad \text{then} \qquad q \oint_C \vec{E} \bullet d\vec{l} = 0$$

RF Cavities



New FNAL Booster Cavity

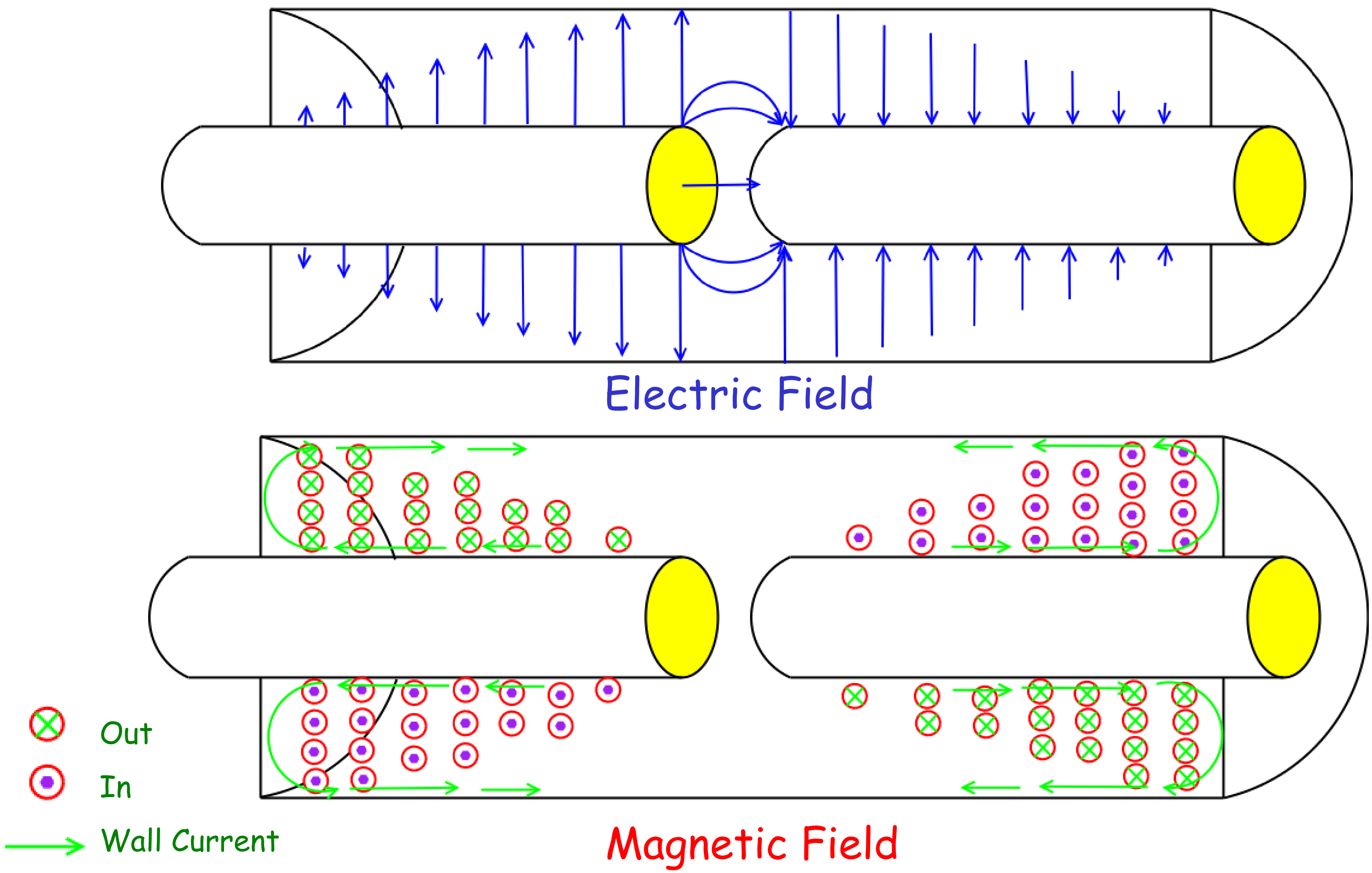


Transmission Line Cavity

Multi-cell
superconducting
RF cavity

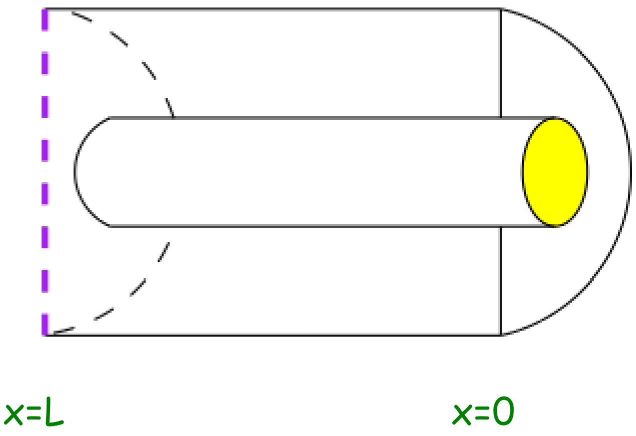
Cavity Field Pattern

For the fundamental mode at one instant in time:



Cavity Modes

We need to solve only ½ of the problem



For starters, ignore the gap capacitance.

The cavity looks like a shorted section of transmission line

$$V = V^{+}e^{-j\beta x} + V^{-}e^{+j\beta x}$$

$$Z_o I = V^{+}e^{-j\beta x} - V^{-}e^{+j\beta x}$$

where Z_o is the characteristic impedance of the transmission line structure of the cavity

Cavity Boundary Conditions

Boundary Condition 1:

At x=0: V=0

$$V = V_o \sin(\beta x)$$

$$Z_o I = -jV_o \cos(\beta x)$$

Boundary Condition 2:

At x=L: I=0

$$\cos(\beta L) = 0$$

$$\beta_n L = (2n+1)\frac{\pi}{2}$$

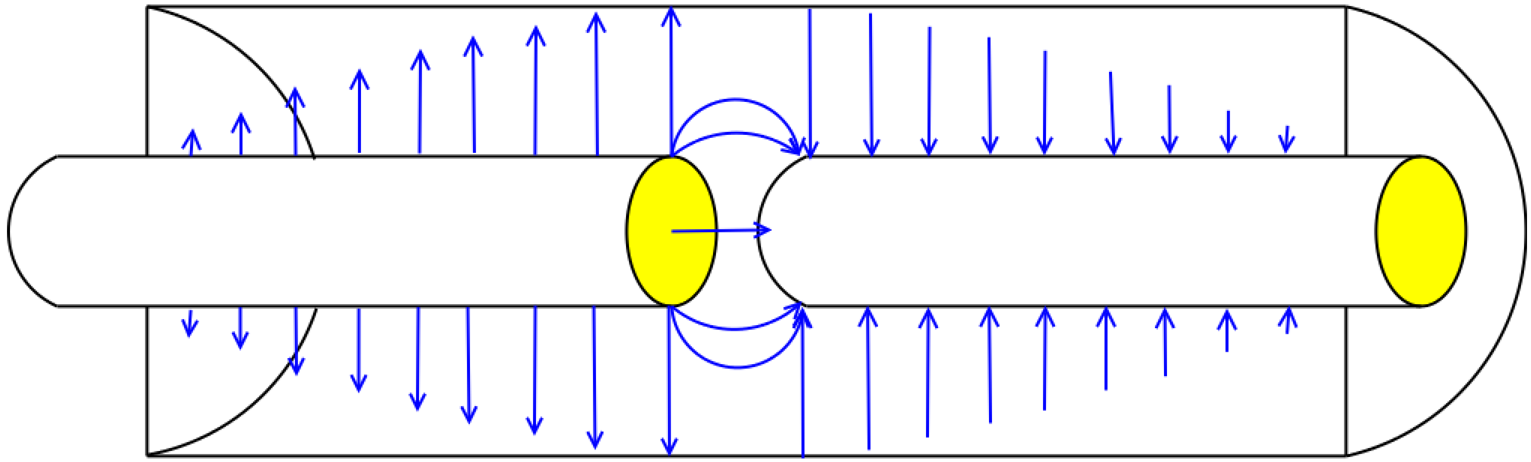
$$n = 0,1,2,3...$$

$$f_n = (2n+1)\frac{c}{4L}$$

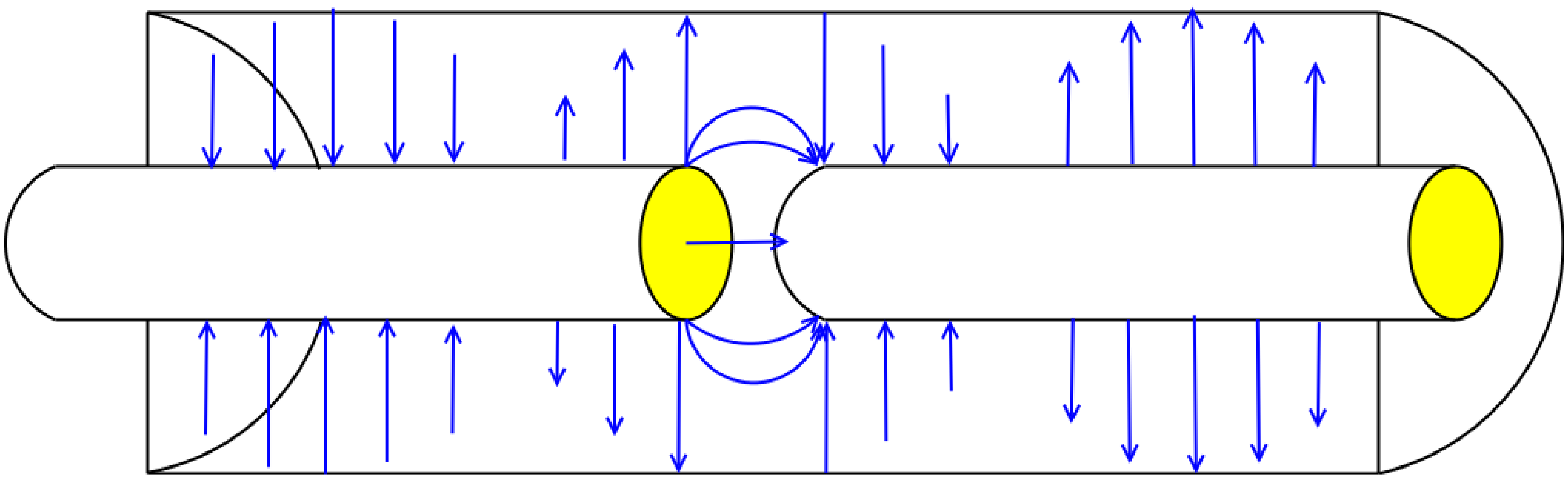
$$L = (2n+1)\frac{\lambda_n}{4}$$

Different values of n are called modes. The lowest value of n is usually called the fundamental mode

Cavity Modes

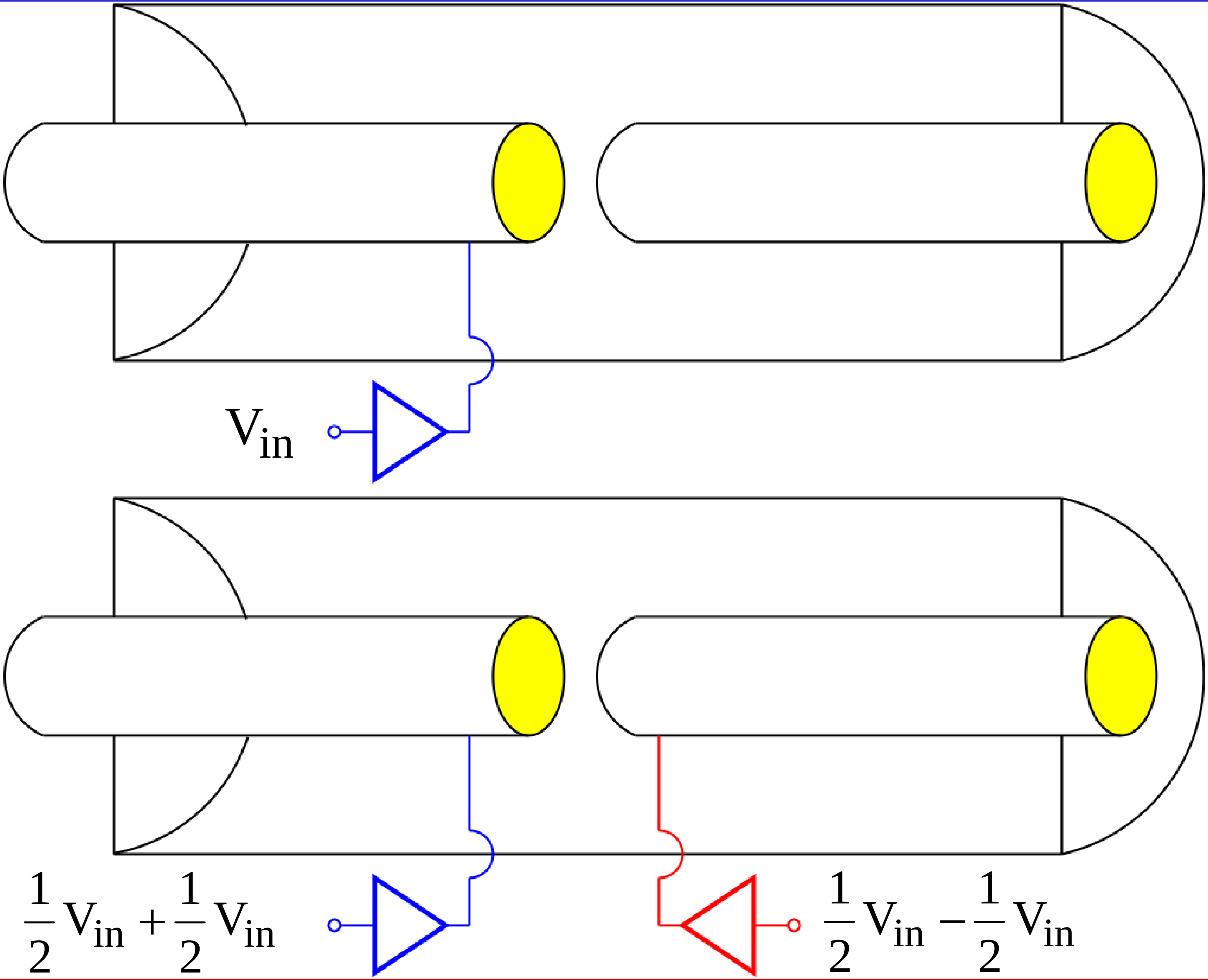


n=0

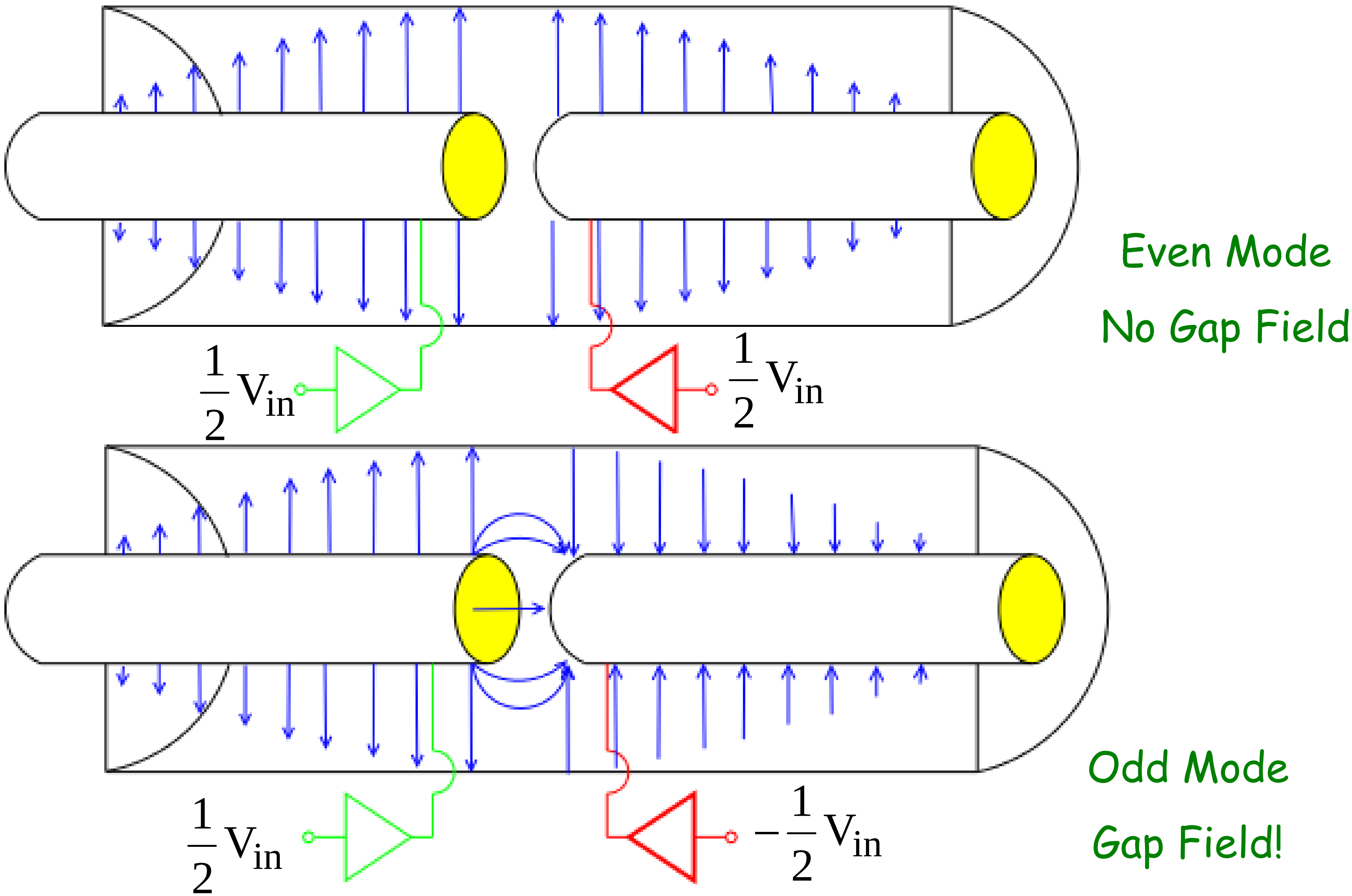


n=1

Even and Odd Mode De-Composition

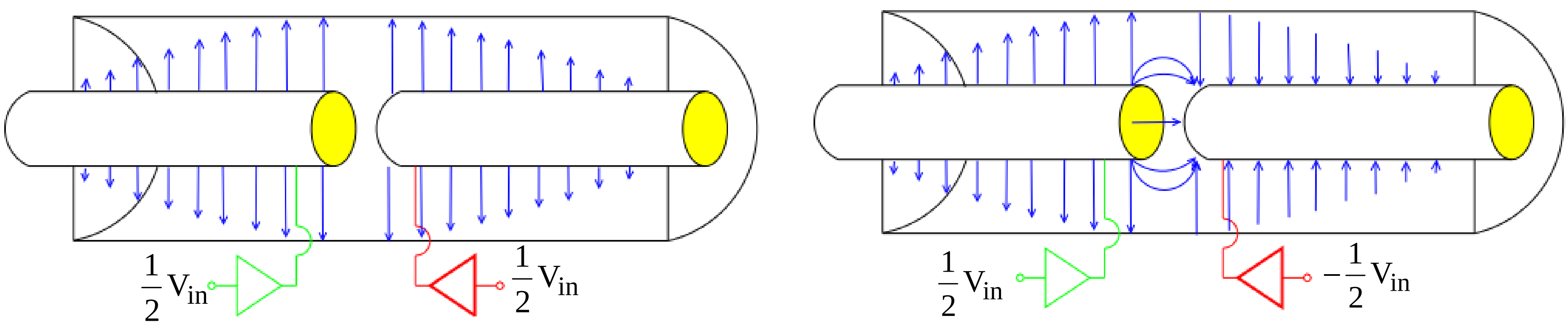


Even and Odd Mode De-Composition



Degenerate Modes

- The even and odd decompositions have the same mode frequencies.
- Modes that occur at the same frequency are called degenerate.
- The even and odd modes can be split if we include the gap capacitance.
- In the even mode, since the voltage is the same on both sides of the gap, no capacitive current can flow across the gap.
- In the odd mode, there is a voltage difference across the gap, so capacitive current will flow across the gap.



Gap Capacitance

Boundary Condition 1:

At x=0: V=0

$$V = V_o \sin(\beta x)$$

$$Z_o I = -jV_o \cos(\beta x)$$

Boundary Condition 2:

At x=L:

$$I = j\omega C_g V$$ where C_g is the gap capacitance

$$\omega C_g Z_o = \frac{\cos(\beta L)}{\sin(\beta L)}$$

RF Cavity Modes

Consider the first mode only (n=0) and a very small gap capacitance.

$$\beta L = \frac{\pi}{2} + \delta$$

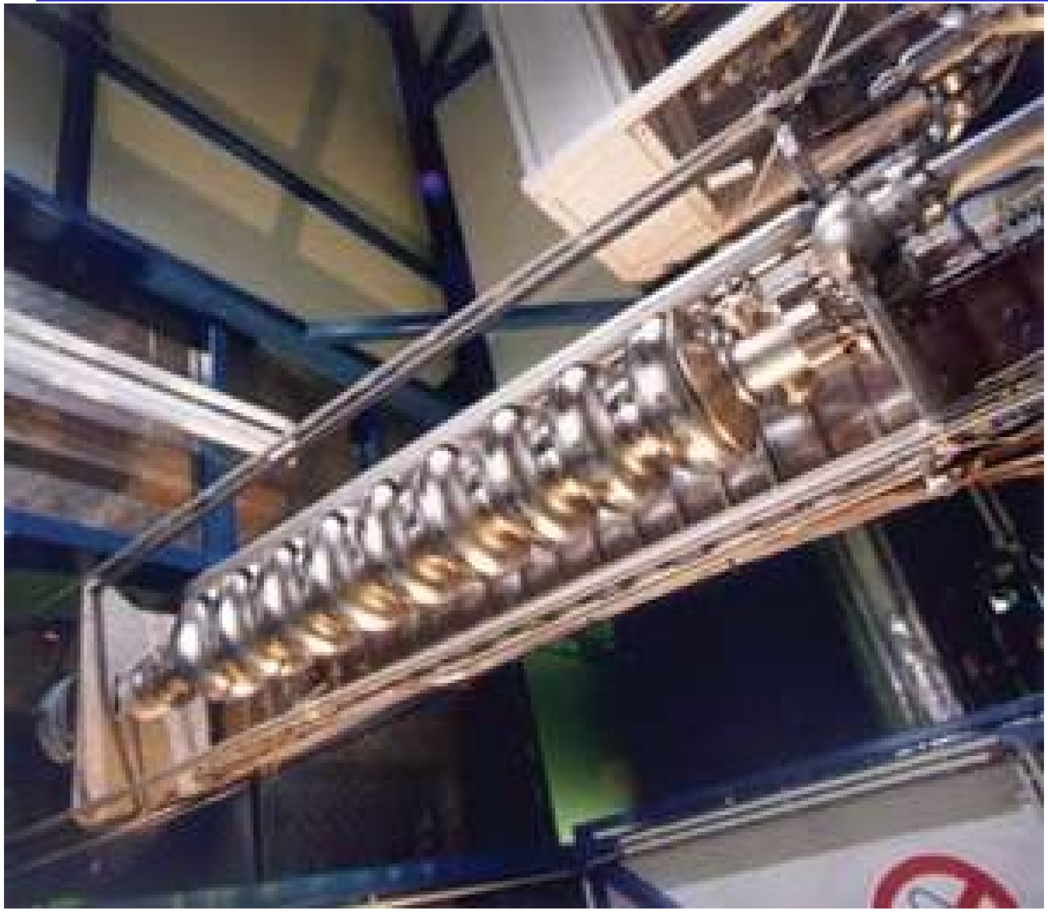
$$\frac{\cos(\beta L)}{\sin(\beta L)} \approx -\delta$$

$$\delta = -\omega_o C_g Z_o$$

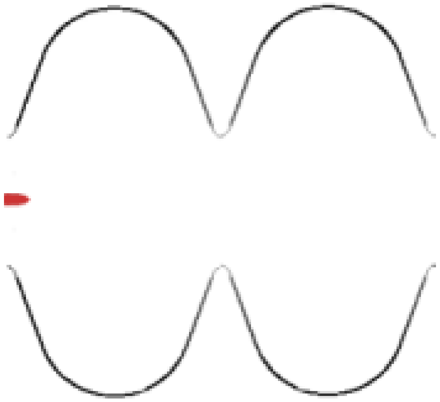
$$\frac{\Delta\omega_o}{\omega_o} = \frac{2}{\pi}\delta$$

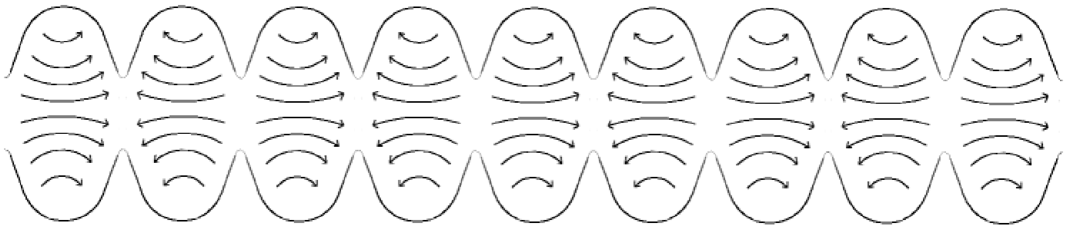
The gap capacitance shifts the odd mode down in frequency and leaves the even mode frequency unchanged

Multi-Celled Cavities



- Each cell has its own resonant frequency
- For n cells there will be n degenerate modes
- The cavity to cavity coupling splits these n degenerate modes.
- The correct accelerating mode must be picked





Cavity Q

- If the cavity walls are lossless, then the boundary conditions for a given mode can only be satisfied at a single frequency.
- If the cavity walls have some loss, then the boundary conditions can be satisfied over a range of frequencies.
- The cavity Q factor is a convenient way the power lost in a cavity.
- The Q factor is defined as:

$$Q = \frac{W_{stored}}{W_{lost/cycle}}$$

$$= \omega_o \frac{W_E + W_H}{P_L}$$

Transmission Line Cavity Q

We will use the fundamental mode of the transmission line cavity as an example of how to calculate the cavity Q.

Electric Energy:

$$W_E = \frac{1}{4}\iiint_{vol} \varepsilon \left|\vec{E}\right|^2 dvol$$

$$= 2\frac{1}{4}\int_0^L C_l \left|V(x)\right|^2 dx$$

Both Halves:

$$= \frac{\pi}{8\omega_o}\frac{V_o^{\,2}}{Z_o}$$

Magnetic Energy:

$$W_H = \frac{1}{4}\iiint_{vol} \mu \left|\vec{H}\right|^2 dvol$$

$$= 2\frac{1}{4}\int_0^L L_l \left|I(x)\right|^2 dx$$

$$= \frac{\pi}{8\omega_o}\frac{V_o^{\,2}}{Z_o}$$

Transmission Line Cavity Q

Assume a small resistive loss per unit length $r_L\Omega$/m along the walls of the cavity.

Also assume that this loss does not perturb the field distribution of the cavity mode.

$$P_{loss} = 2\frac{1}{2}\int_0^L r_l \left|I(x)\right|^2 dx$$

Time average

$$= \frac{1}{2} r_l L \frac{{V_o}^2}{Z_o}$$

The cavity Q for the <u>fundamental mode</u> of the transmission line cavity is:

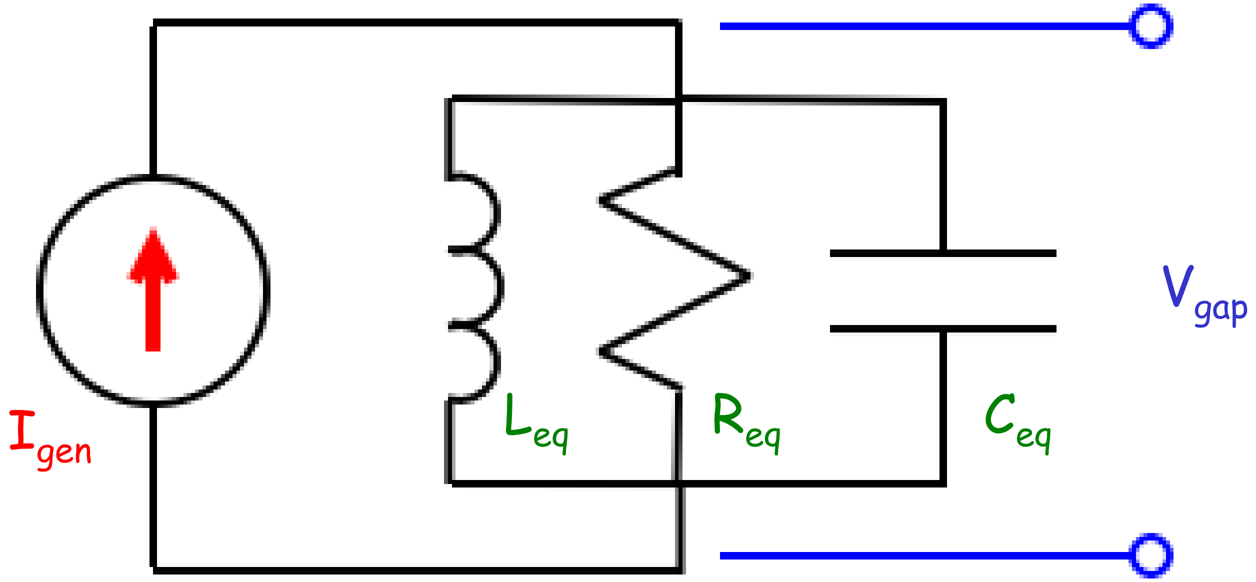
$$Q = \frac{\pi}{2}\frac{Z_o}{r_l L}$$

Less current flowing along walls

Less loss in walls

RLC Model for a Cavity Mode

Around each mode frequency, we can describe the cavity as a simple RLC circuit.



R_{eq} is inversely proportional to the energy lost

L_{eq} is proportional to the magnetic stored energy

C_{eq} is proportional to the electric stored energy

RLC Parameters for a Transmission Line Cavity

For the fundamental mode of the transmission line cavity:

$$P_{loss} = \frac{1}{2}\frac{V_{gap}^{2}}{R_{eq}}$$

$$R_{eq} = 4\frac{Z_o^{2}}{r_l L}$$

$$W_E = \frac{1}{4}C_{eq}V_{gap}^{2}$$

$$C_{eq} = \frac{\pi}{8}\frac{1}{\omega_o Z_o}$$

$$W_H = \frac{1}{4}\frac{V_{gap}^{2}}{\omega_o^{2}L_{eq}}$$

$$L_{eq} = \frac{8}{\pi}\frac{Z_o}{\omega_o}$$

The transfer impedance of the cavity is:

$$Z_c = \frac{V_{gap}}{I_{gen}}$$

$$\frac{1}{Z_c} = \frac{1}{R_{eq}} + \frac{1}{j\omega L_{eq}} + j\omega C_{eq}$$

Cavity Transfer Impedance

Since:

$$\frac{1}{\omega_o} = \sqrt{L_{eq} C_{eq}}$$

Function of geometry only

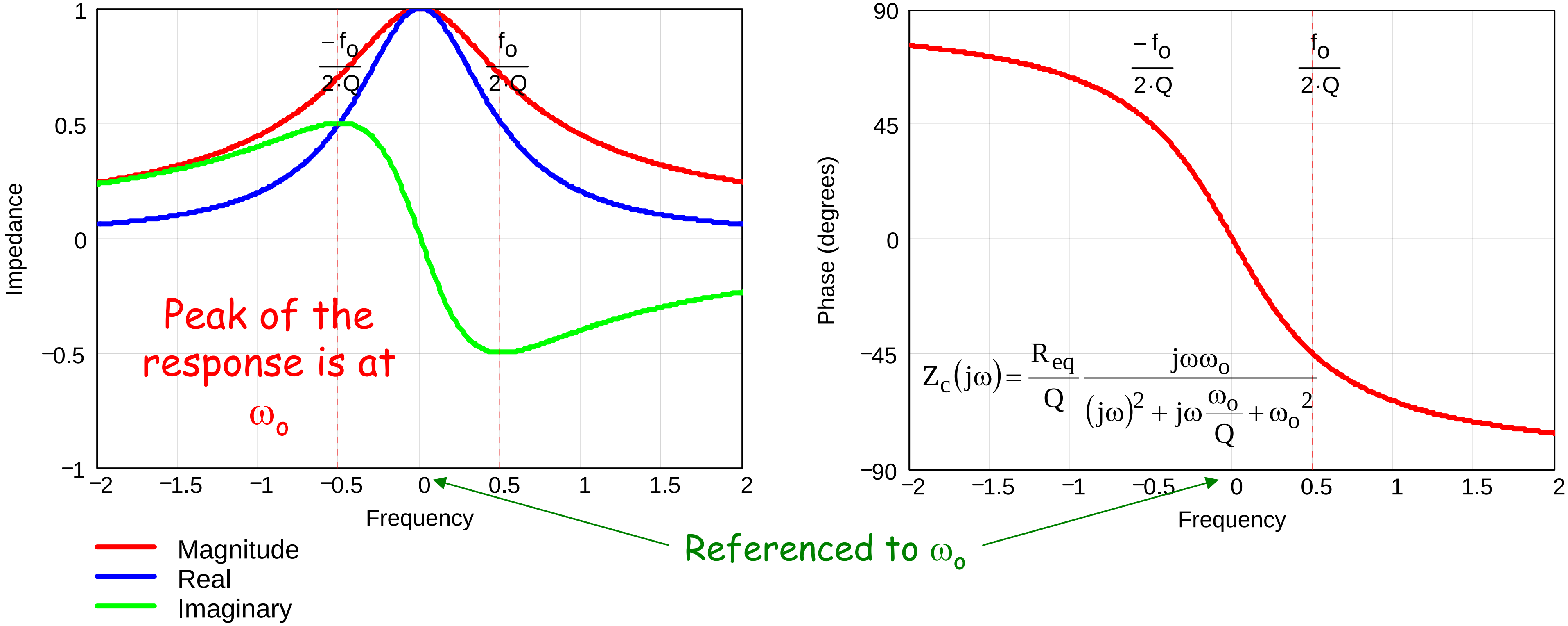
$$\frac{R_{eq}}{Q} = \sqrt{\frac{L_{eq}}{C_{eq}}}$$

Function of geometry and cavity material

$$Q = \omega_o R_{eq} C_{eq}$$

$$Z_c(j\omega) = \frac{R_{eq}}{Q} \frac{j\omega\omega_o}{(j\omega)^2 + j\omega\frac{\omega_o}{Q} + \omega_o{}^2}$$

Cavity Frequency Response

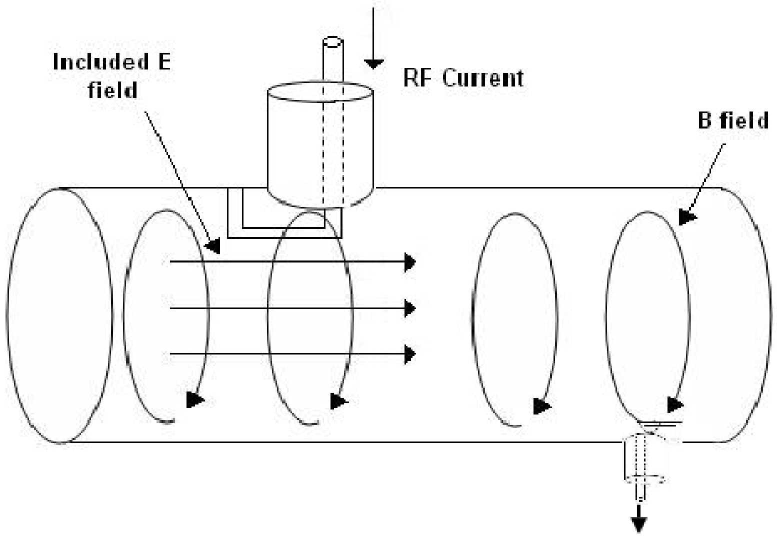


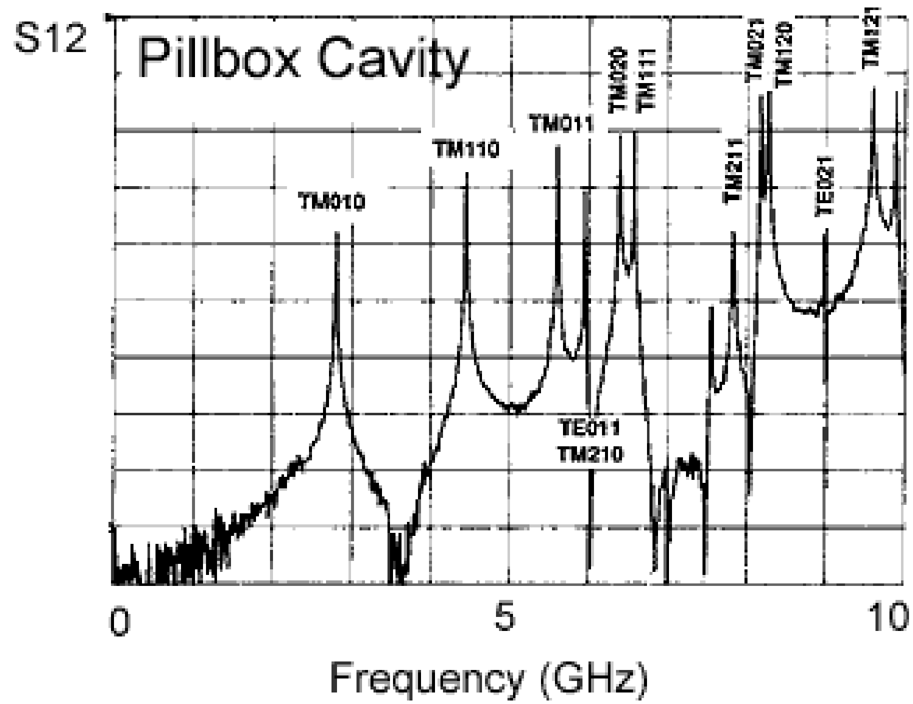
$$\left|Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right|^2 = \frac{1}{2}\left|Z(\omega_o)\right|^2$$

$$\mathrm{Im}\left\{Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right\} = \mp \mathrm{Re}\left\{Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right\}$$

$$\arg\left\{Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right\} = \mp 45°$$

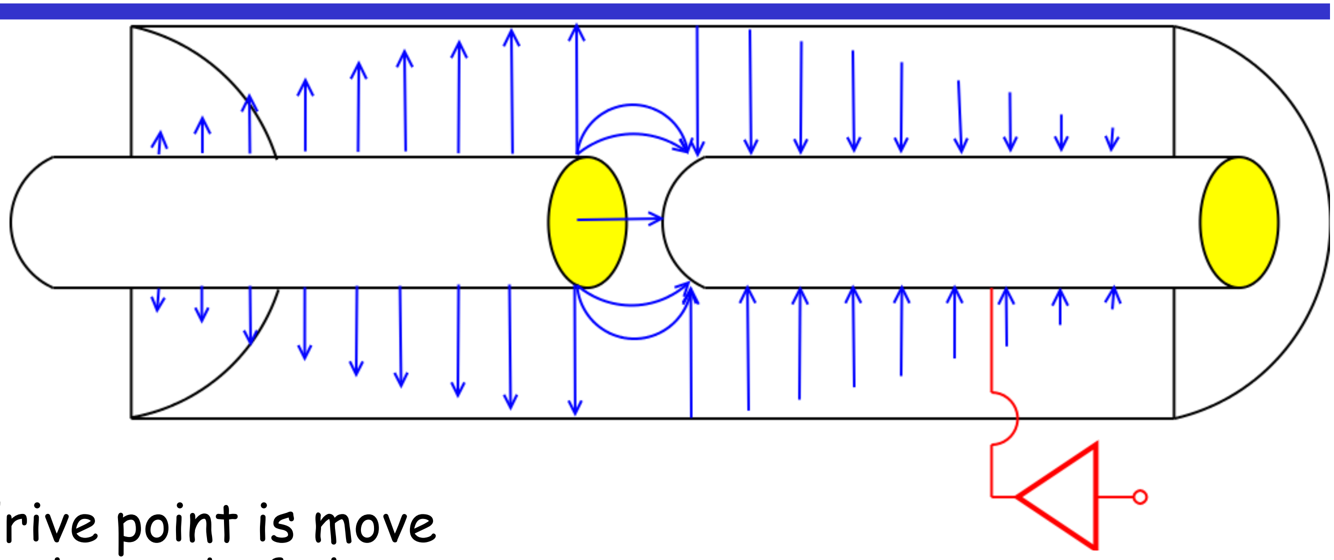
Mode Spectrum Example – Pill Box Cavity



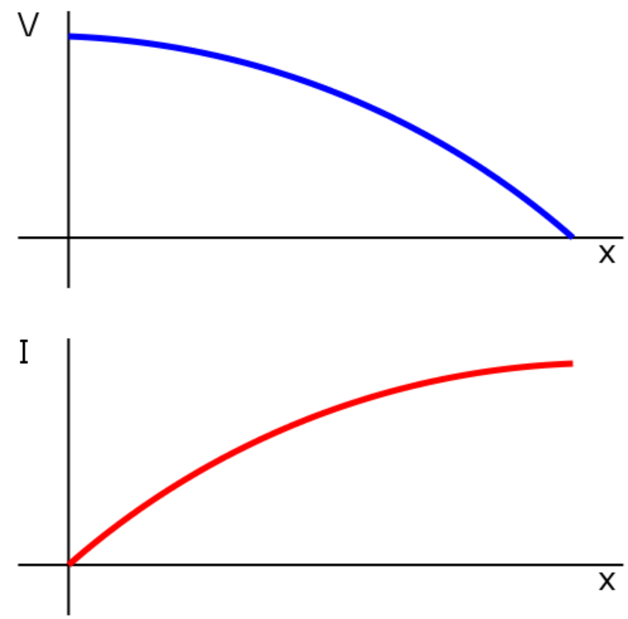


- The RLC model is only around a given mode
- Each mode will a different value of R,L, and C

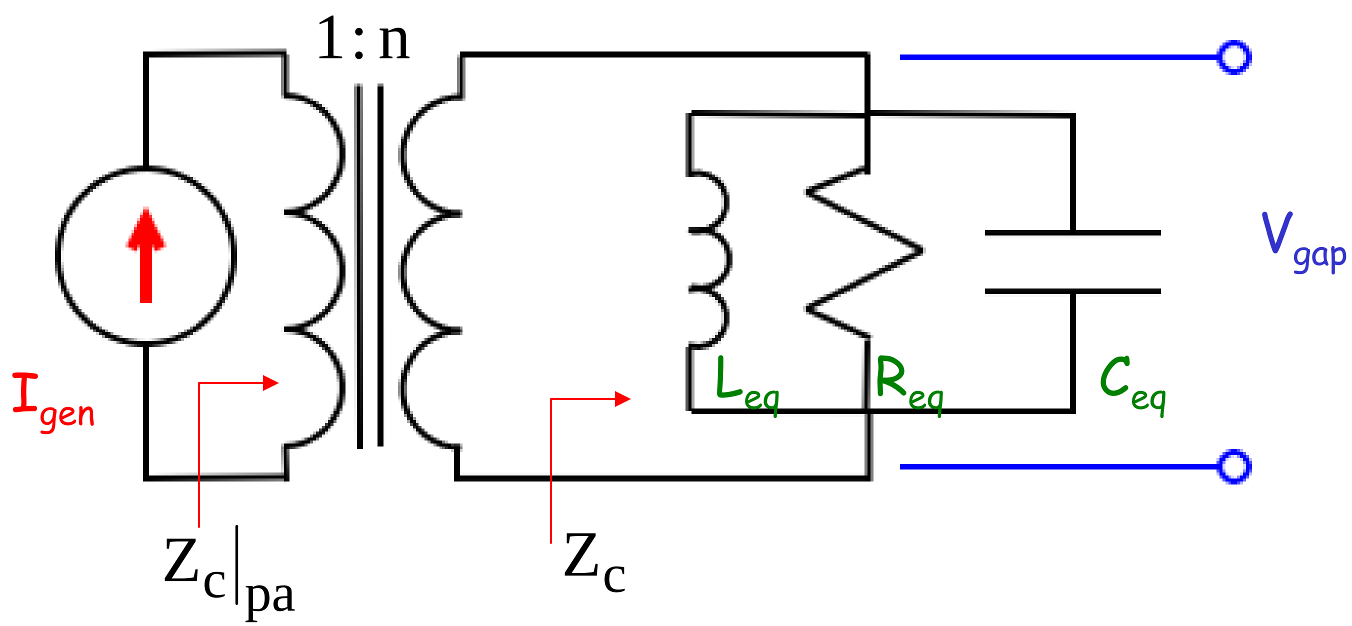
Cavity Coupling – Offset Coupling



- As the drive point is move closer to the end of the cavity (away from the gap), the amount of current needed to develop a given voltage must increase
- Therefore the input impedance of the cavity as seen by the power amplifier decreases as the drive point is moved away from the gap



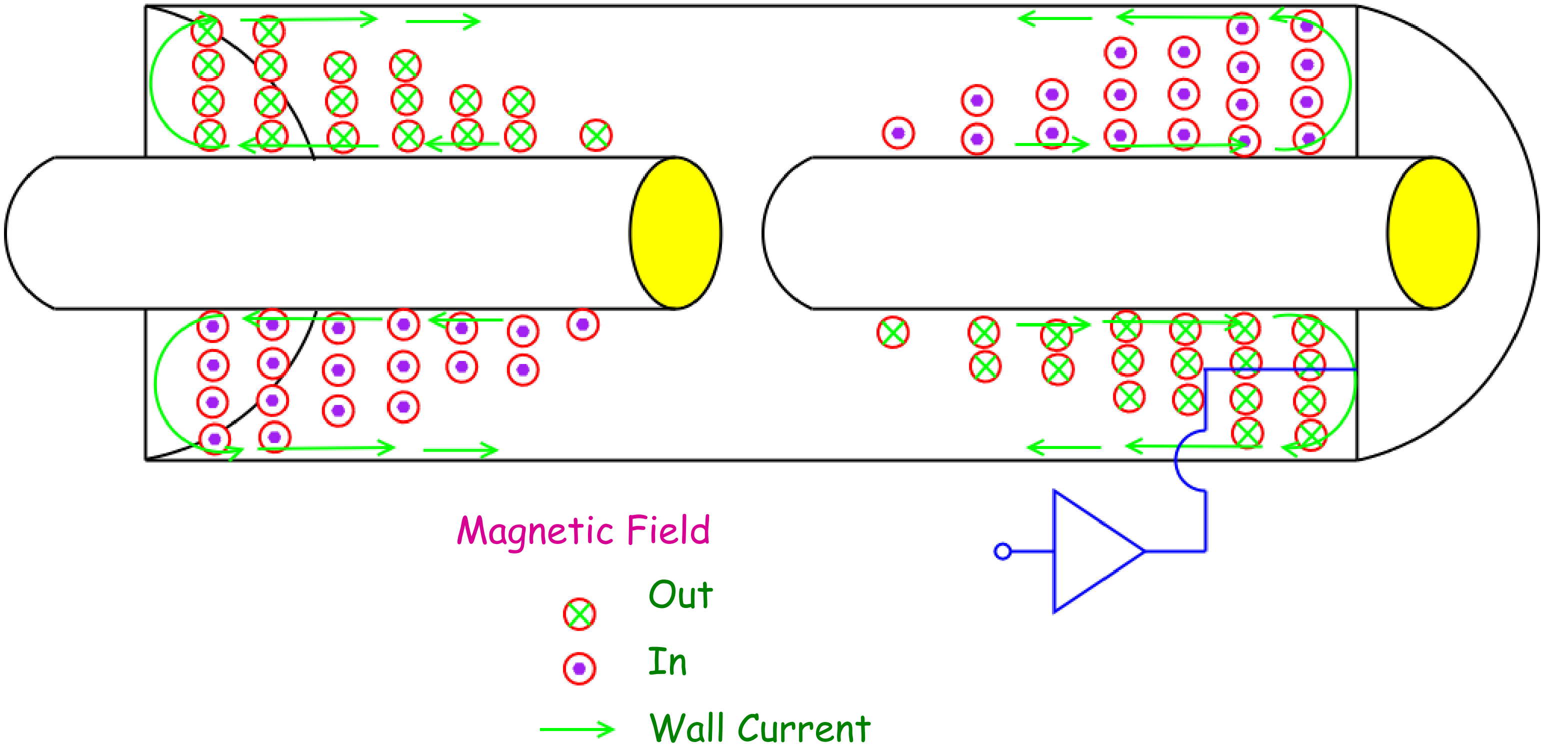
Cavity Coupling



$$Z_c\big|_{pa} = \frac{1}{n^2} Z_c$$

- We can model moving the drive point as a transformer
- Moving the drive point away from the gap increases the transformer turn ratio (n)

Inductive Coupling

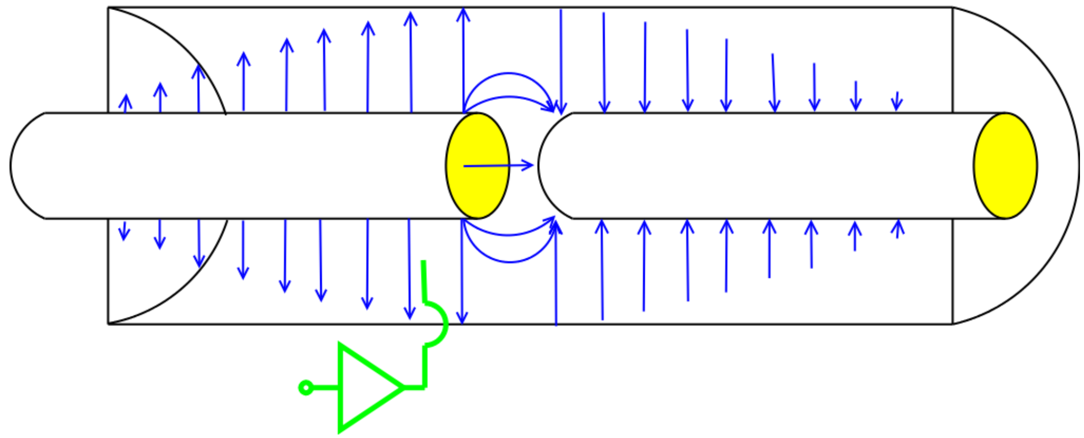


- For inductive coupling, the PA does not have to be directly attached to the beam tube.
- The magnetic flux thru the coupling loop couples to the magnetic flux of the cavity mode
- The transformer ratio n = Total Flux / Coupler Flux

Accelerator Division

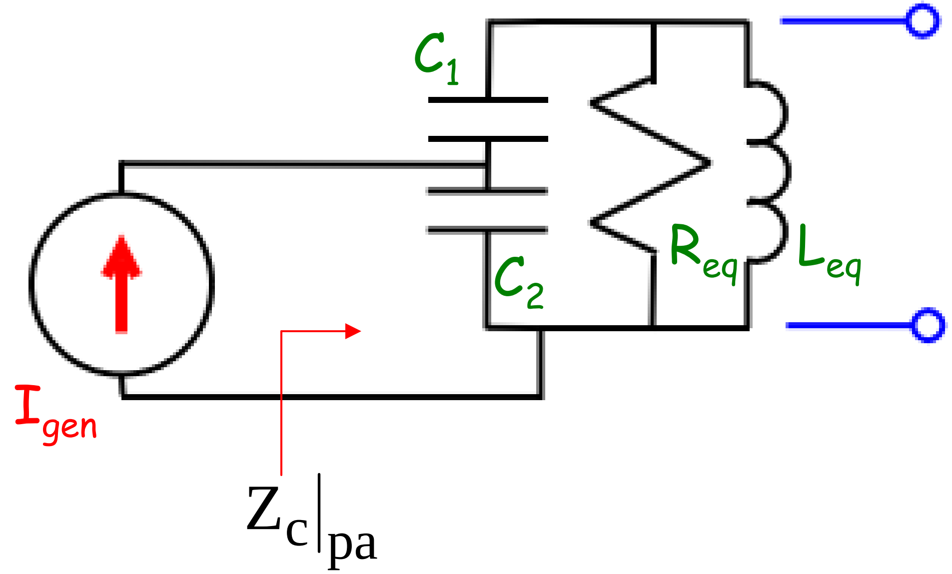
Capacitive Coupling

If the drive point does not physically touch the cavity gap, then the coupling can be described by breaking the equivalent cavity capacitance into two parts.



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

As the probe is pulled away from the gap, C_2 increases and the impedance of the cavity as seen by the power amp decreases

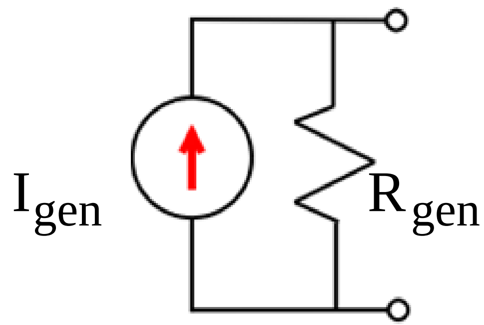


$$Z_c|_{pa} \approx \frac{1}{C_2 / C_{eq}} Z_c$$

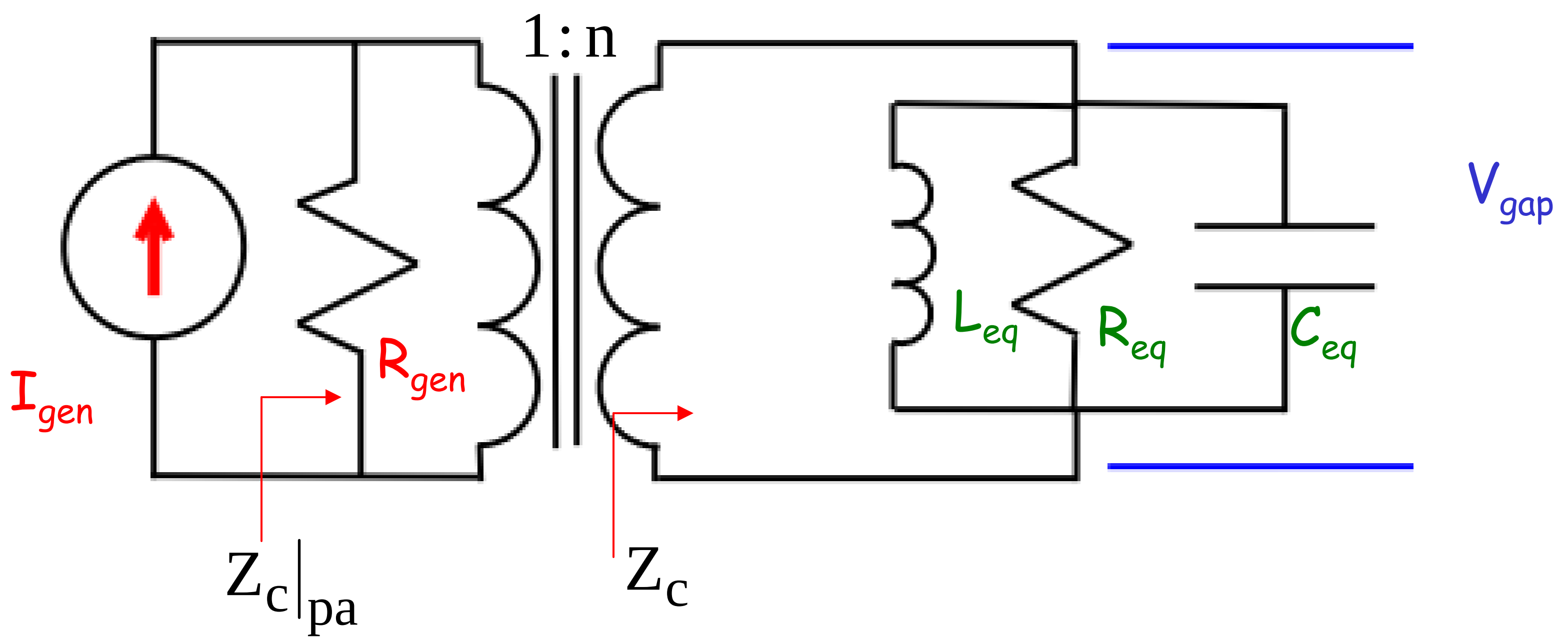
Power Amplifier Internal Resistance

- So far we have been ignoring the internal resistance of the power amplifier.
  - This is a good approximation for tetrode power amplifiers that are used at Fermilab in the Booster and Main Injector
  - This is a bad approximation for klystrons protected with isolators
- Every power amplifier has some internal resistance

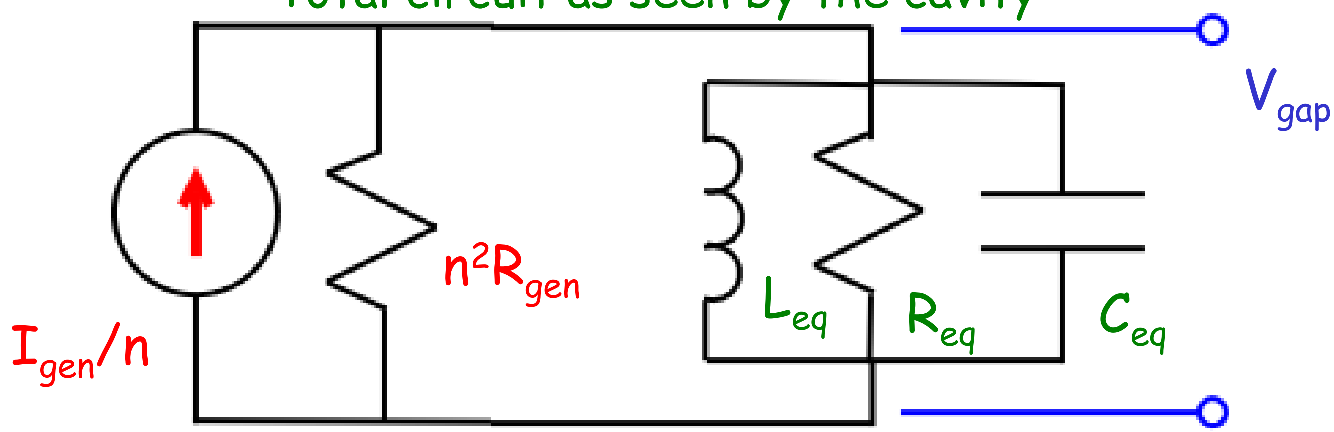
$$R_{gen} = \left.\frac{\Delta V_{gen}}{\Delta I_{gen}}\right|_{I_L = 0}$$



Total Cavity Circuit



Total circuit as seen by the cavity



Loaded Q

- The generator resistance is in parallel with the cavity resistance.
- The total resistance is now lowered.

$$\frac{1}{R_L} = \frac{1}{R_{eq}} + \frac{1}{n^2 R_{gen}}$$

- The power amplifier internal resistance makes the total Q of the circuit smaller (d'Q)

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

$$Q_L = \omega_o R_L C_{eq}$$ **Loaded Q**

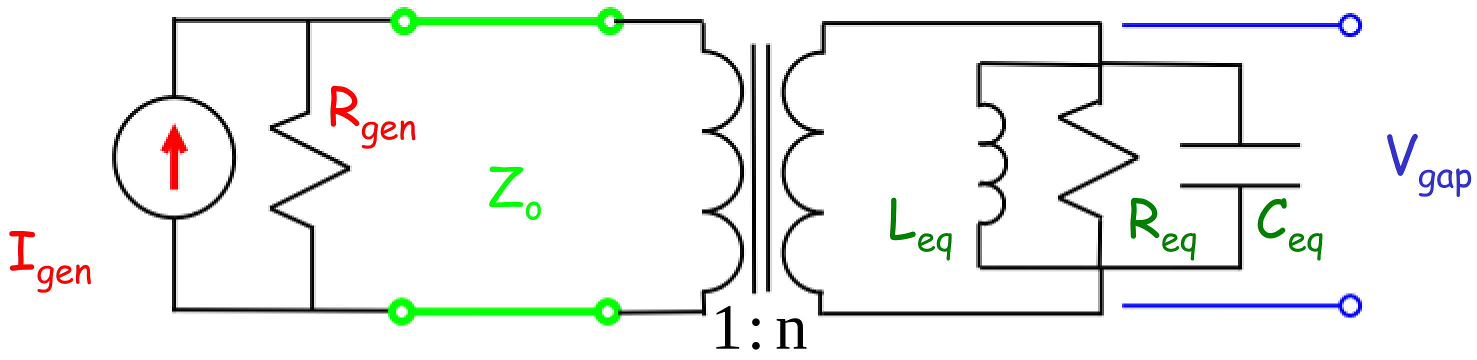
$$Q_o = \omega_o R_{eq} C_{eq}$$ **Unloaded Q**

$$Q_{ext} = \omega_o n^2 R_{gen} C_{eq}$$ **External Q**

Cavity Coupling

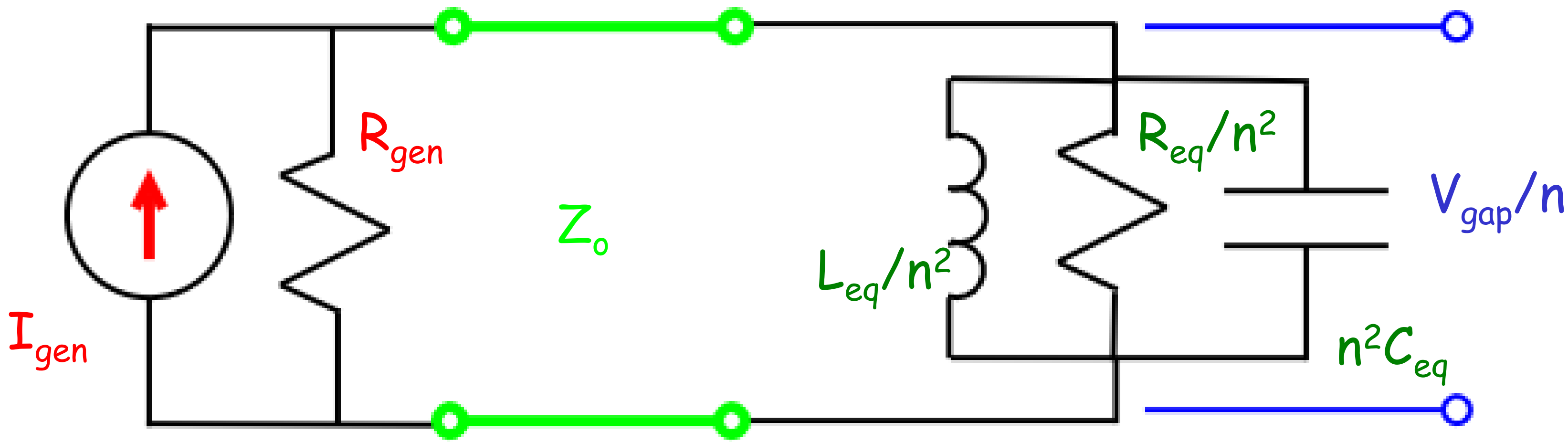
- The cavity is attached to the power amplifier by a transmission line.
  - In the case of power amplifiers mounted directly on the cavity such as the Fermilab Booster or Main Injector, the transmission line is infinitesimally short.
- The internal impedance of the power amplifier is usually matched to the transmission line impedance connecting the power amplifier to the cavity.
  - As in the case of a Klystron protected by an isolator
  - As in the case of an infinitesimally short transmission line

$$R_{gen} = Z_o$$



Cavity Coupling

Look at the cavity impedance from the power amplifier point of view:



Assume that the power amplifier is matched ($R_{gen}=Z_o$) and define a coupling parameter as the ratio of the real part of the cavity impedance as seen by the power amplifier to the characteristic impedance.

$$r_{cpl} = \frac{R_{eq}/n^2}{Z_o}$$

$r_{cpl} < 1$ **under-coupled**

$r_{cpl} = 1$ **Critically-coupled**

$r_{cpl} > 1$ **over-coupled**

Which Coupling is Best?

- Critically coupled would provide maximum power transfer to the cavity.
- However, some power amplifiers (such as tetrodes) have very high internal resistance compared to the cavity resistance and the systems are often under-coupled.
  - The limit on tetrode power amplifiers is dominated by how much current they can source to the cavity
- Some cavities a have extremely low losses, such as superconducting cavities, and the systems are sometimes over-coupled.
- An intense beam flowing though the cavity can load the cavity which can effect the coupling.

Measuring Cavity Coupling

The frequency response of the cavity at a given mode is:

$$Z_c(j\omega) = \frac{R_{eq}}{Q} \frac{j\omega\omega_o}{(j\omega)^2 + j\omega\frac{\omega_o}{Q} + \omega_o{}^2}$$

which can be re-written as:

$$Z_c(j\omega) = R_{eq}\cos(\phi)e^{j\phi}$$

$$\tan(\phi) = Q\frac{\omega_o{}^2 - \omega^2}{\omega_o\omega}$$

Cavity Coupling

The reflection coefficient as seen by the power amplifier is:

$$\Gamma = \frac{Z_c - n^2 Z_o}{Z_c + n^2 Z_o}$$

$$\Gamma = \frac{r_{cpl}\cos(\phi)e^{j\phi} - 1}{r_{cpl}\cos(\phi)e^{j\phi} + 1}$$

This equation traces out a circle on the reflection (u,v) plane

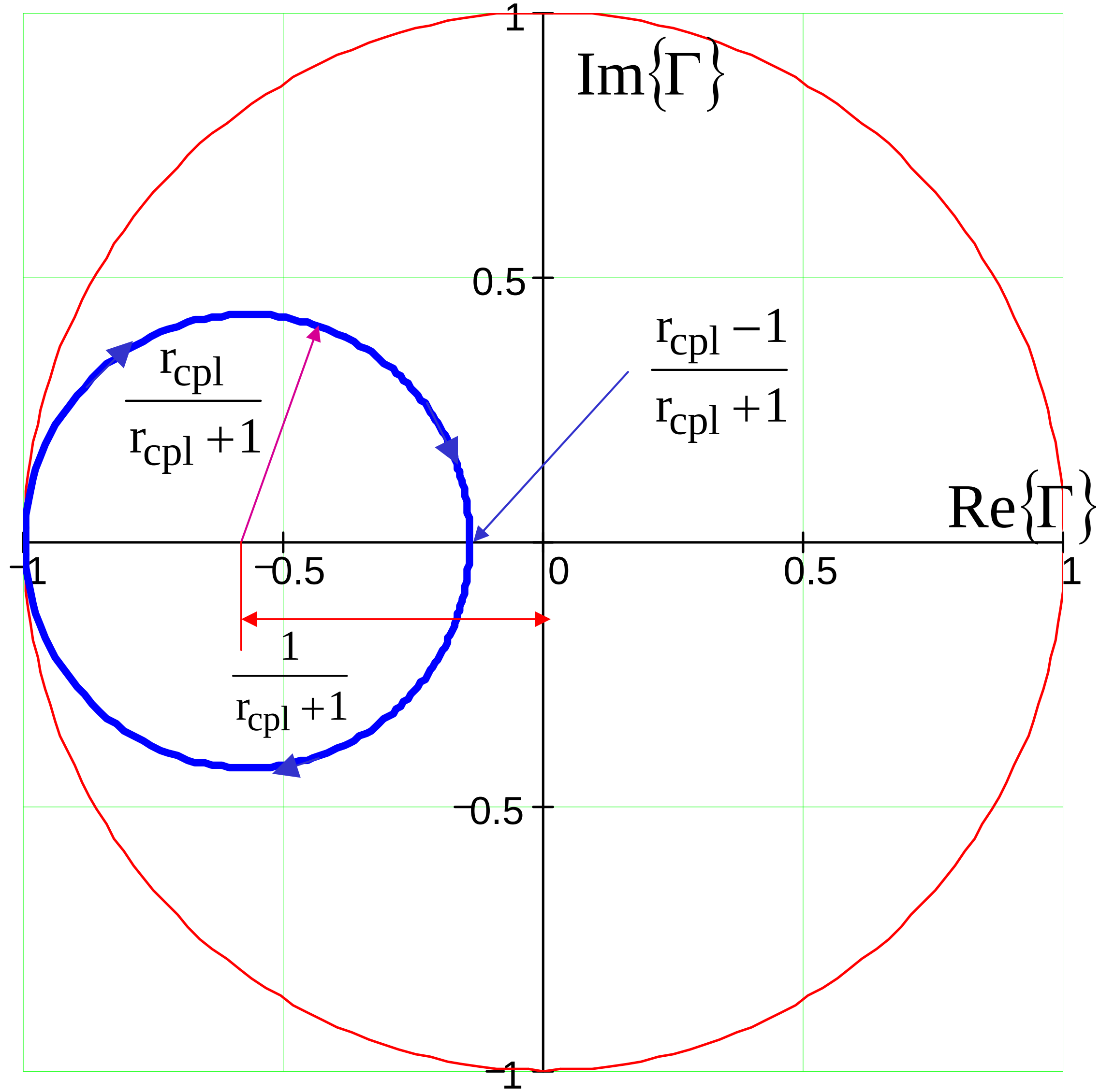
Cavity Coupling

$$\text{Radius} = \frac{r_{cpl}}{r_{cpl}+1}$$

$$\text{Center} = \left(\frac{-1}{r_{cpl}+1}, 0\right)$$

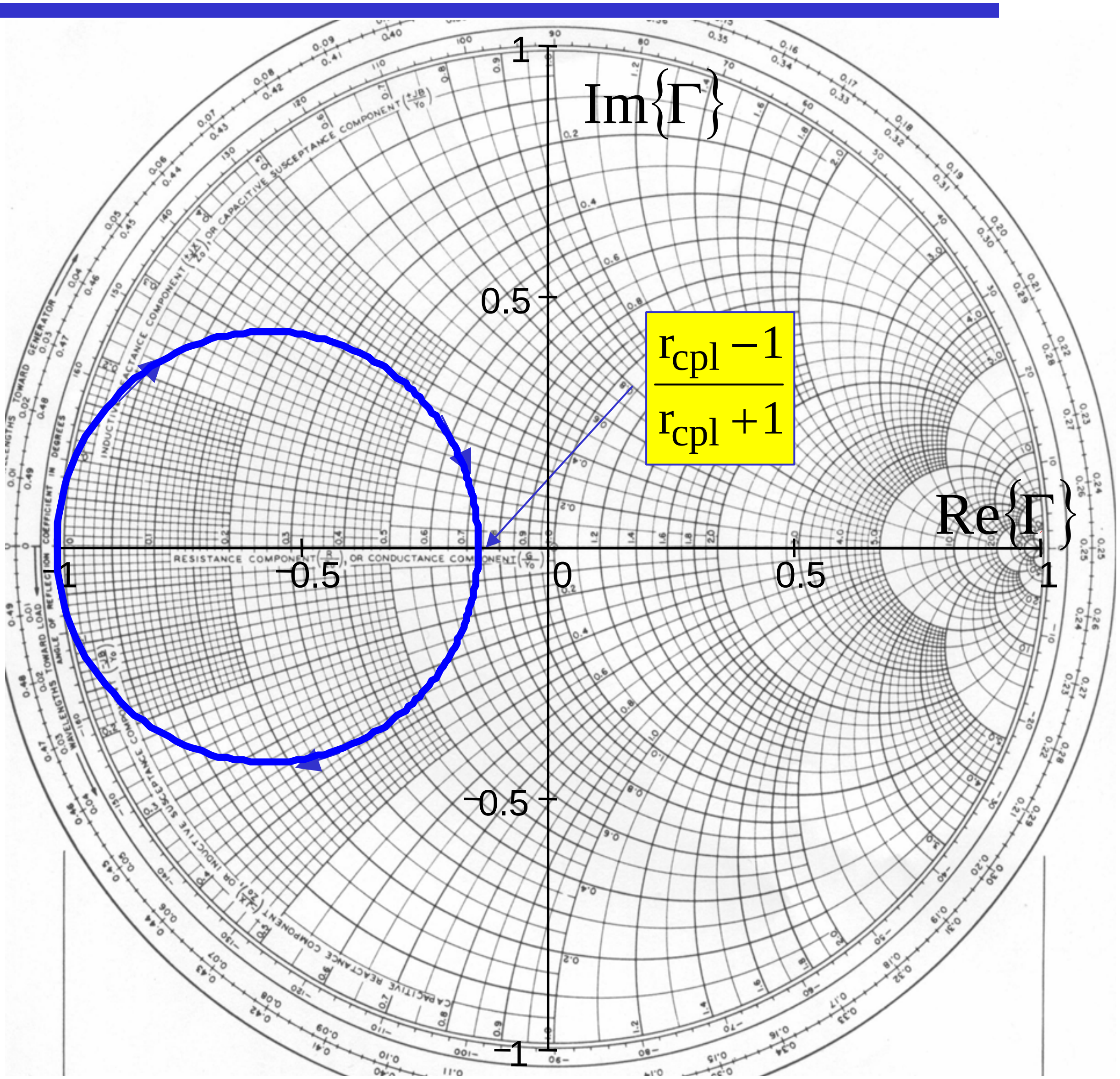
Left edge $(\phi = +/- \pi/2) = (-1, 0)$

Right edge $(\phi = 0) = \left(\frac{r_{cpl}-1}{r_{cpl}+1}, 0\right)$

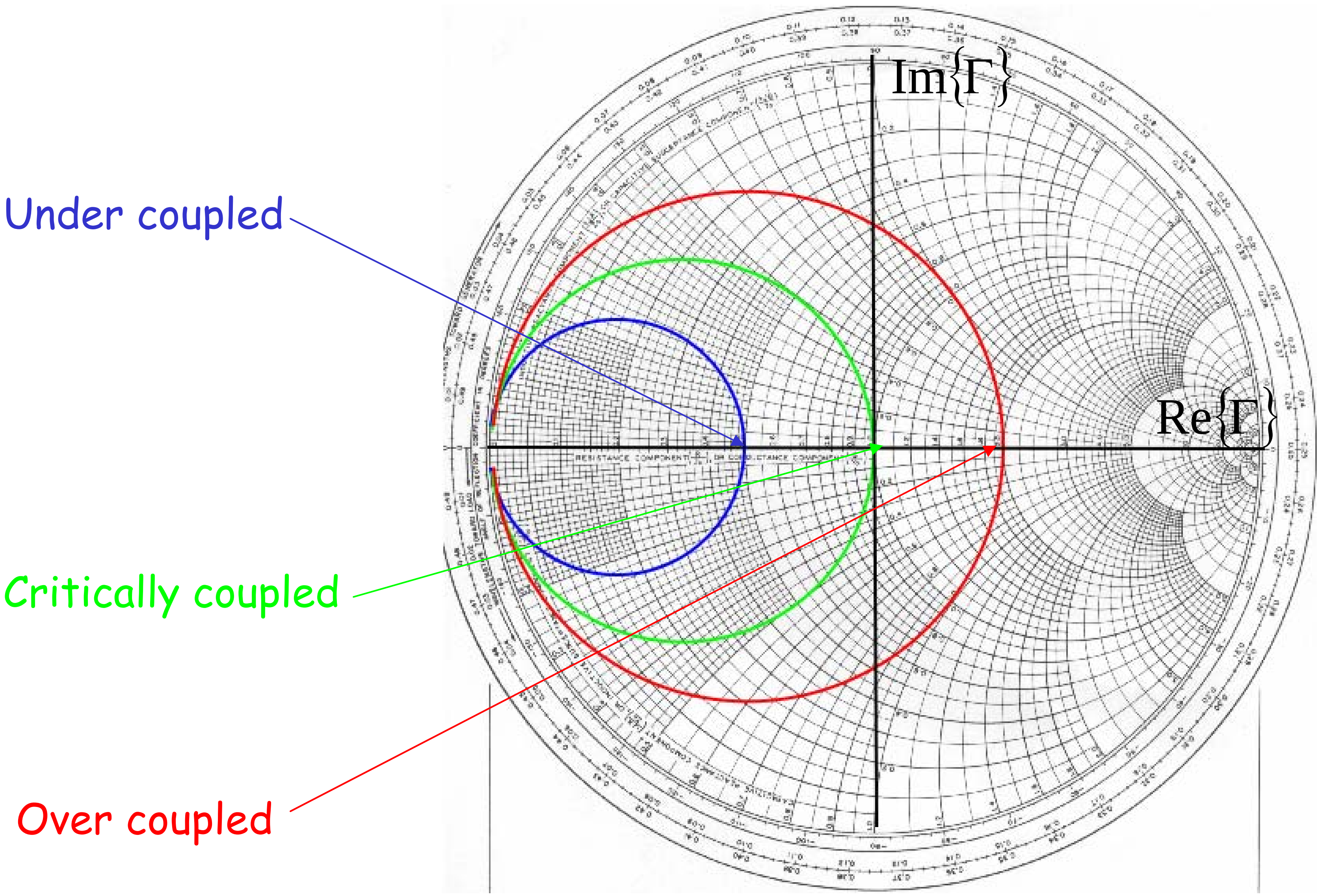


Cavity Coupling

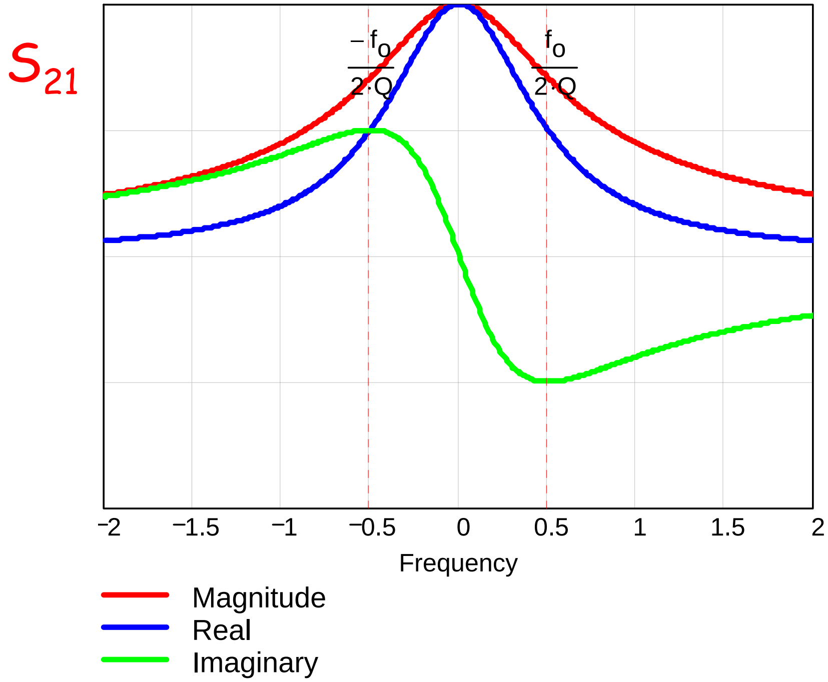
- The cavity coupling can be determined by:
  - measuring the reflection coefficient trajectory of the input coupler
  - Reading the normalized impedance of the extreme right point of the trajectory directly from the Smith Chart

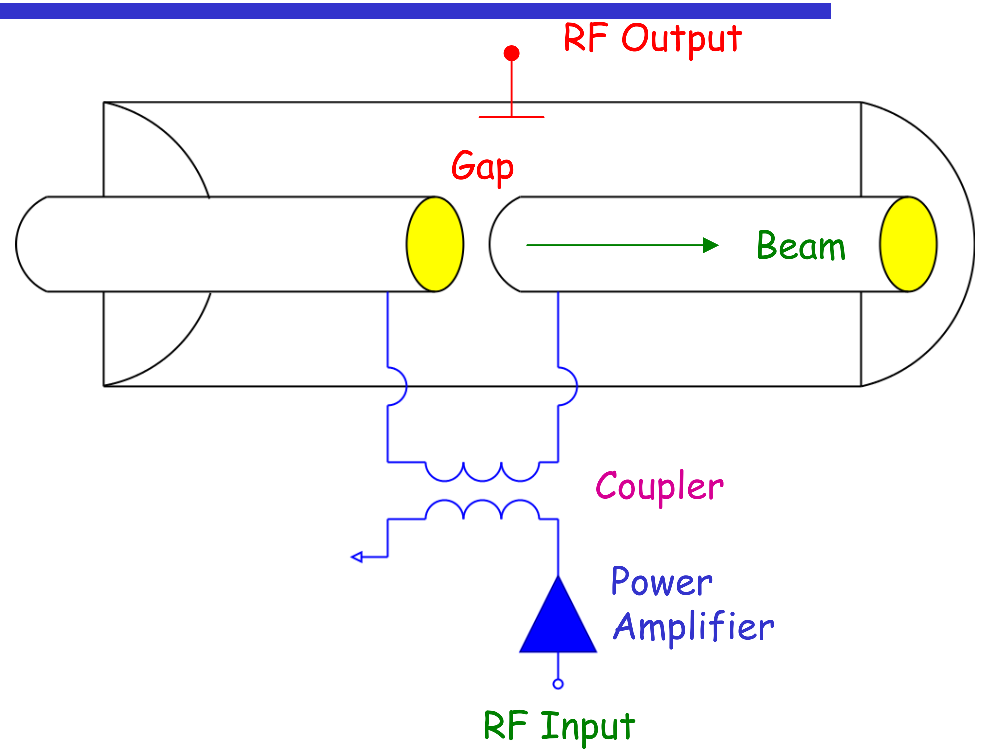


Cavity Coupling



Measuring the **Loaded** Q of the Cavity





- The simplest way to measure a cavity response is to drive the coupler with RF and measure the output RF from a small detector mounted in the cavity.
- Because the coupler "loads" the cavity, this measures the loaded Q of the cavity
  - which depending on the coupling, can be much different than the unloaded Q
  - Also note that changing the coupling in the cavity, can change the cavity response significantly

Measuring the **Unloaded** Q of a Cavity

- If the coupling is not too extreme, the loaded and unloaded Q of the cavity can be measured from reflection (S_{11}) measurements of the coupler.

At:

$$\omega = \omega_o \mp \frac{\omega_o}{Q_o}$$

Unloaded Q!

$$\phi = \pm \frac{\pi}{4}$$

$$\text{Im}\{Z_c\} = \pm \text{Re}\{Z_c\}$$

For:

$$\text{Im}\{Z_c\} = \pm \text{Re}\{Z_c\}$$

$$u^2 + (v \pm 1)^2 = 2$$

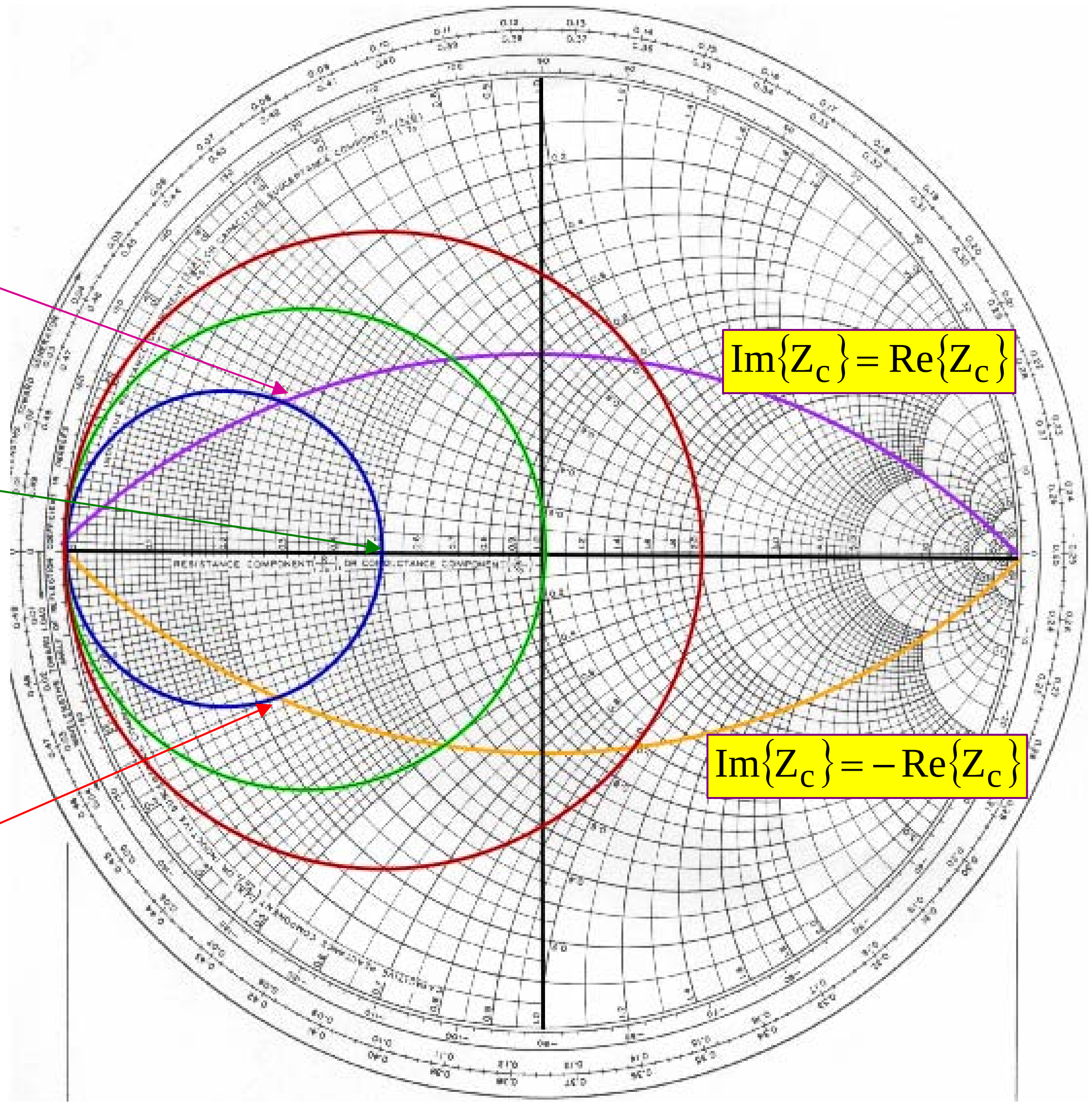
Circles on the Smith Chart

where:

$$\Gamma = u + jv$$

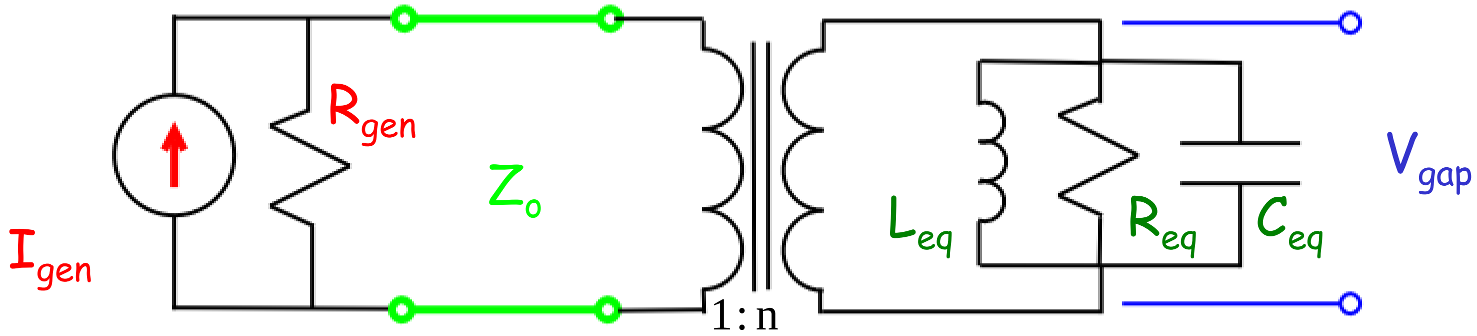
Measuring the **Unloaded** Q of a Cavity

- Measure frequency (ω_-) when:
  $$\mathrm{Im}\{Z_c\} = \mathrm{Re}\{Z_c\}$$
- Measure resonant frequency (ω_o)
- Measure frequency (ω_+) when:
  $$\mathrm{Im}\{Z_c\} = \mathrm{Re}\{Z_c\}$$
- Compute
  $$Q_o = \frac{\omega_o}{\omega_+ - \omega_-}$$



Measuring the **Loaded** Q of a Cavity

- Measure the coupling parameter (r_{cpl})
- Measure the unloaded Q (Q_o)



$$R_{eq} = n^2 r_{cpl} Z_o$$

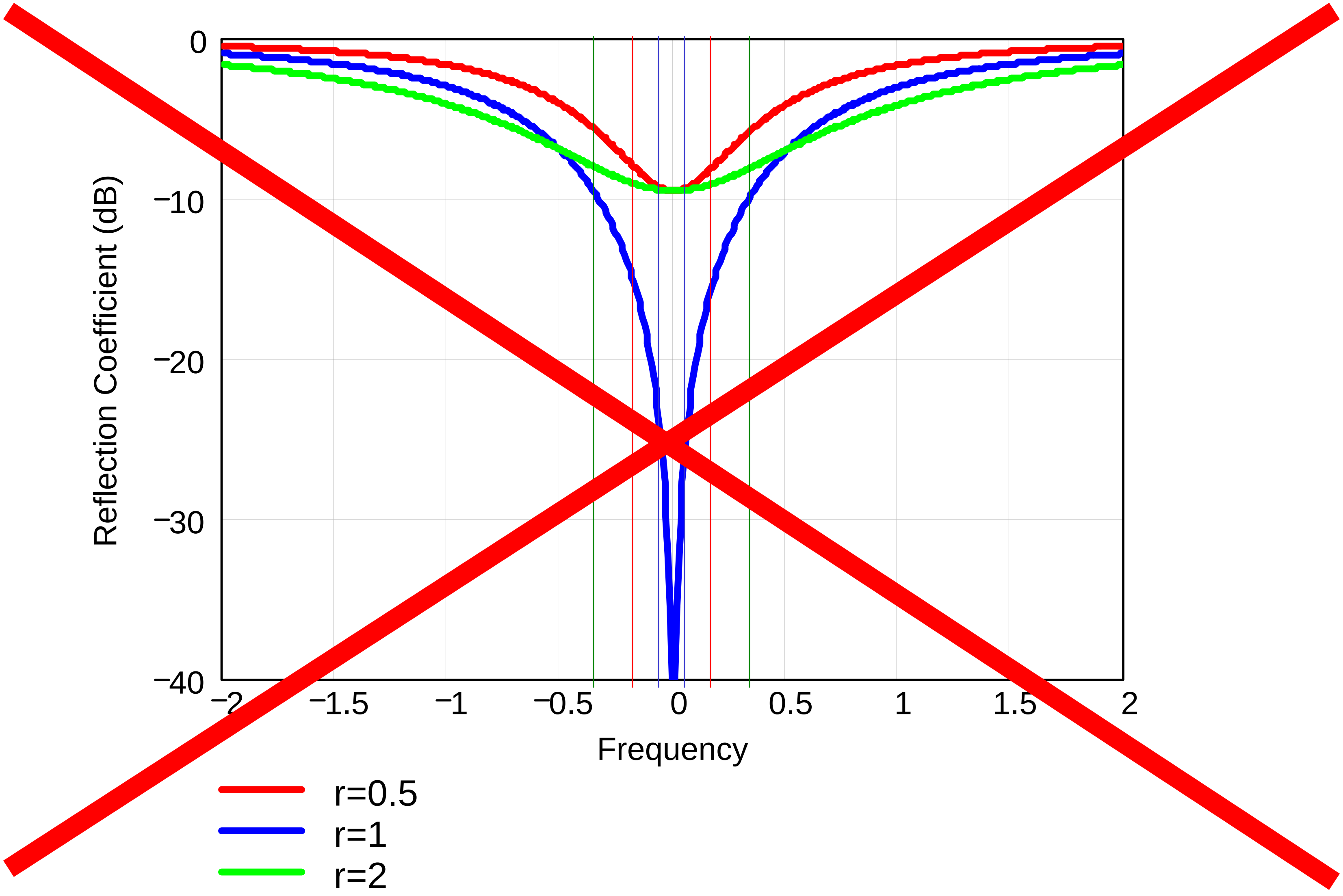
$$Q_o = \omega_o n^2 r_{cpl} Z_o C_{eq}$$

$$Q_{ext} = \omega_o n^2 Z_o C_{eq}$$

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

$$Q_L = \frac{Q_o}{r_{cpl} + 1}$$

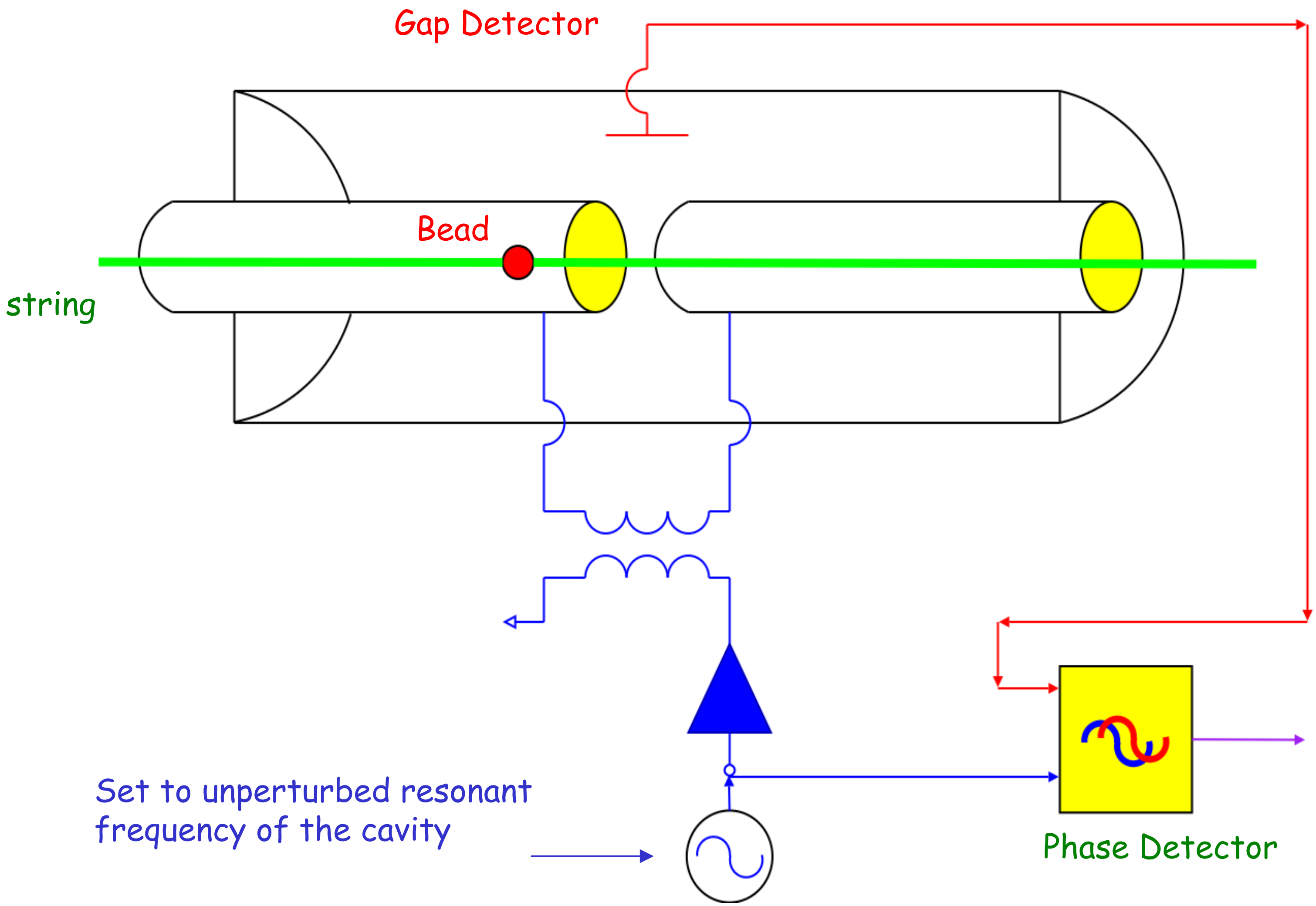
NEVER - EVER!!


0
−10
−20
−30
−40
Reflection Coefficient (dB)
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
Frequency
r=0.5
r=1
r=2

Bead Pulls

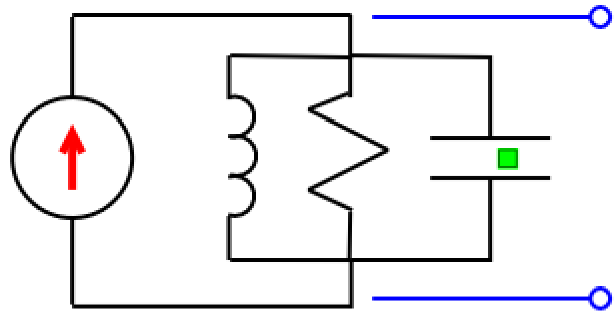
- The Bead Pull is a technique for measuring the fields in the cavity and the equivalent impedance of the cavity as seen by the beam
  - In contrast to measuring the impedance of the cavity as seen by the power amplifier through the coupler

Bead Pull Setup



Bead Pulls

- In the capacitor of the RLC model for the cavity mode consider placing a small dielectric cube
  - Assume that the small cube will not distort the field patterns appreciably
- The stored energy in the capacitor will change



$$W_E = \frac{1}{4} C_{eq} V_{gap}^{\;2} - \frac{1}{4} \varepsilon_o E_c^{\;2} dv + \frac{1}{4} \varepsilon_r \varepsilon_o E_c^{\;2} dv$$

Total original Electric energy

Original Electric energy in the cube

new Electric energy in the cube

Where E_c is the electric field in the cube

dv is the volume of the cube

ε_o is the permittivity of free space

ε_r is the relative permittivity of the cube

$$W_E = \frac{1}{4} \iiint_{vol} \varepsilon \left|\vec{E}\right|^2 dv$$

Bead Pulls

The equivalent capacitance of the capacitor with the dielectric cube is:

$$W_E = \frac{1}{4} C V_{gap}^{\;2} = \frac{1}{4}\left(C_{eq} + \Delta C\right) V_{gap}^{\;2}$$

$$\Delta C = \varepsilon_o \left(\varepsilon_r - 1\right) dv \left(\frac{E_c}{V_{gap}}\right)^2$$

The resonant frequency of the cavity will shift

$$\left(\omega_o + \Delta\omega\right)^2 = \frac{1}{L_{eq}\left(C_{eq} + \Delta C\right)}$$

Bead Pulls

For $\Delta\omega \ll \omega_o$ and $\Delta C \ll C_{eq}$

$$\frac{\Delta\omega}{\omega_o} = \frac{1}{2}\frac{\Delta C}{C_{eq}}$$

$$= \frac{\frac{1}{4}\varepsilon_o(\varepsilon_r - 1)dvE_c{}^2}{\frac{1}{2}C_{eq}V_{gap}{}^2}$$

$$\frac{\Delta\omega}{\omega_o} = \frac{\Delta W_E}{W_T}$$

Bead Pulls

- Had we used a metallic bead (μ_r>1) or a metal bead:

$$\frac{\Delta\omega}{\omega_o} = \frac{\Delta W_E - \Delta W_H}{W_T}$$

- Also, the shape of the bead will distort the field in the vicinity of the bead so a geometrical form factor must be used.
- For a small dielectric bead of radius a

$$\frac{\Delta\omega}{\omega_o} = -\pi a^3 \varepsilon_o \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2} \right) \frac{{E_b}^2}{W_T}$$

- For a small metal bead with radius a

$$\frac{\Delta\omega}{\omega_o} = -\frac{\pi a^3}{W_T} \left[\varepsilon_o {E_b}^2 + \frac{\mu_o}{2} {H_b}^2 \right]$$

A metal bead can be used to measure the E field only if the bead is placed in a region where the magnetic field is zero!

Bead Pulls

- In general, the shift in frequency is proportional to a form factor *F*

$$\frac{\Delta\omega}{\omega_o} = -F\frac{{E_b}^2}{W_T}$$

$$F = \pi a^3 \varepsilon_o \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)$$ Dielectric bead

$$F = \pi a^3 \varepsilon_o$$ Metal bead

Bead Pulls

From the definition of cavity Q:

$$Q = \omega_o \frac{W_T}{P_L}$$

$$P_L = \frac{1}{2}\frac{V_{gap}^{\;2}}{R_{eq}}$$

$$W_T = \frac{1}{2\omega_o}\frac{V_{gap}^{\;2}}{R_{eq}/Q}$$

$$\left(\frac{E_b(x,y,z)}{V_{gap}}\right)^2 = \frac{1}{F}\frac{1}{2\omega_o}\frac{1}{R_{eq}/Q}\frac{\Delta\omega(x,y,z)}{\omega_o}$$

Bead Pulls

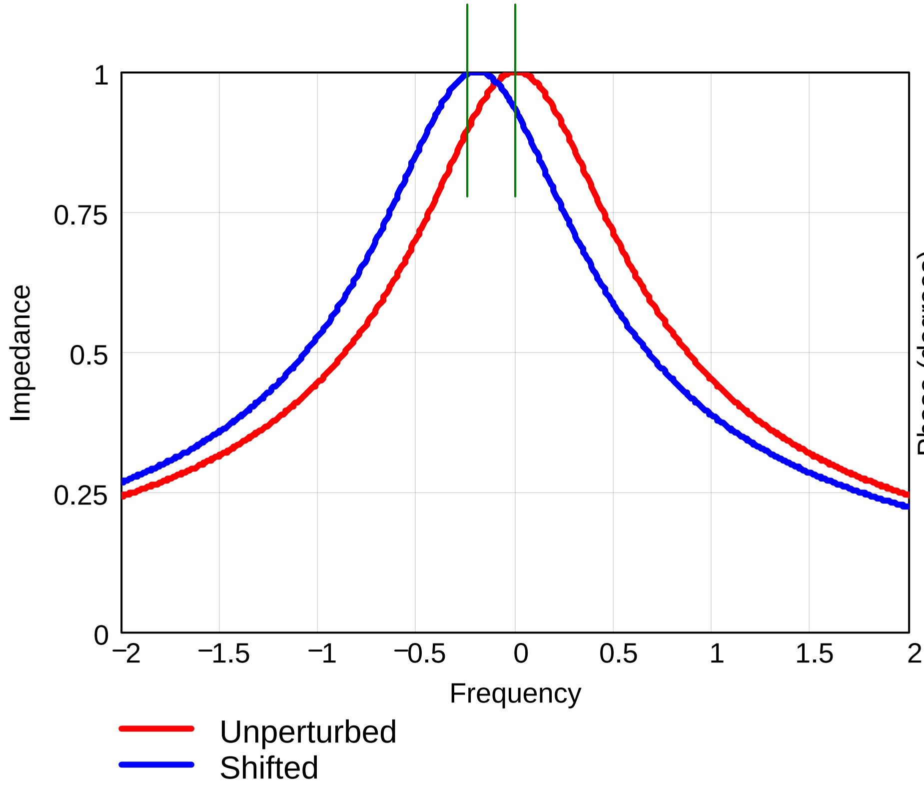
Since:

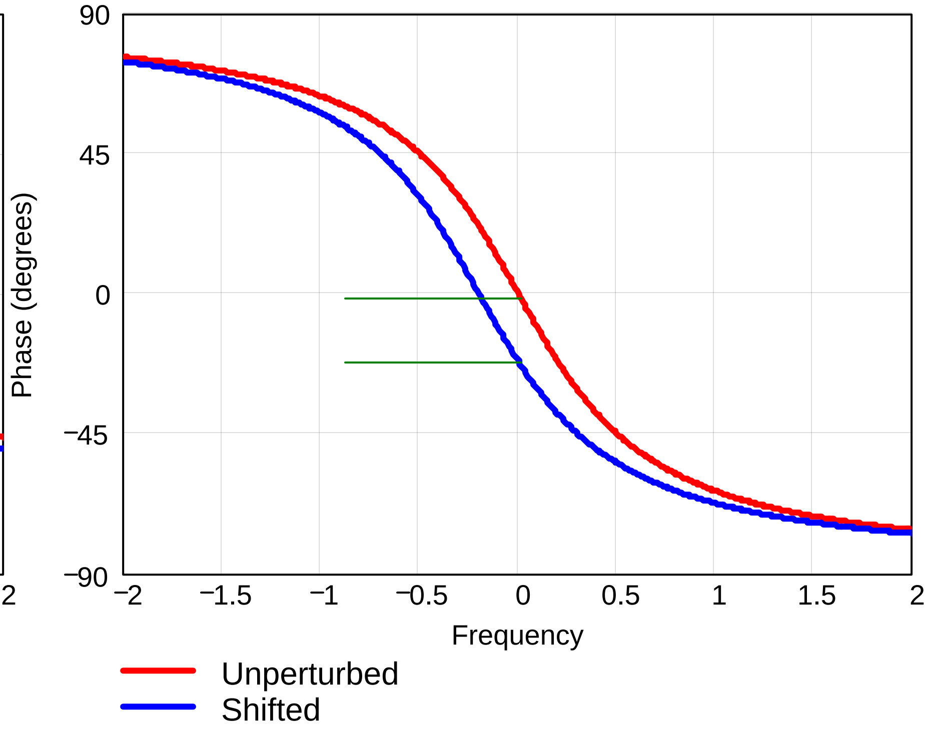
$$\int_{gap} E\left(x_{gap}, y_{gap}, z\right) dz = V_{gap}$$

$$\frac{R_{eq}}{Q} = \frac{1}{F}\frac{1}{2\omega_o}\left[\int_{gap}\sqrt{\frac{\Delta\omega\left(x_{gap}, y_{gap}, z\right)}{\omega_o}}dz\right]^2$$

Bead Pulls

- For small perturbations, shifts in the peak of the cavity response is hard to measure.
- Shifts in the phase at the unperturbed resonant frequency are much easier to measure.





Bead Pulls

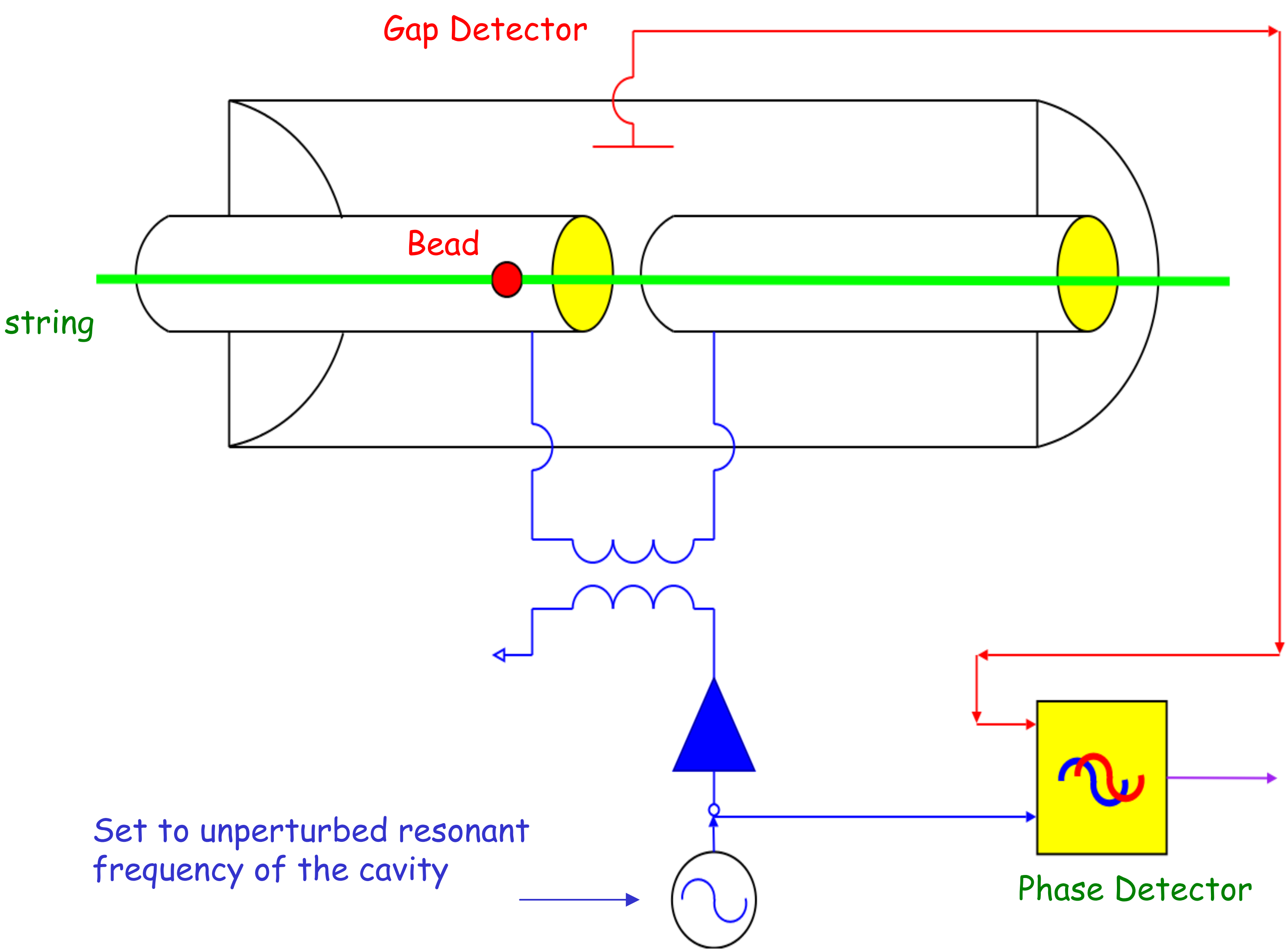
Since:

$$\tan(\phi) = Q\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right)$$

$$\approx 2Q\frac{\Delta\omega}{\omega_o}$$

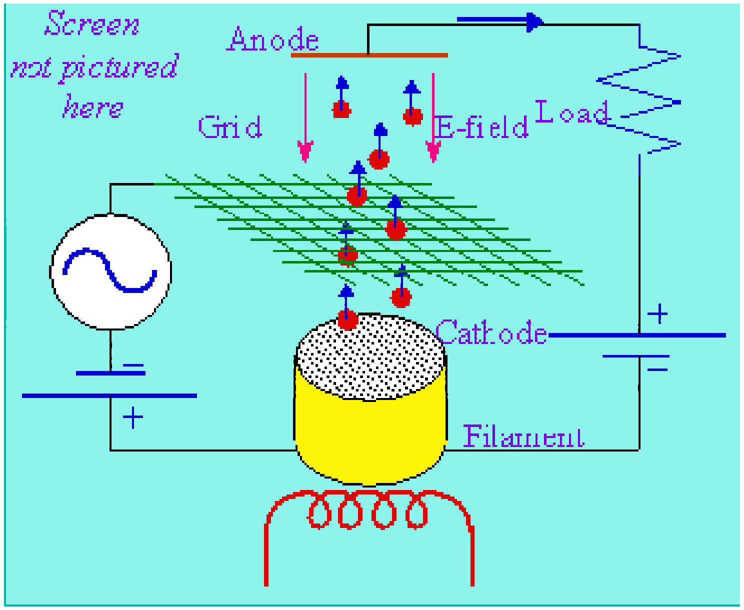
$$R_{eq} = \frac{1}{F}\frac{1}{2\omega_o}\left[\int_{gap}\sqrt{\frac{1}{2}\tan\left(\phi\left(x_{gap}, y_{gap}, z\right)\right)}dz\right]^2$$

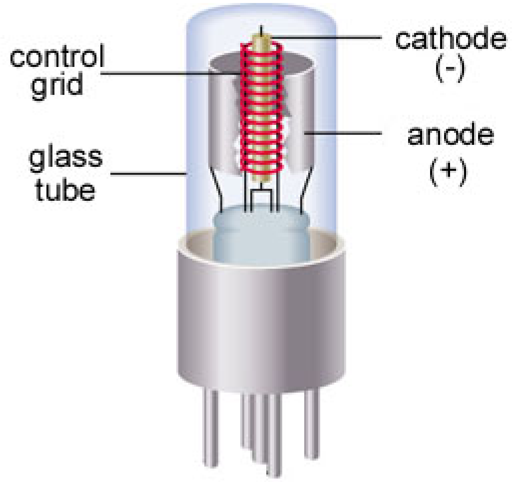
Bead Pulls


Gap Detector
Bead
string
Set to unperturbed resonant
frequency of the cavity
Phase Detector

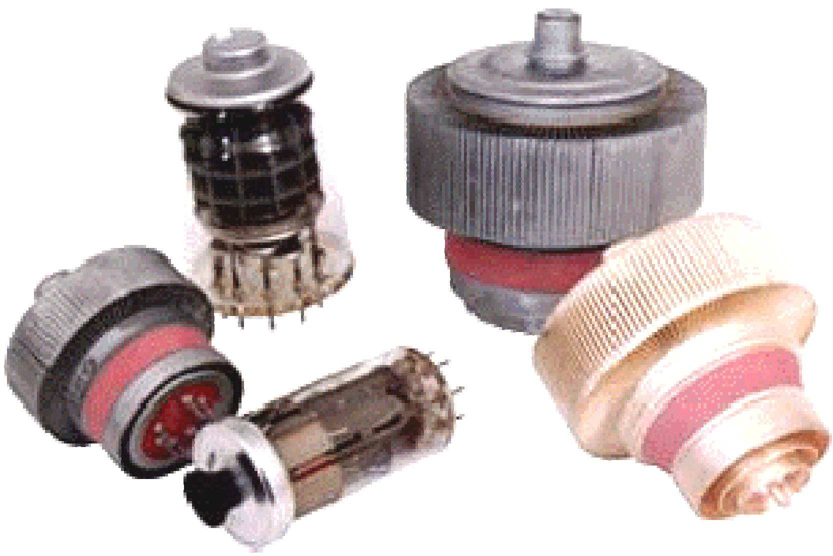
Power Amplifiers - Triode

- The triode is in itself a miniature electron accelerator
- The filament boils electrons off the cathode
- The electrons are accelerated by the DC power supply to the anode
- The voltage on the grid controls how many electrons make it to the anode
- The number of electrons flowing into the anode determines the current into the load.
- The triode can be thought of a voltage controlled current source
- The maximum frequency is inversely proportional to the transit time of electrons from the cathode to the anode.
  - Tetrodes are typically used at frequencies below 300 MHz

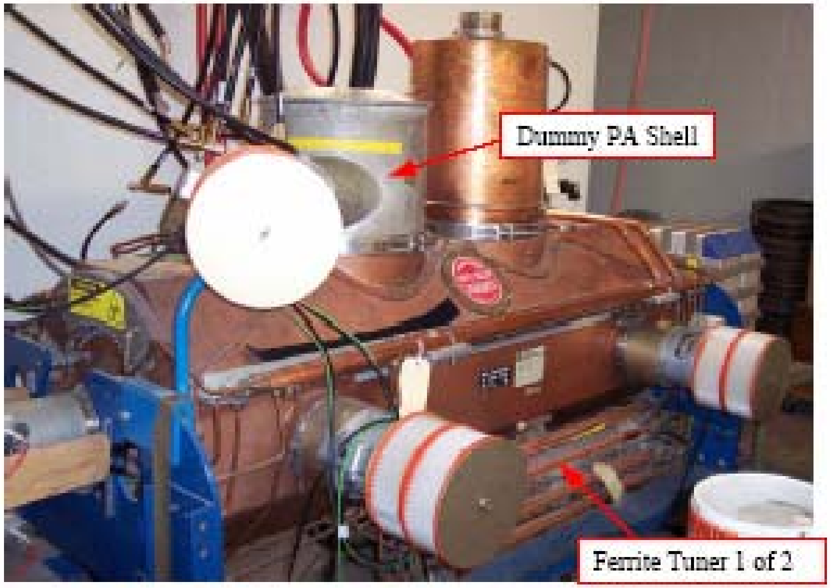


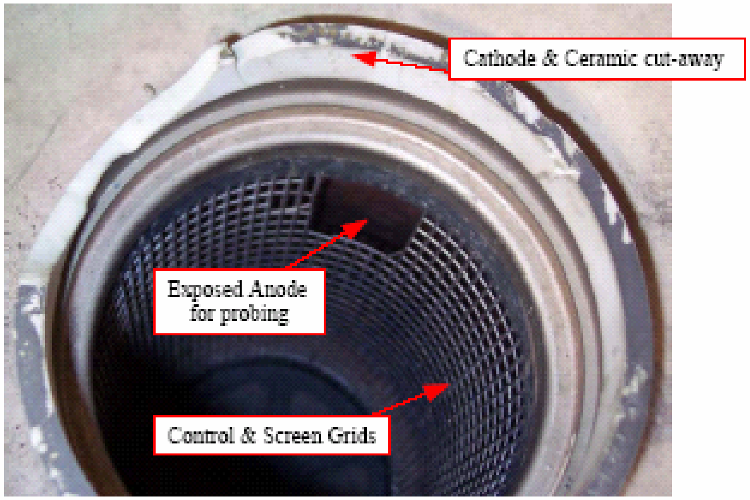


Tetrodes



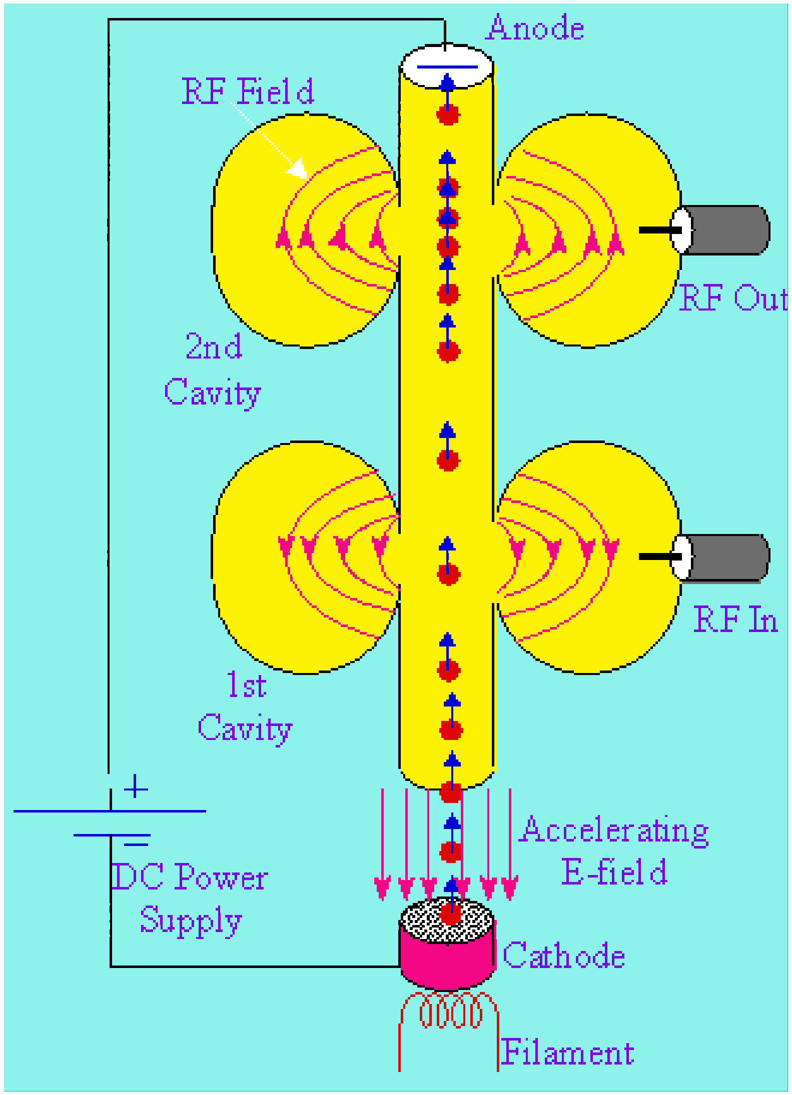
Courtesy of Tim Berenc



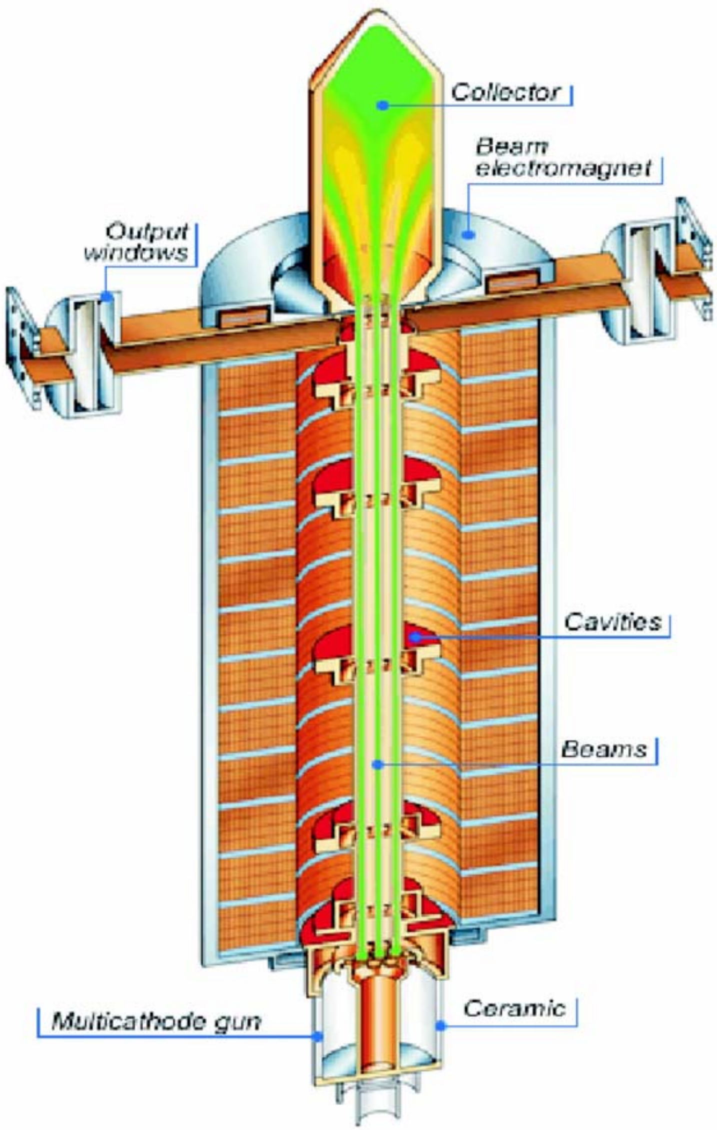


Klystrons

- The filament boils electrons off the cathode
- The velocity (or energy) of the electrons is modulated by the input RF in the first cavity
- The electrons drift to the cathode
- Because of the velocity modulation, some electrons are slowed down, some are sped up.
- If the output cavity is placed at the right place, the electrons will bunch up at the output cavity which will create a high intensity RF field in the output cavity
- Klystrons need a minimum of two cavities but can have more for larger gain.
- A Klystron size is determined by the size of the bunching cavities.
  - Klystrons are used at high frequencies (>500 MHz))

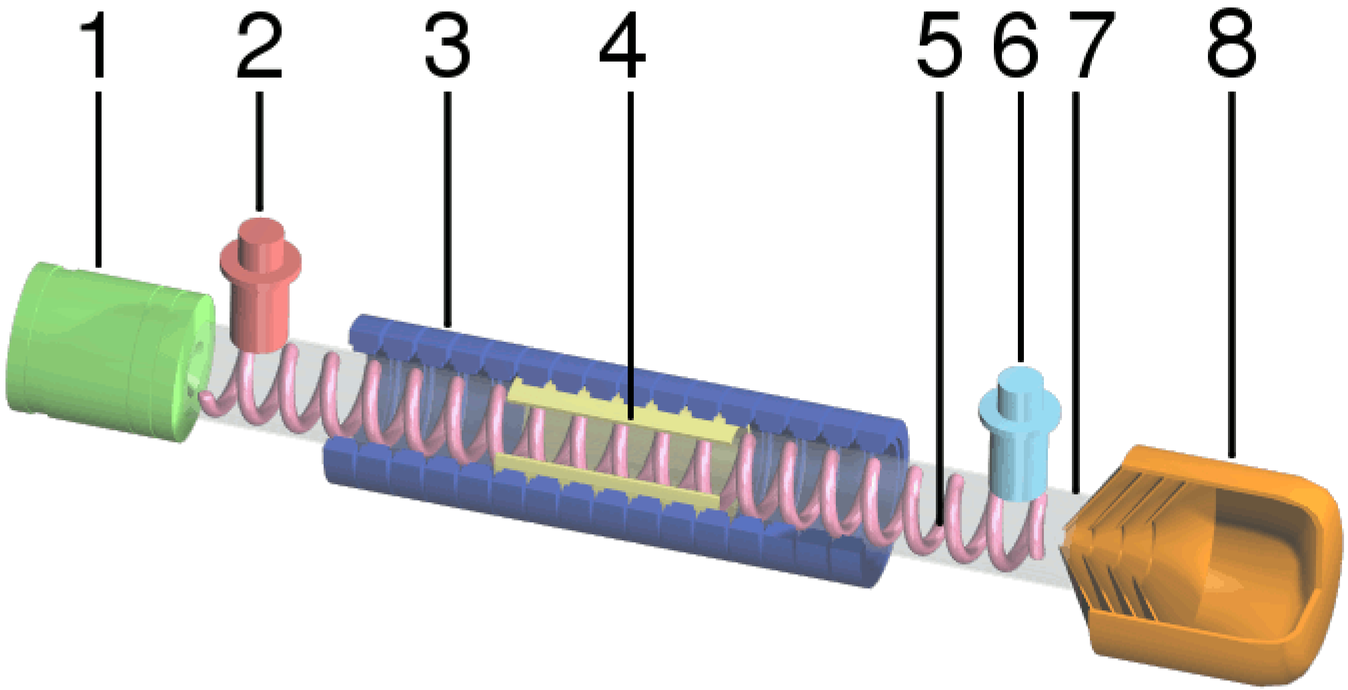


Klystrons


Collector
Beam electromagnet
Output windows
Cavities
Beams
Multicathode gun
Ceramic

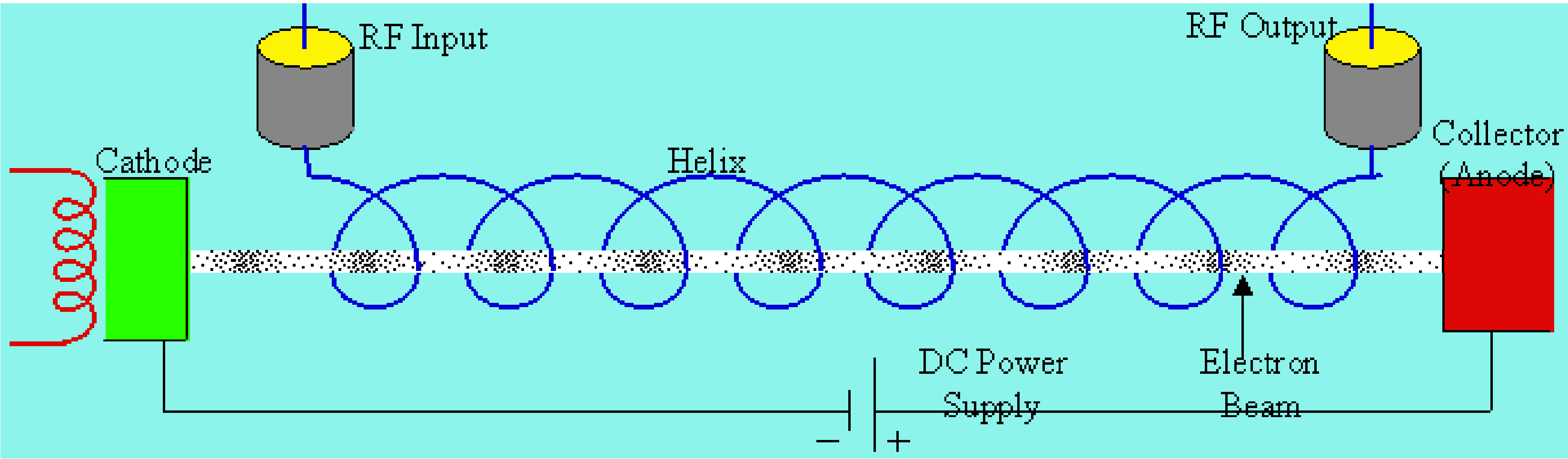
Traveling Wave Tube

Cutaway view of a TWT. (1) Electron gun; (2) RF input; (3) Magnets; (4) Attenuator; (5) Helix coil; (6) RF output; (7) Vacuum tube; (8) Collector.



Traveling Wave Tubes

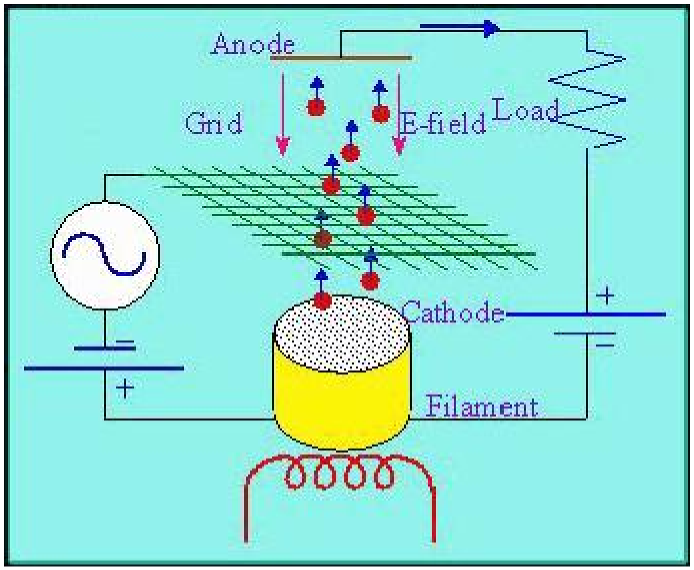
- Traveling wave tubes (TWTs) can have bandwidths as large as an octave ($f_{max} = 2 \times f_{min}$)
- TWTs have a helix which wraps around an electron beam
  - The helix is a slow wave electromagnetic structure.
  - The phase velocity of the slow wave matches the velocity of the electron beam
- At the input, the RF modulates the electron beam.
- The beam in turn strengthens the RF
- Since the velocities are matched, this process happens all along the TWT resulting in a large amplification at the output (40dB = 10000 x)

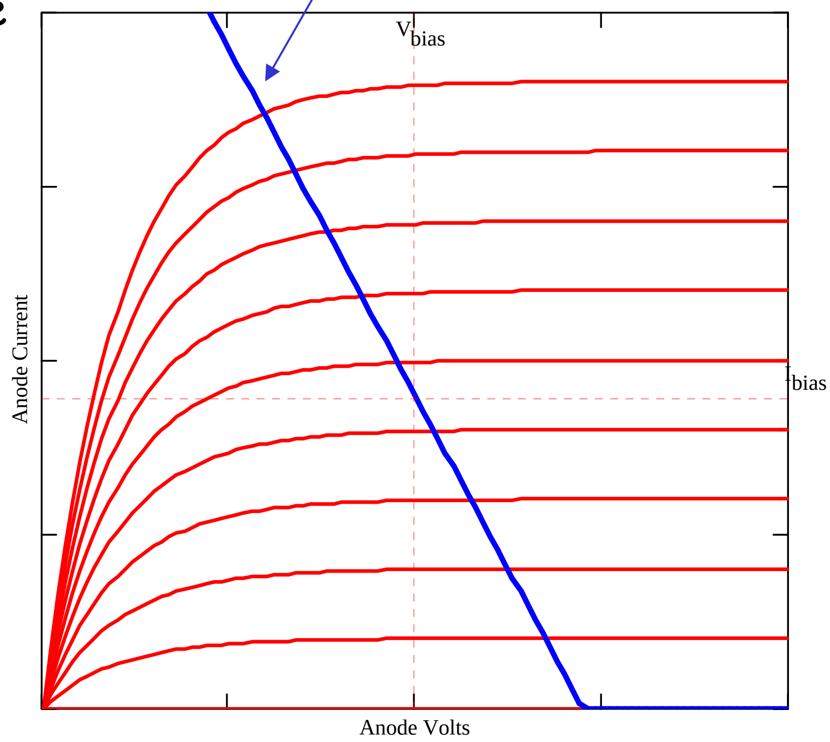


Power Amplifier Bias

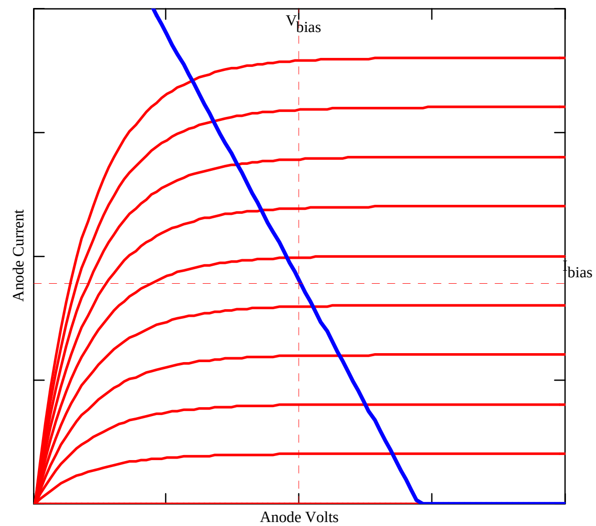
- The power amplifier converts DC energy into RF energy.
- With no RF input into the amplifier, the Power amplifier sits at its DC bias.
- The DC bias point is calculated from the intersection of the tube characteristics with the outside load line

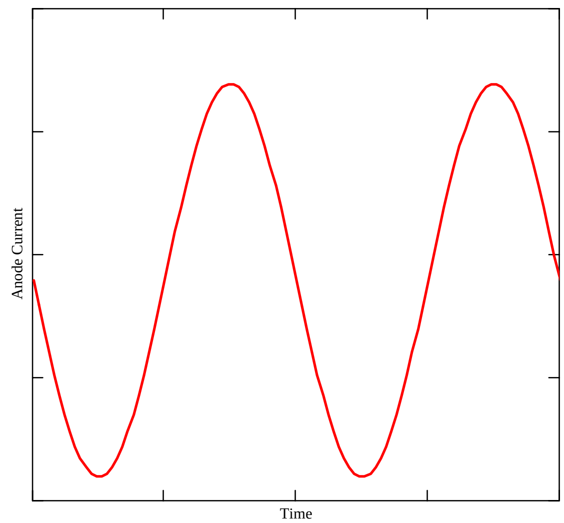
$$I_A = \frac{V_{A_o} - V_A}{R} + I_{A_o}$$

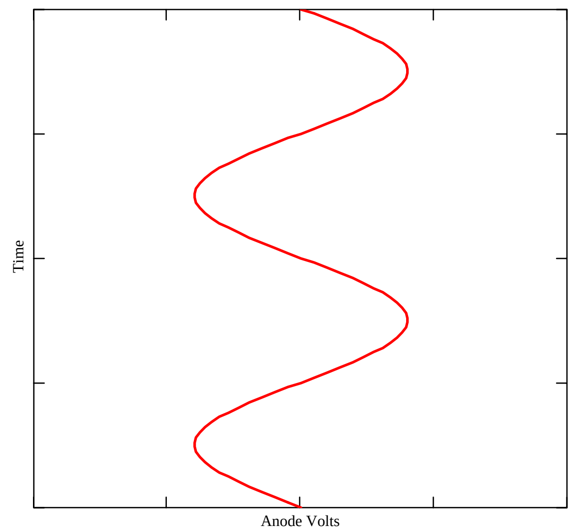




Class A Bias





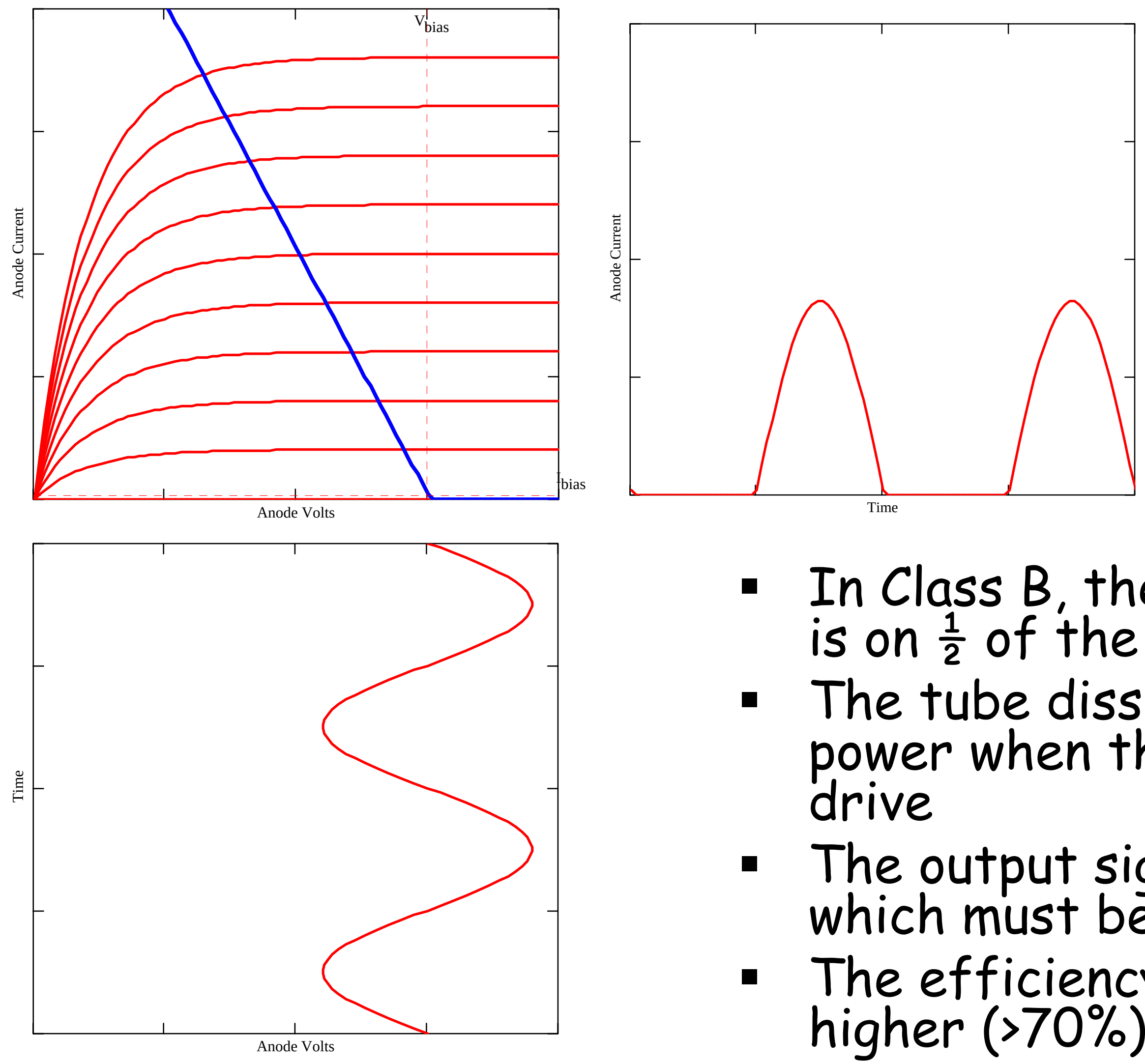


- In Class A, the tube current is on all the time – even when there is no input.
- The tube must dissipate

$$P_{dis} = I_{A_{dc}} V_{A_{dc}} - \frac{1}{2}{}_{A_{ac}} V_{A_{ac}}$$

- The most efficient the power amplifier can be is 50%

Class B Bias



- In Class B, the tube current is on ½ of the time
- The tube dissipates no power when the there is no drive
- The output signal harmonics which must be filtered
- The efficiency is much higher (>70%)