Time to think

- Why are the weak interactions weaker than the EM?
- Why are the strong interactions stronger than the EM?
- Can you write down the standard model particles?

Why is the interaction weak? Do you recall?

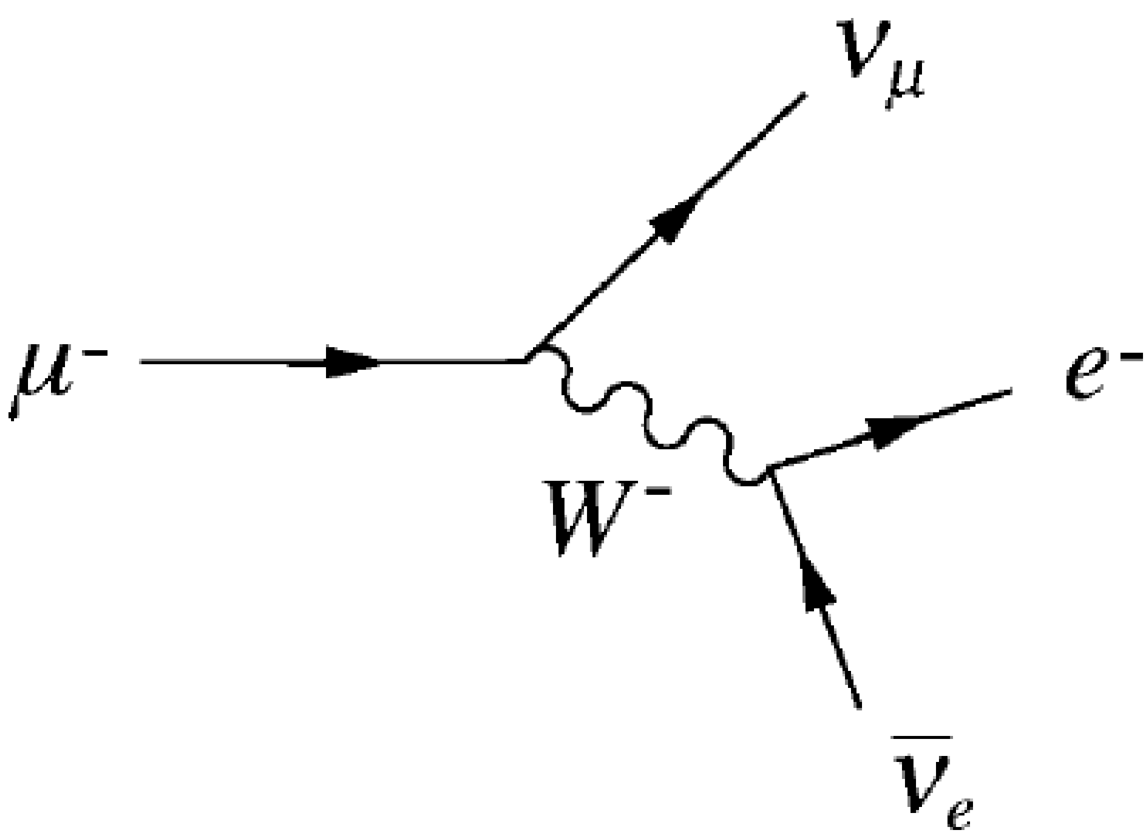


Figure 2.5 Dominant Feynman diagram for muon decay.

Propagator is very small!

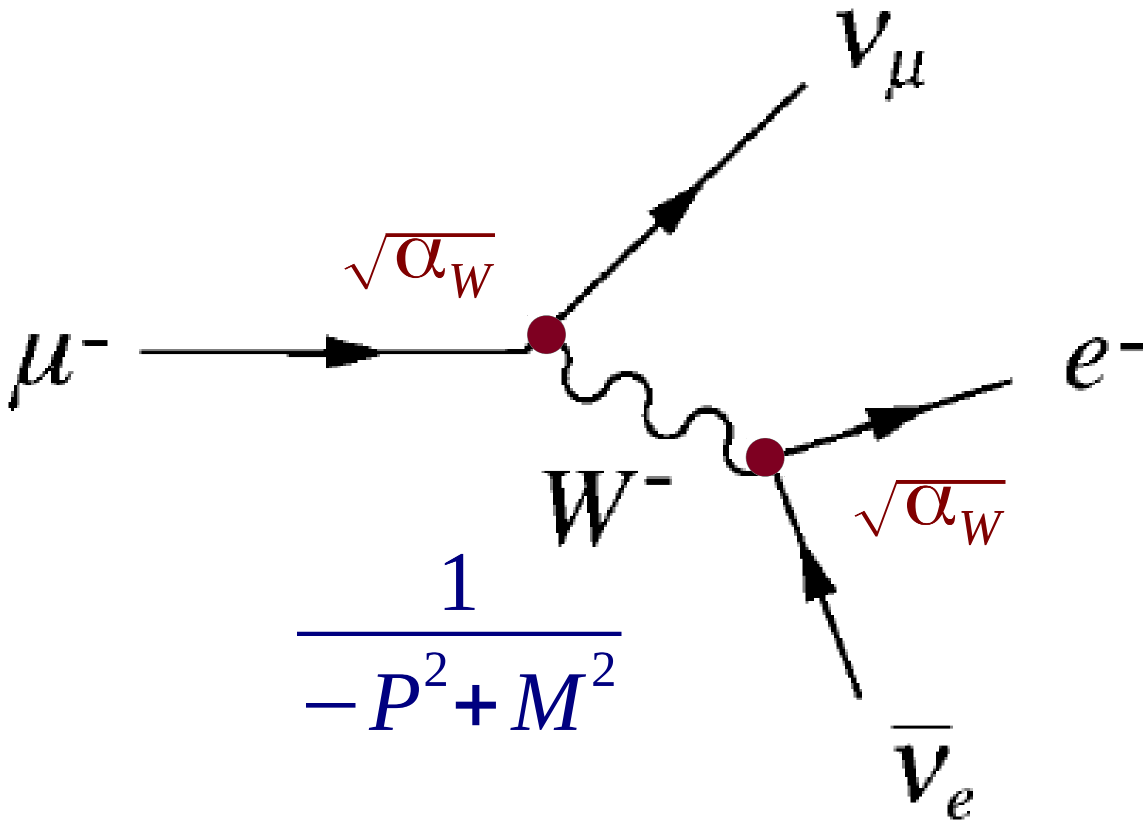


Figure 2.5 Dominant Feynman diagram for muon decay.

Almost constant

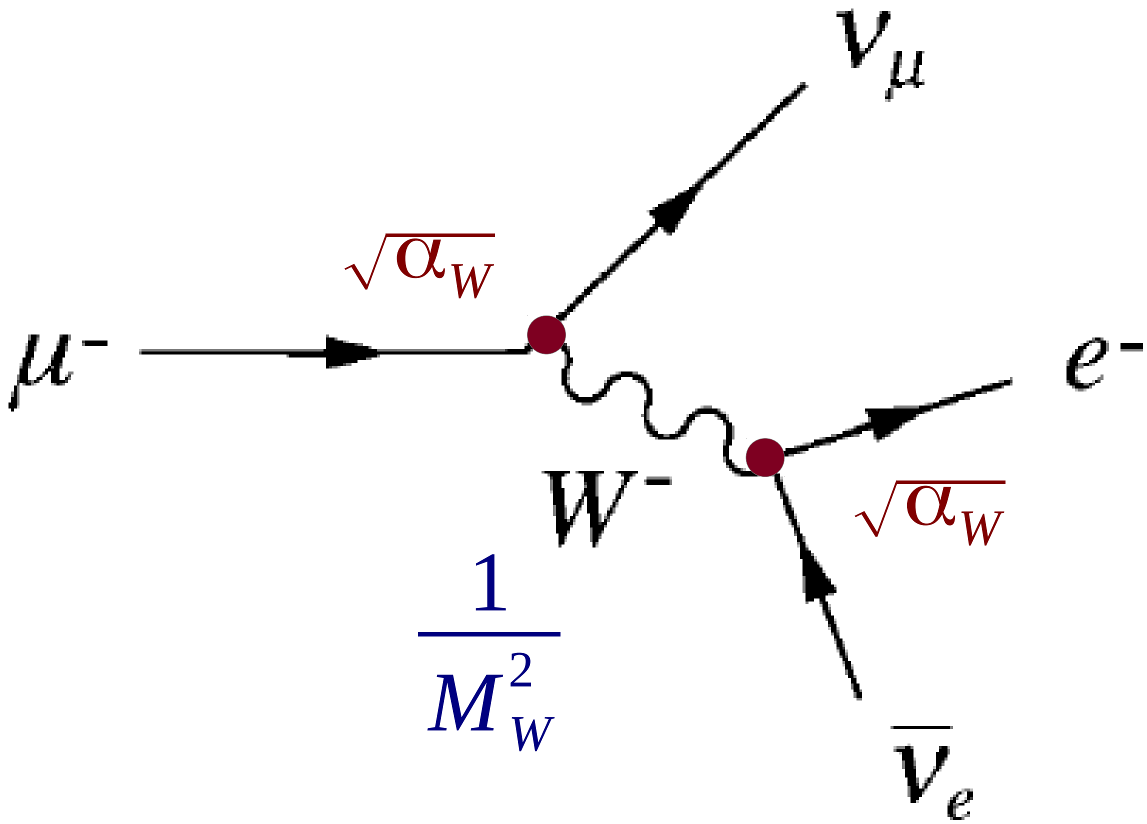


Figure 2.5 Dominant Feynman diagram for muon decay.

Good approximation: point interaction!

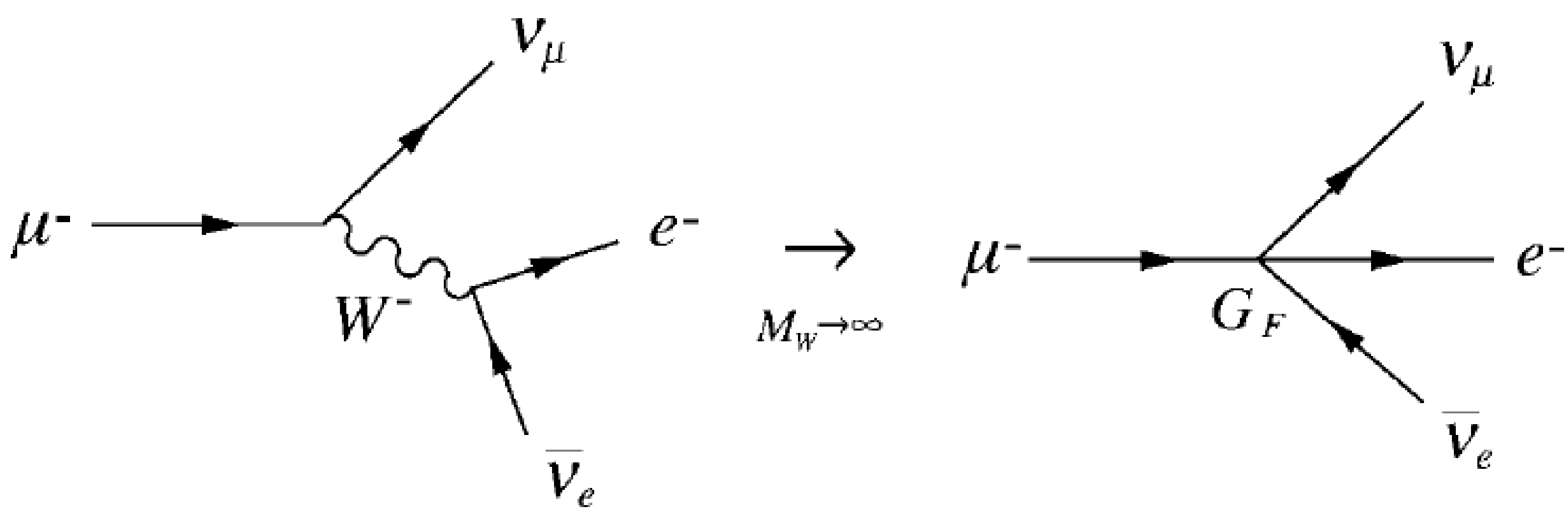


Figure 2.8 Origin of the low-energy zero-range interaction in muon decay (2.10).

Estimate α_W from G_F

- $G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$
- Relation:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} = \frac{4\pi\alpha_W}{M_W^2} \qquad (2.17)$$

- So $\alpha_W \sim 0.0042$, so that $\alpha_W \sim 0.58*\alpha$
- Often today we show $g_W = 2\pi\sqrt{\alpha_W}$ which is the coupling in a single vertex

Weak interaction of leptons and quarks

- Review weak interactions of leptons
- Weak interactions of quarks
- 2 important concepts
  - Lepton ↔ quark symmetry
  - Quark mixing

The basic building blocks

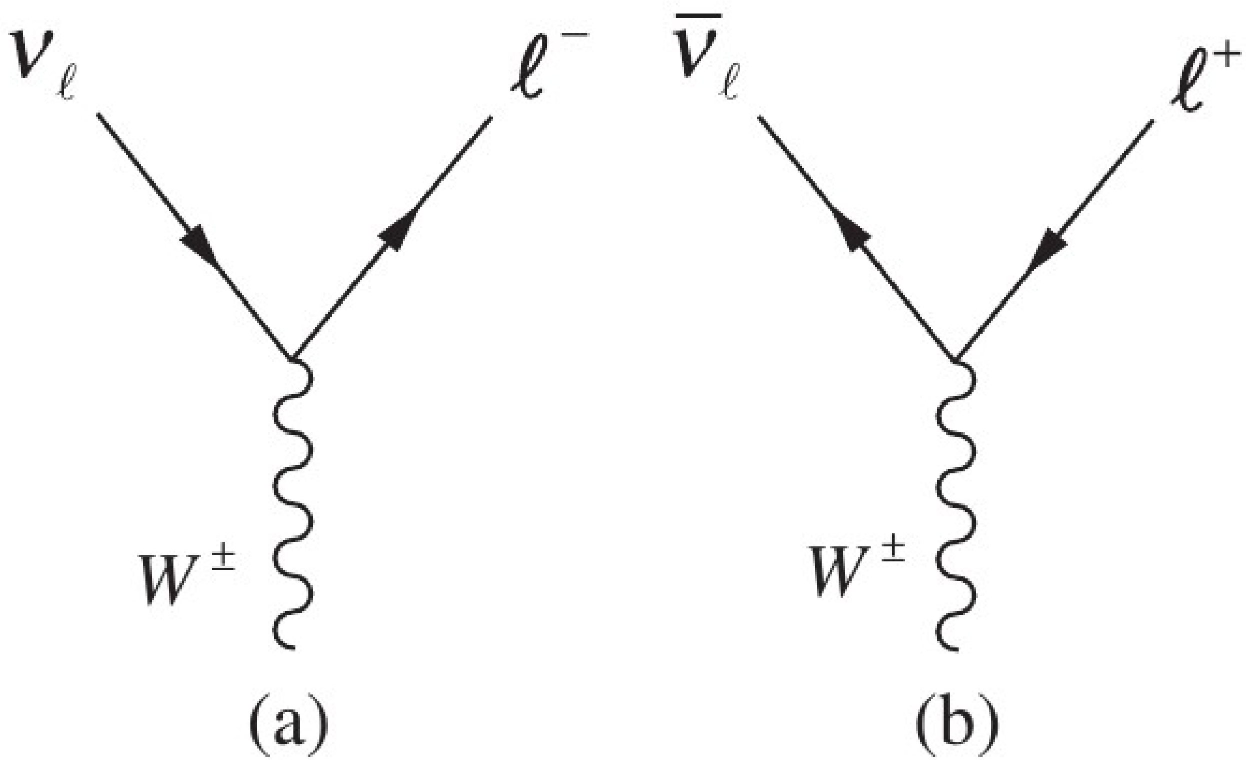
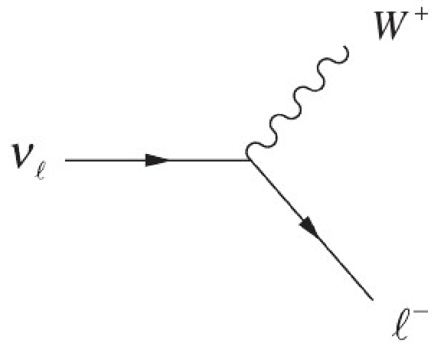
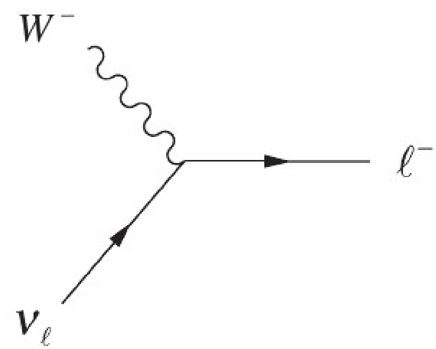


Figure 8.4 The two basic vertices for $W^\pm$–lepton interactions.

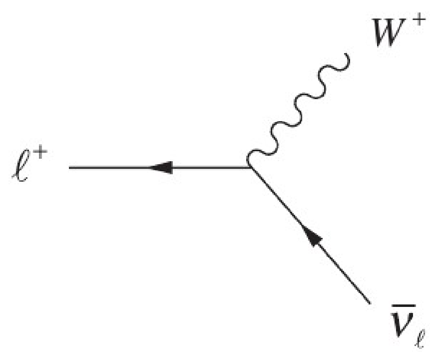
1-4/8 basic diagrams



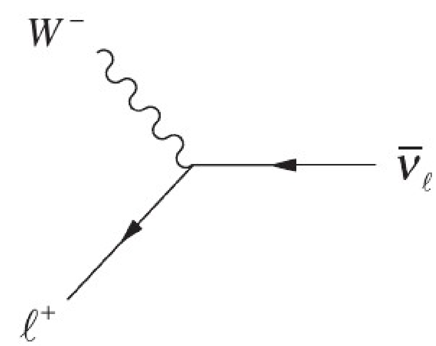
(a) $\nu_\ell \to \ell^- + W^+$



(b) $W^- + \nu_\ell \to \ell^-$



(c) $\ell^+ \to \overline{\nu}_\ell + W^+$



(d) $W^- + \ell^+ \to \overline{\nu}_\ell$

5-8/8 basic diagrams

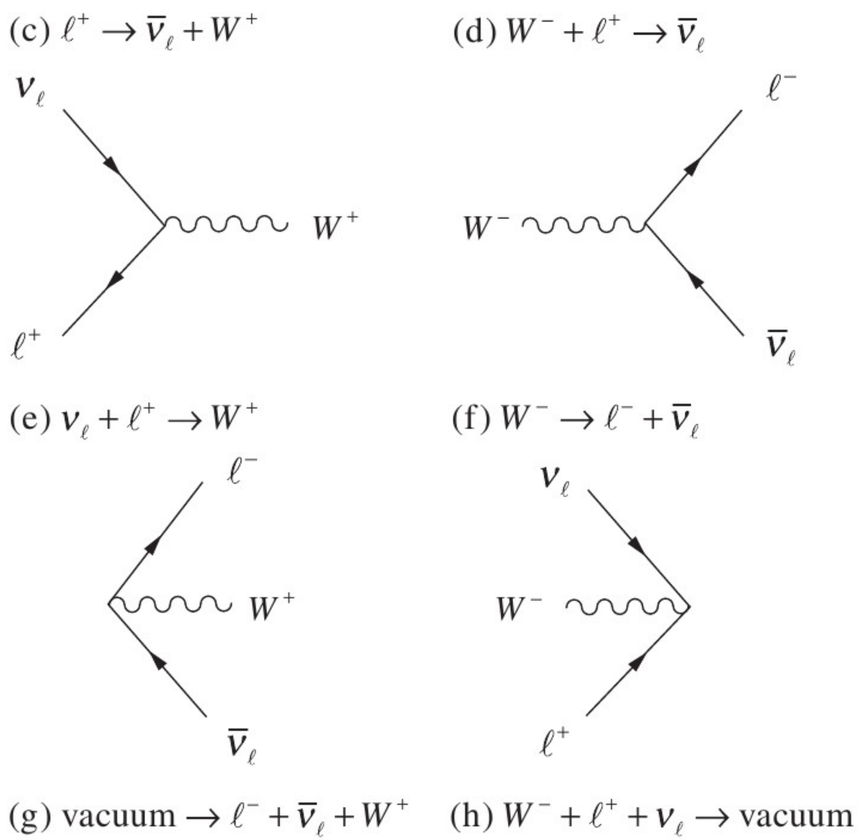


Figure 8.5 The eight basic processes derived from the vertex of Figure 8.4(a).

NB!there are also 8 anti-diagrams

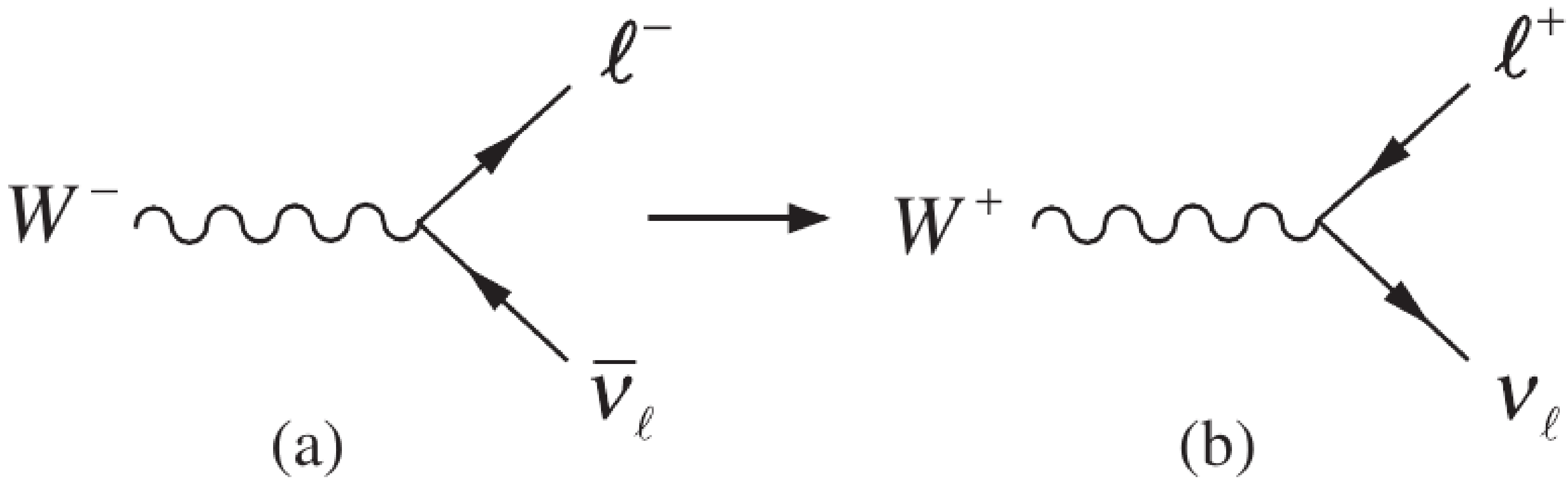


Figure 8.6 A pair of weak interaction processes, which are related by replacing all particles by their antiparticles. The two diagrams (a) and (b) arise from the vertices of Figures 8.4(a) and (b), respectively.

Why choose these diagrams and not only use W^+?

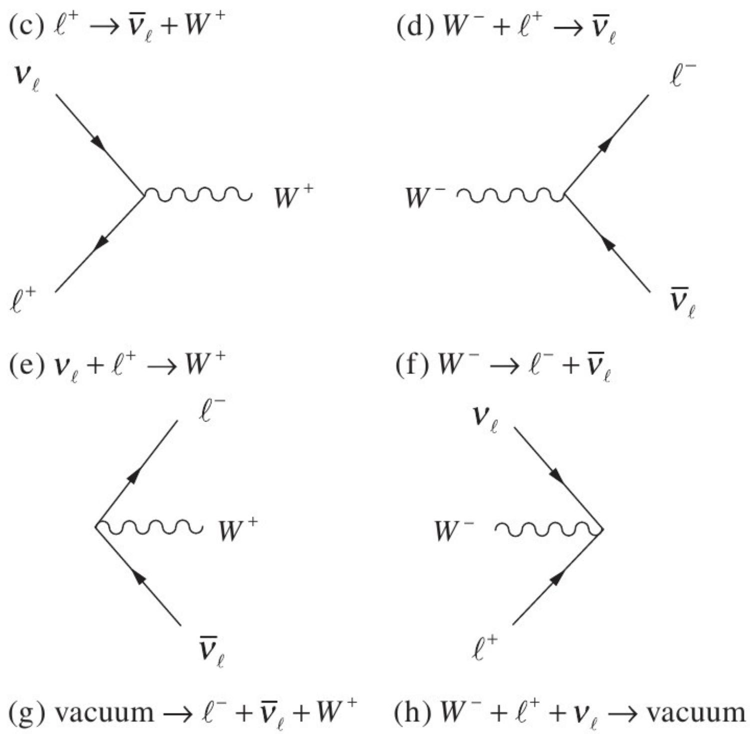


Figure 8.5 The eight basic processes derived from the vertex of Figure 8.4(a).

- Crossing symmetry:

  Exchange particles:

  initial ↔ final state

  particle ↔ anti-particle
- Example: W- intial state becomes W+ in final state
- Xsymmetry states that amplitudes are related

Weak interaction of quarks

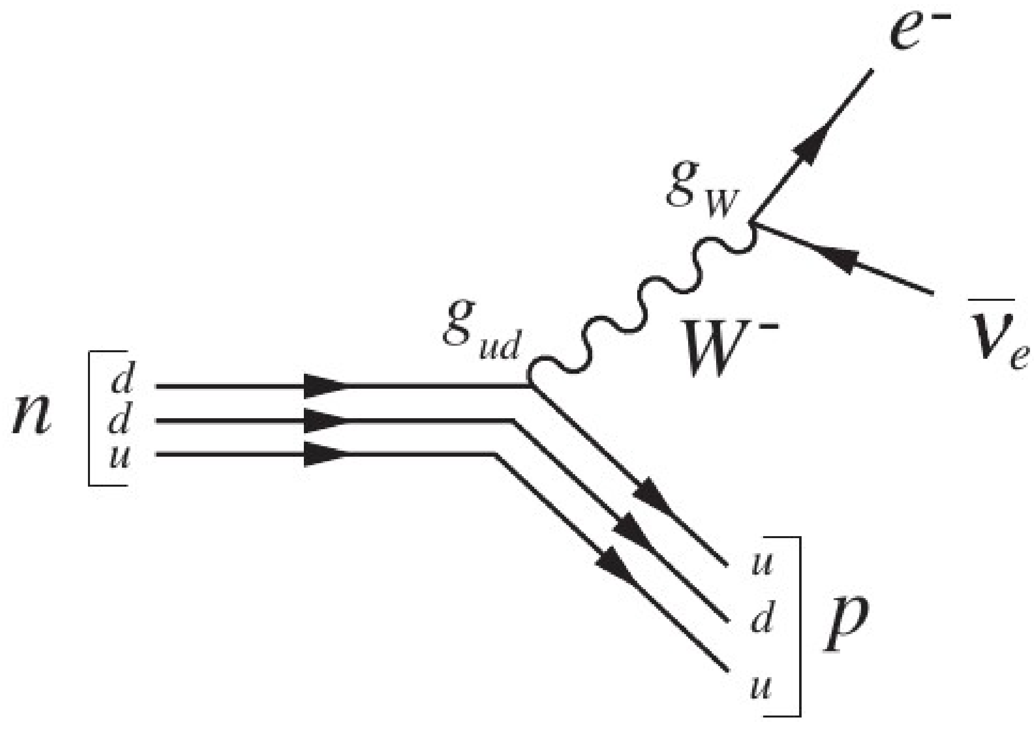


Figure 8.8 Dominant diagram for neutron decay (8.6).

Question: What is g_{ud}?

Lepton-quark symmetry v1

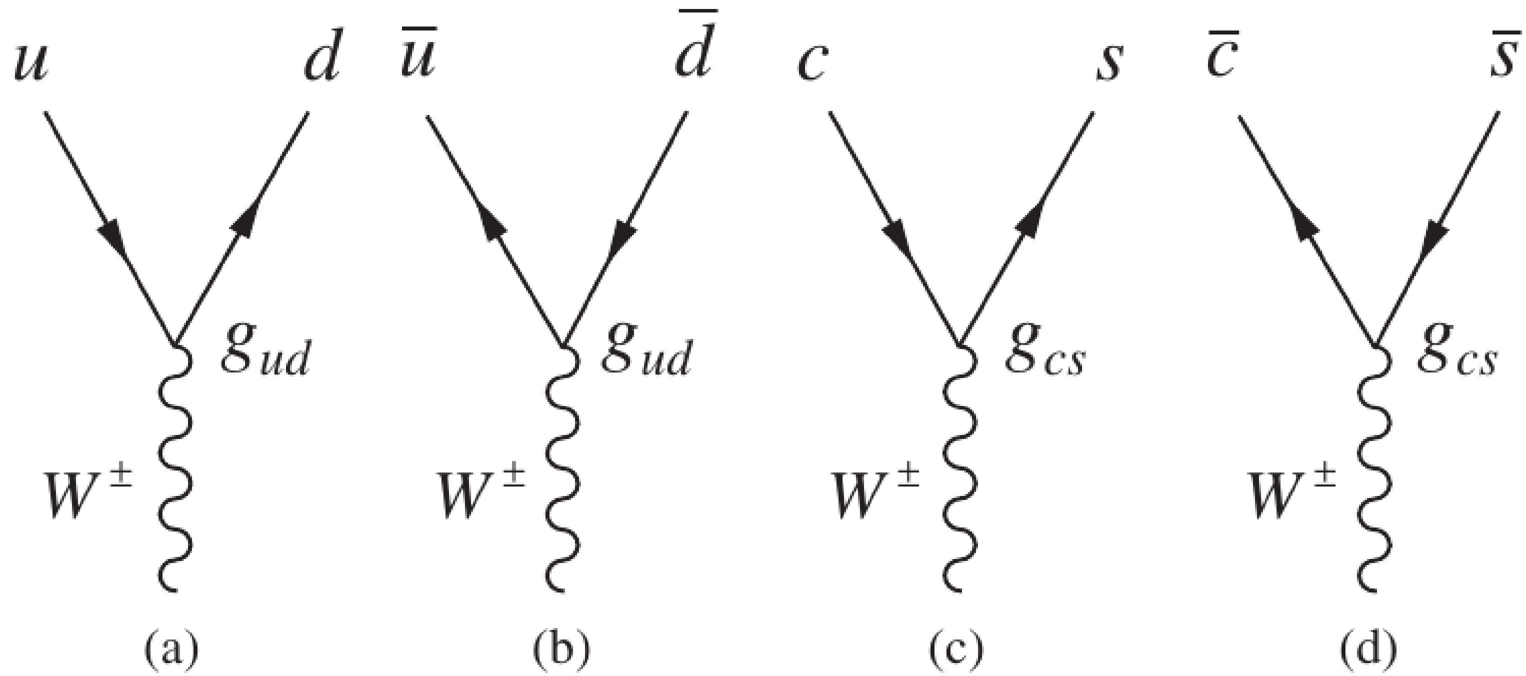


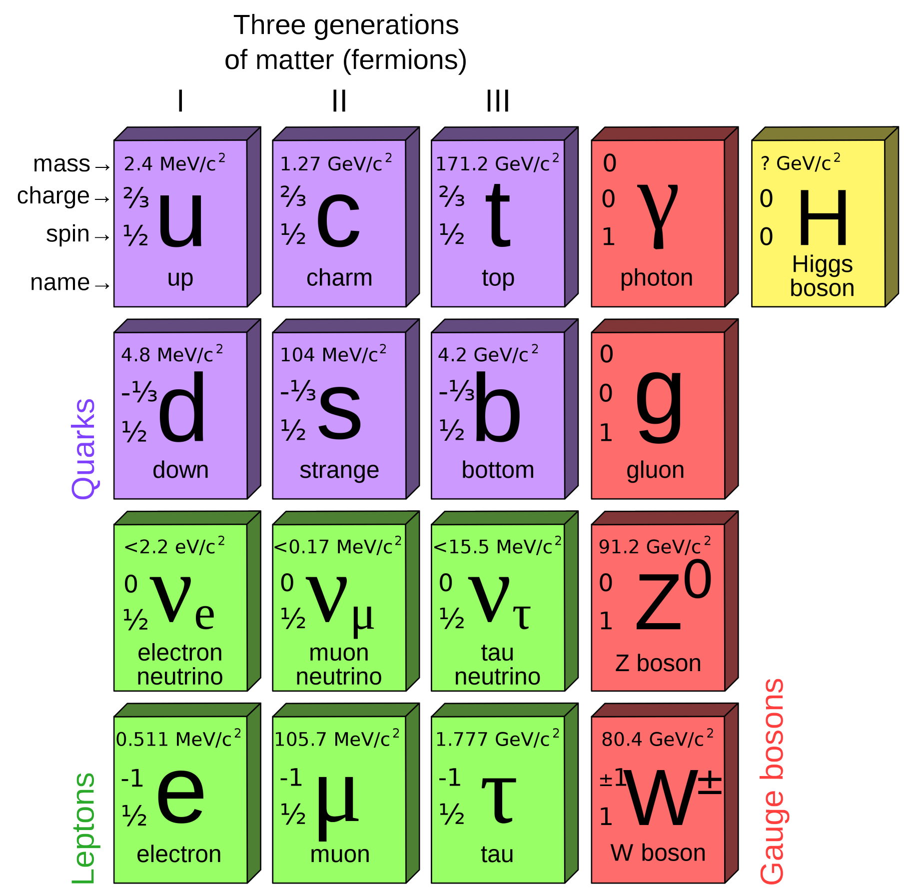
Figure 8.10 The $W^{\pm}$ quark vertices obtained from lepton–quark symmetry when quark mixing is ignored.

$$g_{ud} = g_{cs} = g_W . \qquad (8.12)$$

Fantastic experimental result
Same coupling

Emit W-
Absorb W+

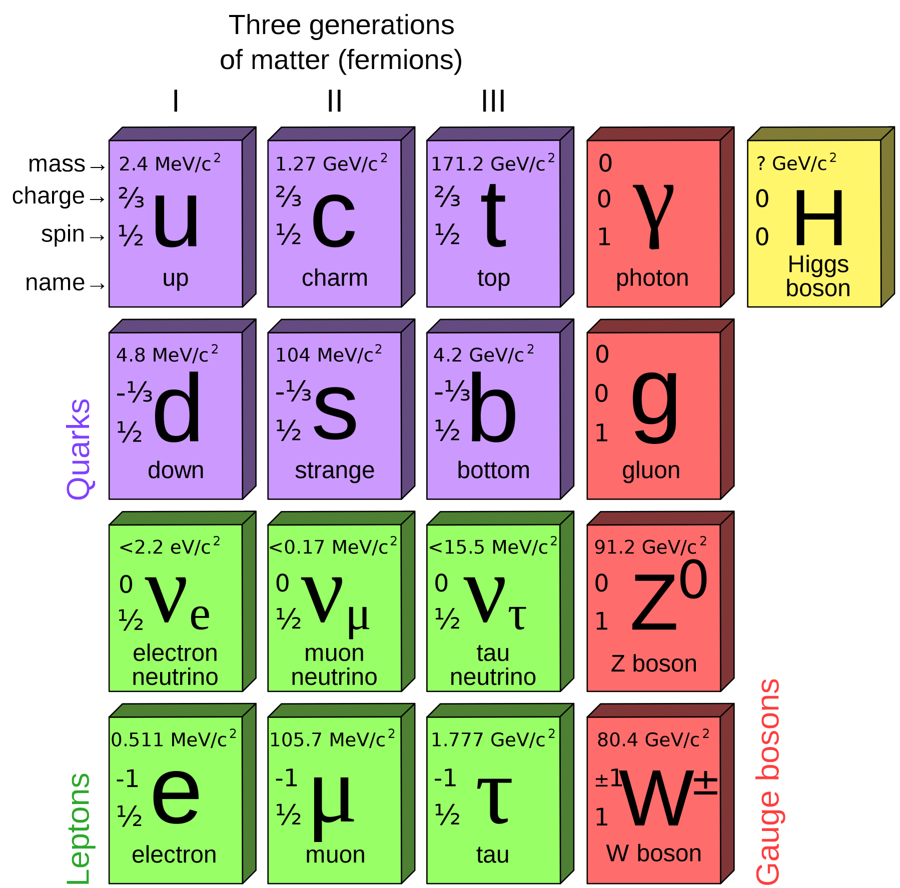
Emit W-
Absorb W+



Fantastic experimental result
Same coupling

Emit W+
Absorb W-

Emit W+
Absorb W-



That is why one draws the neutrino above the electron!

But there is a small detail

OK!

Forbidden in simplest theory.
But observed in nature!

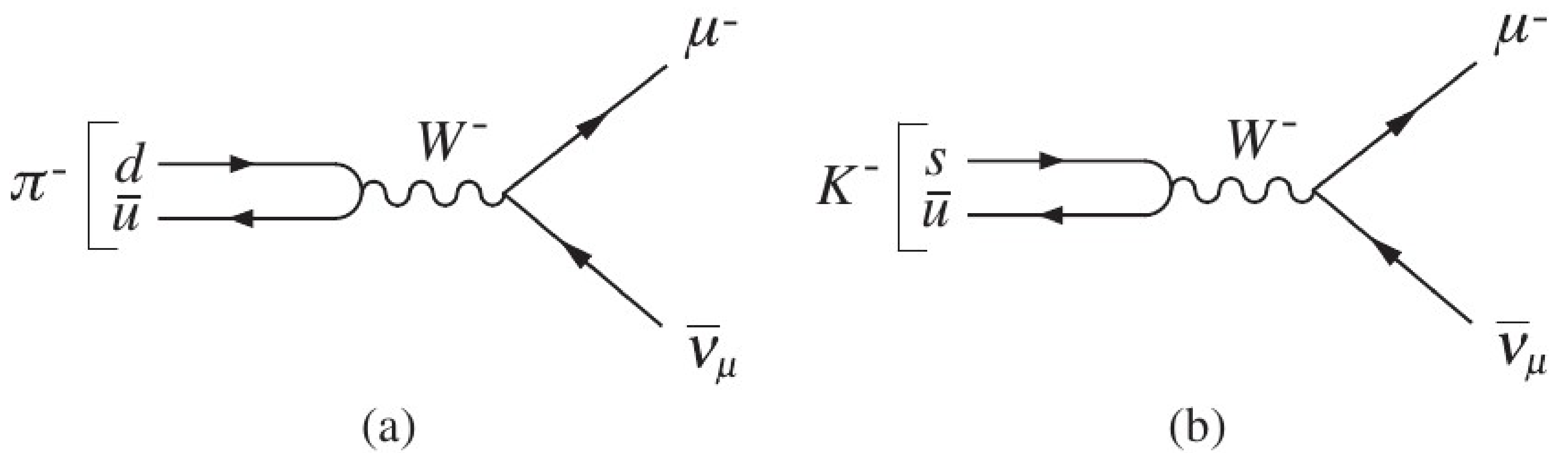
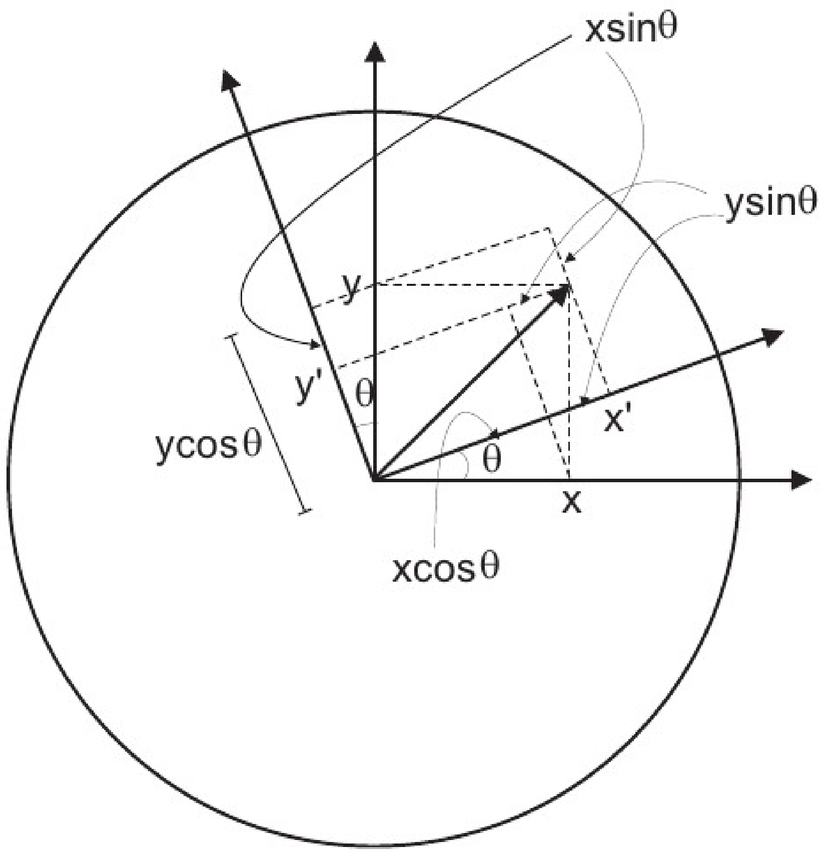


Figure 8.11 Feynman diagrams for the semileptonic decays: (a) $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ and (b) $K^- \rightarrow \mu^- + \bar{\nu}_\mu$.

Neutrino oscillations
Quark mixing



$$x' = x \cos\theta + y \sin\theta$$

$$y' = y \cos\theta - x \sin\theta$$

Weak interaction quark eigenstates are not mass eigenstates

- The weak interaction couples to “rotated” states:

  |d'> = |d> * cos θc + |s> * sin θc

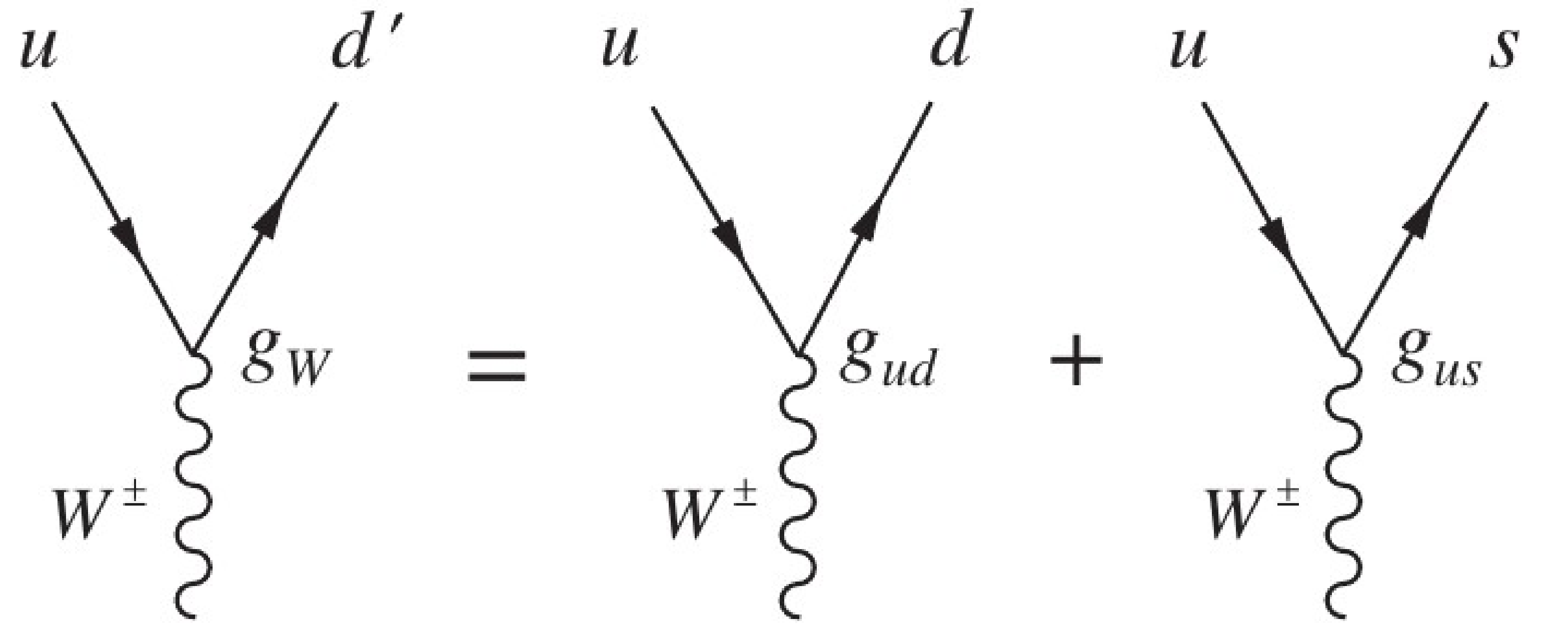
  |s'> = - |d> * sin θc + |s> * cos θc

- The angle θc~13 degrees is called the Cabibbo angle

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}. \qquad (8.39)$$

The physics picture

Weak interaction states | Quark mass eigenstates



$d' = d\cos\theta_C + s\sin\theta_C$ $\quad g_{ud} = g_W \cos\theta_C$ $\quad g_{us} = g_W \sin\theta_C$

Figure 8.12 The $ud'W$ vertex and its interpretation in terms of udW and usW vertices.

Estimating θc

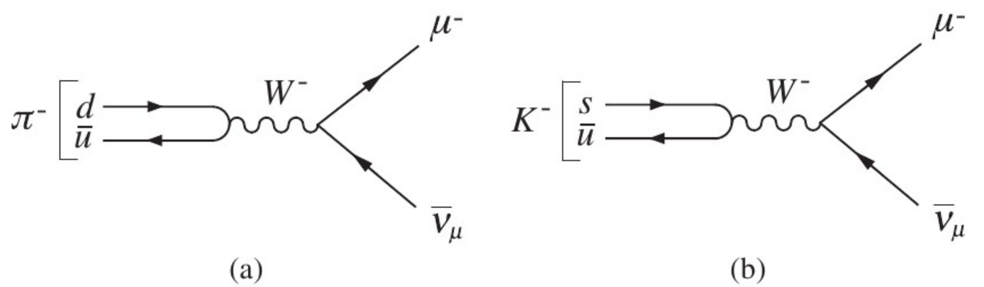


Figure 8.11 Feynman diagrams for the semileptonic decays: (a) $\pi^- \to \mu^- + \bar{\nu}_\mu$ and (b) $K^- \to \mu^- + \bar{\nu}_\mu$.

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2\theta_C,$$

by (8.17). Of course, the difference in the d and s quark masses will also have an effect on the rates, and this must be taken into account. We omit the details and merely quote the mean value obtained from this and other similar determinations, which is

$$g_{us}/g_{ud} = \tan\theta_C = 0.232 \pm 0.002, \tag{8.18}$$

corresponding to a Cabibbo angle of

$$\theta_C = 13.1 \pm 0.1 \text{ degrees.} \tag{8.19}$$

A similar value is obtained by comparing the rates for neutron and muon decay, which depends on the ratio

$$(g_{ud}/g_W)^2 = \cos^2\theta_C,$$

Lepton-quark symmetry v2

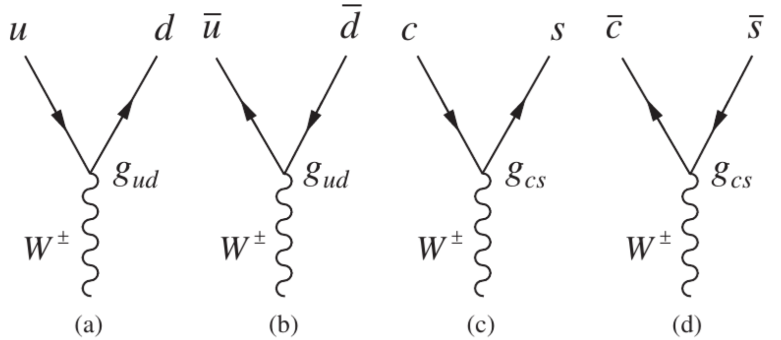


Figure 8.10 The $W^\pm$ quark vertices obtained from lepton–quark symmetry when quark mixing is ignored.

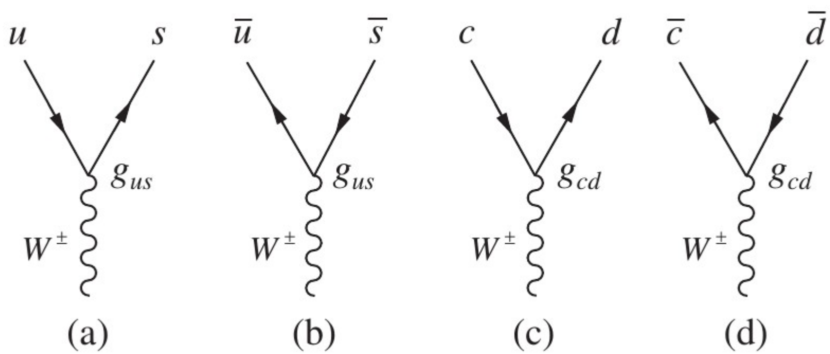


Figure 8.13 The additional vertices arising from lepton–quark symmetry when quark mixing is taken into account.

$$g_{us}^2/g_{ud}^2 = g_{cd}^2/g_{cs}^2 = \tan^2\theta_C = 1/20 \qquad (8.20)$$

Weak decay selection rules

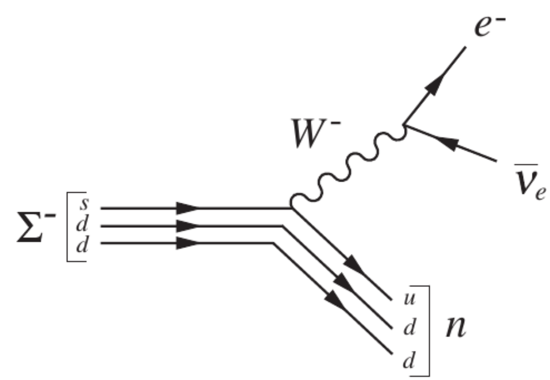


Figure 8.17 Mechanism for the decay $\Sigma^- \to n + e^- + \bar{\nu}_e$.

- <u>**Focus mainly on semi-leptonic decays**</u> i.e. decays where there is a lepton pair in the final state
  - NB! This is always the decay products of the W!
    - So when we use ΔQ in the following we mean for the hadrons!
      - ΔQ = Qfinal – Qinitial = +1 here

Weak decay selection rules (derived from W diagrams)

$$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e \tag{8.27}$$

$$\Sigma^+ \rightarrow n + e^+ + \nu_e \tag{8.28}$$

$$\frac{\Gamma(\Sigma^+ \rightarrow n + e^+ + \nu_e)}{\Gamma(\Sigma^- \rightarrow n + e^- + \bar{\nu}_e)} < 5 \times 10^{-3}.$$

$$uus \rightarrow udd + e^+ + \nu_e$$

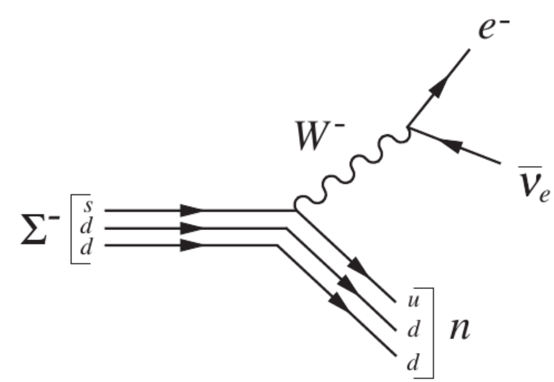


Figure 8.17 Mechanism for the decay $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$.

Because the weak interaction is so weak only single W exchange diagrams are allowed!

Exercise

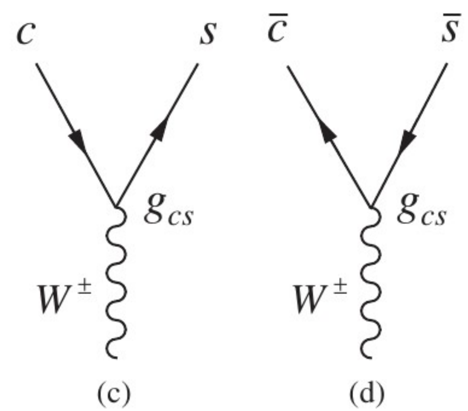
8.5 Classify the following semileptonic decays of the $D^+(1869) = c\bar{d}$ meson as Cabibbo-allowed, Cabibbo-suppressed or forbidden in lowest-order weak interactions, ~~by finding selection rules for the changes in strangeness, charm and electric charge in such decays:~~

(a) $D^+ \to K^- + \pi^+ + e^+ + \nu_e$
(b) $D^+ \to K^+ + \pi^- + e^+ + \nu_e$
(c) $D^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}_e$
(d) $D^+ \to \pi^+ + \pi^- + e^+ + \nu_e$

- D+ = $c\bar{d}$
- K+ = $u\bar{s}$, K- = $s\bar{u}$
- Pions: π+ = $u\bar{d}$, π- = $d\bar{u}$
- Hint: Try to understand what c decays are possible

Trying to make a rule for c decays:

Cabibbo favored

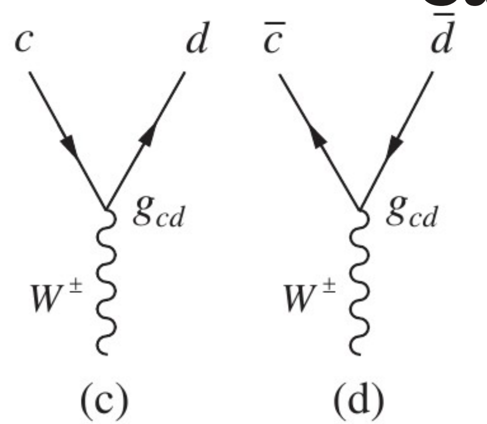


- $\Delta C = \Delta S = \Delta Q = \pm 1$

  S = Nsbar-Ns

  C = Nc - Ncbar

Cabibbo suppressed



- $\Delta C = \Delta Q = \pm 1$, $\Delta S=0$

Exercise

8.5 Classify the following semileptonic decays of the $D^+(1869) = c\bar{d}$ meson as Cabibbo-allowed, Cabibbo-suppressed or forbidden in lowest-order weak interactions, by finding selection rules for the changes in strangeness, charm and electric charge in such decays:

$$
\begin{array}{ll}
\text{(a)} & D^+ \to K^- + \pi^+ + e^+ + \nu_e \\
\text{(b)} & D^+ \to K^+ + \pi^- + e^+ + \nu_e \\
\text{(c)} & D^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}_e \\
\text{(d)} & D^+ \to \pi^+ + \pi^- + e^+ + \nu_e
\end{array}
$$

- D+ = $c\bar{d}$, K+ = $u\bar{s}$, K- = $s\bar{u}$, π+ = $u\bar{d}$, π+ = $d\bar{u}$
- (a): ΔC = -1, ΔS = -1, ΔQ = -1 favored
- (b): ΔC = -1, ΔS = +1, ΔQ = -1 not allowed
- (c): ΔC = -1, ΔS = 0, ΔQ = +1 not allowed
- (d): ΔC = -1, ΔS = 0, ΔQ = -1 suppressed

What about s decays?

- Favored: s → c
  - Not possible due to mass!
- Suppressed: s → u (s-bar → u-bar)
- Rule: $\Delta S = \pm 1$
  - For semi-leptonic decays we would also have $\Delta S = \Delta Q = \pm 1$
  - We can only loose one unit of strangeness at a time!

The full Ω (sss) decay chain

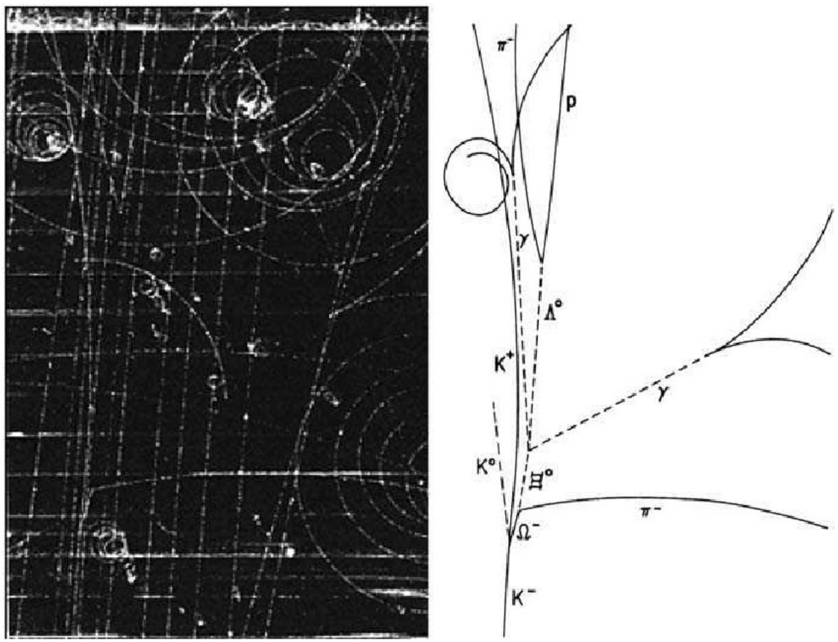


Figure 8.18 Characteristic pattern of tracks produced by the production and decay of the Ω^-. (From Barnes *et al.*, 1964, photo courtesy of Brookhaven National Laboratory.)

$K^- + p \rightarrow \Omega^- + K^+ + K^0$

$\quad \hookrightarrow \Xi^0 + \pi^- (\Delta S = 1 \text{ weak decay})$

$\qquad \hookrightarrow \pi^0 + \Lambda (\Delta S = 1 \text{ weak decay})$

$\qquad\quad \hookrightarrow \pi^- + p \; (\Delta S = 1 \text{ weak decay})$

$\qquad \hookrightarrow \gamma + \gamma (\text{electromagnetic decay})$

$\qquad\quad \downarrow \quad \downarrow$

$\qquad\quad e^+e^- \; e^+e^-$