VIII. Weak Interactions: W and Z bosons

❖ The force carriers in weak interactions are (as in QED and QCD) spin-1 bosons that couple to quarks and leptons.

→ The force carriers of weak interactions are three *intermediate vector bosons*: W^+ and W^- (mass 80.4 GeV) and Z^0 (91.2 GeV). Since the W^+, W^- and Z^0 bosons are **very massive** particles, the weak interactions have a very short range (order of 2×10^{-3} fm).

❖ Before the Electroweak Theory was developed, all observed weak processes were *charged current* reactions (e.g. β-decay) mediated by W^+ or W^- bosons.

→ The Electroweak theory predicted that *neutral current reactions* caused by the Z^0 boson should exist.

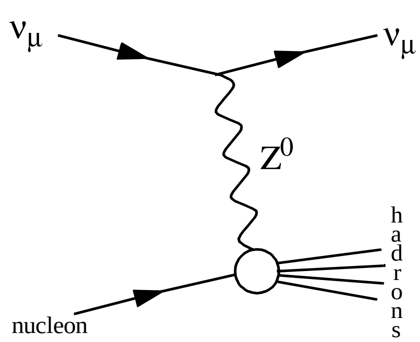


Figure 78: A predicted neutral current reaction which is characterized by no muon in the final state

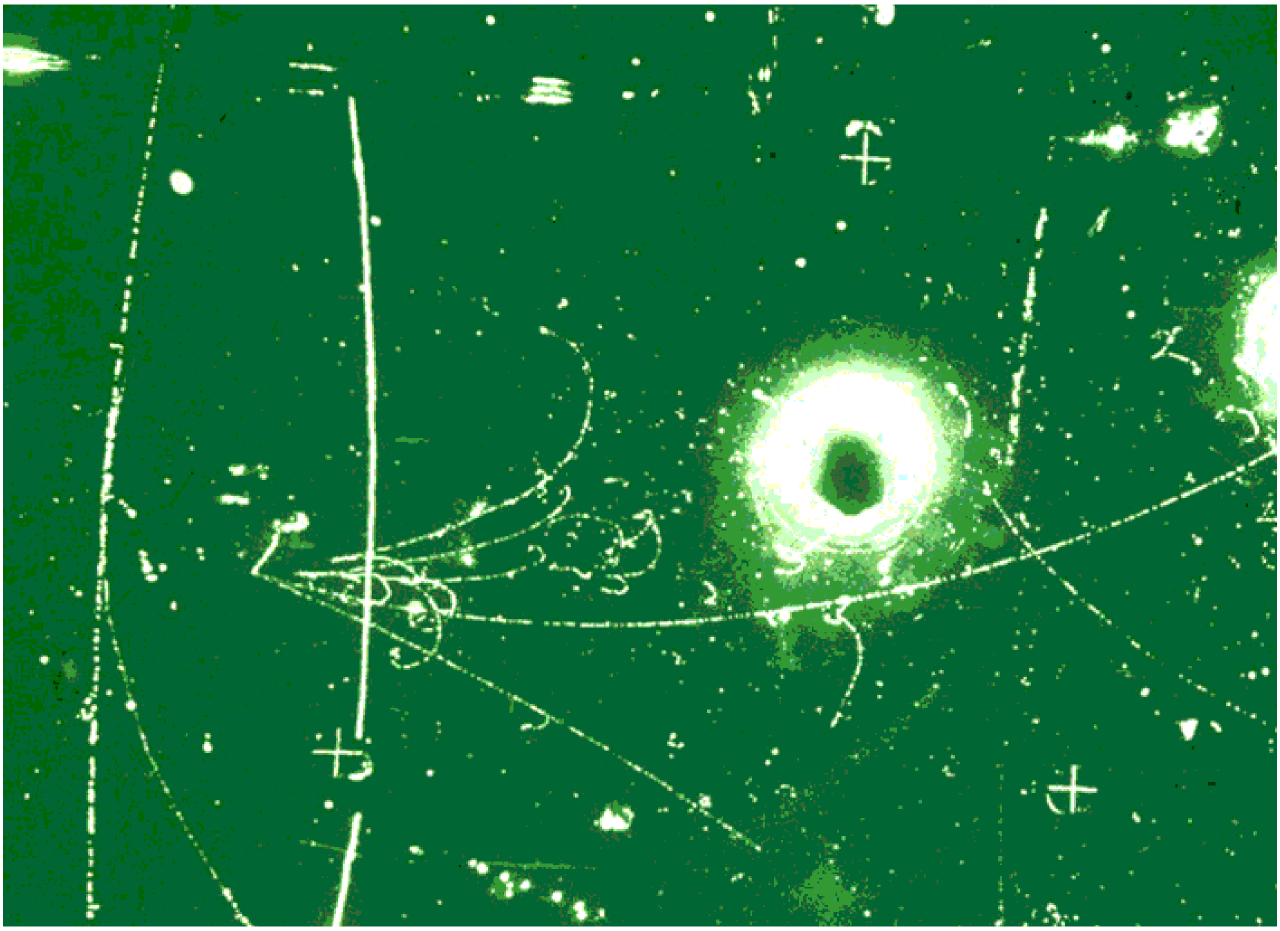


Figure 79: One of the first neutral current reactions as seen by the Gargamelle bubble chamber experiment in 1973.

Discovery of the W and Z bosons.

❖ The first study of direct production and decay of the W and Z vector bosons were made by the UA1 and UA2 experiments at the SPS proton-antiproton collider at CERN.

→ PRODUCTION: In proton colliders the W and Z bosons are produced by quark-antiquark annihilations:

$$u + \bar{d} \to W^+ \ , \qquad d + \bar{u} \to W^- \qquad (94)$$

$$u + \bar{u} \to Z^0 \ , \qquad d + \bar{d} \to Z^0 \qquad (95)$$

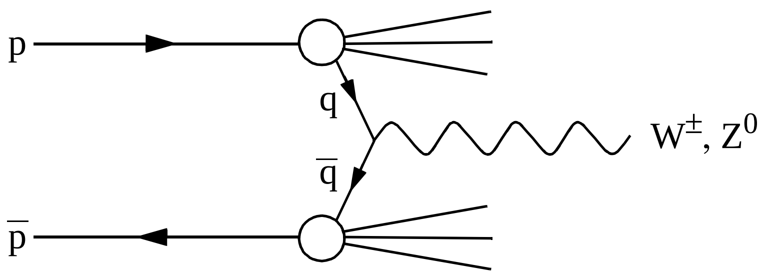


Figure 80: The mechanism of $W^{\pm}$ and Z production in $p\bar{p}$ annihilation.

→ DECAY: The W and Z bosons decay in most cases to hadrons but these decays cannot be identified among all the other hadrons created in $\bar{p}p$ collisions. Instead one looks for decays to leptons:

$$\bar{p} + p \rightarrow W^{+} + X, \quad W^{+} \rightarrow q' + \bar{q}$$

$$\bar{p} + p \rightarrow W^{+} + X, \quad W^{+} \rightarrow l^{+} + \nu_{l}$$

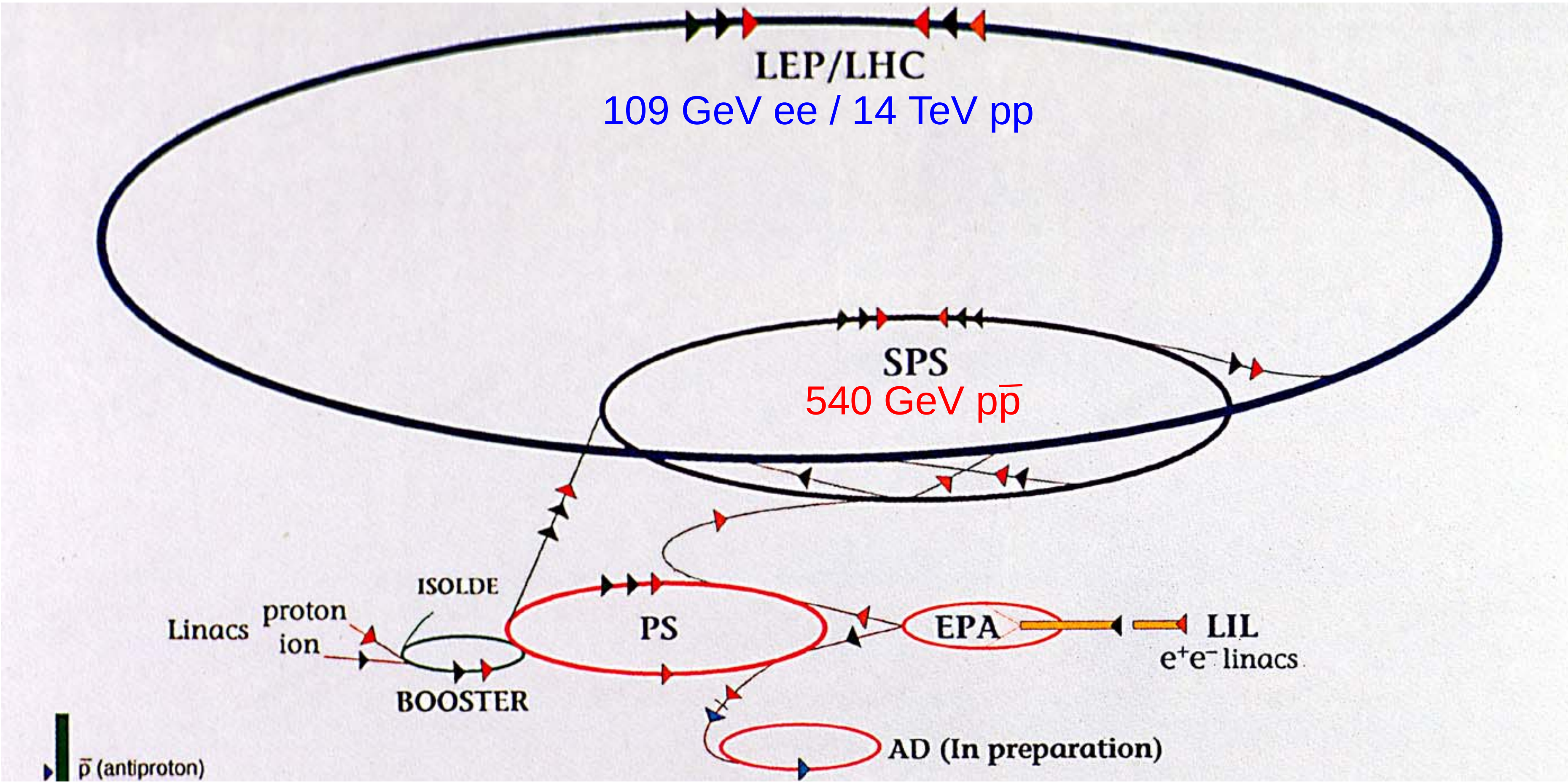
$$\bar{p} + p \rightarrow W^{-} + X, \quad W^{-} \rightarrow q' + \bar{q}$$

$$\bar{p} + p \rightarrow W^{-} + X, \quad W^{-} \rightarrow l^{-} + \bar{\nu}_{l}$$

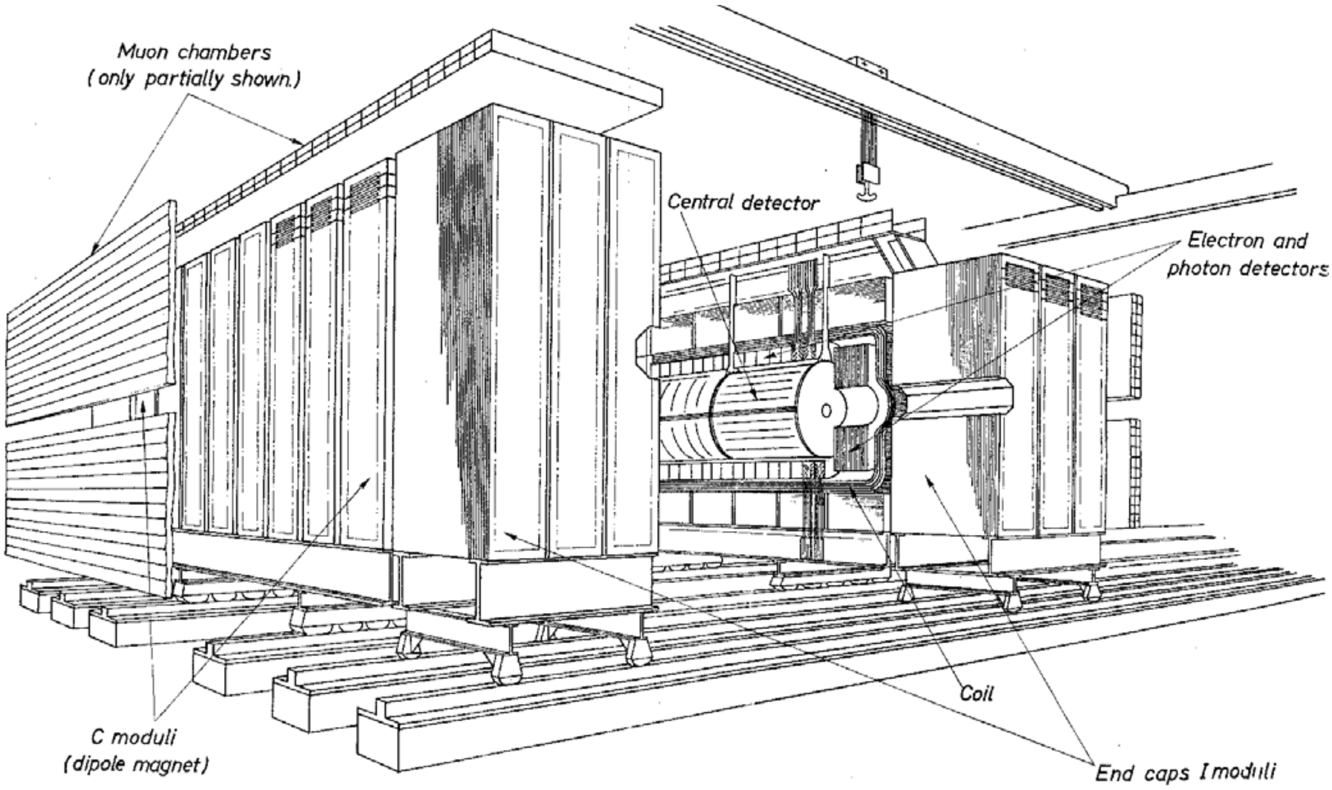
$$\bar{p} + p \rightarrow Z^{0} + X, \quad Z^{0} \rightarrow q + \bar{q}$$

$$\bar{p} + p \rightarrow Z^{0} + X, \quad Z^{0} \rightarrow l^{+} + l^{-}$$

→ The lifetime of both the W and the Z is about 3×10^{-25} s and particles with such a short lifetime are never seen directly in the experiments.



In hadron colliders, the energy of the quarks and gluons that participate in the interaction is only a fraction of the energy of the colliding protons. To produce W- and Z-bosons with a mass of 80-90 GeV it was therefore needed to build a accelerator where the protons had an energy of 270 GeV.



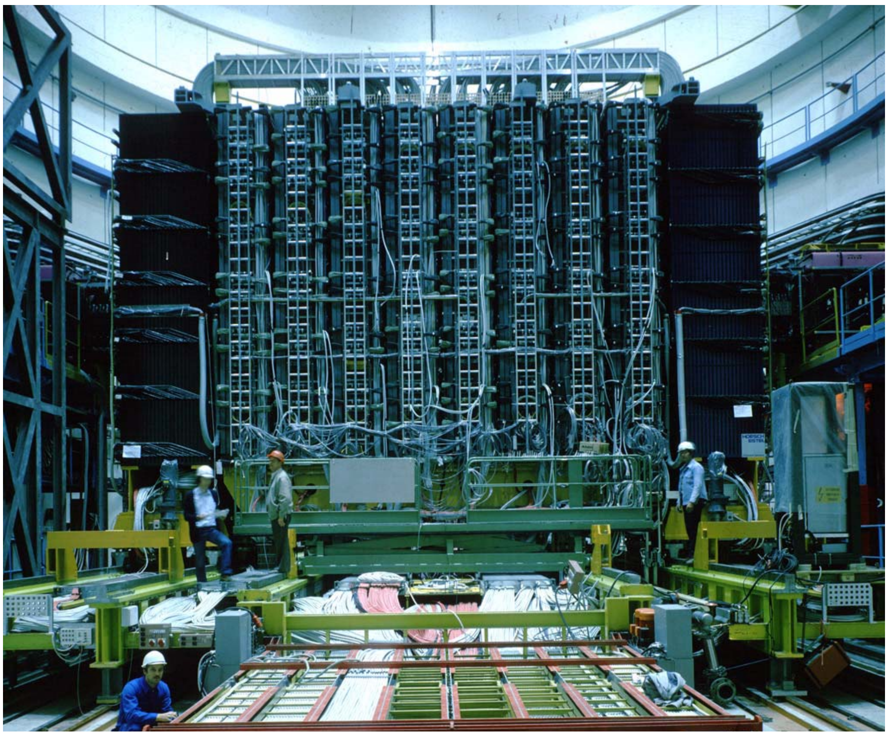


Figure 81: The UA1 experiment where the W and Z bosons were discovered.

DISCOVERY OF THE W BOSON

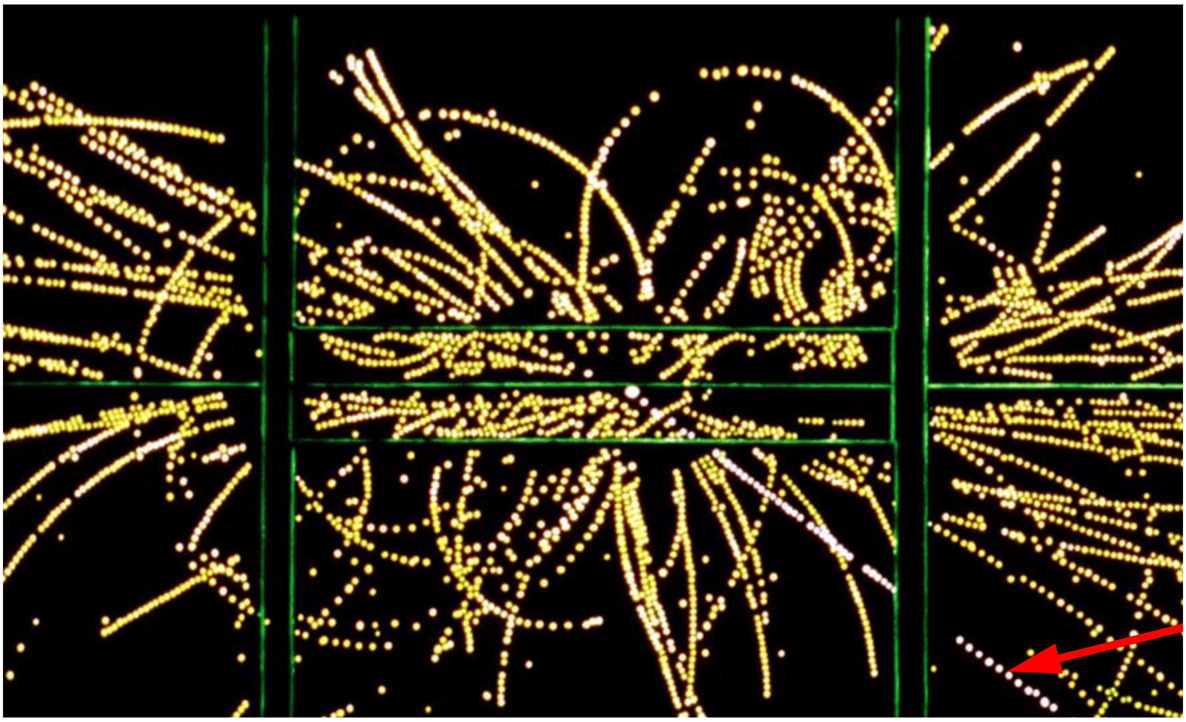


Figure 82: A W event observed by the UA1 detector in 1982. A high transverse momentum electron is indicated by the arrow.

The signature of a W boson is:

– a lepton with large momentum (>10 GeV/c) emitted at a wide angle to the beam ($>5^{\circ}$).

– large "*missing transverse momentum*" carried away by the neutrino.

Transverse Energy: $E_T = E \sin\theta$

Tranverse Momentum: $P_T = P \sin\theta$

where θ is the angle to the colliding beams.

→ $E_T = P_T$ if the mass of a particle is small compared to its energy.

→ If the momentum of all particles in a collision is added up the result should be zero (momentum conservation). Neutrinos can, however, not be detected and if the total momentum is different from zero the event is said to have missing momentum (or missing energy).

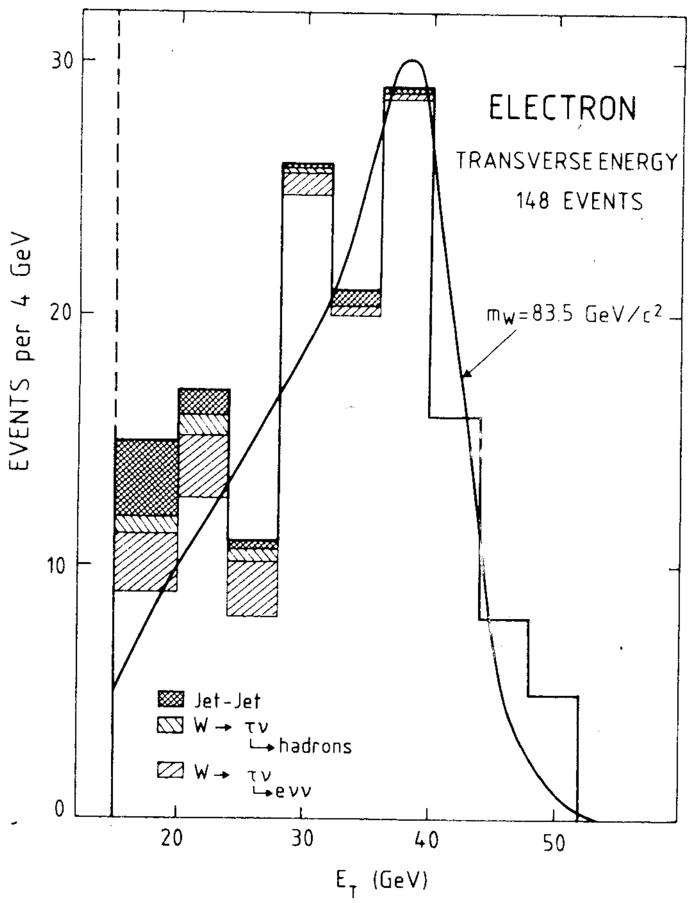


Figure 83: The UA1 transverse energy distribution of electrons in events with more than 15 GeV missing energy.

→ From the first 148 electron and 47 muon events recorded by the UA1 it was estimated that

$$M_W = 83{,}5 \pm 2{,}8\ GeV \qquad \Gamma_W \le 6{,}5\ GeV$$

→ W bosons can be pair-produced in e^+e^- annihilation. From measurement at the LEP accelerator, the W mass and width is now estimated to be

$$M_W = 80{,}43 \pm 0{,}05\ GeV \qquad \Gamma_W = 2{,}1 \pm 0{,}1\ GeV$$

→ The branching ratios of the leptonic decay modes of the $W^{\pm}$ have been measured to be about 11% for each lepton generation.

STUDIES OF THE W AT LEP.

The DELPHI experiment was one of the four experiments at LEP which studied W-production in e+e- collisions.

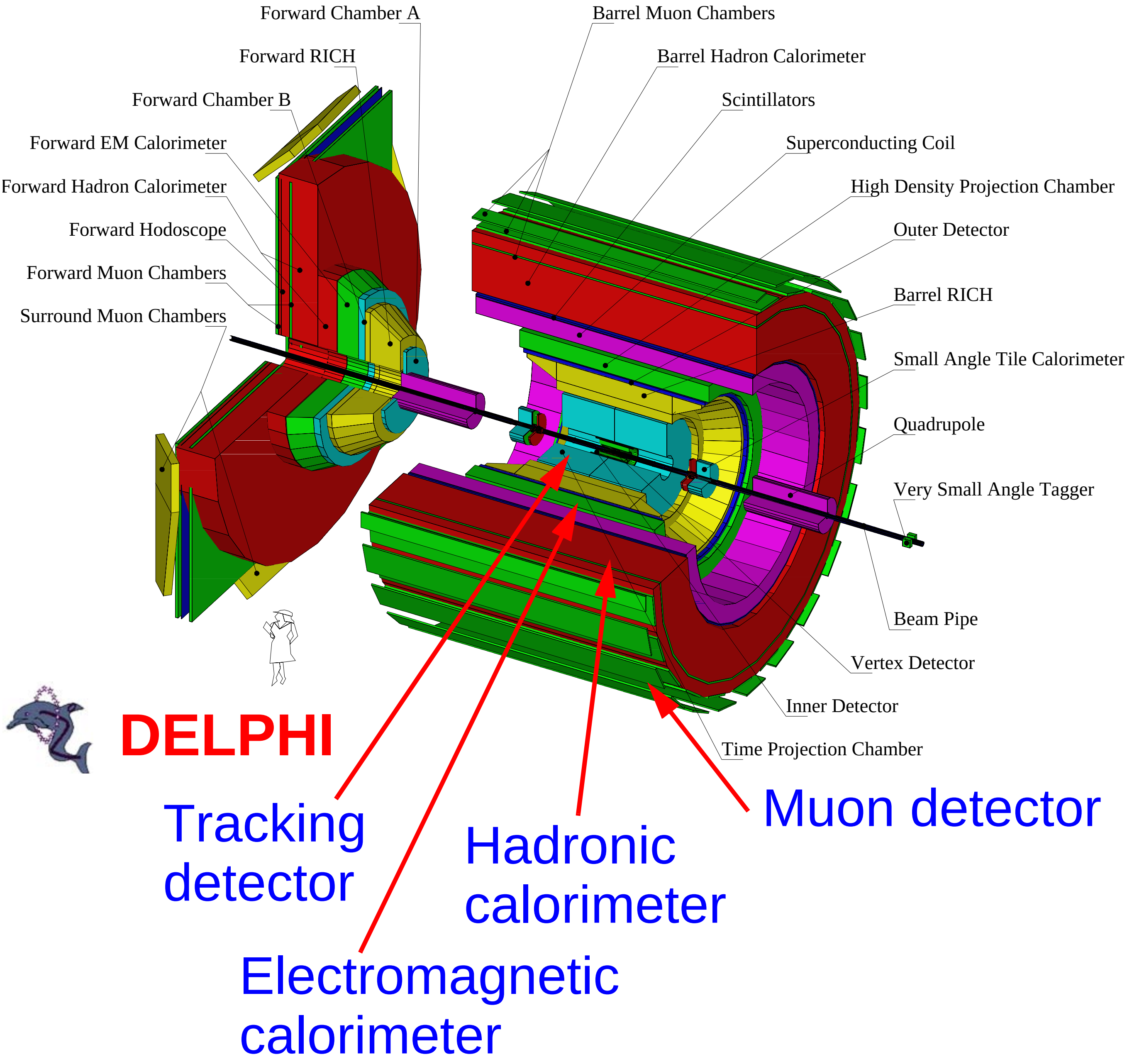
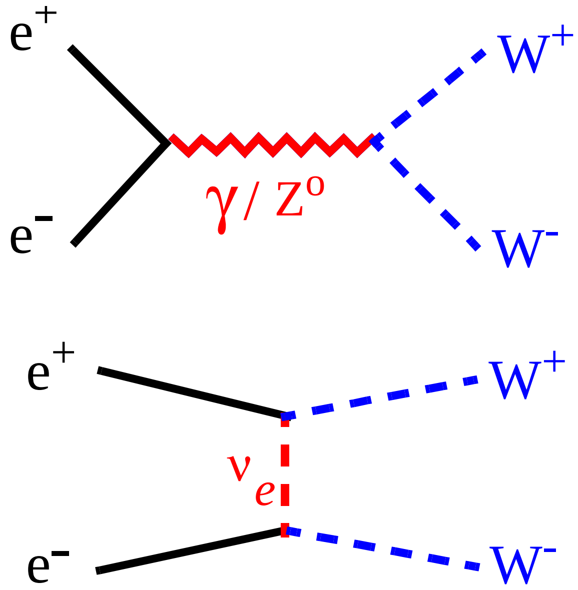
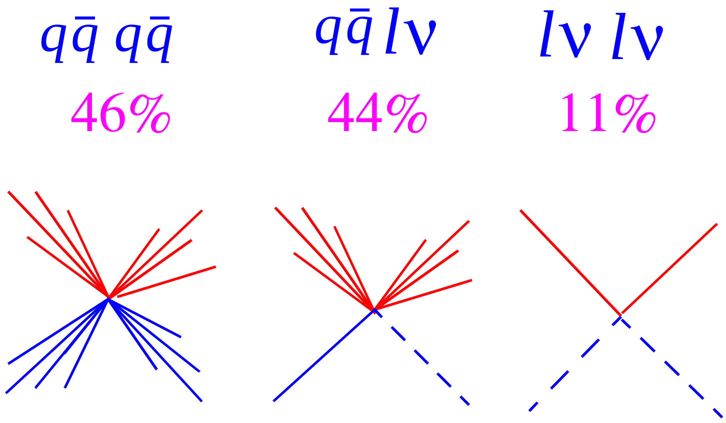


Figure 84: The DELPHI experiment at LEP.

- The W bosons were produced in pairs at LEP:





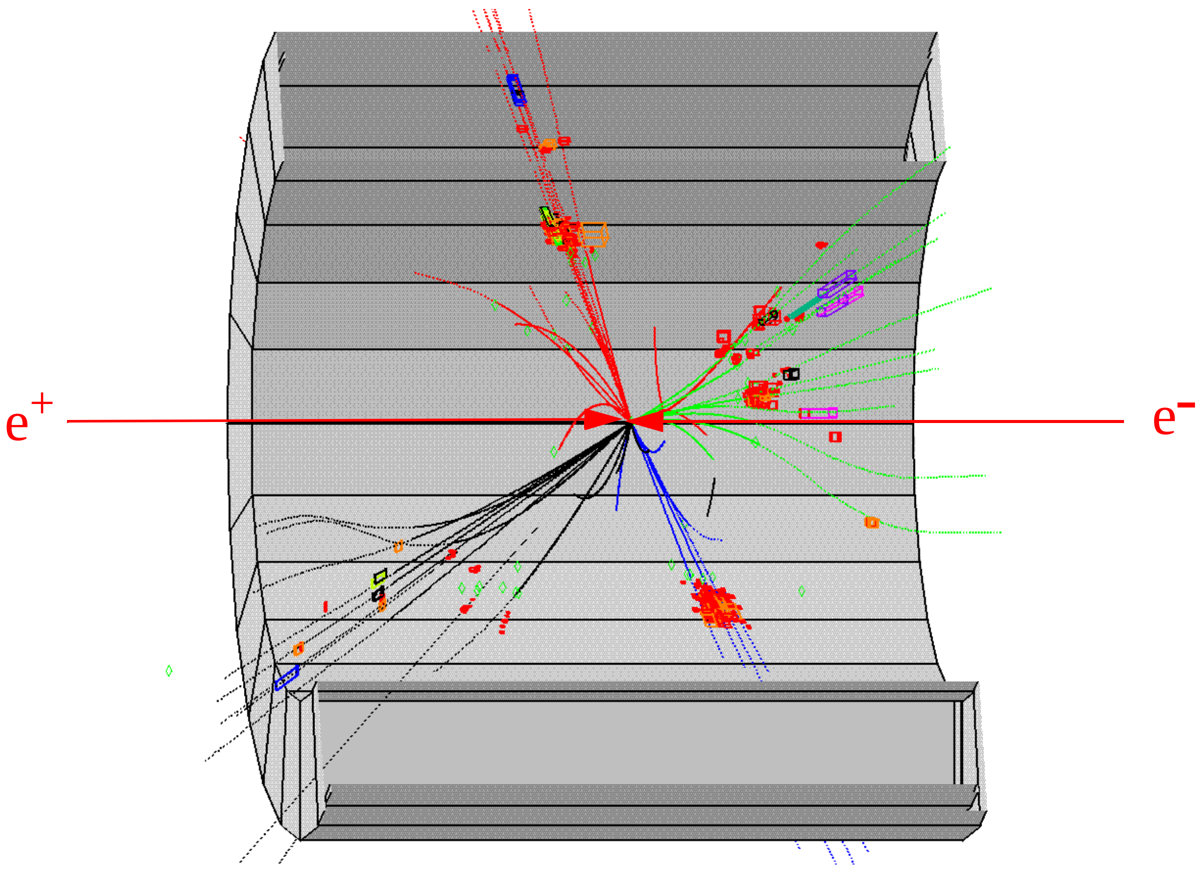
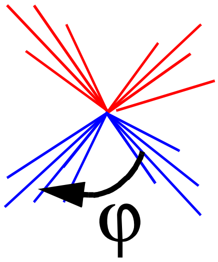


Figure 85: A WW-event with 4 jets in the DELPHI experiment.

The mass of the W can be calculated from the energy and the direction of the jets:

$q\bar{q}\ q\bar{q}$

46%



$$M_W^2 = (\vec{P}_q + \vec{P}_{\bar{q}})^2 \quad \text{(4-vectors)}$$

$$M_W^2 = 2\, E_q E_{\bar{q}} \, (1 - \cos\varphi)$$

if $m_q = 0$

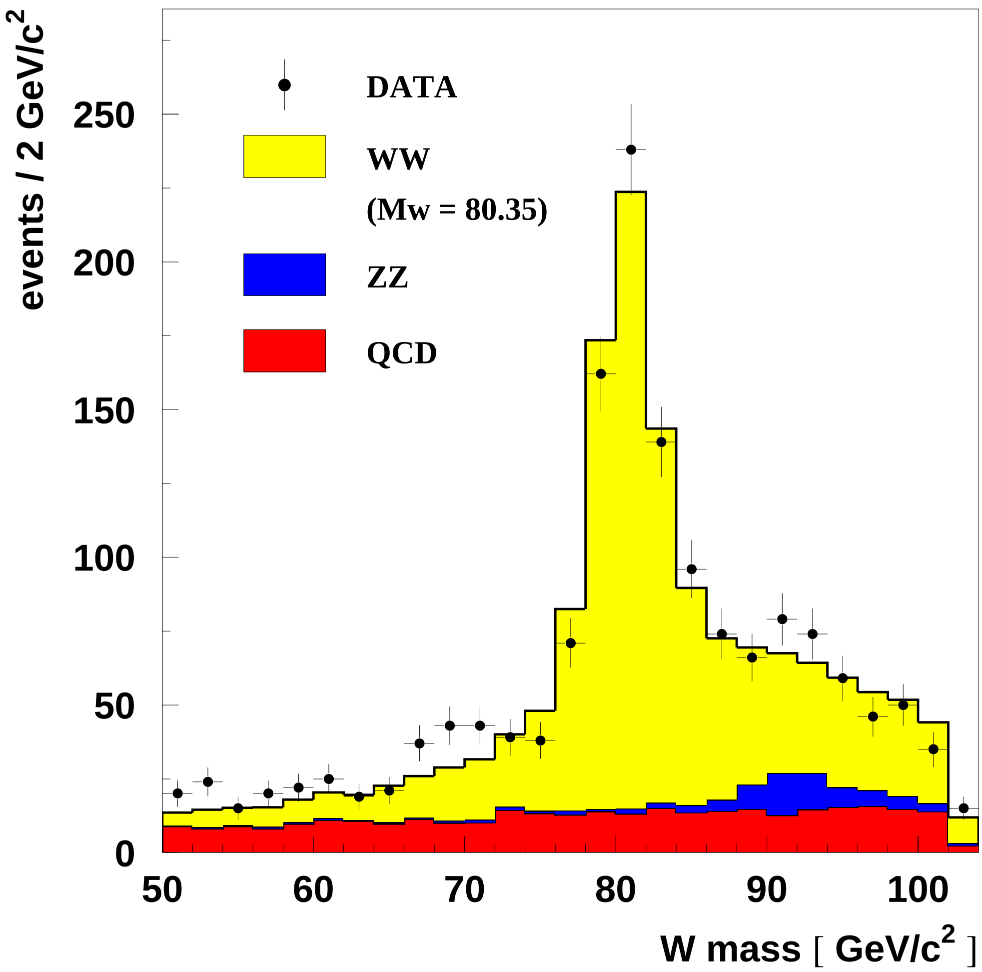


Figure 86: The mass distribution from jet-pairs in WW events selected by the DELPHI experiment.

DISCOVERY OF THE Z BOSON

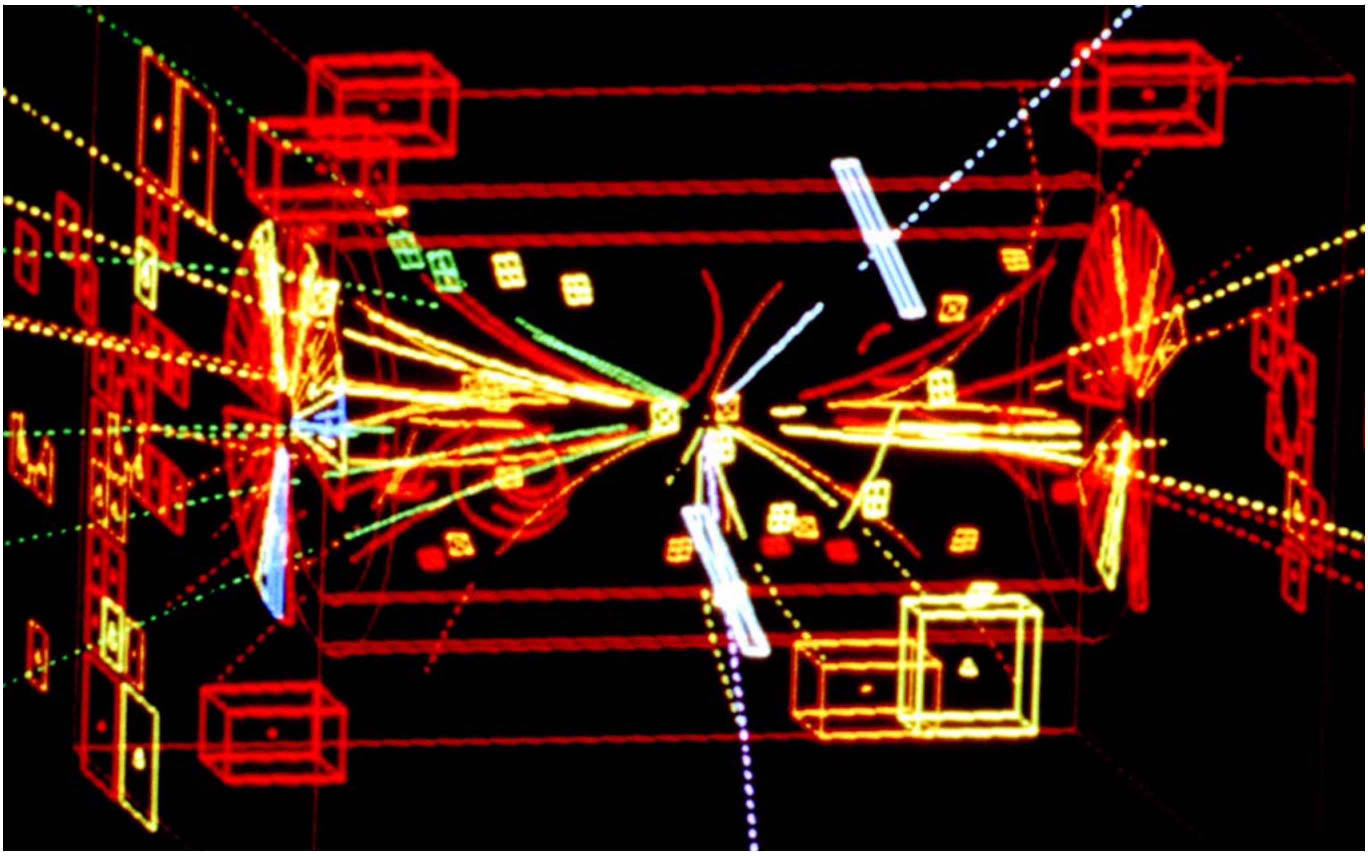
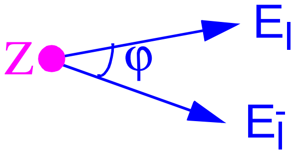


Figure 87: The production of a Z^0 event in the UA1 detector.

- The signature of a Z^0 boson created in $p\bar{p}$ collision is a pair of leptons with large transverse momenta.

- The mass of the Z^0 is given by the invariant mass of the leptons:



if $m_l = 0$

$$M_Z^2 = 2 E_l E_{\bar{l}} (1 - \cos\varphi)$$

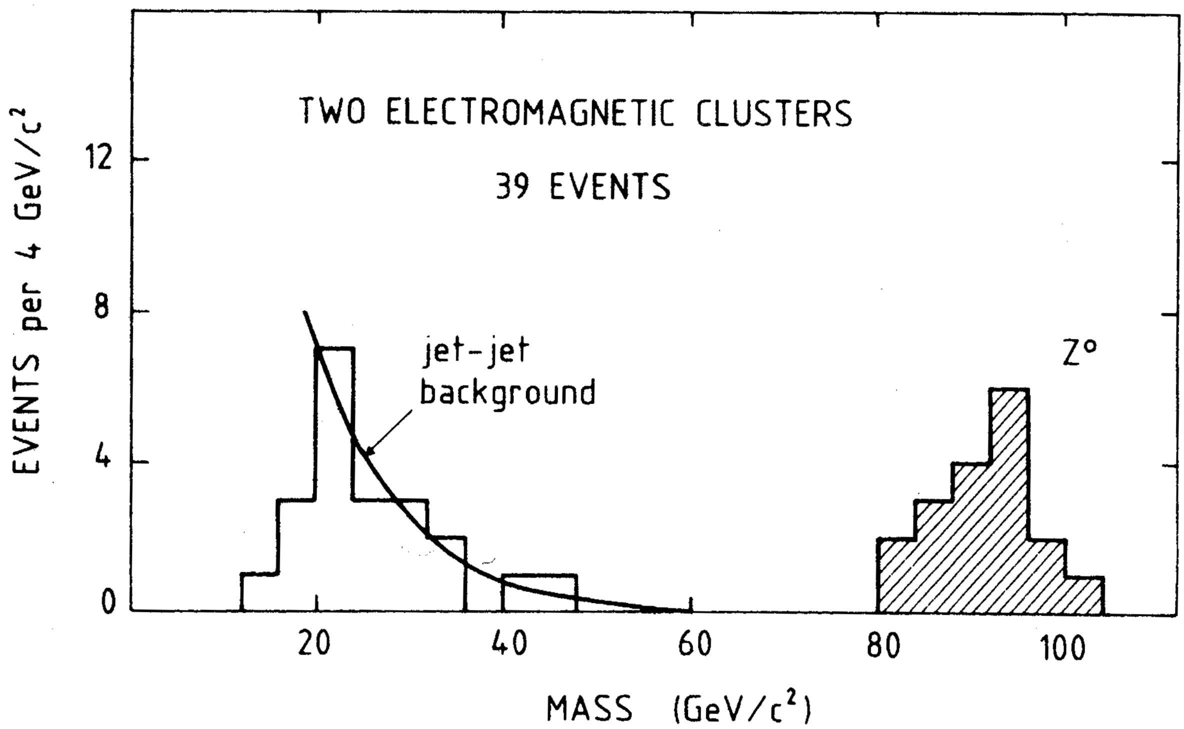


Figure 88: The UA1 mass distribution of pairs of electrons where each electron has E_T > 8 GeV.

→ From the first 18 electron and 10 muon events recorded by the UA1 it was estimated that

$$M_Z = 93{,}0 \pm 1{,}4\ GeV \qquad \Gamma_Z \leq 8{,}1\ GeV$$

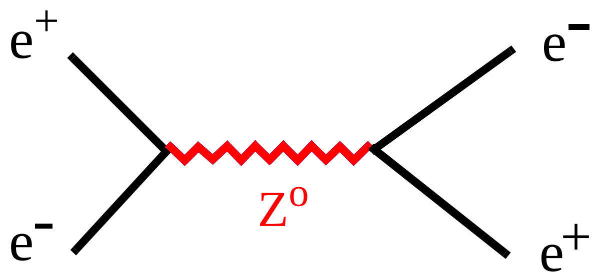
→ From measurement at the LEP accelerator, the Z mass and width is now estimated to be

$$M_Z = 91{,}188 \pm 0{,}002 GeV \qquad \Gamma_Z = 2{,}495 \pm 0{,}002\ GeV$$

The branching ratios of the leptonic decay modes of the Z are measured to be about 3% for each lepton.

STUDIES OF THE Z AT LEP.

Example of events in which Z bosons are produced:



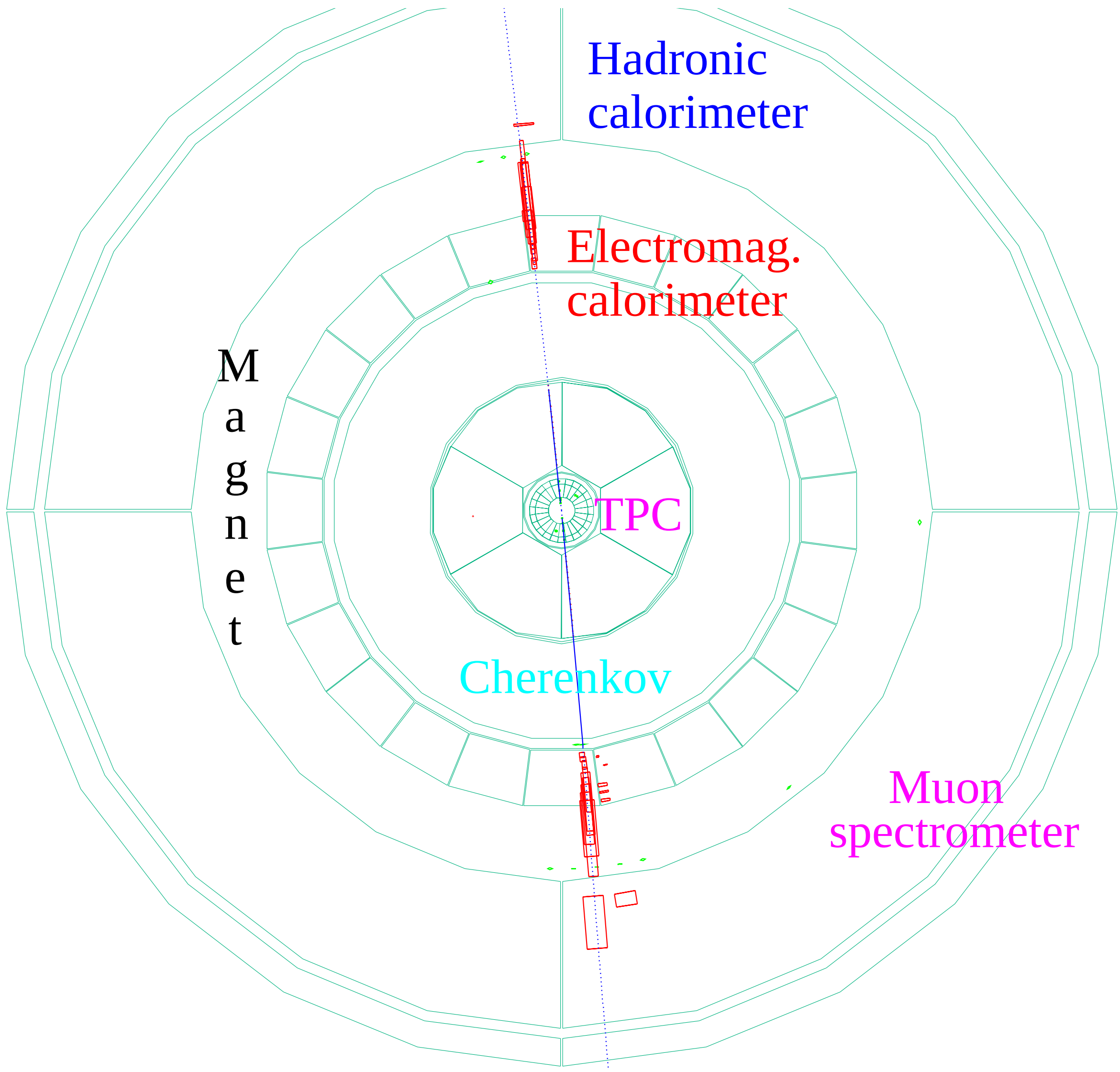
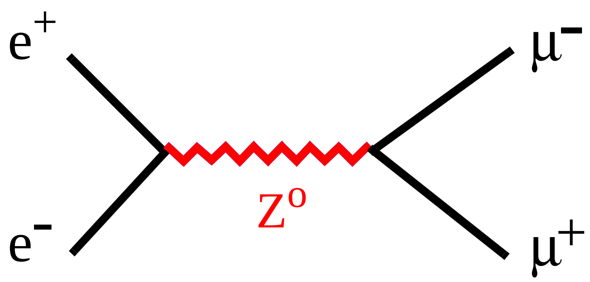


Figure 89: A Z boson decays to an electron pair in DELPHI.



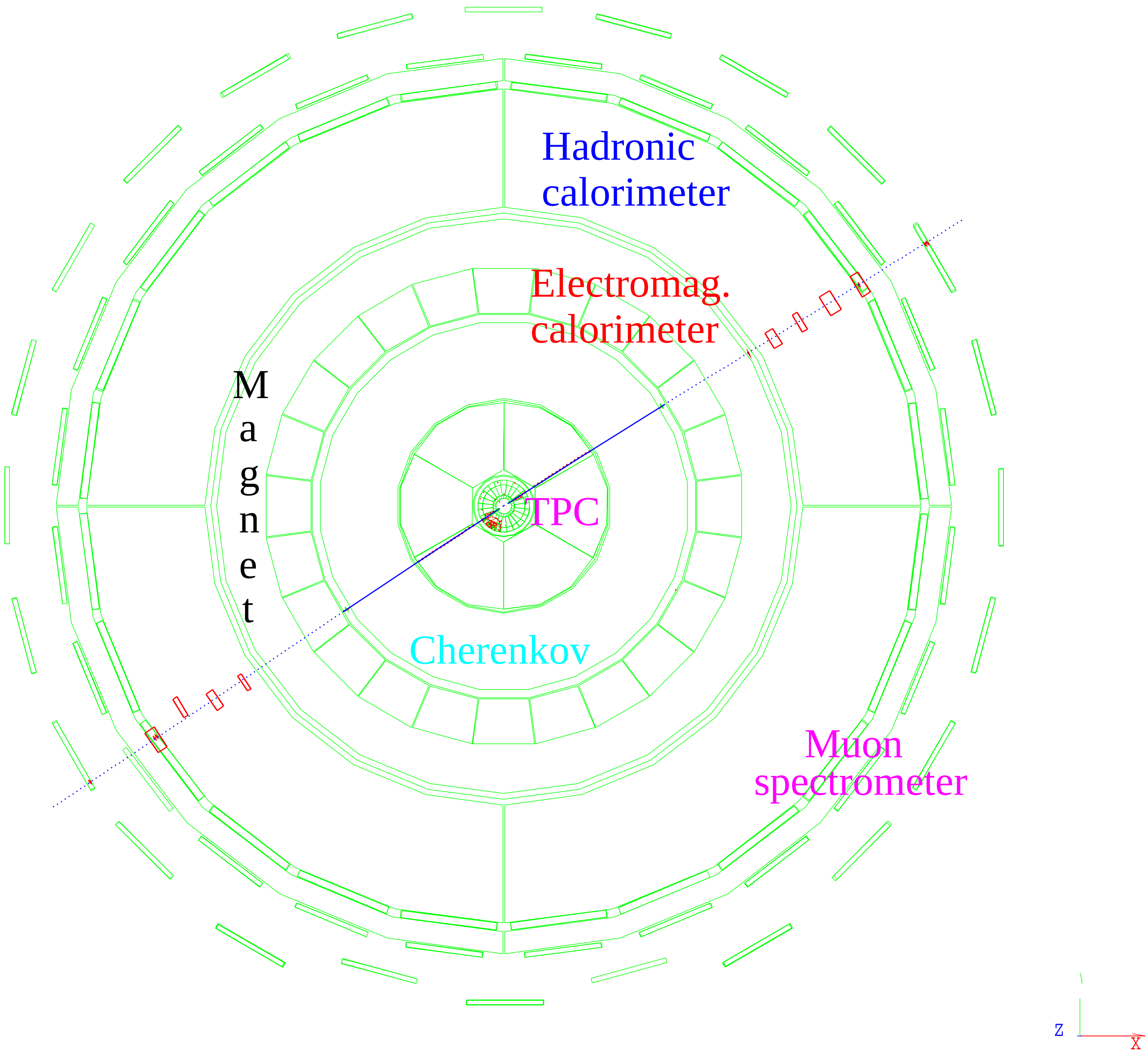
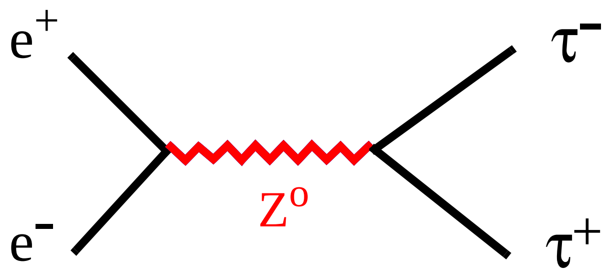


Figure 90: A Z boson decays to a muon pair in DELPHI.



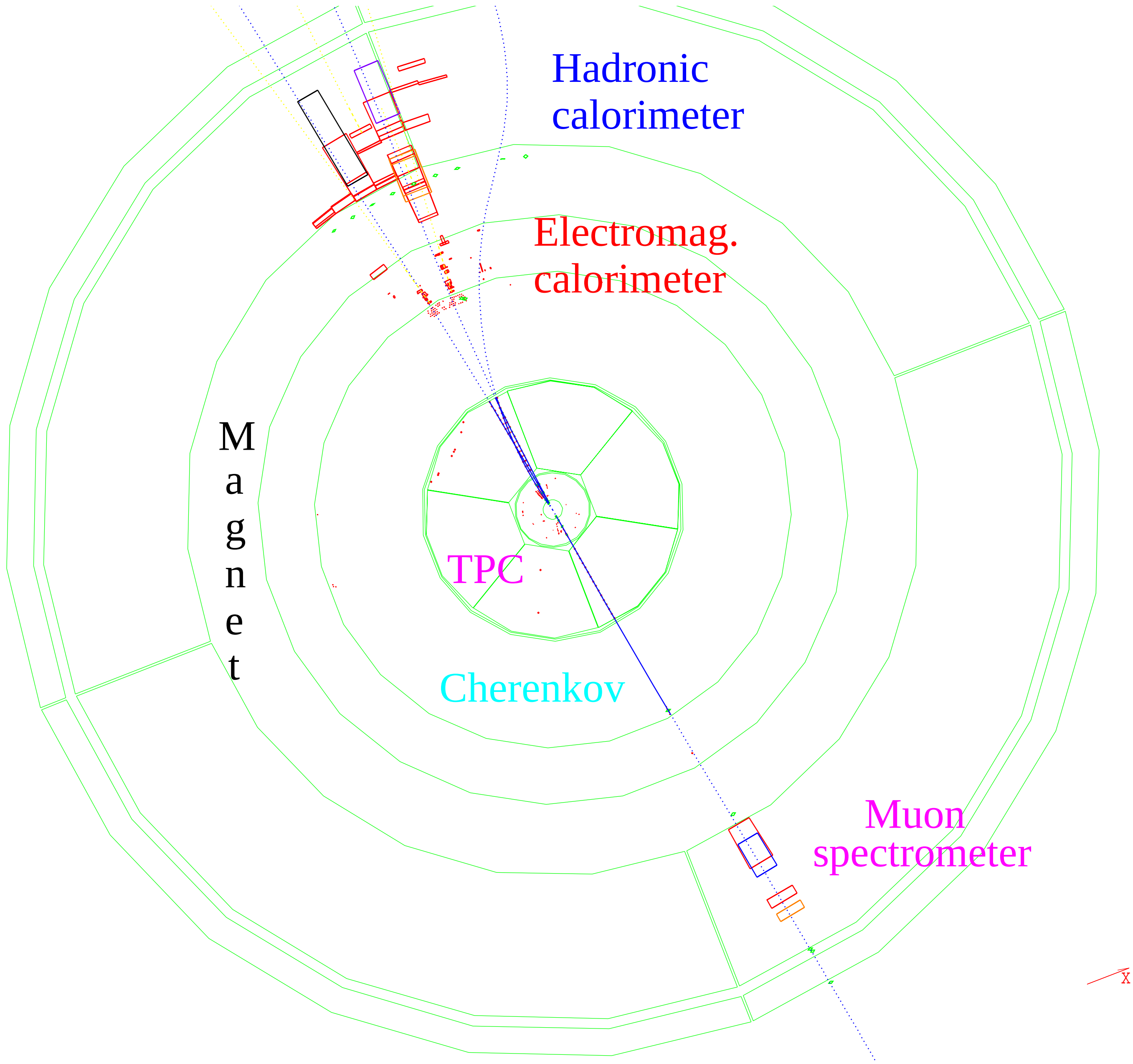


Figure 91: A Z boson decays to a tau pair in DELPHI.

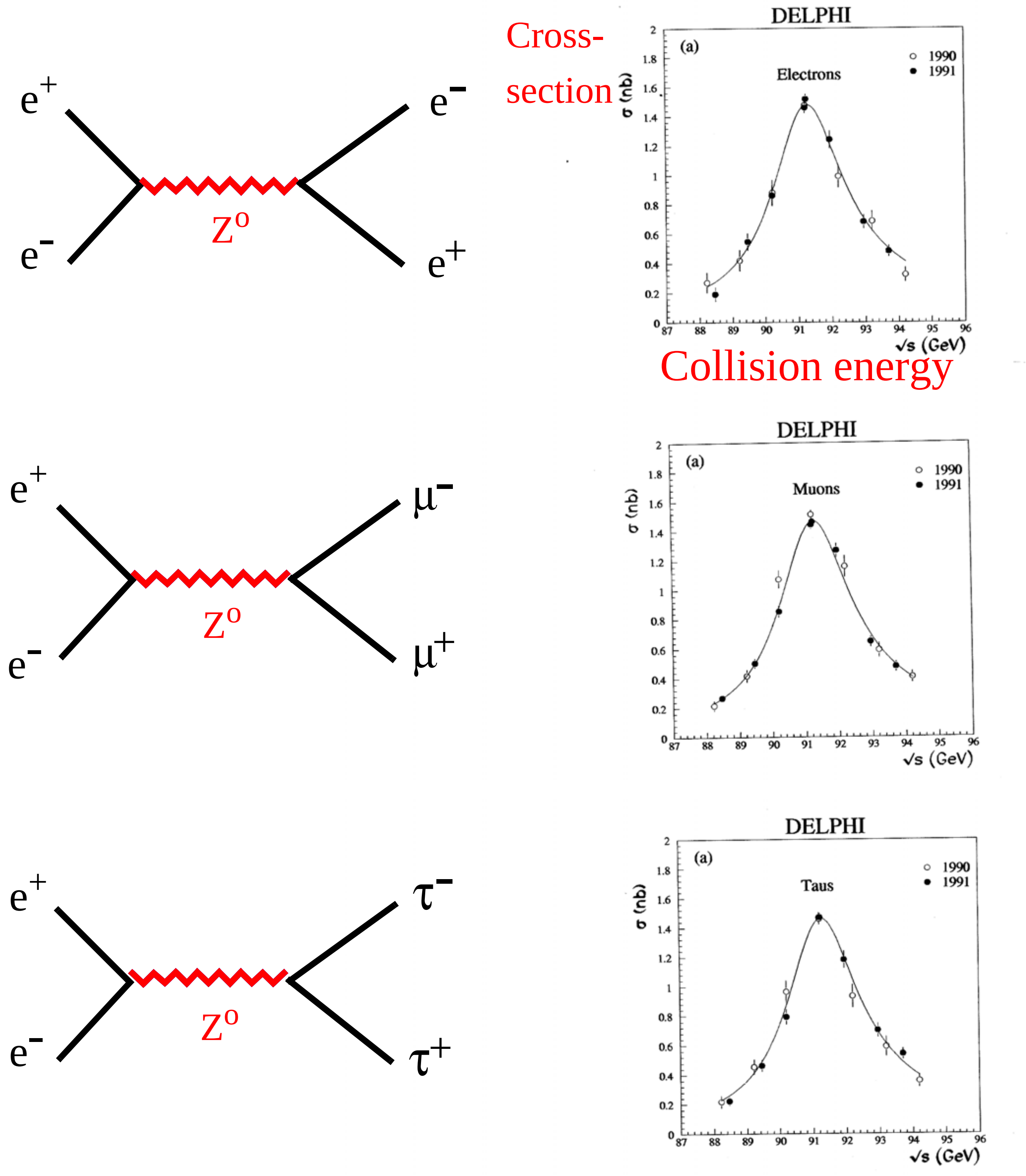
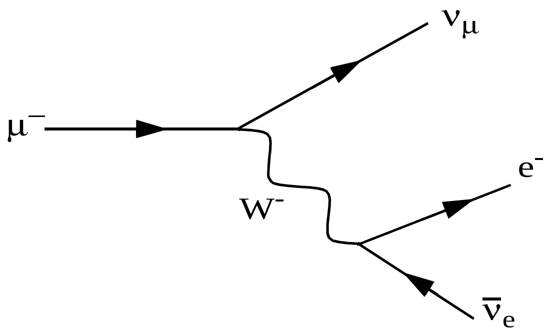


Figure 92: A cross section measurement of the leptonic decay of the Z^0.

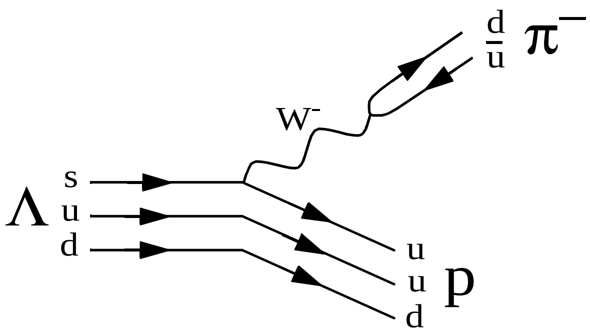
Charged current reactions

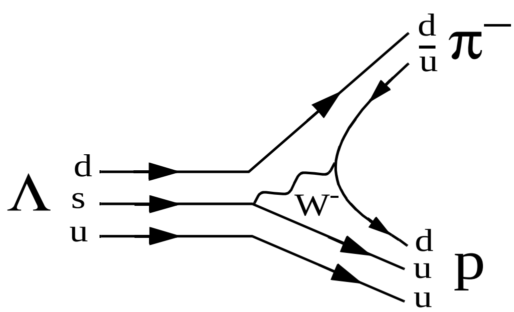
❖ Charged current reaction are reactions mediated by the charged W-bosons. They can be divided up into:

1) purely *leptonic* processes: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

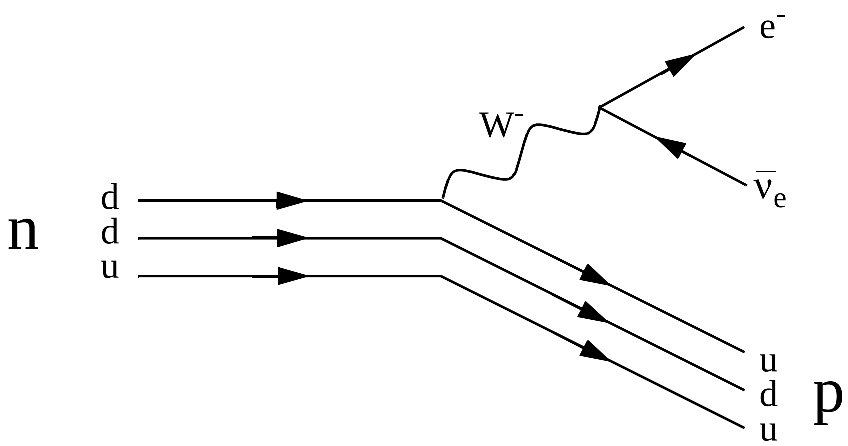


2) purely *hadronic* processes: $\Lambda \rightarrow \pi^- + p$





3) *semileptonic* reactions: $n \rightarrow p + e^- + \bar{\nu}_e$



Reminder: All the electromagnetic interactions can be built from eight basic interactions:

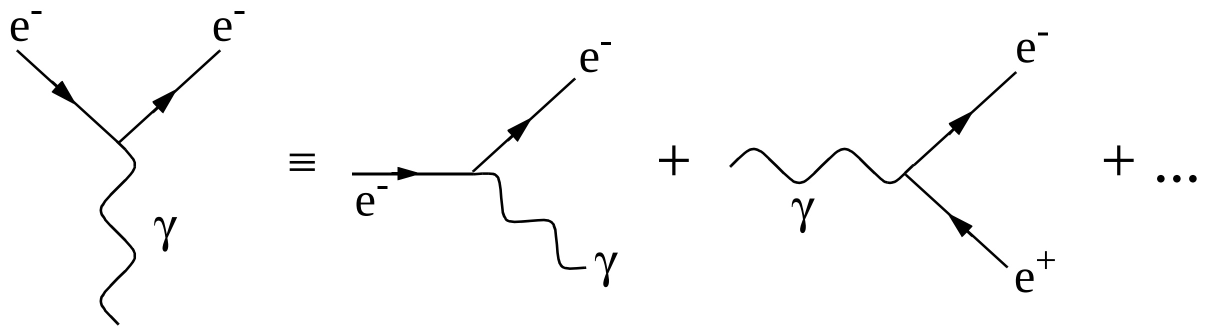


Figure 93: The basic vertex for electron-photon interactions.

- In a similar way, leptonic weak interaction processes can be built from a certain number of reactions corresponding to basic vertices:

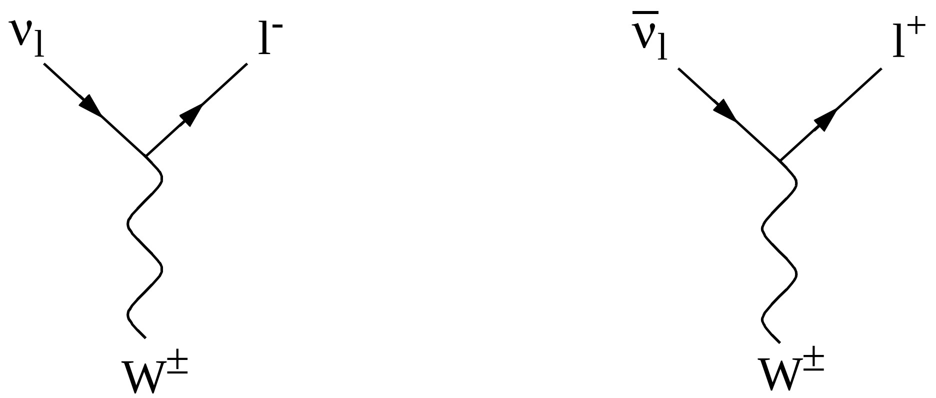
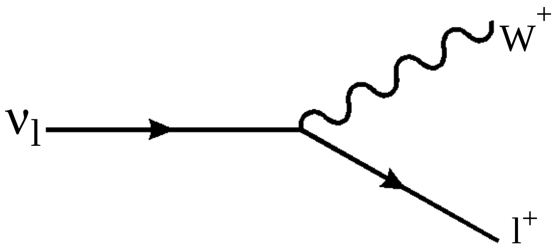
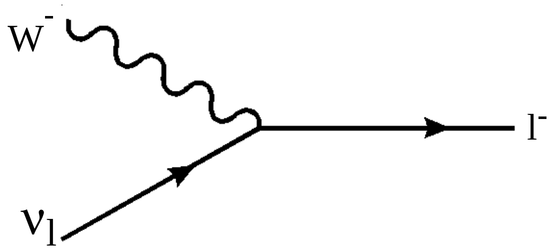


Figure 94: The two basic vertices for $W^\pm$-lepton interactions.

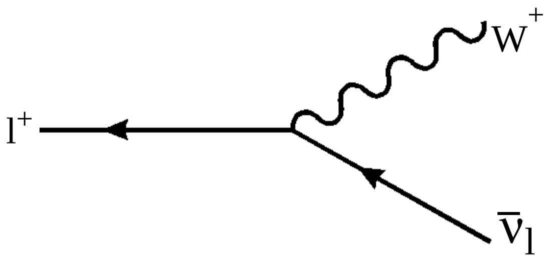
a) $\nu_l \rightarrow l^- + W^+$



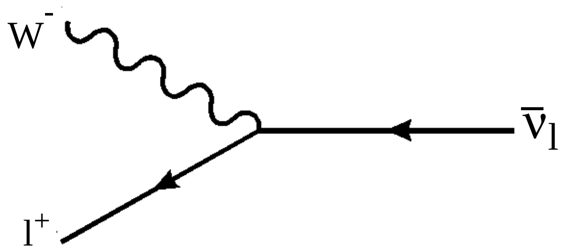
b) $W^- + \nu_l \rightarrow l^-$



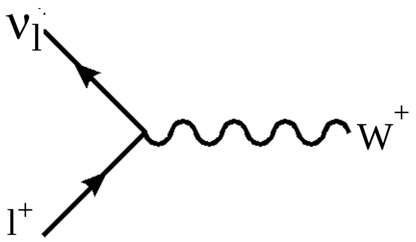
c) $l^+ \rightarrow W^+ + \bar{\nu}_l$



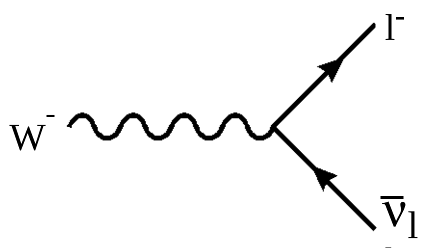
d) $W^- + l^+ \rightarrow \bar{\nu}_l$



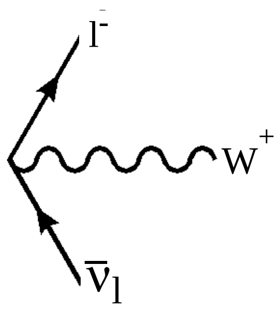
e) $l^+ + \nu_l \rightarrow W^+$



f) $W^- \rightarrow l^- + \bar{\nu}_l$



g) vacuum $\rightarrow l^- + \bar{\nu}_l + W^+$



h) $l^+ + \nu_l + W^- \rightarrow$ vacuum

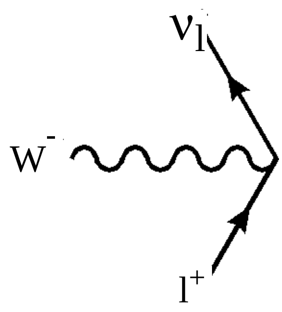


Figure 95: Eight basic reactions derived from the two basic W vertices. None of these processes, with the exception of e) and f) can occur by themselves since they do not conserve momentum and energy. e) and f) can occur by themselves if $M_W > M_l + M_{\nu l}$ ($l = e, \mu, \tau$) in the W restframe.

❖ Weak interactions always conserve the lepton numbers: L_e , L_μ , L_τ .

Diagram-wise this conservation is guaranteed by:

– at each vertex, there is one arrow pointing in and one pointing out.

– lepton indices “l” are the same on both lines.

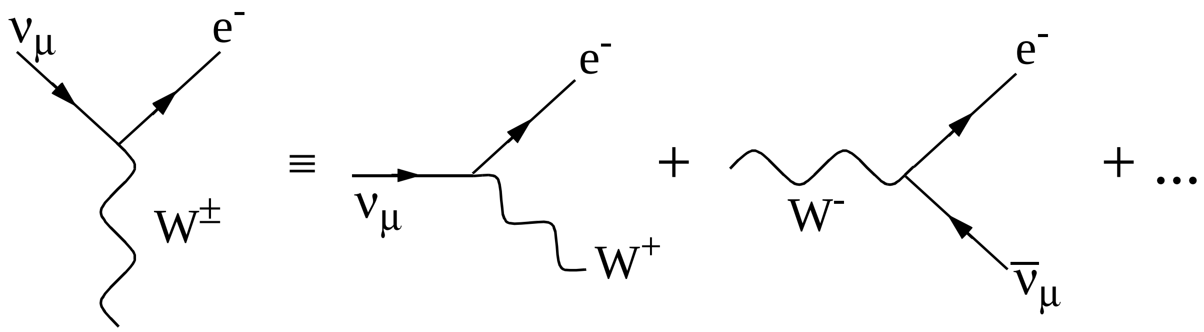


Figure 96: Vertices **violating** lepton number conservation.

❖ Leptonic vertices are characterized by the corresponding weak strength parameter α_W which do not depend on the lepton type involved.

→ The decay rate of $W \to e\nu$, can be estimated to the first order to be

$$\Gamma(W \to e\nu) \approx \alpha_W M_W \approx 80\alpha_W\ GeV$$

since the process involves only one vertex and lepton masses are negligible.

→ A meaurement of the decay rate gives

$$\Gamma(W \to e\nu) \approx 0{,}2\ GeV$$

which translates into a value of the weak strength parameter α_W of

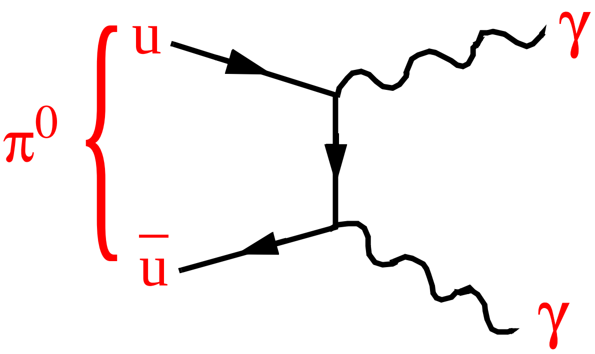
$$\alpha_W \approx 0{,}003 \tag{96}$$

hence the strength of the weak interaction is comparable with that of the electromagnetic interaction for which $\alpha_{em} \approx 0{,}007$.

❖ Since α_W and α_{em} is of a similar size, why is the weak interaction weak ?

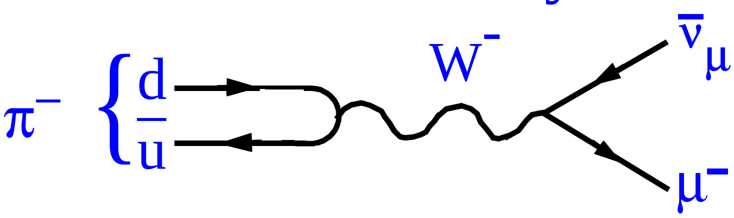
Compare the decay of charged and neutral pions:

Electromagnetic decay



Lifetime = 8 x 10^{-17} s

Weak decay



Lifetime = 3000000000 x 10^{-17} s

(Lifetime of a real W = 0.00000003 x 10^{-17}s)

→ The apparent weakness of the weak interaction is due to the the very large W and Z masses !

❖ An analogue of electron-electron scattering by photon exchange is the inverse muon decay:

$$\nu_\mu + e^- \to \mu^- + \nu_e$$

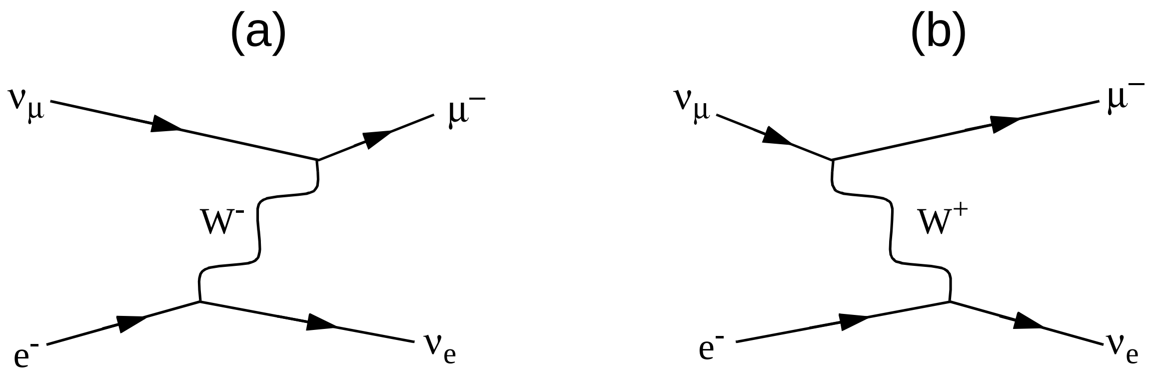
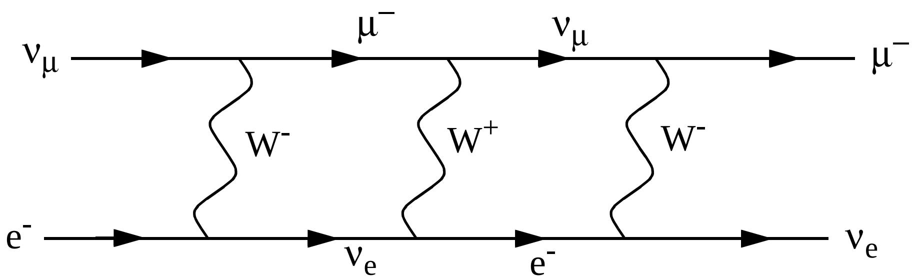


Figure 97: Time-ordered diagrams for inverse muon decay.

Time ordering implies changing the sign of the current !

→ The above diagrams are of second order while the following diagram is of the sixth order



This diagram gives a contribution of order α_W^6 to the total cross section while the second order diagram contributes α_W^2.

❖ Since W bosons are very heavy, inverse muon decay can be approximated by a zero-range interaction:

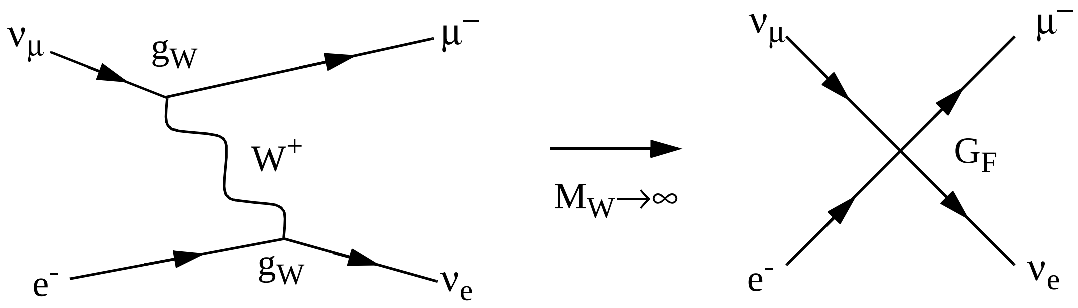


Figure 98: A low-energy zero-range interaction in muon decay.

→ The strength of the zero-range interaction is given by the Fermi coupling constant G_F.

→ Taking into account spin effects, the relation between α_W and G_F in the zero-range approximation is:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{M_W^2} = \frac{4\pi\alpha_W}{M_W^2}$$

where g_W is a coupling constant which characterize the strength at the charge current vertex: $\alpha_W \equiv g_W^2 / 4\pi = 0{,}004$ to be compared with $\alpha = 0{,}007$.

Weak interaction of hadrons.

- Weak interactions of hadrons: constituent quarks emit or absorb W bosons

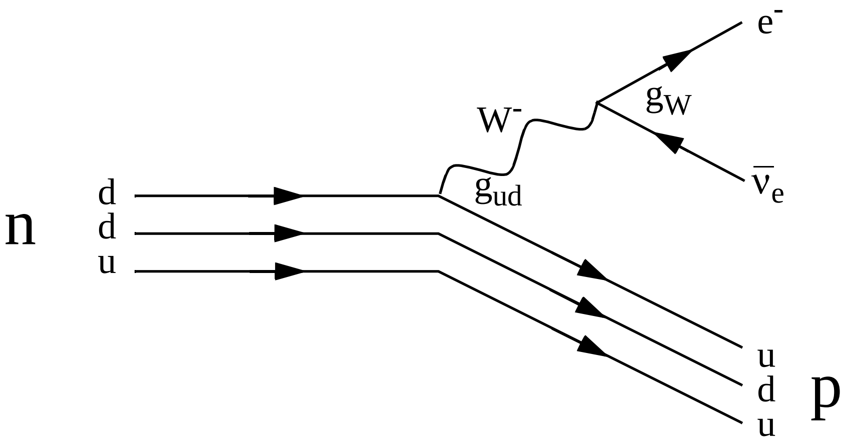


Figure 99: Neutron β-decay.

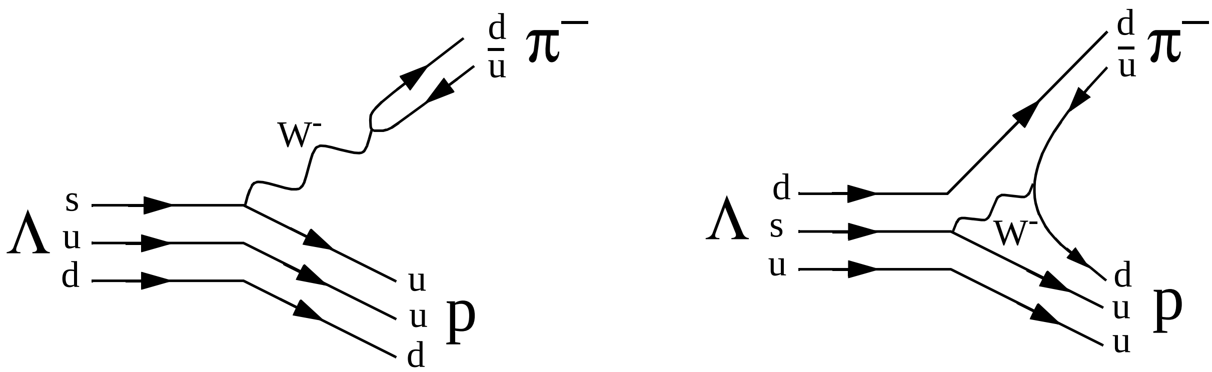


Figure 100: The dominant quark diagrams for Λ decay.

❖ *Lepton-quark symmetry*: assumption that corresponding generations of quarks and leptons have identical weak interactions:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \leftrightarrow \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \leftrightarrow \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \leftrightarrow \begin{pmatrix} t \\ b \end{pmatrix}$$

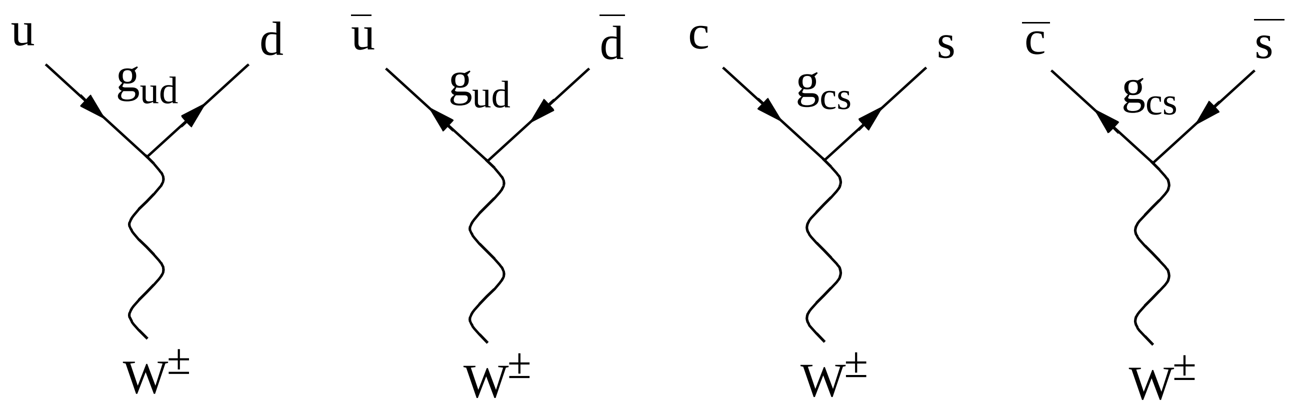


Figure 101: The W-quark vertices obtained from lepton-quark symmetry (if quark mixing and the third family is ignored).

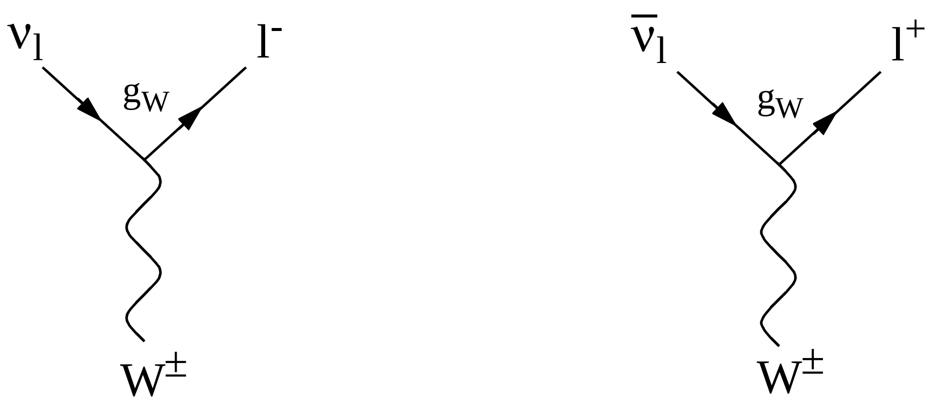


Figure 102: REMINDER: The two basic vertices for $W^{\pm}$-lepton interactions.

→ Examples of reactions that are allowed and not allowed according to the lepton-quark symmetry scheme are pion and kaon decay:

$\pi^- \to \mu^- + \bar{\nu}_\mu$ $\quad d\bar{u} \to \mu^- + \bar{\nu}_\mu$ $\quad$ allowed !

$K^- \to \mu^- + \bar{\nu}_\mu$ $\quad s\bar{u} \to \mu^- + \bar{\nu}_\mu$ $\quad$ not allowed !

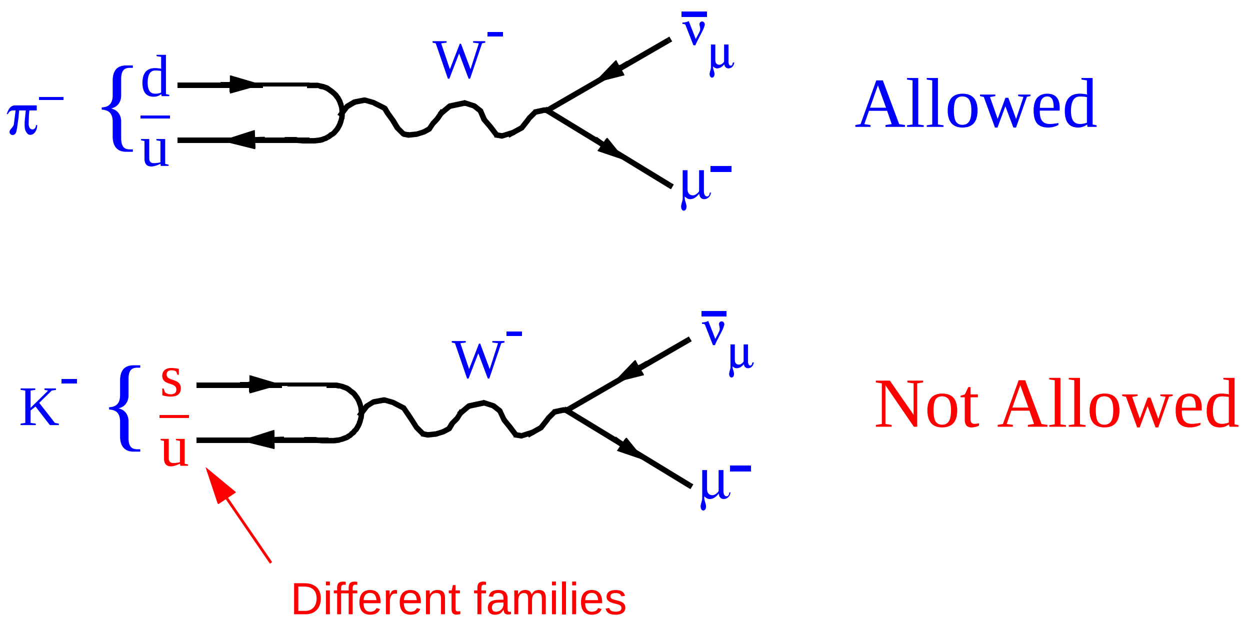


Figure 103: Decays that are allowed and not allowed if there is a lepton-quark symmetry in nature.

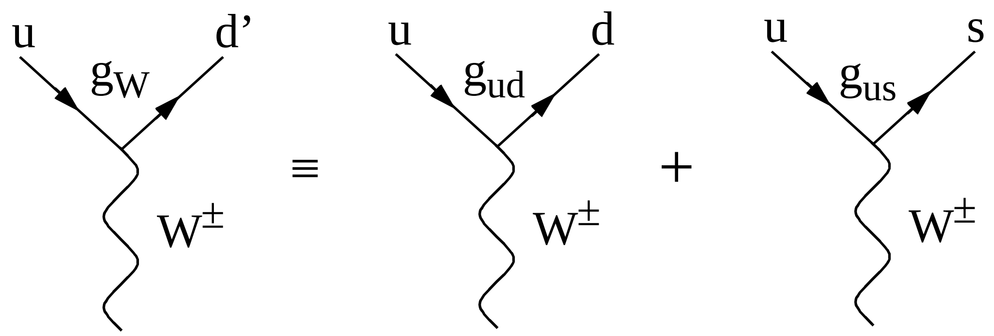
→ The kaon decay can be made allowed by introducing quark mixing (originally proposed by Cabibbo).

→ According to the quark mixing scheme, d- and s-quarks participate in the weak interactions via the linear combinations:

$$d' = d\cos\theta_C + s\sin\theta_C$$
$$s' = -d\sin\theta_C + s\cos\theta_C$$

where the parameter θ_C is called the *Cabibbo angle*.

→ With quark mixing the quark-lepton symmetry applies to doublets like $\begin{pmatrix} u \\ d' \end{pmatrix}$ *and* $\begin{pmatrix} c \\ s' \end{pmatrix}$



$g_{ud}=g_W\cos\theta_C$ $g_{us}=g_W\sin\theta_C$

Figure 104: The ud´W vertex can be interpreted as a sum of the udW and usW vertices.

→ With the quark mixing hypothesis some more W-quark vertices are allowed:

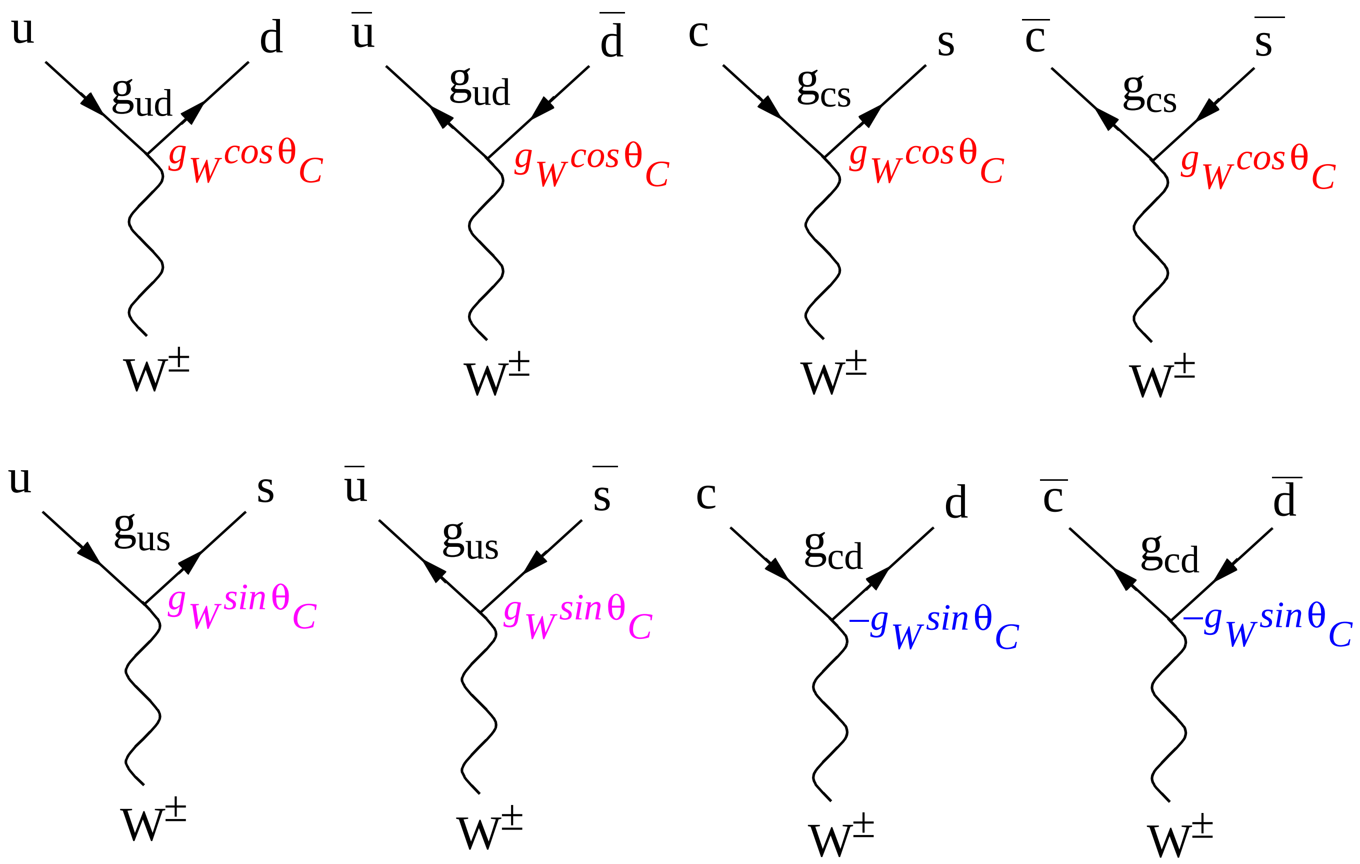


Figure 105: The basic W-quark vertices if quark mixing is taken into account.

The top row of diagrams have couplings given by

$$g_{ud} = g_{cs} = g_W \cos\theta_C \tag{97}$$

while the bottom row diagrams have the couplings

$$g_{us} = -g_{cd} = g_W \sin\theta_C \tag{98}$$

❖ The Cabibbo angle is measured experimentally by comparing decay rates like:

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2\theta_C$$

which give the result

$$\theta_C = 12{,}7° \pm 0{,}1° \qquad (99)$$

$$g_W \cos\theta_C = 0{,}98 g_W$$

$$g_W \sin\theta_C = 0{,}22 g_W$$

→ The charmed quark couplings g_{cd} and g_{cs} are measured in neutrino scattering experiments and this give the same result:

$$\theta_C = 12° \pm 1°$$

❖ Experimentally it has been observed that charmed hadrons almost always decays into strange hadrons.

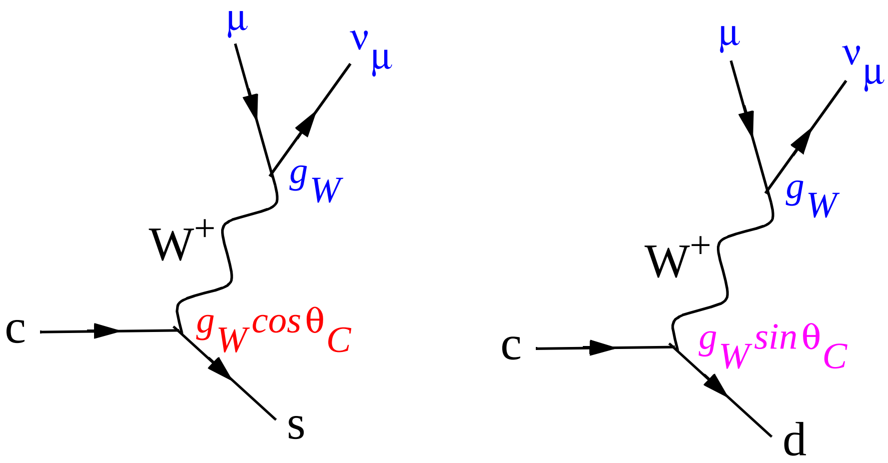


Figure 106: Cabbibo-allowed and Cabbibo-suppressed semi-leptonic decay of a charmed quark.

→ The ratio of semi-leptonic decays with Cabbibo-allowed and Cabibbo surpressed couplings are given by

$$\frac{g_{cd}^2}{g_{cs}^2} = \tan^2\theta_C = \frac{1}{20}$$

The third generation

→ The existence of the c-quark was first predicted from the lepton-quark symmetry.

→ After the discovery of τ, ν_τ, and b, the sixth quark was predicted to complete the symmetry and the top-quark was discovered in 1994 with a mass of about 180 GeV/c^2.

❖ The two generation quark mixing is conveniently written in matrix form as:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \qquad (100)$$

→ A third generation: gives rise to the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{\alpha\beta}$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad (101)$$

→ The coupling constants are then:

$$g_{\alpha\beta} = g_W V_{\alpha\beta} \qquad (\alpha = u, c, t; \beta = d, s, b) \tag{102}$$

→ If the mixing between the b quark and (d,s) quarks can be neglected, the CKM matrix is reduced to

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{103}$$

and hence b'=b.

→ Since the two-generation mixing model agree well with data V_{ub}, V_{cb}, V_{td} and V_{ts} must be small.

→ The b-quark is heavy (mass=4.5 GeV) and it can decay to the lighter u and c quarks as in the following decay:

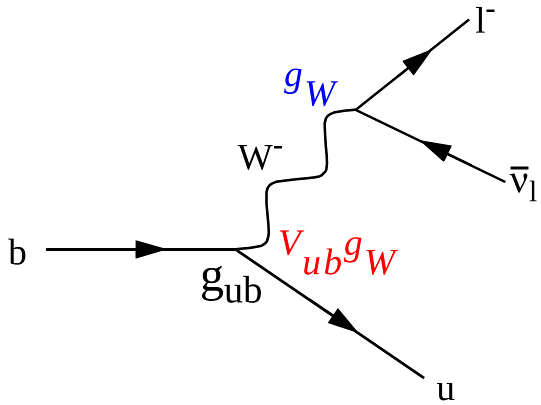


Figure 107: The semi-leptonic decay of the b-quark to a lighter quark.

→ This decay modes has a rate proportional to the squared couplings:

$$\left|g_{ub}\right|^2 = \left|V_{ub}\right|^2 g_W^2 \tag{104}$$

→ If V_{ub} and V_{cb} are 0, b-quark must be stable. Experimentally, it is clear that the b lifetime is long but it is NOT stable.

- The weak b-quark decay can be compared to that of the τ-lepton which have a decay rate proportional to g_W^2.

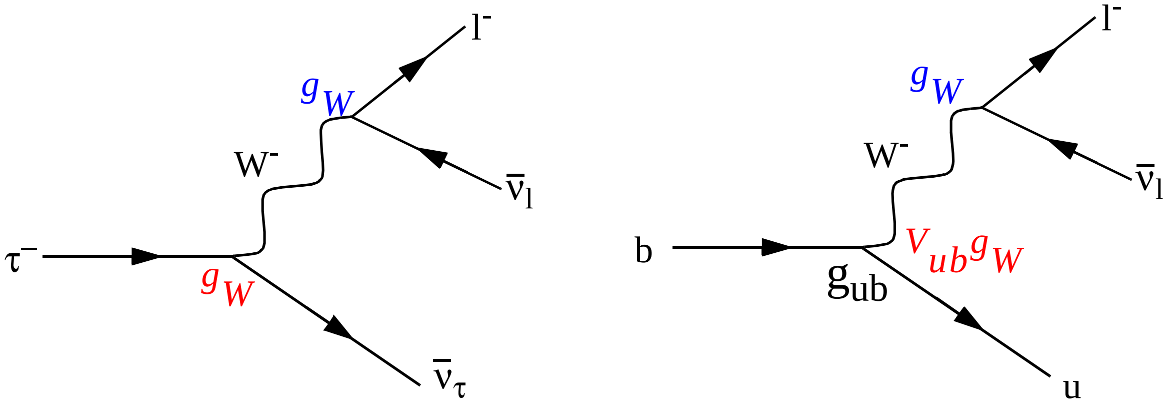


Figure 108: The decay of the τ-lepton can be compared to that of the b-quark.

If one assumes that $|V_{ub}| = 1$ then one can predict what lifetime this would result in for the b-meson:

$$\tau_b \approx \frac{1}{N}\left(\frac{m_\tau}{m_b}\right)^5 \bullet \tau_\tau \approx 10^{-15} s$$

where $\tau_\tau \approx 3 \times 10^{-13}$ s is the lifetime of the τ-lepton, N is the number of possible b-quark decays per analogous τ-decays (3 or 4) and m_τ and m_b are the masses of the τ and the b.

Conclusion:

$$|V_{ub}| = |V_{cb}| = 0 \qquad \tau_b \approx \infty$$

$$|V_{ub}| = 1 \qquad \tau_b \approx 10^{-15} s$$

Experimentally: $\tau_b \approx 10^{-12} s$

The most precise measurements at present yield

$$|V_{ub}| \approx 0,004 \quad and \quad |V_{cb}| \approx 0,04 \qquad (105)$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0,004 \\ -\sin\theta_C & \cos\theta_C & 0,04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0,98 & 0,22 & 0,004 \\ -0,22 & 0,98 & 0,04 \\ \sim 0 & \sim 0 & \sim 1 \end{pmatrix}$$

The top-quark

❖ The top-quark is much heavier than even W bosons and can decay by

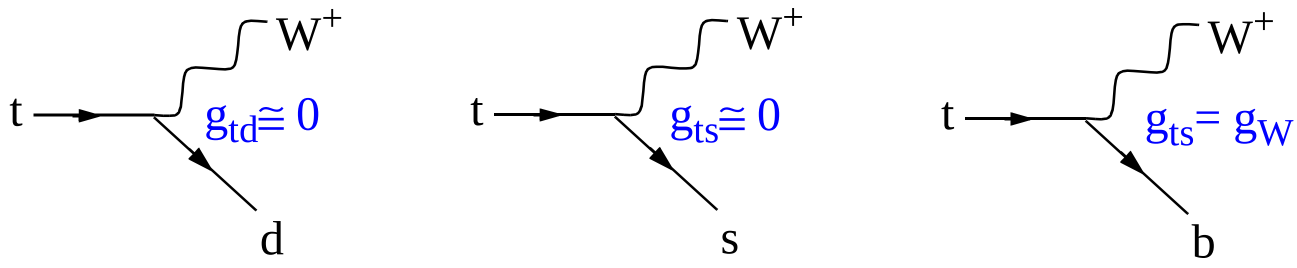


Figure 109: Weak interactions involving the top-quark.

❖ Since $g_{td} \cong 0$ and $g_{ts} \cong 0$ the only significant decay mode of the t-quark is

$$t \to W^+ + b \tag{106}$$

with a rate proportional to

$$\alpha_W = g_W^2 / 4\pi \approx 4{,}2 \times 10^{-3}$$

Estimation of the decay width: $\Gamma \sim \alpha_W m_t \sim 1\ GeV$ suggests a very short lifetime for the top-quark:

$$\tau_t \approx 4 \times 10^{-25} s \tag{107}$$

Discovery of the top quark.

❖ The top-quark was discovered in $p\bar{p}$-collisions in the Collider Detector at Fermilab (CDF) in 1994. The Tevatron accelerator with a collision energy = 1.8 TeV was used.

→ PRODUCTION: In proton colliders, pairs of top quarks are produced by the quark-antiquark annihilation process:

$$q + \bar{q} \to g \to t + \bar{t} \tag{108}$$

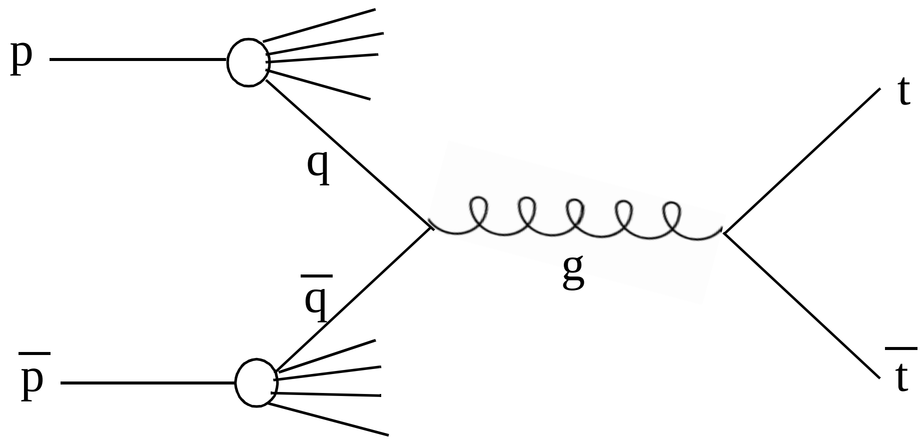


Figure 110: The production of top quarks in $p\bar{p}$ annihilation.

→ DECAY: The top quark decay in most cases to a b-quark and to a W which in turn decays to leptons or quarks:

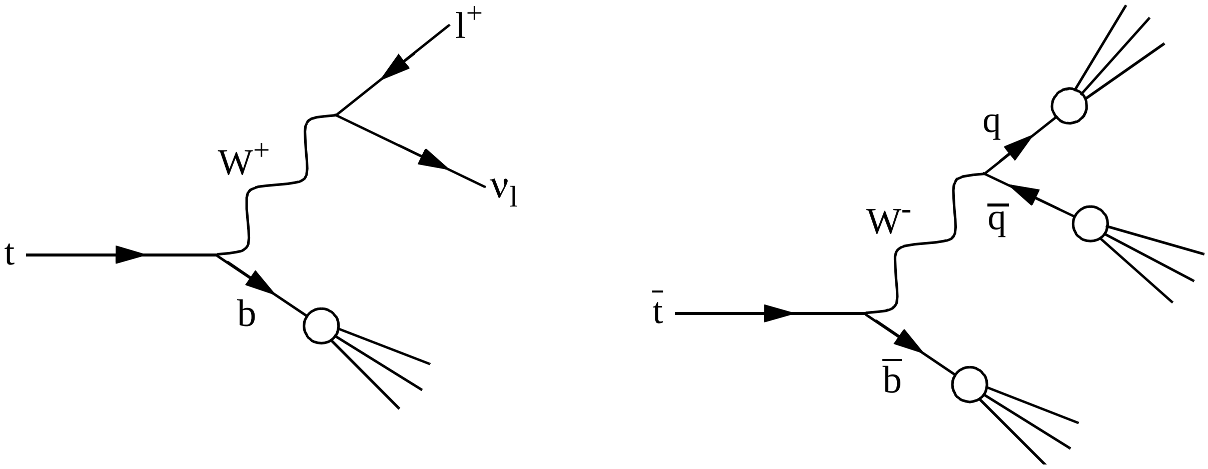


Figure 111: The decay of top quarks.

The final state is a complex mix of jets and leptons:

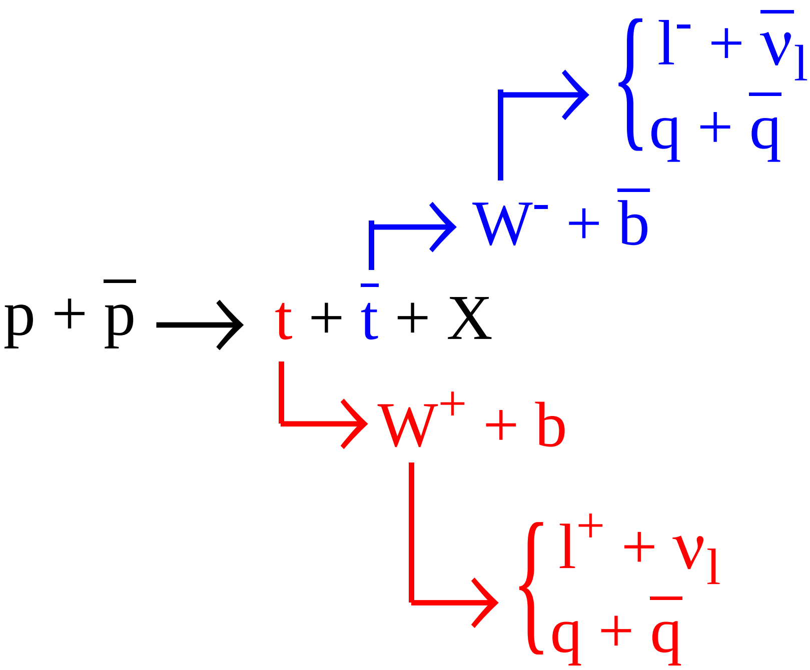
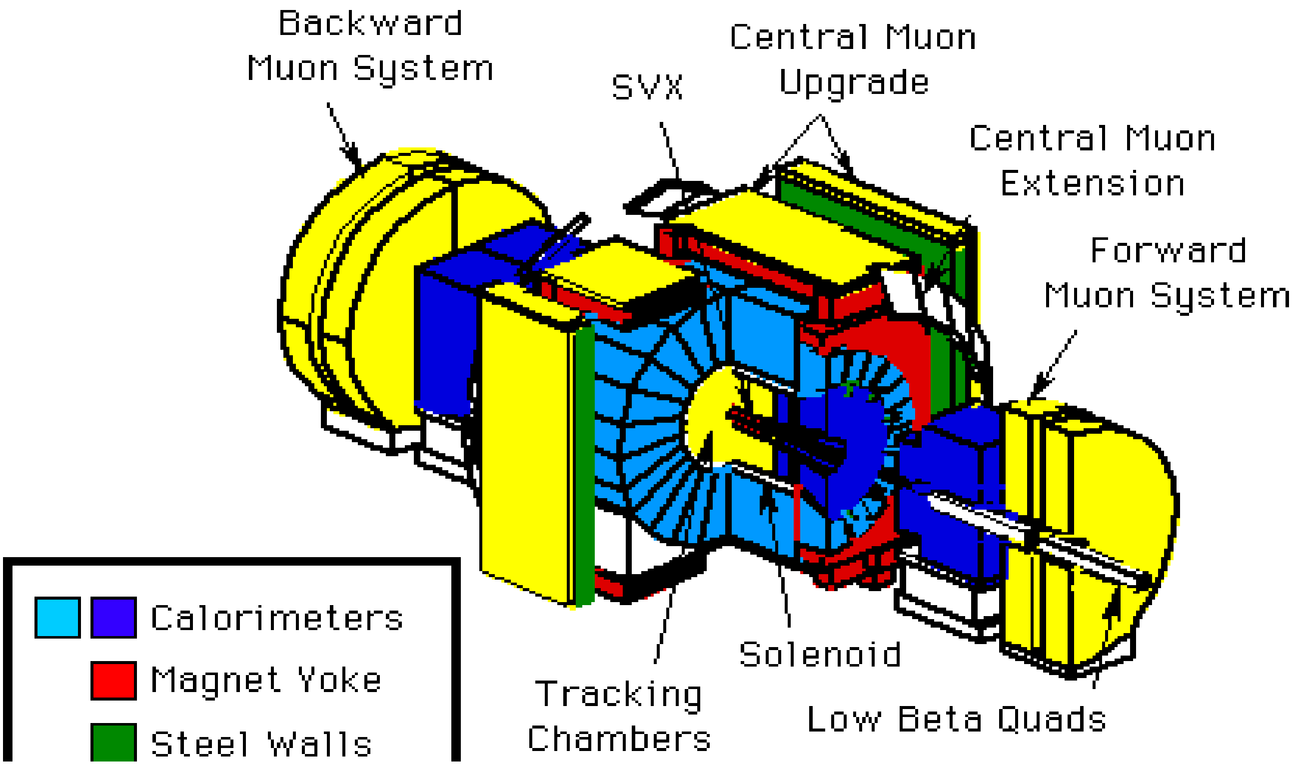


Figure 112: The production of $t\bar{t}$-pairs in $p\bar{p}$-collisions.



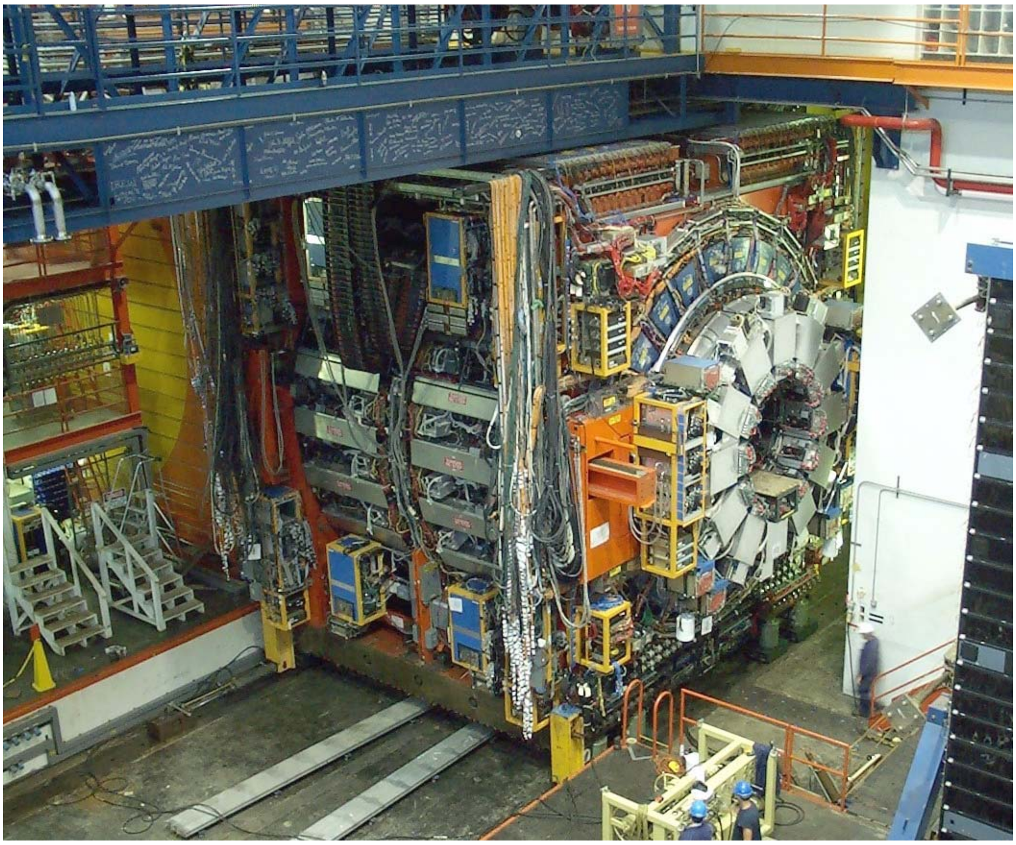


Figure 113: The CDF experiment which discovered the top quark.

→ The events that the CDF collaboration searched for were:

$$t + \bar{t} \to b + l + \nu + \bar{b} + q + \bar{q}$$

where the lepton l was either an electron or a muon.

→ SIGNATURE:

Two b-jets
Two light quark jets
Missing energy from the neutrino
Lepton with large transverse energy

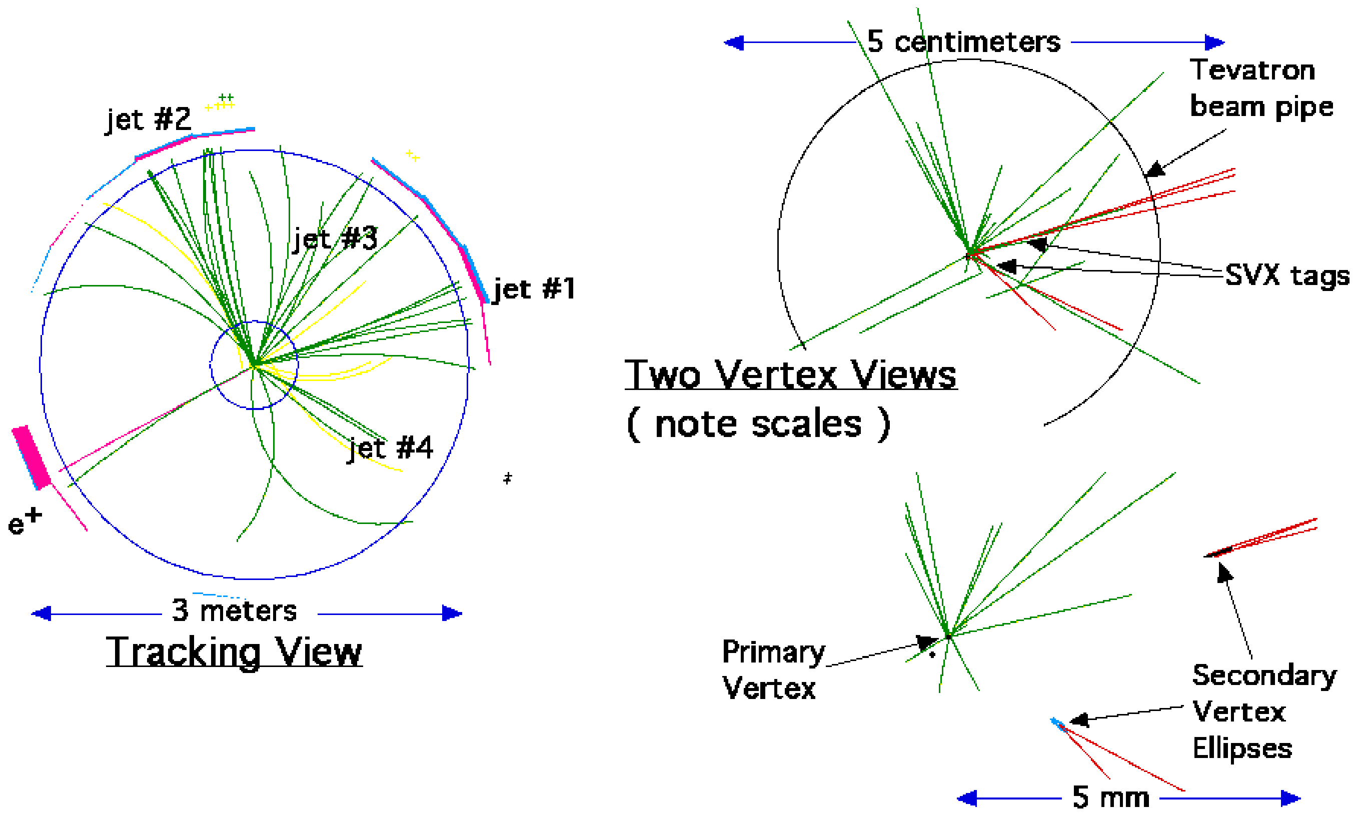


Figure 114: A top event in the CDF experiment.

→ The measured mass distribution (by the CDF experiment) of the events with the required signature was:

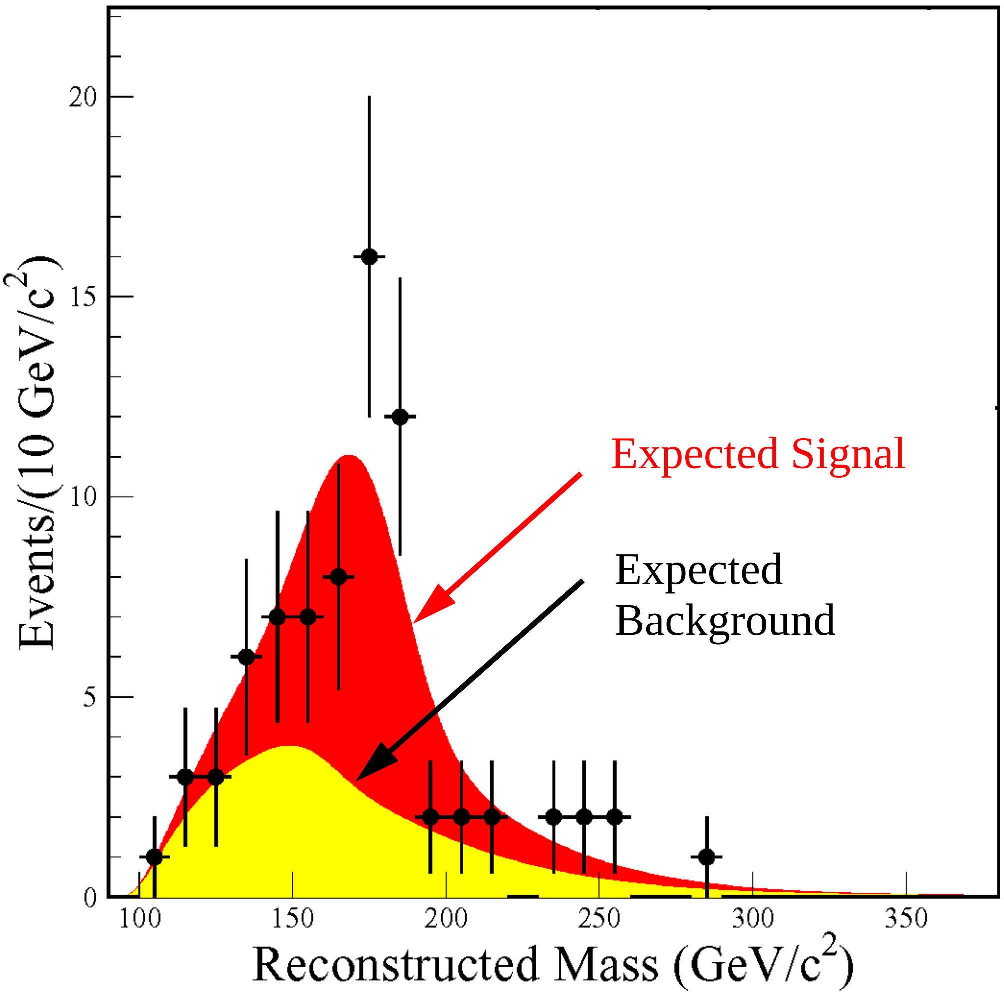


Figure 115: Mass distribution of top events.

→ The top-quark's mass was measured to be

$$M_t = 176 \pm 5\ GeV$$

Summary

- **W and Z bosons.**

a) W and Z are the force carriers in weak interaction.

b) They are very massive spin-1 bosons.

c) Processes with W exchange are called charged current reactions and processes with Z exchange neutral current reactions.

- **Discovery of the W and Z bosons.**

d) The W and Z were discovered in $p\bar{p}$-collisions by the UA1 and UA2 experiments at CERN.

e) The W and Z decays to pairs of leptons or quarks.

f) One has measured the W and Z mass with a large accuracy at the LEP accelerator: M_W=80GeV and M_Z=91 GeV.

• Charged current reactions.

g) Leptonic weak interactions can be built from a set of basic vertices.

h) The weak strength parameter is of the same size as that of the electromagnetic interaction.

• Weak interactions of hadrons.

i) There is a lepton-quark symmetry between particles in the same generation.

j) The quark mixing scheme describe how quarks with different flavours interact with each other.

• The third generation.

k) Quark mixing is described by the CKM-matrix.

l) The mixing is much smaller for the heavy quarks than the light quarks.

• The top quark.

m) The top decay in most cases to a W and a b.

n) It was discovered in the CDF experiment.